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SOME ASPECTS OF DOUBLE LAYER FORMATION IN A PLASMA CONSTRAINED BY A MAGNETIC MIRROR

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ABSTRACT

The discussion of parallel electric fields in the Earth's magnetosphere has undergone a notable shift of emphasis in recent years, away from wave-generated anomalous resistivity toward the more large-scale effects of magnetic confinement of current carrying plasmas. This shift has been inspired in large part by the more extensive data on auroral particle distribution functions that have been made available, data that may often seem consistent with a dissipation-free acceleration of auroral electrons over an extended altitude range.

Efforts to interpret these data have brought new vigor to the concept that a smooth and static electric field can be self-consistently generated by suitable pitch angle anisotropies among the high-altitude particle populations, different for electrons and ions, and that such an electric field is both necessary and sufficient to maintain the plasma in a quasi-neutral steady state. This paper reviews and criticizes certain aspects of this concept, both from a general theoretical standpoint and from the standpoint of what we know about the magnetospheric environment. It is argued that this concept has flaws and that the actual physical problem is considerably more complicated, requiring a more complex electric field, possibly including double layer structures.

I. INTRODUCTION

Few topics in space plasma physics have been as controversial as that of "parallel electric fields," that is electric fields with a static or quasi-static component aligned along the Earth's magnetic field lines and strong enough to substantially alter the velocity distribution of the charged particles. Much of this controversy has centered on the interpretation of auroral particle data, especially the data on precipitating electrons, and has evolved along with developments in measurement technology (e.g., Swift, 1965; Block, 1967; O'Brien, 1970; Evans, 1974; Lennartsson, 1976; Papadopoulos, 1977; Hudson et al., 1978; Chiu and Schulz, 1978; Goertz, 1979; Lyons et al., 1979; Smith, 1982; and references therein).

Possibly the first truly compelling evidence of parallel electric field was presented by Evans (1974), who was able to account in a rather convincing fashion for the different parts of a typical auroral electron spectrum. The type of data presented by Evans is illustrated in a condensed form in Figure 1, which is taken from a more recent study by Kaufmann and Ludlow (1981). The two principal parts of this spectrum are a virtually isotropic low-energy part, including the central peak and most of the plateau, and a high-energy part on the flanks, which is essentially isotropic in the downward hemisphere (positive $v_{\|}$) but strongly reduced in the upward hemisphere (negative $v_{\|}$). According to Evans' interpretation, only the high-energy part in the downward hemisphere consists of precipitating primary electrons, accelerated by an upward parallel electric field at higher altitude. Only these primary electrons can contribute to a field-aligned (upward) current at this point in space. The low-energy part consists of backscattered and energy-degraded primary electrons and of electrons of atmospheric origin, many of which are secondary electrons generated by the impact of primary electrons. All of these low-energy electrons are trapped below the electric field and cannot contribute to the field-aligned current. Any additional contribution must be from upward-moving ions.

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As noted by Evans (and by other investigators before him) the primary electrons ("p") on the downward flanks of the distribution typically have a velocity distribution f_p that is reminiscent of a Maxwell-Boltzmann distribution that has been displaced in energy:

$$f_p(\vec{v}) \approx C \exp[-(m|\vec{v}|^2/2 - U)/kT] \quad , \tag{1}$$

where C is a normalization constant, m the electron mass, kT a thermal energy, and the positive quantity U is independent of $\vec{v}$ and may be equated to a certain difference in electric potential energy eV:

$$U = e\Delta V \quad . \tag{2}$$

This quantity corresponds to the kinetic energy of the electrons on the downward edge of the plateau in Figure 1 and, by inference, corresponds to primary electrons with zero initial energy (at high altitude).

If the distribution in Figure 1 is integrated in terms of a net field-aligned current density $i_{\|}$, only the electrons on the flanks make a significant contribution because of the near isotropy at energies smaller than U. If the distribution of these flank electrons f_p ("primary electrons") is approximated by (1) at pitch angles $\alpha \leq \alpha_{max}$ (where α_{max} is slightly larger than 90° in this figure) and approximated by zero at $\alpha > \alpha_{max}$, then the integration of $-ef_p(\vec{v})v\cos\alpha$ readily yields:

$$i_{\|} \approx -eC2\pi(kT/m)^2\sin^2\alpha_{max}\,(1 + U/kT) \quad , \tag{3}$$

which is a linear function of U for constant values of C, kT, and α_{max} (the latter corresponding to a local atmospheric "loss cone" angle of 180° $-\alpha_{max}$). Some comparisons of auroral electron spectra with the associated field-aligned currents (inferred from other data) have confirmed that the precipitating primary electrons do in fact account for a large or dominant portion of upward field-aligned currents, and the current density is sometimes fairly well approximated by (3) (Burch et al., 1976; Lyons, 1981; Yeh and Hill, 1981).

Although the right-hand side in (3) can be derived on purely empirical grounds, as an approximation of observed electron fluxes, the same type of expression can also be "predicted" if the primary electrons are assumed to originate at high altitude (a few Earth radii, or more), with an isotropic Maxwell-Boltzmann distribution with a temperature T, and fall through a static parallel electric field with a total potential difference $\Delta V = U/e$ (e.g., Knight, 1973; Lemaire and Scherer, 1974; Lennartsson, 1976, 1980; Lyons et al., 1979, Lyons, 1981; Chiu and Schulz, 1978; Chiu and Cornwall, 1980; Stern, 1981). The electric field distribution is not uniquely defined by (3), but to assure the maximum degree of isotropy of the precipitating electrons at low altitude, in accordance with Figure 1, and thus the closest approximation of a linear dependence between $i_{\|}$ and ΔV, it is necessary to assume that the electric potential V varies with the magnetic field strength B in such a fashion that

$$V(B) - V(B_o) \geq (B - B_o)\,\Delta V/\Delta B \quad , \tag{4}$$

where o refers to the high-altitude origin of the electrons and ΔB refers to the total difference in magnetic field strength between this origin and the low-altitude point of observation (Lennartsson, 1977, 1980). Among the possible solutions of (4) are various double layer configurations, single or multiple.

The fact that (3) can be derived under such simple assumptions and yet give a fair approximation of upward field-aligned currents, at least in some studies, has helped in focusing attention on the subject of magnetic confinement of current carrying plasmas. The theoretical implications of this fact are still obscure, however, and there is no consensus yet on the actual properties of the parallel electric field. This paper reviews a few aspects of this complex problem, including the possible role of double layers.

II. NATURAL BOUNDARY CONDITIONS

A rather traditional approach to magnetospheric plasma dynamics at non-relativistic energies is to consider adiabatic single-particle motion, assuming that at least the first adiabatic invariant is preserved for both ions and electrons. This approach has proved fruitful in numerous applications but does have intrinsic problems in many others. To illustrate the latter it is assumed that the particle dynamics is dominated by magnetic and electric force fields, $\vec{B}$ and $\vec{E}$, respectively. To save space the symbols M and Q are used for the mass and charge, respectively, of either ions or electrons. The first invariant (in MKS units) can thus be expressed as

$$\mu = Mv_g^2/2/B \approx \text{constant} \quad , \tag{5}$$

where the gyro velocity v_g equals $|\vec{v}_\perp - \vec{E}\times\vec{B}/B^2|$, apart from a small perturbations velocity $\vec{v}_\perp'$ defined by:

$$\vec{v}_\perp' = (M/Q/B^2)(d\vec{E}_\perp/dt + v_g^2(\vec{B}\times\nabla B)/2/B + v_\parallel^2\,\vec{B}\times(1/B)\vec{B}\cdot\nabla(\vec{B}/B) \quad , \tag{6}$$

where the time derivative is taken in the frame of reference of the moving particle (e.g., Alfvén and Fälthammar, 1963; Longmire, 1963). This velocity represents the mass and charge dependent part of the gyro center drift, which is added to the common $\vec{E}\times\vec{B}$ drift. The parallel velocity is likewise defined by

$$M\,(d\vec{v}/dt)_\parallel \approx QE_\parallel - Mv_g^2(\vec{B}\cdot\nabla B)/2/B^2 \quad . \tag{7}$$

The intrinsic problem in these equations lies in the second and third terms on the right-hand side of (6), which have opposite directions for ions and electrons and are generally non-zero in the Earth inhomogeneous magnetosphere. These terms thus translate into electric currents which flow across the magnetic field lines and must be part of closed current loops in a stationary state. Otherwise the assumption in (5) cannot be a valid description of the particle dynamics.

As far as (5) is valid, equations (6) and (7) should provide a valid description of the interaction between the solar wind plasma and the Earth's magnetic field. In this case the currents associated with (6) can, at least in principle, close through the Earth's ionosphere, as indicated schematically in Figure 2. The field-aligned portions of such a current loop may be carried in part by terrestrial particles, but the flow density of these particles is limited by the maximum possible escape rates (e.g., Lemaire and Scherer, 1974). This restriction is less severe for the downward current, since the terrestrial electrons may escape at a higher rate than the ions if allowed to flow freely.

If the demand for upward current exceeds the flow rate of terrestrial ions, the additional contribution must be carried by precipitating solar electrons. The flow density of these electrons is on the other hand limited by the "magnetic mirror" force on the right-hand side of (7), and can only be increased by a parallel electric field. In fact, if these electrons have a Maxwell-Boltzmann distribution with a temperature T and density n, the flow density is limited by (3), where $U = e\Delta V$ and $C = n\sqrt{kT/(2\pi m)}$ (Lennartsson, 1980). This approach thus leads in a natural fashion to the subject of magnetic confinement. The fact that auroral electrons are observed to have a significantly higher temperature than solar electrons (cf. Fig. 1), may suggest, however, that (5) is not entirely valid.

III. A "CLASSICAL" APPROACH TO MAGNETIC CONFINEMENT

Since particles with different pitch angles mirror at different locations in an inhomogeneous static magnetic field, the number density n of these particles is a function of B, unless the velocity distribution is completely isotropic (according to Liouville's theorem). If the magnetic field strength has a single minimum B_o and increases monotonically away from this minimum, in at least one direction, then the density n is known at any $B > B_o$, if the distribution function is known at B_o. This is still true in the presence of a parallel electric field (assuming a one-dimensional geometry), provided the electric field is also time independent:

$$dE/dt = 0 \quad , \tag{8}$$

and the electric potential is sufficiently monotonic, for example (Chiu and Schulz, 1978):

$$dV/dB > 0 \tag{9}$$

$$d^2V/dB^2 \leq 0 \quad . \tag{10}$$

The last condition is much stronger than (4); it precludes double layer structures and implies that the electron and ion densities are very nearly equal at all points. Under these three conditions, and assuming that (5) holds and the ions are all positive and singly charged, the quasi-neutrality may be expressed in a somewhat "classical" form as:

$$n_e\,(V,B,f_{eo}) \approx n_i\,(V,B,f_{io}) \quad , \tag{11}$$

where f_{eo} and f_{io} are the electron and ion distribution functions, respectively, at B_o. With a careful selection of f_{eo} and f_{io} this relation will yield a solution for V in the form $V = V(B)$ (e.g., Alfvén and Fälthammar, 1963; Persson, 1963, 1966; Block, 1967; Lemaire and Scherer, 1974; Chiu and Schulz, 1978; Stern, 1981). Whether this also yields a self-consistent solution of Poisson's equation is a rather intricate question, however.

A comparatively simple and analytically tractable case is illustrated in Figure 3, which is adapted from the works of Persson (1966) and Block (1967). The shaded areas repesent the only populated regions of velocity space. Within these regions the particle distributions are assumed to be isotropic but may have arbitrary functional dependence on the energy and may be different for electrons and ions. The ions are also assumed to have energies larger than $e(V_a - V(B))$, which ensures that no part of the ion energy distribution is entirely excluded from low altitude ($B \approx B_a$). The electron energies are only limited by the acceleration ellipsoid and by the loss hyperboloid. As discussed by Persson and Block, these ion and electron populations can be made to have equal densities everywhere, $n_i = n_e$, if and only if:

$$E_{\parallel} = -((V_a - V_o)/(B_a - B_o))dB/ds \quad , \tag{12}$$

where s is a distance coordinate running along $\vec{B}$ (downward). The conventional physical interpretation of this case is the following (cf. Persson and Block): Since the ion distribution at $B = B_o$ includes smaller pitch angles than the electron distribution, the ion density tends to exceed the electron density at $B > B_o$, thereby creating an upward electric field that drags the electrons along, modifies the electron and ion distributions, and maintains $n_i \approx n_e$ at all $B_o \leqslant B < B_a$ (and $n_i = n_e = 0$ at $B \geqslant B_a$).

Although this case may be considered more of a textbook example than a description of typical magnetospheric conditions, it has generally been thought to illustrate a sound physical principle. However, on closer inspection this physical principle may not seem entirely sound. If the right-hand side in (12) is differentiated once more with respect to s, assuming the magnetic field is a dipole field, it follows that:

$$dE_{\parallel}/ds < 0 \quad . \tag{13}$$

Hence, the small net charge required to maintain $n_i \approx n_e$ cannot be provided by the ions. In fact, there is no net positive charge at any location along the magnetic field line where $n_i > 0$, and there are no ions to support the electric stress at $B \geqslant B_a$. It can thus be argued that this simple case rather illustrates the difficulty of satisfying all of the conditions in (8)-(11) at the same time.

A much more elaborate and perhaps more realistic case has been presented by Chiu and Schulz (1978) and Chiu and Cornwall (1980). Their case also considers an ion population at high altitude which is isotropic outside of the loss hyperboloids in Figure 3, but the corresponding electron population is required to be anisotropic, with a wider distribution in $v_\perp$ and in $v_\parallel$ (bi-Maxwellian). Their case further includes particles within the loss hyperboloids, some of which have a terrestrial origin, and thus includes a net current. They reach the condition in (11) not by analytical methods alone, but by iterative numerical approximations, and their solution is far too complex to be evaluated here. A few comments with bearing on their case will be made below, however.

IV. POSSIBLE ROLE OF DOUBLE LAYERS

The studies of quasi-neutrality in a model magnetic mirror configuration show that it is mathematically possible to satisfy $n_i \approx n_e$ in a time-independent parallel electric field that extends over large distances and does not contain any double layer structures, provided the particle distribution functions are carefully designed. It is not clear from these studies, however, that such electric fields are realistic, or even physically possible. One argument to that effect was made in the preceding section, applied to a simple case where all particles are trapped by the combined electric and magnetic fields. Other arguments to the same effect may be applied to the more general case where the loss hyperboloids are also populated, and thus a current flows (e.g., Chiu and Schulz, 1978). In that case it can be argued, for instance, that the parallel electric field is made subject to potentially conflicting conditions; on one hand the model electric field is designed to satisfy $n_i \approx n_e$ everywhere, based on the entire pitch angle distributions of all particles, while on the other hand the electric field in reality must also be subject to the external condition that the current be of the appropriate magnitude, and the current only involves particles within the loss hyperboloids.

The aforementioned studies, however, do point to an unambiguous condition for the non-existence of electric fields; in order for the parallel electric field to vanish over a large distance along a magnetic flux tube, the pitch angle distributions of ions and electrons, when integrated over all energies, must be identical (cf. Persson, 1963). As a consequence, it may not be possible, given realistic particle distributions, to have the electric field entirely contained within a single stable double layer, or even within multiple double layers. The double layers naturally generate different pitch angle distributions for the ions and the electrons, and these in turn will affect the quasi-neutrality at all other altitudes. In other words, a stable double layer may not be nature's replacement for an extended electric field, but may perhaps be part of it (cf. Stern, 1981). Such a configuration cannot be modeled, however, if the condition in (10) is part of the assumptions.

A possibly fundamental shortcoming of the classical approach to magnetic confinement is its disallowance of temporal variations in the electric field, including rapid and small-scale fluctuations. The assumption in (8) is needed to make a tractable problem, but may not be supported by data. Close scrutiny of Figure 1, for example, fails to produce the sharp boundaries of Figure 3 (with $B \approx B_a$). This and other published illustrations of auroral electron spectra have in fact a rather blurred appearance, suggesting that the electrons have traversed a "turbulent" electric field. Numerous reports of intense plasma wave turbulence at various altitudes along auroral magnetic field lines (e.g., Fredricks et al., 1973; Gurnett and Frank, 1977; Mozer et al., 1980; and references therein) lend additional support to that kind of interpretation.

Allowing the electric field to have temporal fluctuations of a small scale size may render an untractable computational problem, but provides for a more realistic description of the collective behavior of the particles. From a qualitative point of view this may also seem to make the magnetic mirror a more favorable environment for the formation of double layers, as illustrated schematically in Figure 4. This figure assumes that the increase in kinetic energy of individual electrons is not a unique function of location in space, but varies somewhat randomly about an average increase, due to temporal fluctuations in the electric field. Only the average increase is a function of location and has the sharp boundaries in velocity space. An electron that has a kinetic energy slightly inside of the acceleration boundary when passing point P, either on the way down or after mirroring in the magnetic field below, is likely to be trapped by the average electric field on the way up, thereby adding to the local concentration of negative charge (during part of its oscillation), at the expense of the negative charge at higher altitude. This in turn further widens the acceleration boundary in the transverse direction, enabling electrons with a larger perpendicular energy to be trapped as well. Electrons inside the acceleration boundary may be removed again after a slight increase in the energy, but the net diffusion is assumed inward as long as the density of particles is higher on the outside. A conceivable end result may be some form of double layer, thin enough to harbor a significant charge imbalance in a stable fashion (cf. Lennartsson, 1980).

Whether trapping of electrons between magnetic and electric mirror points will produce a stable double layer, or merely add to the plasma turbulence, cannot be decided from this simplistic exercise alone. A redistribution of the electric field from higher to lower altitude carries with it a redistribution of the ion density as well, and that is not considered. It is worth noting, however, that the shape and size of the electron acceleration boundary depends on the angle of the double layer, and is the smallest for a double layer with the electric field nearly perpendicular to $\vec{B}$. In that case the boundary may be almost circular (cf. Figure 3 with $B >> B_o$), and can trap the fewest number of electrons. This kind of structure is perhaps the most likely to materialize and is, in fact, reminiscent of the "electrostatic shocks" commonly observed in the auroral regions (e.g., Mozer et al., 1977; see also Swift, 1979; Lennartsson, 1980; Borovsky and Joyce, 1983). It also has a favorable geometry for satisfying (4), thus producing a large electron current.

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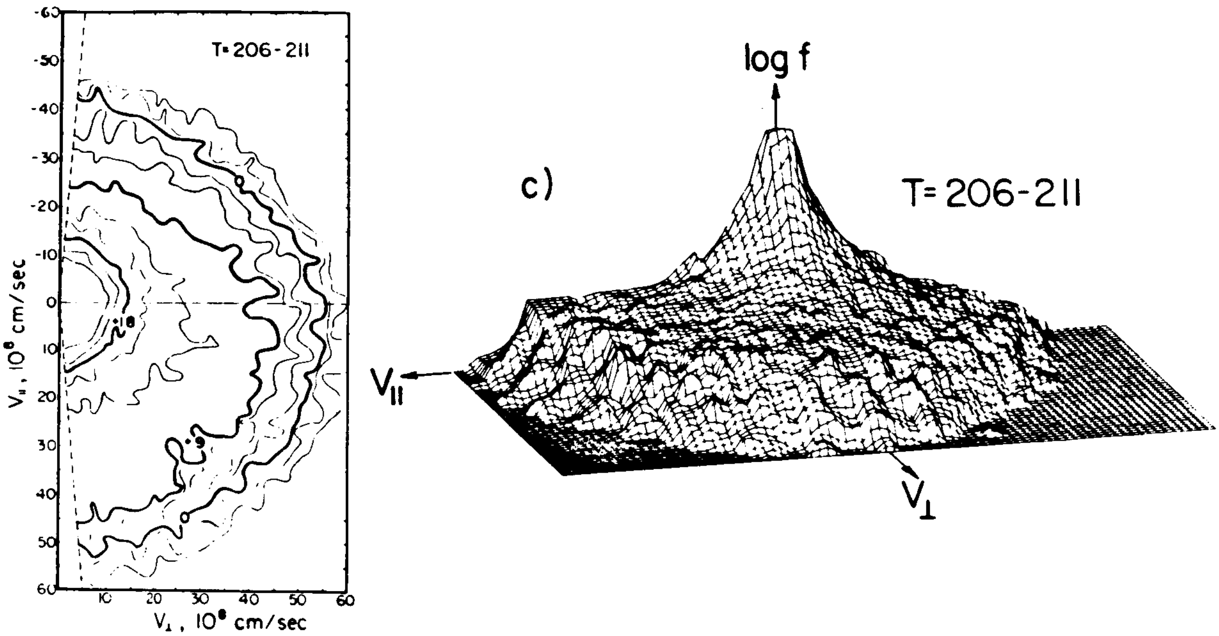

Figure 1. Contour and three-dimensional plot of auroral electron distribution function, in the energy range 25eV to 15 keV, measured from a rocket at about 240 km altitude. Downgoing electrons have positive $v_{\|}$. Curves of constant $f(\vec{v})$ on the contour plot are labeled by the common logarithm of $f(\vec{v})$ in s^3/km^6. This distribution is typical of electrons producing discrete auroral arcs (from Kaufmann and Ludlow, 1981).

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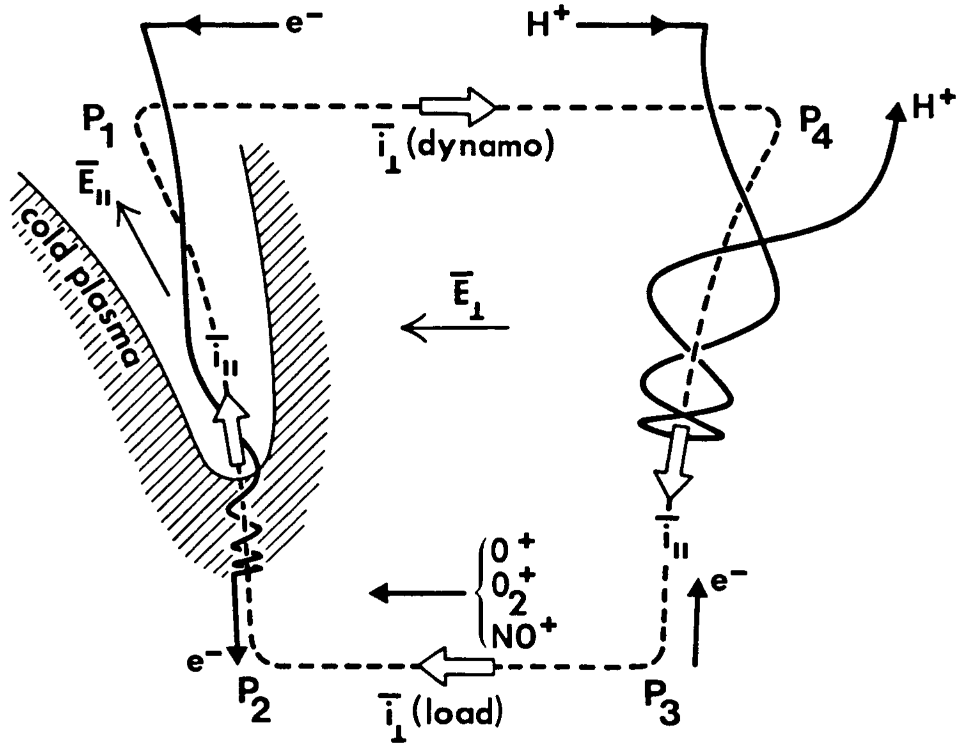
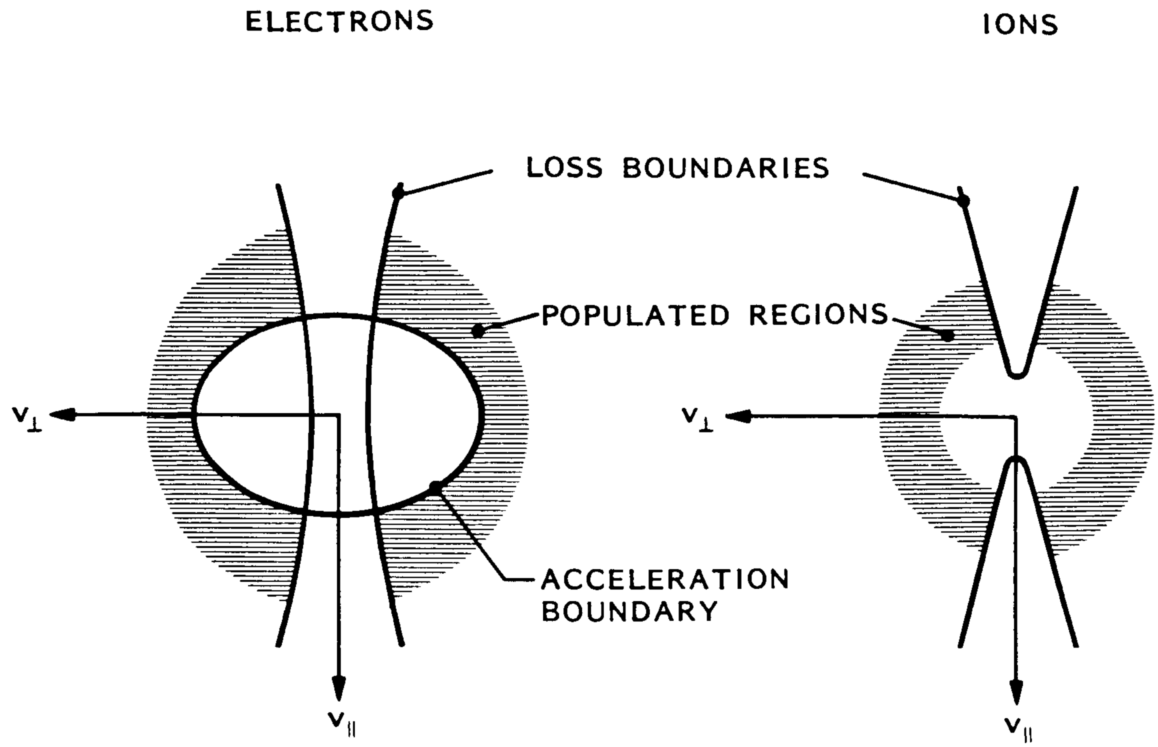


Figure 2. Schematic magnetosphere-ionosphere current system. The dynamo current $P_1 - P_4$ is assumed to be caused by the differential drift of hot protons and electrons. The downward parallel current $P_4 - P_3$ may be carried mainly by escaping ionospheric electrons, while the upward parallel current $P_2 - P_1$ is carried to a large extent by downflowing hot electrons. Point P_2 is at a high positive potential with respect to point P_1, which enables the downflowing electrons to overcome the magnetic mirror. The current $P_3 - P_2$ is a Pedersen current (from Lennartsson, 1976).



Figue 3. Hypothetical case of plasma confinement by a magnetic mirror in the presence of a parallel electric field, directed away from the magnetic mirror (upward). Only the shaded regions are assumed populated (see text). The loss boundaries (hyperboloids) are defined by $(B_a/B - 1)v_\perp^2 - v_\parallel^2 = 2H\,(V_a - V)$, where the subscript a refers to atmospheric (loss) altitude and $H = e/m_e$ for electrons and $H = -e/m_i$ for ions. The acceleration boundary (ellipsoid) is defined by $(1 - B_o/B)v_\perp^2 + v_\parallel^2 = 2\,(e/m_e)\,(V - V_o)$, where the subscript o refers to a high altitude ($B_o <$ B) (adapted from Persson, 1966).

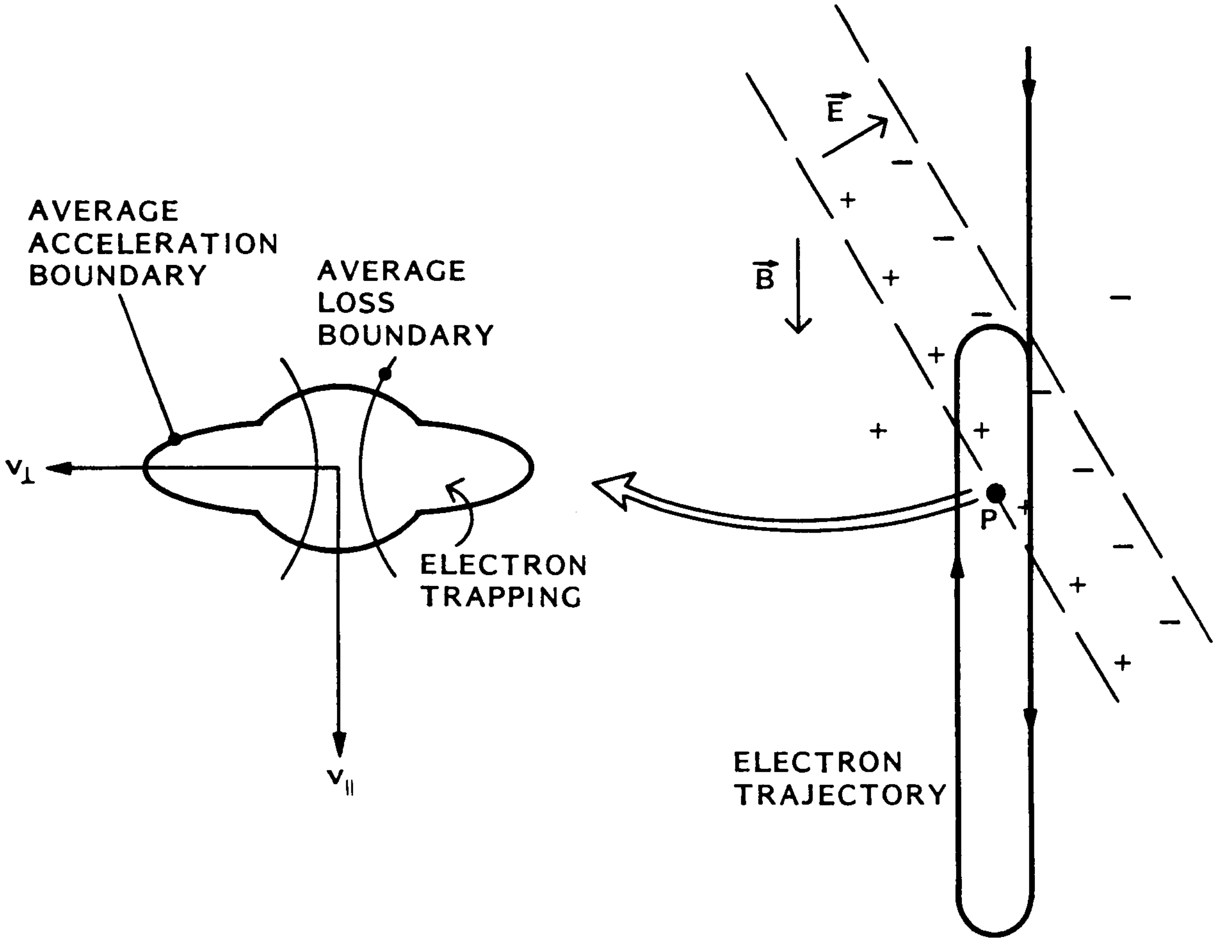


Figure 4. Hypothetical case of electron trapping by a locally enhanced electric field (right panel), associated with diffusion in velocity space (left panel). The diffusion is assumed to result from small-scale fluctuations in the electric field. The acceleration boundary at point P refers to an average acceleration and is the combined effect of the weak electric field at higher altitudes and the stronger field nearby (see text) (adapted from Lennartsson, 1980).