

## Dynamics of Satellites, Asteroids, and Rings

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Work is in progress on: (a) determining the shapes and the internal structures of satellites (with Peter Thomas of Cornell University); (b) investigating the tidal heating of Miranda (with Renu Malhotra of Cornell University and Carl Murray of Queen Mary College, London); (c) investigating the dynamics of arc-like rings (with Carl Murray); and (d) determining the structure of the zodiacal cloud as revealed by IRAS (with Philip Nicholson of Cornell University). Significant progress has been made in (a) the determination of the shape and the internal structure of Mimas and (b) understanding the dynamical evolution of Miranda's orbit.

### (a) The Shape and Internal Structure of Mimas

Limb profiles from the six best Voyager images have been used to determine the shape of the satellite. Correction of image distortions allows coordinates on the limbs to be located with an accuracy of approximately one-half picture element: about 0.5 km for the two best images and between 1 and 2 km for the other images. Ellipses fit to the limbs show that the shape of Mimas is well-represented by a triaxial ellipsoid: it is the smallest satellite observed for which this is possible. The ratio of the differences of the axes,  $(b - c)/(a - c)$ , is  $0.27 \pm 0.04$ , indicating that the satellite is close to hydrostatic equilibrium. This is the first observation of a satellite in the solar system with a triaxial equilibrium figure. Using the satellite mass determined by Kozai (1957) from observations of the libration period and the libration amplitude of the Mimas-Tethys resonance, and a second-order theory for the ellipsoidal figure of equilibrium, we deduce that the satellite has a mean radius  $\langle R \rangle$  of  $198.9 \pm 0.6$  km, a mean density of  $1.137 \pm 0.018$  g/cm<sup>3</sup> and that the difference between the long and short axes,  $a - c$ , is  $17.0 \pm 0.7$  km. The expected value of  $a - c$  for a comparable, but homogeneous satellite in hydrostatic equilibrium is  $20.3 \pm 0.3$  km. We conclude that Mimas is probably differentiated and may have a rocky core of radius  $(0.43 \pm 0.10) \langle R \rangle$ . The material outside the core probably has a mean density of  $0.98 \pm 0.08$  g/cm<sup>3</sup>, consistent with that of uncompressed water-ice. The rock/ice ratio (by weight) of Mimas is probably a factor of 2 lower than the cosmic ratio: Mimas is markedly deficient in rock. This work represents the first determination of the internal structure of a satellite in the solar system, other than the Moon, and is likely to shed light on the accretion of satellites. Preliminary considerations favor the idea of heterogeneous accretion (Dermott and Thomas, 1986).

### (b) Tidal Heating of Miranda

This work is part of a continuing effort to understand the dynamics of the Uranian satellite system. We showed that the theory previously used to find the masses of the Uranian satellites from their orbital precession rates contained a fundamental error (Dermott and Nicholson,

1986). Thus, we were able to predict that the pre-Voyager masses of the satellites would prove to be incorrect. This prediction has now been proved to be right.

We have now analyzed the evolution of the Uranian satellite orbits due to tidal dissipation in the planet and have calculated the change in the orbital elements due to passage through low-order orbit-orbit resonances. We have succeeded in showing that the orbital eccentricity of Miranda would have been dramatically increased by these passages and that the subsequent tidal damping of this eccentricity would have heated the satellite. We have also calculated the change in the orbital inclination of Miranda on passage through the same set of resonances. The calculated change is large enough to account for the present very high orbital inclination of Miranda. Thus, we consider that we have found a dynamical solution for both the bizarre appearance of Miranda observed by Voyager and the anomalous orbital inclination (Malhotra, Dermott and Murray, 1986).

### **(c) Dynamics of Arc-Like Rings**

The location and the stability of the Lagrangian equilibrium points in the restricted circular three-body problem have been examined under a variety of drag forces. Linear stability analysis and numerical integration confirm that, contrary to what might be expected from simple energy arguments, the  $L_4$  and  $L_5$  points can be asymptotically stable under the action of certain drag forces, despite being points of potential maxima. The results have been extended to the horseshoe regime where the radial oscillations of the particle are small compared with the width of the horseshoe path in the rotating reference frame. In this case, the behavior of the Jacobi constant averaged over the horseshoe path determines the stability and the sense of evolution of the particle. If the drag force varies as  $v^p$ , where  $v$  is the velocity of the particle in the inertial frame, then the value  $p = 2$  is critical for both the tadpole and the horseshoe regimes. Stability is ensured if  $\text{sign}(a) \times \text{sign}(2 - p)$  is negative.

A similar analysis can be applied to any particle in a co-rotating arc. These results may have important implications for the stability of arcs of ring material where the dynamical effects of drag can counteract the spreading due to particle collisions (Murray and Dermott, 1986).

### **(d) Structure of the Zodiacal Cloud as Revealed by IRAS**

The IRAS Zodiacal History File, which contains the all-sky survey data, is now to hand at Cornell. Software has been written to Fourier analyse the data and thereby separate the smooth large-scale zodiacal background from the narrower dust bands. Our preliminary results were described in a paper read at the Uppsala Asteroid, Comets and Meteors meeting (Dermott, Nicholson, and Wolven, 1986).

We have previously shown that the dust bands may be debris associated with the Hirayama asteroid families, in particular, the Eos and Themis families, and we have predicted (a) that the ecliptic latitudes of the dust bands should vary with ecliptic longitude and (b) that the central dust band should be split (Dermott et al., 1984, and Dermott et al., 1985). We now have evidence supporting both of these predictions. In particular, we have evidence showing that the central dust band is indeed split and that the separation in latitude of the two components is consistent with that expected for debris derived from the Themis family. This has important implications for the origin of the particles in the zodiacal cloud.

### References

1. Dermott, S. F., Nicholson, P. D., and Wolven, B. (1986). Preliminary Analysis of the IRAS Dust Data. In "Asteroids, Comets and Meteors II" (eds. C-I. Lagerkvist and H. Rickman) Uppsala, 583-594.
2. Dermott, S. F., Nicholson, P. D., Burns, J. A., and Houck, J. R. (1985). An Analysis of IRAS' Solar System Dust Bands. IAU Colloquium No. 85, "Properties and Interaction of Interplanetary Dust" (ed. Giese, R.H. and P. Lamy), D. Reidel Pub. Co., 395-409.
3. Dermott, S. F., Nicholson, P. D., Burns, J. A., and Houck, J. R. (1984). Origin of the Solar System Dust Bands discovered by IRAS, Nature, 312, 505-509.
4. Dermott, S. F. and Nicholson, P. D. (1985). Masses of the Satellite of Uranus. Nature 319, 115-120.
5. Dermott, S. F., and Thomas, P. C. (1986). The Shape and Internal Structure of Mimas. B.A.A.S. 18, 761.
6. Malhotra, R., Dermott, S. F., and Murray, C. D. (1986). Tidal Heating of the Uranian Satellites. B.A.A.S. 18, 785.
7. Murray, C. D., and Dermott, S. F. (1986). Dynamical Effects of Drag on Particles in Corotational Resonances. B.A.A.S. 18, 778.