Hamiltonian Theory of Nonlinear Waves in Planetary Rings[*]

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Nonlinear spiral density waves have been observed at several locations in Saturn's rings. Our theoretical understanding of these waves was substantially advanced by the recent derivation by Shu et al. (1985) and Borderies et al. (1985) of a nonlinear dispersion relation which generalizes the linear relation found earlier by Lin and Shu.[1,2] The purpose of this abstract is to report the derivation of a Hamiltonian field theory for nonlinear density waves. Starting from the Hamiltonian for a discrete system of gravitating streamlines, an averaged Hamiltonian is obtained by successive application of Lie transforms. The transformation may be carried out to any desired order in q, where q is the nonlinearity parameter defined in the above references. Subsequent application of the WKB approximation yields an asymptotic field Hamiltonian. Both the nonlinear dispersion relation and the wave action transport equation are easily derived from the corresponding Lagrangian by the standard variational principle.

[1]Shu et al., *Ap. J.* 291 (1985) 356

[2]Borderies et al., *Icarus* 63 (1985) 406