

## ASTEROID LIGHTCURVE INVERSION

Steven J. Ostro, Jet Propulsion Laboratory, Pasadena, CA 91109  
Robert Connelly, Cornell University, Ithaca, NY 14853

One of the most fundamental physical properties of any asteroid is its shape, i.e., its dimensions. Lightcurves provide the only source of shape information for most asteroids. Unfortunately, the functional form of a lightcurve is determined by the viewing/illumination geometry ("VIG") and the asteroid's light-scattering characteristics as well as its shape, and in general it is impossible to determine an asteroid's shape from lightcurves (1). The best one can do is to derive a shape constraint that is useful and takes advantage of all the information in the lightcurve.

We have introduced a technique called convex-profile inversion (CPI) that obtains a convex profile,  $\underline{P}$ , from any lightcurve (2). If certain ideal conditions are satisfied, then  $\underline{P}$  is an estimator for the asteroid's "mean cross section",  $\underline{C}$ , a convex set defined as the average of all cross sections  $C(z)$  cut by planes a distance  $z$  above the asteroid's equatorial plane.  $\underline{C}$  is therefore a 2-D average of the asteroid's 3-D shape. The ideal conditions are that (A) all  $C(z)$  are convex; (b) the asteroid's scattering law is geometric, so brightness is proportional to the projected visible, illuminated area; (C) the VIG is equatorial, i.e., the asteroid-centered declinations  $\delta_E$  and  $\delta_S$  of the Earth and Sun are zero; (D) the solar phase angle  $\phi \neq 0$ .

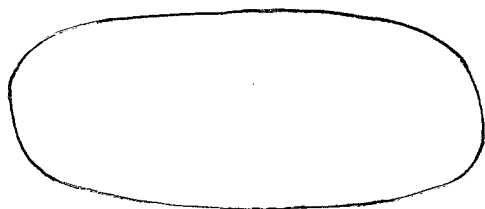
The first three conditions are unlikely to ever be satisfied exactly, but the issue here is the extent to which their violation degrades the validity of  $\underline{P}$  as an estimator for  $\underline{C}$ . Laboratory simulations suggest that modest, "topographic" concavities play a relatively minor role in determining the form of a lightcurve (3). Similarly, numerical experiments indicate that systematic errors introduced by small (several-degree) violation of Condition C are not severe (4). The bulk of available polarimetric and spectrophotometric data show that whereas "hemispheric" albedo variations can be detected at about the several percent level for several asteroids [e.g., (5)], the forms of most broadband optical lightcurves seem less sensitive to surface heterogeneity than to gross asteroidal shape.

Geometric scattering is expected to be an excellent approximation close to opposition (6), but a poor approximation far from opposition (7). This is an unfortunate circumstance, because CPI's ability to reveal odd harmonics in  $\underline{C}$  improves as  $\phi$  increases. The systematic error introduced by non-geometric scattering will depend on the 3-dimensional shape as well as on the VIG. Hence, the nature and magnitude of this error will be to invert several lightcurves obtained under nearly ideal VIG but at a variety of solar phase angles, use the weighted mean profile as an estimate of  $\underline{C}$ , and use the variance in the profiles to gauge the severity of systematic error.

What can an opposition lightcurve tell us about an asteroid's shape? At  $\phi = 0$ , CPI yields a profile  $\underline{P}_S$  as an estimator for  $\underline{C}_S$ , the "symmetrization" and the ratio  $\beta$ , of the profile's maximum breadth to its minimum breadth, remain intact.) Moreover, Condition A need not hold at opposition, and if it does not, then  $\underline{C}_S$  is the symmetrization of the asteroid's mean convex envelope, or "hull".

Opportunities for reliable estimation of  $\underline{C}_S$  should be much more abundant than those for estimation for  $\underline{C}$  for several reasons. First, the VIG required for reliable estimation of  $\underline{C}_S$  occurs much more frequently than that for estimation of  $\underline{C}$ . Second, to assess how close the VIG is to ideal, we want to know the direction of the asteroid's spin vector when estimating  $\underline{C}$ , but we just need to know the direction of the asteroid's line of equinoxes when estimating  $\underline{C}_S$ . Third, as noted above, geometric scattering is most valid close to opposition. Finally, as shown by Russell (1), it is easy to test the hypothesis that the ideal conditions required for reliable estimation of  $\underline{C}_S$  ( $\delta_E = \delta_S = \phi = 0$  and geometric scattering) actually hold; if the lightcurve has any odd harmonics, the conditions are violated.

$\underline{C}_S$  is the symmetrized average of all  $C(z)$  and constitutes the optical extraction of shape information from an opposition lightcurve, just as  $\underline{C}$  constitutes the optimal extraction of shape information from a non-opposition lightcurve. If an estimate of  $\underline{C}$  were free of systematic errors, its symmetrization would look the same as an estimate of  $\underline{C}_S$ , so we can use opposition lightcurves to qualify the interpretation of non-opposition lightcurves.



This profile is our estimate of  $\underline{C}_S$  for asteroid 624 Hektor from CPI of a lightcurve obtained by Dunlap and Gehrels (8) at  $\phi = 4^\circ$ . The pole directions estimated by those authors and by Magnusson (9) indicate that  $|\delta_E|$ ,  $|\delta_S| < 10^\circ$ . The constancy of Hektor's color indices with rotational phase (8) results of Russell's Fourier test, and the goodness of fit of CPI's model lightcurve to the data concur in supporting the expected high reliability of this estimate.

The profile has  $\beta = 2.5$  and is distinctly non-elliptical. Since the mean cross section of an ellipsoid rotating about a principal axis is an ellipse, our results suggest that neither the asteroid nor its convex hull are ellipsoids. On the other hand, our results are quite consistent with many other models for Hektor's 3-D shape, including a cylinder with rounded ends (8), a dumbbell (10), and various binary configurations.

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