Why No Orbital Resonances Among the Satellites of Uranus?

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Most of the orbital resonances among the satellites of the major planets are thought to have been assembled by differential tidal expansion of their orbits. We can investigate why this has not occurred for the Uranus system by determining the resonances which would have been encountered for various values of Uranus' tidal effective Q. If the minimum Q = 17,000 is assumed for Uranus (from Ariel's proximity to Uranus), Miranda would have passed through the 4:3 and 3:2 orbital resonances with Ariel. As Ariel's orbit expands faster than Miranda's, capture into the Miranda-Ariel resonances is impossible. Althrough the passage of Miranda through the resonances in the wrong direction for possible capture would have increased Miranda's orbital eccentricity and inclination, the latter increase cannot explain the current 4° inclination for Miranda's orbit since the corresponding 6:4 and 8:6 inclination resonance passages would have increased the inclination only a few tenths of a degree. For the same minimum Q, Ariel would have passed through the 2:1 and 5:3 orbital resonances with Umbriel. But Ariel and Umbriel would have been captured into either of the lowest order 2:1 eccentricity resonances with certainty unless $e_A > 0.0224$ and $e_U > 0.0219$ at the time of encounter. As the time constant for damping Ariel's eccentricity from tidal dissipation within the satellite is only about 5×10^7 years (with Q = 100 and rigidity $\mu = 4 \times 10^{10}$ assumed), and Ariel's current eccentricity being only 0.0034, it is very unlikely that Ariel could have avoided capture into the 2:1 libration if the system in fact had passed through this resonance. The Q of Uranus need only be increased to 30,000 to keep Ariel always outside the 2:1 resonance with Umbriel. To avoid certain capture of Ariel–Umbriel in the 5:3 resonance, $e_A > 0.0036$ and $e_U > 0.0059$ at the time of passage. With current eccentricities at 0.0034 and 0.005 respectively, and the fact that eccentricities are *reduced* from initial values upon a probalistic escape from the resonance, it is possible that the system passed through the 5:3 resonance. But if the tidal Q of Uranus is larger than 200,000, Ariel would have started even outside the 5:3 resonance. Since the Q of Jupiter is in the range 6×10^4 to 2×10^6, 200,000 is not an outrageous value of Q for Uranus. This comparison and the possible escape of Ariel-Umbriel from the 5:3 resonance, means that it is not unreasonable that we find no orbital resonances among these satellites.