

**N87 - 22746**

**NOTES ON IMPLEMENTATION OF COULOMB  
FRICTION IN COUPLED DYNAMICAL SIMULATIONS**

**R. J. VanderVoort**

**R. P. Singh**

**DYNACS Engineering Company, Inc.  
CLEARWATER, FLORIDA 33575**

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## SYSTEM TOPOLOGY

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A coupled dynamical system is defined as an assembly of rigid/flexible bodies that may be coupled by kinematical connections. Large relative displacement and rotation are permitted. The interfaces between bodies are modeled using hinges having 0 to 6 degrees of freedom. A hinge is defined as a pair of two material points, one on each of two adjoining bodies. A reference body is arbitrarily selected and it is assumed for convenience that the reference body is connected to an imaginary inertially fixed body. For consistency, a fictitious hinge is assigned to the reference body by assuming  $P_0$  (See Figure), an inertial point. Thus the number of hinges equals the number of bodies in the system (as shown in Figure 2).

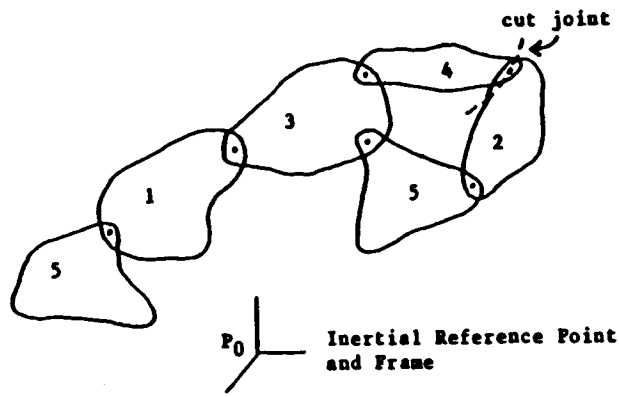


Figure 1 An Arbitrary Topology

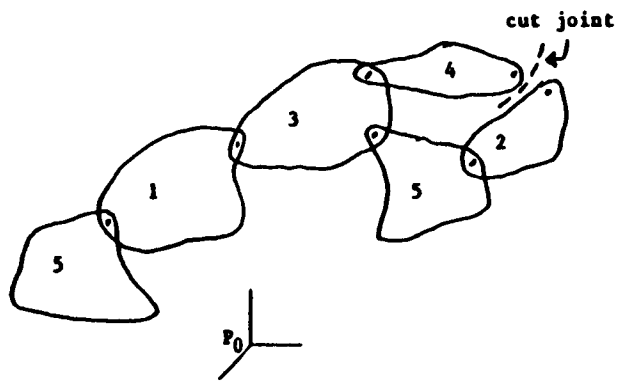


Figure 2 Equivalent Tree Topology

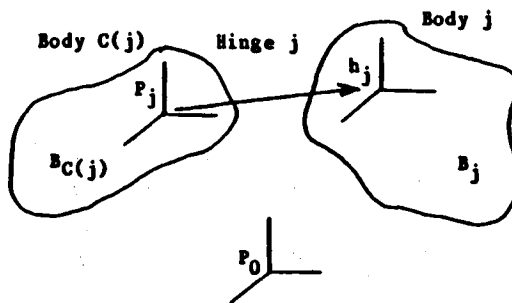


Figure 3 A Typical Hinge

## CONSTRAINED DYNAMICAL SYSTEM

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For a mechanical system of  $n$  flexible bodies in a topological tree configuration, the equations of motion are presented in Reference 1. Lagrange's form of D'Alembert's principle was employed to derive the equations. A detailed discussion of the approach is available in References 1 and 2. Equations (14), (15) in Reference 1 are the motion equations for the system of Figure 2. These equations are augmented by the kinematical constraint equations. This augmentation is accomplished via the method of singular value decomposition (see Reference 3).

## CONSTRAINED DYNAMICAL SYSTEMS

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- LET  $q_1, \dots, q_n$  DEFINE THE CONFIGURATION OF UNCONSTRAINED SYSTEM
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- CONSTRAINT EQUATIONS

$$A \dot{q} = B$$

$A_{m \times n}$   
 $m < n$

- NUMBER OF INDEPENDENT COORDINATES

$$= n - \text{RANK } A$$

- EQUATIONS OF MOTION

$$M \ddot{q} = f + f_c$$

$f_c$  - FORCES/MOMENTS OF CONSTRAINT

- PROBLEMS OF PRACTICAL INTEREST

- SIMPLE NON HOLONOMIC OR HOLONOMIC CONSTRAINTS

-  $n+m$             2nd ORDER D.E.

OR  $2n+m$         1st ORDER D.E.

## SINGULAR VALUE DECOMPOSITION

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- $L$   $m \times n$  MATRIX

THERE EXIST ORTHOGONAL MATRICES  $U_{m \times m}$  AND  $V_{n \times n}$

SUCH THAT  $U^T L V = \left[ \begin{array}{c|c} X & 0 \\ \hline 0 & 0 \end{array} \right] = S$

$X = \text{DIAG} (\lambda_1 > \lambda_2 > \dots > \lambda_r)$        $r = \text{RANK } L$

- $\lambda_i^2$  - NONZERO EIGENVALUES OF  $L^T L$

$L = U S V^T$

$$\Rightarrow [ U_1 \mid U_2 ] \left[ \begin{array}{c|c} X & 0 \\ \hline 0 & 0 \end{array} \right] \left[ \begin{array}{c} V_1^T \\ \hline V_2^T \end{array} \right]$$

$V_2$  SPANS NULL SPACE OF  $L$

$U_2$  SPANS NULL SPACE OF  $L^T$

$L V_2 = 0$        $L^+ = V \left[ \begin{array}{c|c} X^{-1} & 0 \\ \hline 0 & 0 \end{array} \right] U^T$

$L^+ = \text{PSEUDO INVERSE OF } L$

## APPLICATION OF SVD TO CONSTRAINED SYSTEMS

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- $A\dot{q} = B$

$$\dot{q} = A^+B + V_2\dot{Z} \quad \dot{Z} \text{ COLUMN VECTOR } (n-r)$$

$$V_2\dot{Z} \text{ SOLUTION OF } A\dot{q} = 0$$

$$A^+B \text{ PARTICULAR SOLUTION}$$

- DIFFERENTIATE

$$A\ddot{q} = \dot{B} - \dot{A}\dot{q}$$

$$A\ddot{q} = B'$$

$$\ddot{q} = A^+B' + V_2\ddot{Z}$$

- $Z'$ 's ARE REDUCED SET OF  $n-r$  COORDINATES

$$V_2 \text{ IS THE DESIRED TRANSFORMATION}$$

- $M\ddot{q} = F + F^c$

PROJECTION ON NULL SPACE OF A

$$V_2^T M \ddot{q} = V_2^T F + V_2^T F^c$$

SUBSTITUTE FOR  $\ddot{q}$

$$V_2^T M V_2 \dot{z} = V_2^T F + V_2^T F^c - V_2^T M A^+ B$$

- $F^c = A^T \lambda$                        $\lambda$  - LAGRANGE MULTIPLIERS

$$V_2^T F^c = V_2^T A^T \lambda$$

$$= [ A V_2 ]^T \lambda = 0$$

- $[V_2^T M V_2] \dot{z} = V_2^T F - V_2^T M A^+ B$       GOVERNING DIFF. EQN.

$$\dot{q} = A^+ B + V_2 \dot{z}$$



## DAMPING LAW

- ACTUAL DAMPING LAW:

$$F_f(t_1) = f(\text{LOAD OVER CONTACT AREA};$$

$$x(t), 0 < t < t_1, t_1, \dots)$$

$$F_f = \text{FRICTION FORCE}$$

$x(t), 0 < t < t_1$ , REPRESENTS THE HISTORY OF MOTION  
PRIOR TO  $t_1$

- IN GENERAL THE DAMPING LAW IS VERY COMPLICATED

- COULOMB DAMPING

$$F_f(t_1) = f(N, \text{SIGN}(\dot{x}(t_1)), \mu_s, \mu_d)$$

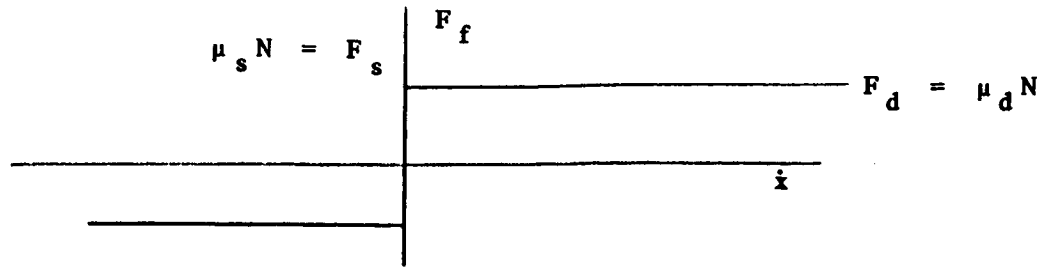
$N$  - NORMAL LOAD

$\mu_s$  - STATIC COEFFICIENT OF FRICTION

$\mu_d$  - DYNAMIC COEFFICIENT OF FRICTION

$$\text{SIGN}(x(t_1)) = \begin{cases} 1 & \dot{x}(t_1) > 0 \\ 0 & \dot{x}(t_1) = 0 \\ -1 & \dot{x}(t_1) < 0 \end{cases}$$

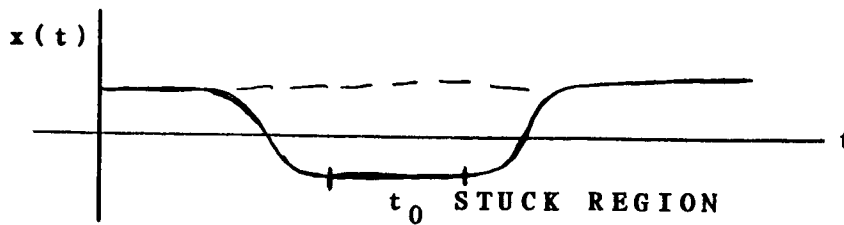
COULOMB DAMPING (CONT.)



- THE MOST COMMON DRY FRICTION DAMPING LAW

$$\mu_s = \mu_d \quad \text{OR} \quad F_s = F_d$$

- MANY TIME DOMAIN AND FREQUENCY DOMAIN STUDIES HAVE BEEN PERFORMED FOR HARMONICALLY EXCITED SYSTEMS WITH DRY FRICTION DAMPING. THERE ARE THREE BASIC METHODS OF ANALYSIS
  - EXACT
  - HARMONIC APPROXIMATIONS
  - TIME INTEGRATION
- OUR APPROACH IS TIME INTEGRATION
- STICK/SLIP MOTION



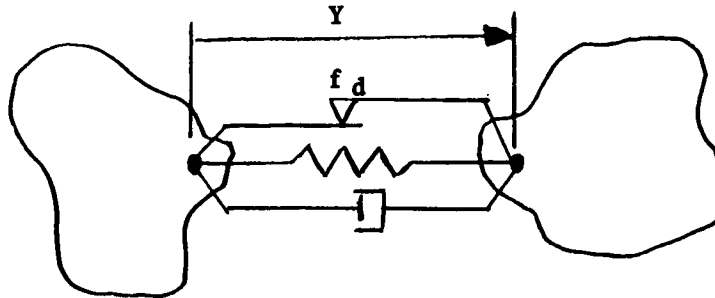
$t_0$  BECOMES AN UNKNOWN.

- STUCK HINGE RESULTS IN ADDITIONAL CONSTRAINTS ON KINEMATICAL VARIABLES.

## COULOMB DAMPER ALGORITHM

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- A TYPICAL HINGE WITH COULOMB DAMPER



- TRACK  $\dot{Y}$  (RELATIVE VELOCITY) FOR SIGN CHANGE

- $\dot{Y}$  CHANGES SIGN

COMPARE  $|\dot{Y}$  DIFFERENCE| TO A PRESCRIBED  $\epsilon \sim 0$

IF GREATER THAN  $\epsilon$  GO BACK TO PREVIOUS STEP AND REDUCE THE STEP SIZE AND REPEAT UNTIL  $|\dot{Y}$  DIFFERENCE|  $< \epsilon$

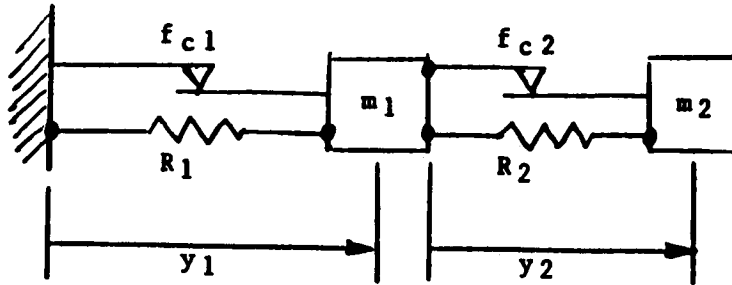
ACTIVATE THE CONSTRAINT  $\dot{Y} = 0$  AND COMPUTE CONSTRAINT FORCE  $f_c$  FOR THIS CONSTRAINT

IF  $f_c$  OVERCOMES  $f_d$  SLIP CONDITION, DEACTIVATE  $\dot{Y} = 0$

IF NOT, STICK CONDITION, RETAIN  $\dot{Y} = 0$  AND KEEP COMPARING  $f_c$  WITH  $f_d$  UNTIL SLIP CONDITION

EXAMPLE

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$$m_1 = 1$$

$$m_2 = 1$$

$$k_1 = 2$$

$$k_2 = 2.5$$

INITIAL  
CONDITION

$$y_1 = 1$$

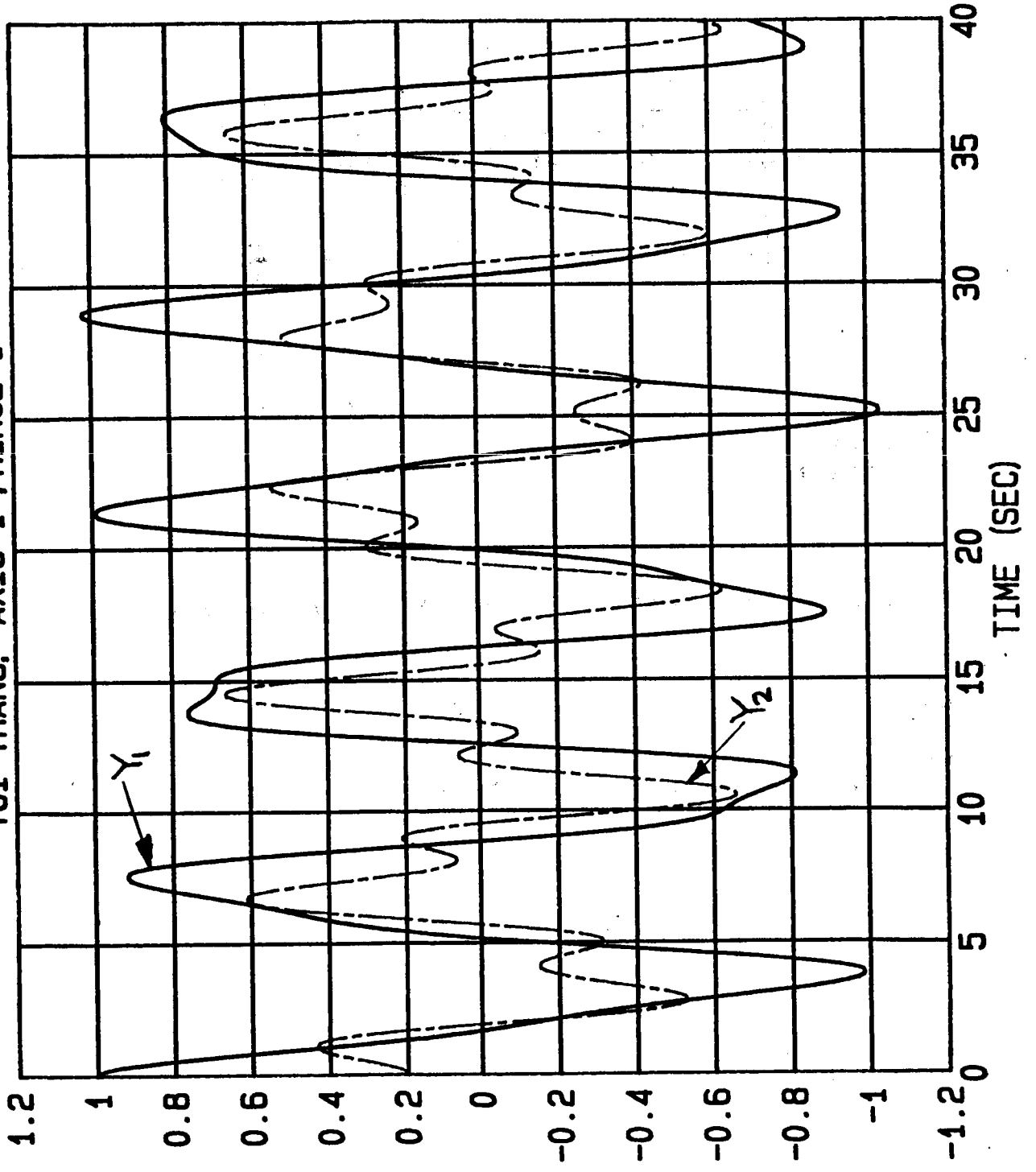
$$y_2 = .2$$

TEST CASES FOR VARIOUS COULOMB DAMPER LEVELS

|        | $f_{c1}$ | $f_{c2}$ |
|--------|----------|----------|
| CASE 1 | 0        | 0        |
| CASE 2 | .1       | 0        |
| CASE 3 | .1       | .1       |
| CASE 4 | .1       | 8000.    |

CASE #1

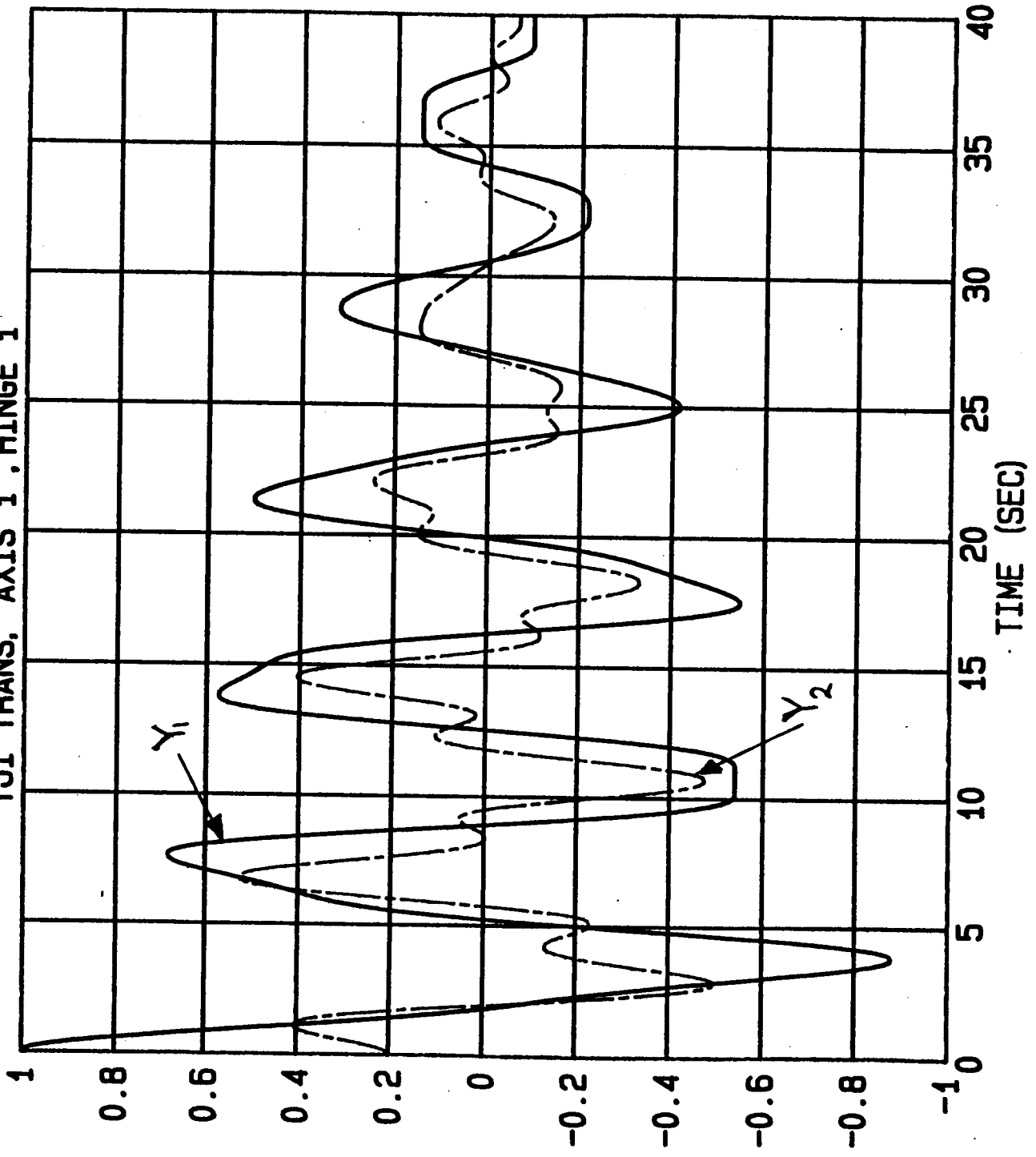
YJI TRANS. AXIS 1, HINGE 2  
YJI TRANS. AXIS 1, HINGE 1



~~Case # 2~~

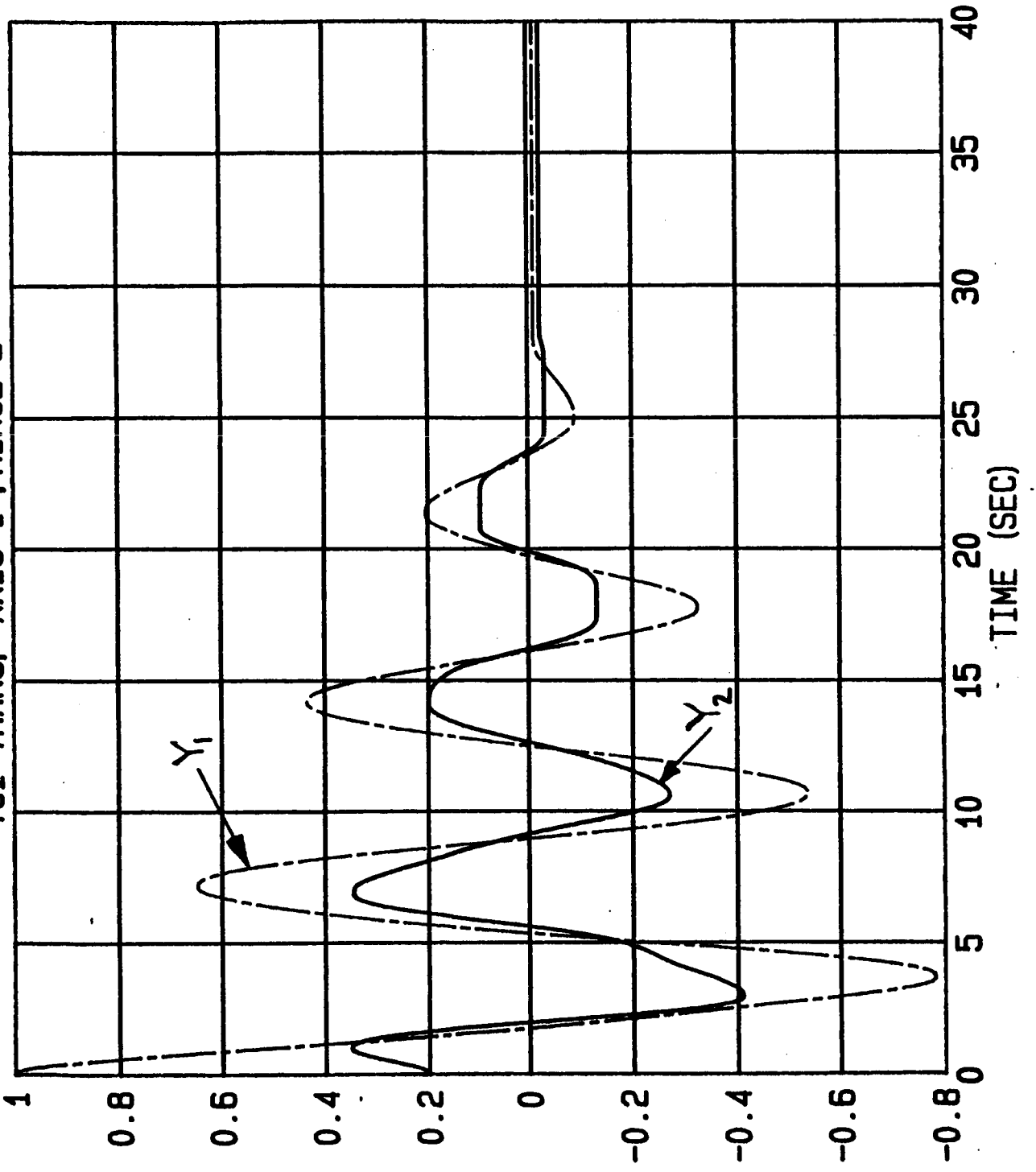
CASE # 2

YJI TRANS. AXIS 1 ; HINGE 2  
YJI TRANS. AXIS 1 ; HINGE 1



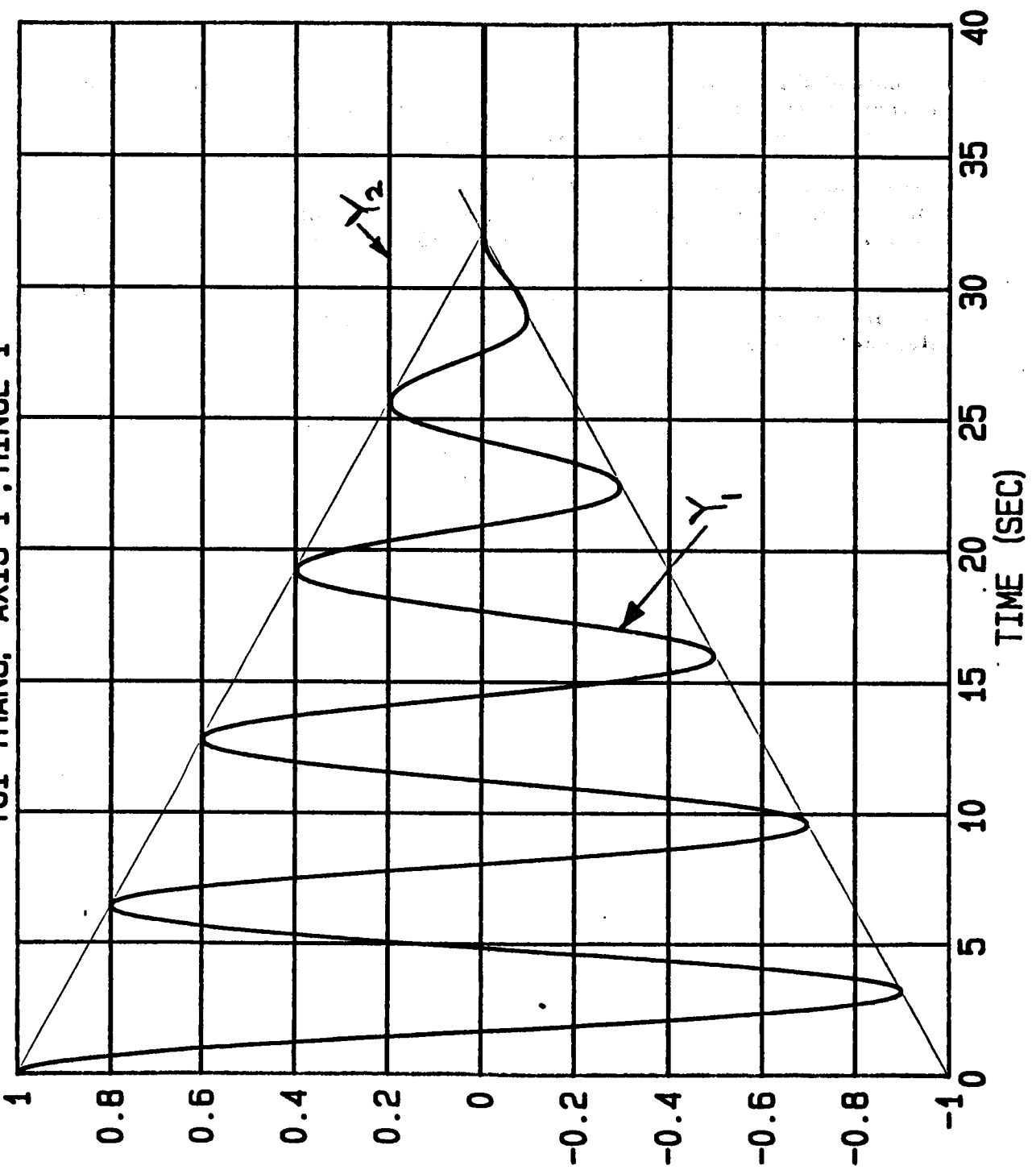
CASE # 3

YJI TRANS, AXIS 1, HINGE 1  
YJI TRANS, AXIS 1, HINGE 2



YJI TRANS. AXIS 1 : HINGE 2  
YJI TRANS. AXIS 1 : HINGE 1

CASE # 11





## REFERENCES

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1. Singh, R. P., VanderVoort, R. J. and Likins, P. W. "Dynamics of Flexible Bodies in Tree Topology--A Computer Oriented Approach," AIAA Journal of Guidance Control and Dynamics, V8, No. 5, September 1985, pp. 584-590.
2. Singh, R. P. and Likins, P. W., "Manipulator Interactive Design with Interconnected Flexible Elements," Proceedings of 1983 Automatic Control Conference, San Francisco, CA, June 1983, pp. 505-512.
3. Singh, R. P. and Likins, P. W., "Singular Value Decomposition for Constrained Dynamical Systems," Transactions of ASME, Journal of Applied Mechanics, V52, No. 4, 1985, pp. 943-948.

### CAPTIONS FOR DON EDBERG'S VIEWGRAPHS

This presentation gives some preliminary results of research on control of flexible structures carried out at Jet Propulsion Laboratory, Pasadena, California. This work has been underway since the summer of 1985. The research is supported by the NASA Office of Aeronautics and Space Technology. Don Edberg and Jay-Chung Chen are Members of the Technical Staff at the Jet Propulsion Laboratory.