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Modeling Digital Control Systems With MA-Prefiltered Measurements

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I. INTRODUCTION

The usual problem posed in books on digital control systems is to find a feedback controller for a continuous-time plant driven by a zero-order-hold with a sampled output as shown in Figure 1 (Jacquot [1], p. 126). Here $\underline{x}(t)$ is an nxl state vector, $\underline{u}(k)$ is an rxl control input vector, $\underline{y}_I(k)$ is an mxl output or measurement vector, F is an nxn system matrix, G is an nxr control matrix, and C_I is an mxn output matrix. Since $\underline{y}_I(k) = C_I\underline{x}(k)$ where k is the usual shorthand notation for time kT, $\underline{y}_I(k)$ represents an instantaneous measure of the system at the sampling instant kT. Hence, the plant in Figure 1 will be regarded as having instantaneous measurements for outputs. It is well known that this system can be modeled at the sampling instants by the set of discrete state equations

$$\underline{x}(k+1) = A\,\underline{x}(k) + B\,\underline{u}(k) \tag{1}$$

$$\underline{y}_I(k) = C_I\,\underline{y}(k) \tag{2}$$

where A and B are constant matrices (Jacquot [1], p. 127).

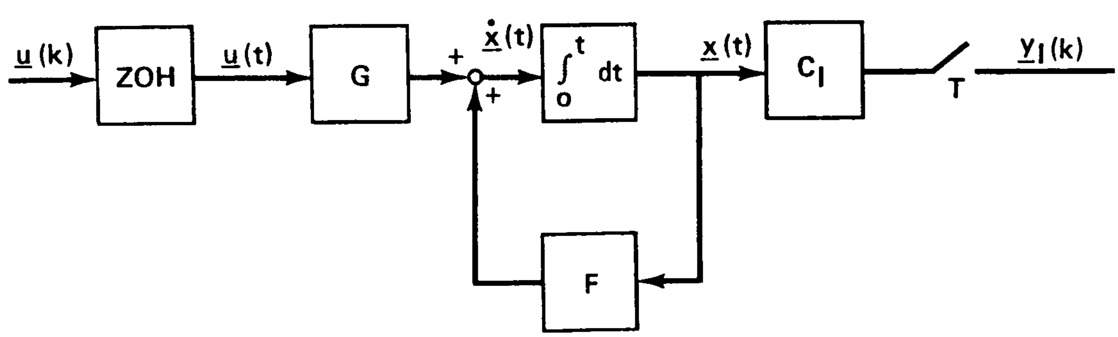

Figure 1. Continuous-time plant driven by a zero-order-hold with instantaneous measurements.

Unfortunately, not all linear time-invariant digital control systems found in the real world have plants which fit the model shown in Figure 1. For example, there exists systems in which the output, rather than being an instantaneous measure of the system at the sampling instants, represents an average measure of it over the time interval between samples. Such systems can be found in the aerospace field wherever startrackers and some state-of-the-art rate gyroscopes and accelerometers are used, to name a few (Polites [2], p. 6). In one system of this type, sets of discrete measurements are averaged to produce discrete outputs as shown in Figure 2. The continuous-time output $\underline{z}(t)$ is sampled every $^{T}/_{N}$ seconds. Every N samples are averaged to generate the averaged measurement vector $\underline{y}_A(kT)$, every T seconds. In Figure 2, $\underline{z}(t)$ and $\underline{y}_A(kT)$ are both px1 vectors; C_A is a pxn output matrix. Note, the plant in Figure 2 allows for the possibility of instantaneous measurements also. $\underline{y}_I(kT)$ is an mx1 instantaneous measurement vector and so C_I is an mxn output matrix. Previously, Polites [2] developed discrete state variable formulations for the plant in Figure 2 which have the same general form as equations (1) and (2).

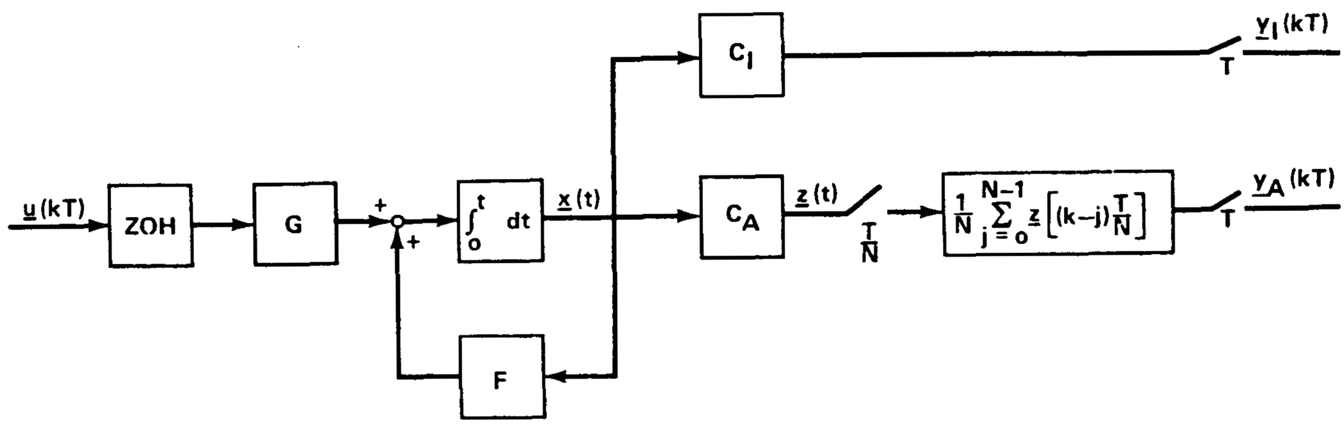

Figure 2. Continuous-time plant driven by a zero-order-hold with instantaneous and averaged measurements.

Now consider the plant in Figure 3, which is a generalization of the one in Figure 2. Here, the continuous-time output $\underline{z}(t)$ is sampled every $^{T}/_{N}$ seconds just as in Figure 2. However, now every N samples are multiplied by the qxp weighting matrices H_j, j = 0,...,N-1, and then summed to generate the qx1 output vector $\underline{y}_F(kT)$, every T seconds. Functionally, this is equivalent to passing the measurements sampled every $^{T}/_{N}$ seconds through a multi-input/multi-output moving average (MA) process with coefficient matrices H_j, j = 0,1,...,N-1 [3]. The output of the MA prefilter is sampled every T seconds to generate $\underline{y}_F(kT)$. In Figure 3, C_F is a pxn output matrix and $\underline{z}(t)$ is a px1 vector. A special case exists when $H_j = (^{1}/_{N})\,\underline{I}$, j = 0,1,...,N-1, where I is a pxp identity matrix. Then, the plant in Figure 3 degenerates to the plant in Figure 2, assuming of course that $C_F = C_A$. Another special case occurs when $C_F = 0$. Then, the plant in Figure 3 degenerates to the plant in Figure 1.

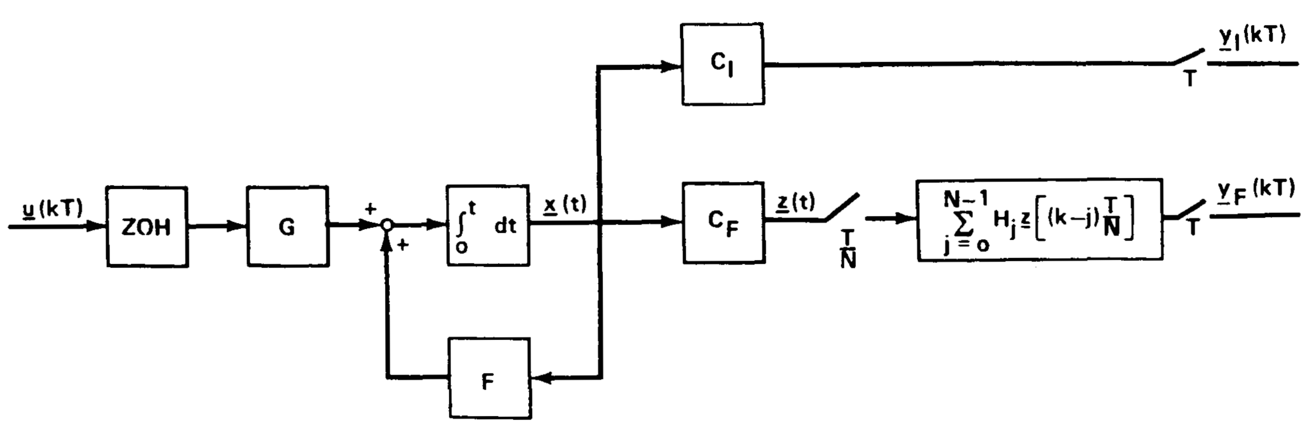

Figure 3. Continuous-time plant driven by a zero-order-hold with instantaneous and MA-prefiltered measurements.

This brings us to a statement of the problem of this paper. Namely, what are discrete state variable representations for the plant with MA-prefiltered measurements shown in Figure 3? With these, the control system engineer can apply existing techniques to accurately design digital feedback controllers for this type plant. Without these, he must resort to approximate methods which could be either less accurate or else very time consuming if iterative. Fortunately, the latter will not be necessary, because the former is made possible by the formulations derived in Section III. Prior to this, Section II reviews some results to date for the plant with instantaneous measurements shown in Figure 1. These will be utilized in Section III. Section IV presents an example which illustrates how to obtain the coefficient matrices for each formulation derived in Section III. Section V contains the conclusions and final comments.

II. PRELIMINARY

The plant in Figure 1 can be modeled at the sampling instants by the discrete state equations (1) and (2) where

$$\phi(t) = L^{-1}[(SI-F)^{-1}] \quad , \tag{3}$$

$$A = \phi(T) \quad , \tag{4}$$

and

$$B = [\int_0^T \phi(\lambda)\, d\lambda]\, G \tag{5}$$

as in Reference 1, p. 126. $\phi(t)$ and A are nxn matrices while B is an nxr matrix. A and B can be determined analytically using equations (3) to (5). An alternative approach, which is also quite suitable for numerical computation, is as follows [4]. $\phi(t)$ and $\int_0^t \phi(\lambda)\, d\lambda$ can be expressed in the form of matrix exponential series as

$$\phi(t) = \sum_{i=0}^{\infty} \frac{F^i t^i}{i!} \tag{6}$$

and

$$\int_0^t \phi(\lambda)\, d\lambda = \sum_{i=0}^{\infty} \frac{F^i\, t^{i+1}}{(i+1)!} \quad , \tag{7}$$

respectively. From equations (6) and (7),

$$\phi(t) = I + F\left[\int_0^t \phi(\lambda)\, d\lambda\right] \quad , \tag{8}$$

where I is an nxn identity matrix. Hence, $\int_0^T \phi(\lambda)\, d\lambda$ can be determined using equation (7) with $t = T$ and this result substituted into equation (8) to get $\phi(T)$. With these results, A and B can be found using equations (4) and (5).

III. STATE VARIABLE REPRESENTATIONS FOR PLANTS WITH MA-PREFILTERED MEASUREMENTS

For the plant in Figure 3,

$$\underline{x}(t) = \phi(t-t_o)\,\underline{x}(t_o) + \int_{t_o}^{t} \phi(t-\tau)\, G\, \underline{u}(\tau)\, d\tau \tag{9}$$

as in Jacquot [1], p. 125. Let

$$\underline{y}_T(k) = \begin{bmatrix} \underline{y}_I(k) \\ \underline{y}_F(k) \end{bmatrix} \tag{10}$$

where

$$\underline{y}_I(k) = C_I\, \underline{x}(k) \tag{11}$$

and

$$\underline{y}_F(k) = \sum_{j=0}^{N-1} H_j\, \underline{z}\left(kT - j\frac{T}{N}\right) . \tag{12}$$

Let $t = kT - j(T/N)$ and $t_o = (k-1)T$ where $k = 0,1,2,\ldots$ and $j = 0,1,\ldots, N-1$. Hence, equation (9) can be written as

$$\underline{x}(kT - j\frac{T}{N}) = \phi(T - j\frac{T}{N})\underline{x}[(k-1)T] + \int_{(k-1)T}^{kT-j(T/N)} \phi(kT - j\frac{T}{N} - \tau)\, G\, \underline{u}(\tau)\, d\tau \quad . \tag{13}$$

Since

$$\underline{z}(t) = C_F \underline{x}(t) \quad , \tag{14}$$

it follows that equation (12) can be written as

$$\underline{y}_F(k) = \sum_{j=0}^{N-1} H_j C_F \underline{x}(kT - j\frac{T}{N}) \quad . \tag{15}$$

From equations (13) and (15),

$$\underline{y}_F(k) = [\sum_{j=0}^{N-1} H_j C_F \phi(T - j\frac{T}{N})]\, \underline{x}[(k-1)T] + \sum_{J=0}^{N-1} H_j C_F \int_{(k-1)T}^{kT-j(T/N)} \phi(kT - j\frac{T}{N} - \tau) G \underline{u}(\tau) d\tau \quad . \tag{16}$$

Since

$$\underline{u}(t) = \underline{u}[(k-1)T] \quad , \qquad (k-1)T \leq t < kT \quad , \tag{17}$$

equation (16) can be written as

$$\underline{y}_F(k) = [\sum_{j=0}^{N-1} H_j C_F \phi(T - j\frac{T}{N})]\, \underline{x}[(k-1)T] + [\sum_{j=0}^{N-1} H_j C_F \int_{(k-1)T}^{kT-j(T/N)} \phi(kT - j\frac{T}{N} - \tau) d\tau] G \underline{u}[(k-1)T] \quad . \tag{18}$$

Consider the second term in the right hand side of equation (18). Using the transformation $\lambda = kT - j(T/N) - \tau$ in it, equation (18) becomes

$$\underline{y}_F(k) = [\sum_{j=0}^{N-1} H_j C_F \phi(T - j\frac{T}{N})]\, \underline{x}[(k-1)T] + [\sum_{j=0}^{N-1} H_j C_F \int_{0}^{T-j(T/N)} \phi(\lambda) d\lambda]\, G\, \underline{u}[(k-1)T] \quad . \tag{19}$$

Let

$$D_+ = \sum_{j=0}^{N-1} H_j C_F \phi(T - j\frac{T}{N}) \tag{20}$$

and

$$E_+ = [\sum_{j=0}^{N-1} H_j C_F \int_0^{T-j(T/N)} \phi(\lambda)d\lambda] G \quad . \tag{21}$$

From equations (19) to (21),

$$\underline{y}_F(k+1) = D_+ \underline{x}(k) + E_+ \underline{u}(k) \quad , \tag{22}$$

using the standard shorthand notation for kT. Letting

$$\underline{\eta}(k+1) = D_+ \underline{x}(k) + E_+ \underline{u}(k) \tag{23}$$

implies that

$$\underline{y}_F(k) = \underline{\eta}(k) \quad . \tag{24}$$

For the plant in Figure 3, equation (1) applies just as it does for the plant in Figure 1. Using it and equations (10), (11), (23), and (24), the discrete state equations for the plant in Figure 3 can be written as

$$\begin{bmatrix} \underline{x}(k+1) \\ \underline{\eta}(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ D_+ & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} + \begin{bmatrix} B \\ E_+ \end{bmatrix} \underline{u}(k) = [A_{T1}] \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} + [B_{T1}]\, \underline{u}(k) \tag{25}$$

$$\underline{y}_T(k) = \begin{bmatrix} \underline{y}_I(k) \\ \underline{y}_F(k) \end{bmatrix} = \begin{bmatrix} C_I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} = [C_{T1}] \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} \quad . \tag{26}$$

A, B, D_+, and E_+ can be evaluated analytically using equations (3), (4), (5), (20), and (21). An alternative approach, which can be either analytical or numerical, is as follows. Let $t = T - j(T/N)$, $j = 0,1,\ldots,N-1$ and use equation (7) to determine $\int^{T-j(T/N)} \phi(\lambda)d\lambda$, $j = 0,1,\ldots,N-1$. Use these results in equation (8) to get $\phi[T-j(T/N)]$, $j = 0,1,\ldots,N-1$. At this point, D_+ and E_+ can be found using equations (20) and (21). A and B can be evaluated using the procedure outlined in Section II.

Notice in the formulation given by equations (25) and (26), there are $(n+q)$ states where n is the number of states in $\underline{x}(t)$, and hence $\underline{x}(k)$, and q is the number of prefiltered measurements in $\underline{y}_F(k)$. Hence, this formulation has $(n+q)$ eigenvalues, which are the eigenvalues of A_{T1} in equation (25). It is straightforward to show that n of these are the eigenvalues of A and the other q are zero.

For the special case in which H_j, $j = 0,1,\ldots,N\text{-}1$ are pxp matrices such that

$$H_j = \frac{1}{N} I \quad , \qquad j = 0,1,\ldots,N\text{-}1 \tag{27}$$

where I is a pxp identity matrix, the MA-prefiltered measurements degenerate to simple averaged measurements. In this case, it follows from equations (20), (21), and (27) that

$$D_+ = C_F \bar{A}_+$$

where

$$\bar{A}_+ = \frac{1}{N} \sum_{j=0}^{N-1} \phi\left(T - j\frac{T}{N}\right)$$

and

$$E_+ = C_F \bar{B}_+$$

where

$$\bar{B}_+ = \left[\frac{1}{N} \sum_{j=0}^{N-1} \int_0^{T-j(T/N)} \phi(\lambda)\, d\lambda\right] G \quad .$$

These results match those previously published by Polites [2] for digital control systems with averaged measurements.

For the plant in Figure 3, an alternative formulation to equations (25) and (26) can be derived by letting $t = kT - j(T/N)$ and $t_o = kT$ in equation (9) where $k = 0,1,2,\ldots$ and $j = 0,1,\ldots,N\text{-}1$. Hence equation (9) becomes

$$\underline{x}\left(kT - j\frac{T}{N}\right) = \phi\left(-j\frac{T}{N}\right)\underline{x}(kT) + \int_{kT}^{kT-j(T/N)} \phi\left(kT - j\frac{T}{N} - \tau\right) G\underline{u}(\tau)d\tau \quad . \tag{28}$$

From equations (12), (14), (17), and (28),

$$\underline{y}_F(k) = \left[\sum_{j=0}^{N-1} H_j C_F \phi\left(-j\frac{T}{N}\right)\right] \underline{x}(kT) + \left[\sum_{j=0}^{N-1} H_j C_F \int_{kT}^{kT-j(T/N)} \phi\left(kT - j\frac{T}{N} - \tau\right) d\tau\right] G\underline{u}[(k-1)T] \quad . \tag{29}$$

Consider the second term in the right hand side of equation (29). Using the transformation $\lambda = kT - j(T/N) - \tau$ in it, equation (29) becomes

$$\underline{y}_F(k) = \left[\sum_{j=0}^{N-1} H_j C_F \phi\left(-j\frac{T}{N}\right)\right] \underline{x}(kT) + \left[\sum_{j=0}^{N-1} H_j C_F \int_0^{-j(T/N)} \phi(\lambda)d\lambda\right] G\underline{u}[(k-1)T] \quad . \tag{30}$$

Let

$$D_- = \sum_{j=0}^{N-1} H_j C_F \phi\left(-j\frac{T}{N}\right) \tag{31}$$

and

$$E_- = \left[\sum_{j=0}^{N-1} H_j C_F \int_0^{-j(T/N)} \phi(\lambda)d\lambda\right] G \quad . \tag{32}$$

From equations (30) to (32),

$$\underline{y}_F(k) = D_-\underline{x}(k) + E_-\underline{u}(k-1) \quad , \tag{33}$$

using the standard shorthand notation for kT. Letting

$$\underline{\eta}(k+1) = \underline{u}(k) \tag{34}$$

implies that

$$\underline{y}_F(k) = D\_\underline{x}(k) + E\_\underline{\eta}(k) \quad . \tag{35}$$

From equations (1), (10), (11), (34), and (35), the alternative set of discrete state equations for the plant in Figure 3 is

$$\begin{bmatrix} \underline{x}(k+1) \\ \underline{\eta}(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \underline{u}(k) = [A_{T2}] \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} + [B_{T2}]\underline{u}(k) \tag{36}$$

$$\underline{y}_T(k) = \begin{bmatrix} \underline{y}_I(k) \\ \underline{y}_F(k) \end{bmatrix} = \begin{bmatrix} C_I & 0 \\ D\_ & E\_ \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} = [C_{T2}] \begin{bmatrix} \underline{x}(k) \\ \underline{\eta}(k) \end{bmatrix} . \tag{37}$$

A, B, D_, and E_ can be evaluated analytically using equations (3), (4), (5), (31), and (32). An alternative approach, which can be either analytical or numerical, is as follows. Let $t = -j(T/N)$, $j = 0,1,\ldots,N\text{-}1$ and use equation (7) to determine $\int_0^{-j(T/N)} \phi(\lambda)d\lambda$, $j = 0,1,\ldots,N\text{-}1$. Use these results in equation (8) to get $\phi[-j(T/N)]$, $j = 0,1,\ldots,N\text{-}1$. At this point, D_ and E_ can be found using equations (31) and (32). A and B can be evaluated using the procedure outlined in Section II.

Notice in the formulation given by equations (36) and (37), there are $(n+r)$ states where n is the number of states in $\underline{x}(t)$, and hence $\underline{x}(k)$, and r is the number of control inputs in $\underline{u}(k)$. Hence, this formulation has $(n+r)$ eigenvalues, which are the eigenvalues of A_{T2} in equation (36). It is straightforward to show that n of these are the eigenvalues of A and the other r are zero.

For the special case in which H_j, $j = 0,1,\ldots,N\text{-}1$, are pxp matrices as in equation (27), the MA-prefiltered measurements degenerate to simple averaged measurements. In this case, it follows from equations (27), (31), and (32) that

$$D\_ = C_F \bar{A}\_$$

where

$$\bar{A}_{-} = \frac{1}{N} \sum_{j=0}^{N-1} \phi\left(-j\frac{T}{N}\right)$$

and

$$E_{-} = C_F \bar{B}_{-}$$

where

$$\bar{B}_{-} = \left[\frac{1}{N} \sum_{j=0}^{N-1} \int_0^{-j(T/N)} \phi(\lambda)d\lambda\right] G \quad .$$

These results match those previously published by Polites [2] for digital control systems with averaged measurements.

Now suppose the plant in Figure 3 has the prefiltered measurement vector modified as shown in Figure 4. From Figure 4,

$$\underline{y}'_F(k) = \underline{y}_F(k) - E_{-}\, \underline{u}(k-1) \quad . \tag{38}$$

Furthermore, equations (36) and (37) also apply to the plant in Figure 4. Eliminating $\underline{\eta}(k)$ and $\underline{y}_F(k)$ in equations (36) to (38) yields the discrete state equations

$$\underline{x}(k+1) = A\, \underline{x}(k) + B\, \underline{u}(k) = [A_{T3}]\, \underline{x}(k) + [B_{T3}]\, \underline{u}(k) \tag{39}$$

$$\underline{y}_{T3}(k) = \begin{bmatrix} \underline{y}_I(k) \\ \underline{y}'_F(k) \end{bmatrix} = \begin{bmatrix} C_I \\ D_{-} \end{bmatrix} \underline{x}(k) = [C_{T3}]\, \underline{x}(k) \quad . \tag{40}$$

Compare equations (39) and (40) for the plant in Figure 4 with equations (1) and (2) for the plant in Figure 1. Notice that the plant equations (1) and (39) are the same. Only the output equations (2) and (40) are different. Obviously, the discrete state equations (39) and (40) for the plant in Figure 4 have n states and n eigenvalues, which are the eigenvalues of A.

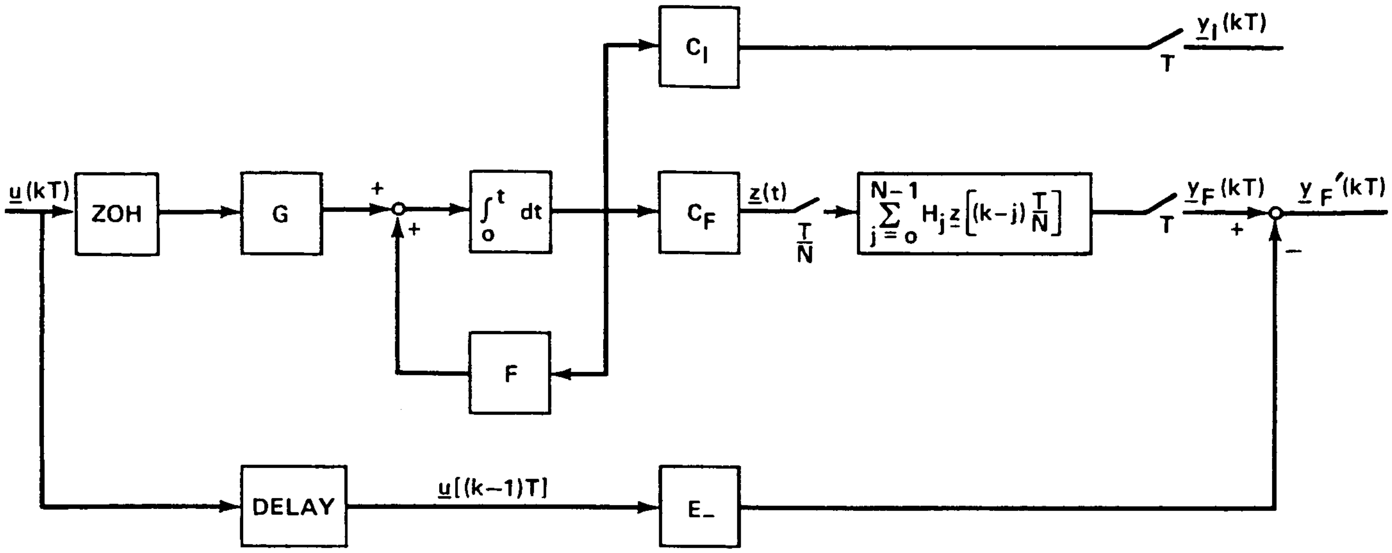

Figure 4. Plant in Figure 3 with a modified prefiltered measurement vector.

IV. AN EXAMPLE

Consider the double integrator plant in Figure 5 where the instantaneous measurement $y_I(kT)$ measures $x_1(kT)$ and the MA-prefiltered measurement $y_F(kT)$ estimates $\dot{x}_1(kT) = x_2(kT)$. This example is rather basic in relation to the kind of problems the formulations in Section III can handle. However, it is chosen for pedagogical reasons to illustrate the basic procedures one uses to obtain discrete state variable models for plants with MA-prefiltered measurements using the formulations in Section III.

Comparing Figures 3 and 5,

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \tag{41}$$

$$G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad , \tag{42}$$

$$C_I = [1 \quad 0] \quad , \tag{43}$$

$$C_F = [1 \quad 0] \quad , \tag{44}$$

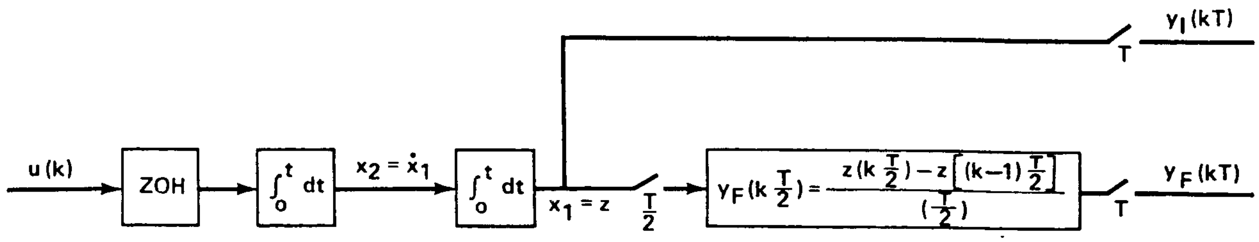

Figure 5. Plant for the example.

$$H_o = \frac{2}{T} \quad , \tag{45}$$

and

$$H_1 = -\frac{2}{T} \quad . \tag{46}$$

Utilizing equations (7), (8), and (41),

$$\int_0^t \phi(\lambda)d\lambda = \begin{bmatrix} t & t^2/2 \\ 0 & t \end{bmatrix} \tag{47}$$

and

$$\phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \quad . \tag{48}$$

From equations (4), (5), (42), (47), and (48),

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{49}$$

and

$$B = \begin{bmatrix} T^2/2 \\ T \end{bmatrix} . \quad (50)$$

Since N = 2 in equations (20) and (21), it follows that

$$D_+ = H_o\, C_F\, \phi(T) + H_1\, C_F\, \phi(T/2) \quad (51)$$

and

$$E_+ = \left[H_o\, C_F \int_0^T \phi(\lambda)\, d\lambda + H_1\, C_F \int_0^{T/2} \phi(\lambda) d\lambda \right] G . \quad (52)$$

From equations (42), (44) to (48), (51), and (52),

$$D_+ = [0 \quad 1] \quad (53)$$

and

$$E_+ = \frac{3}{4}\, T . \quad (54)$$

Utilizing equations (43), (49), (50), (53), and (54), the discrete state equations for the plant in Figure 5 can be expressed in terms of equations (25) and (26) with

$$A_{T1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} ,$$

$$B_{T1} = \begin{bmatrix} \frac{T^2}{2} \\ T \\ \frac{3}{4} T \end{bmatrix} ,$$

and

$$C_{T1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

A check will show these state equations to be observable and controllable for $T > 0$.

Utilizing equations (31) and (32) with $N = 2$,

$$D_{-} = H_o C_F \phi(0) + H_1 C_F \phi(-T/2) \tag{55}$$

and

$$E_{-} = \left[H_o C_F \int_0^0 \phi(\lambda) d\lambda + H_1 C_F \int_0^{-(T/2)} \phi(\lambda) d\lambda \right] G . \tag{56}$$

Making the proper substitutions into equations (55) and (56) yields

$$D_{-} = [0 \quad 1] \tag{57}$$

and

$$E_{-} = - \frac{T}{4} . \tag{58}$$

Utilizing equations (43), (49), (50), (57), and (58), the discrete state equations for the plant in Figure 5 can be expressed in terms of equations (36) and (37) with

$$[A_{T2}] = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} ,$$

$$[B_{T2}] = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 1 \end{bmatrix} ,$$

and

$$[C_{T2}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -T/4 \end{bmatrix} .$$

A check will show these state equations to be observable and controllable for $T > 0$.

If the plant in Figure 5 is modified according to Figure 4 where E_- is given by equation (58), the result is shown in Figure 6. Utilizing equations (43), (49), (50), and (57), the discrete state equations for the plant in Figure 6 can be expressed in terms of equations (39) and (40) with

$$[A_{T3}] = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} ,$$

$$[B_{T3}] = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} ,$$

and

$$[C_{T3}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

Again, a check will show these state equations to be observable and controllable for T > 0. One may recognize these to be the discrete state equations for the double integrator plant with two instantaneous measurements as shown in Figure 7. Hence in Figure 6, $x_2(kT)$ is being observed by sampling $x_1(t)$ every T/2 seconds, prefiltering the samples with a backward difference equation, and modifying the result as shown in Figure 6.

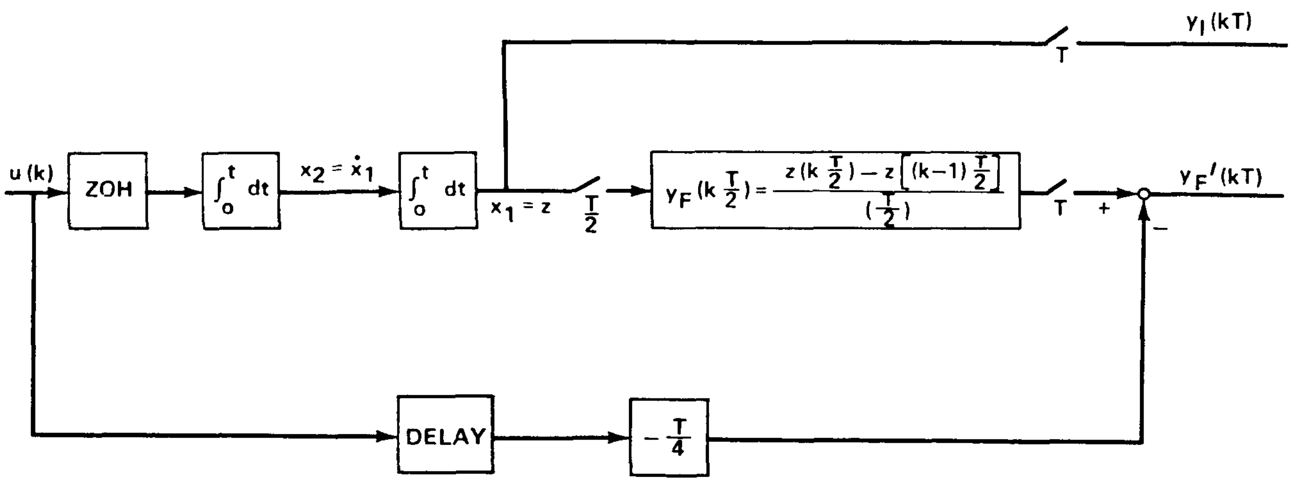

Figure 6. Plant in Figure 5 with a modified prefiltered measurement.

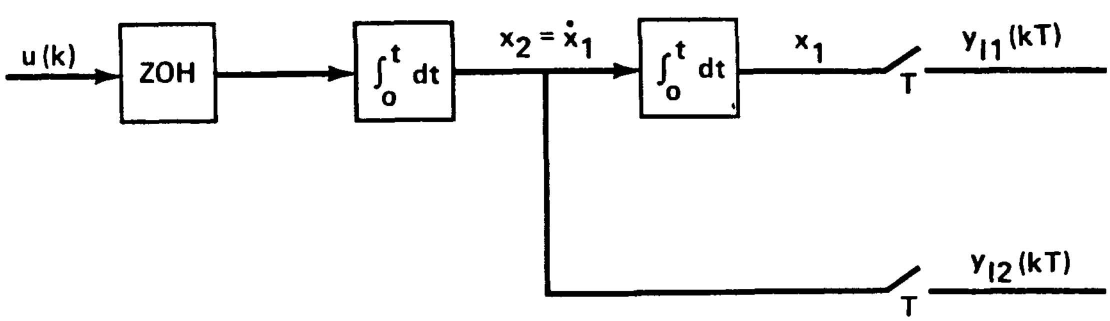

Figure 7. Double integrator plant with two instantaneous measurements.

V. CONCLUSIONS

Two different discrete state variable formulations were derived for the continuous-time plant driven by a zero order hold with a combination of instantaneous and MA-prefiltered measurements. This plant is shown in Figure 3 and the discrete state variable formulations are presented in equations (25), (26), (36), and (37). The first formulation has (n + q) states where n is the number of states in $\underline{x}(t)$, and hence $\underline{x}(k)$, and q is the number of prefiltered measurements in $\underline{y}_F(k)$. The second formulation has (n + r) states where r is the number of control inputs in $\underline{u}(k)$. If the prefiltered measurement vector is modified as shown in Figure 4, a third formulation can be derived as shown in equations (39) and (40). This one has n states and the same plant equation as equation (1) for the plant in Figure 1.

With regard to which formulation is the best to use for modeling a given plant with MA-prefiltered measurements, the following advice is offered. Choose the one which yields the least number of states in the resulting discrete state equations. In general, this is the formulation which has n states, equations (39) and (40). However, this formulation does require modifying the prefiltered measurement vector as shown in Figure 4. If this is either impossible or else undesirable, for whatever reason, then choose the formulation, from the remaining two, which yields the fewest number of states. Hence, if the plant in question has fewer prefiltered measurements than control inputs (i.e., $q < r$), choose the formulation which has (n + q) states, equations (25) and (26). If the plant has fewer control inputs than prefiltered measurements (i.e., $r < q$), choose the formulation which has (n + r) states, equations (36) and (37). If the number of prefiltered measurements equals the number of control inputs (i.e., $q = r$), the choice is arbitrary. By following these guidelines, the resulting set of discrete state equations is more likely to be observable and controllable, plus there will be fewer states and, hence, fewer eigenvalues to contend with in designing the feedback controller.

Having the formulations derived in this paper, the control system engineer can apply standard available design techniques to accurately design a digital feedback controller for any plant which fits the model shown in Figure 3. He does not have to resort to approximate methods which may be either less accurate or else time consuming, if iterative procedures are required.

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16. ABSTRACT

This paper derives three discrete state variable representations for a continuous-time plant driven by a zero-order-hold with a combination of instantaneous measurements and measurements prefiltered by moving-average (MA) digital filters. These representations allow the control system engineer to accurately model plants of this type in a form which permits him to use standard techniques to design digital feedback controllers for them. An example is presented which illustrates how to obtain the coefficient matrices in each representation. Guidelines are presented for choosing the best representation to use for any given plant of this type.

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