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OPTIMIZATION OF PAYLOAD MASS PLACEMENT IN A DUAL KEEL SPACE STATION

Melvin J. Ferebee, Jr.

Robert B. Powers

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NOMENCLATURE

a	ellipse semi-major axis
b	ellipse semi-minor axis
d	length of focal point of ellipse to ellipse center
F	objective function to be minimized
$G_j(X)$	inequality constraint function
$H_k(X)$	equality constraint function
h_i	height (y-direction) of element i
[I]	Inertia matrix
I_{ij}	ij component in inertia matrix
I_{ij_o}	ij component of inertia matrix for initial orientation
l_i	length (x-direction) of element i
m_i	mass of element i
T_g	gravity gradient torque vector
T_{g_i}	gravity gradient torque in i^{th} direction
U_v	unit vector along vertical from Earth's center to spacecraft
U_{v_i}	i^{th} component of u_v
X	vector of variables to be optimized
x_{cg}	x center-of-gravity location for system
x_{cg_o}	initial center of gravity in x-direction for system
x_i	x center of gravity for element i
y_{cg}	y center-of-gravity location for system
y_{cg_o}	initial center of gravity in y-direction for system
y_i	y center of gravity for element i
z_{cg}	z center-of-gravity location for system
z_{cg_o}	initial center of gravity in z-direction for system
z_i	z center of gravity for element i
Θ	angle between spacecraft vertical and local vertical
ω	Earth's orbital angular rate

INTRODUCTION

In order to keep an operational Space Station in a stable low Earth orbit, a control system for station-keeping and attitude control operations will have to be used. This control system will employ control moment gyros for angular momentum storage and reaction control systems for translational attitude control. To minimize the size of the momentum storage system and the amount of propellant required for reaction control and momentum desaturation, certain overall mass properties of the Station will have to be minimized, these being the mass cross products of inertia. With minimized inertias, the Station would also experience minimal torques due to gravity gradient effects. Minimizing the inertias involves relocating externally attached payloads and appendages in a process that finds optimal moment arms within a two-dimensional plane for each payload mass.

Because of their important effects on the gravity induced torques, the mass properties to be minimized in this analysis are the cross products of inertia I_{xy}, I_{xz}, and I_{yz}. Through the use of mathematical programming techniques such as those used in operations research, an optimum arrangement of payload elements can be achieved that will minimize the cross products of inertia and thus the controllability resources.

The methodology has been automated into an interdependent set of four programs that can be used with the NASA IDEAS**2 program to provide a visual representation of the initial mass placement and the final optimized mass placement. These programs are generalized and depend only on the number of payload elements and minimal input from an interactive user. This computer aided engineering tool allows the analyst to arrange payloads and appendages within a Space Station geometry faster and more efficiently than the trial and error techniques previously used by mass properties engineers. The benchmark case of design will be the Dual Keel Space Station, with five payloads. This paper will present the derivation and execution of a methodology in which Space Station elements, primarily externally attached masses representing payloads, can be optimally arranged to minimize the cross products of inertia.

DESCRIPTION OF SPACE STATION ELEMENTS

The Dual Keel configuration, Figure 1, operates in a local vertical-local horizontal (LVLH) orientation, with its vertical keels along the local vertical direction and the solar array boom perpendicular to the orbit plane (POP). The lower horizontal keel of the Space Station contains Earth-viewing payloads. The upper horizontal keel contains solar, stellar, and anti-Earth viewing payloads and communications antennas. Non-viewing payloads are located at various places on the Space Station and the pressurized modules are located at the center of the transverse boom. Servicing equipment is to be located about the Station with front and back sides of the keels kept free for the traverse of the Mobile Remote Manipulator System (MRMS). The servicing and refueling facilities, Orbital Maneuvering Vehicle (OMV), Orbital Transfer Vehicle (OTV) technology demonstration equipment, and satellite storage and equipment areas are located at various places along the structure.

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U_{v_i}	i^{th} component of u_v
X	vector of variables to be optimized
x_{cg}	x center-of-gravity location for system
x_{cg_o}	initial center of gravity in x-direction for system
x_i	x center of gravity for element i
y_{cg}	y center-of-gravity location for system
y_{cg_o}	initial center of gravity in y-direction for system
y_i	y center of gravity for element i
z_{cg}	z center-of-gravity location for system
z_{cg_o}	initial center of gravity in z-direction for system
z_i	z center of gravity for element i
Θ	angle between spacecraft vertical and local vertical
ω	Earth's orbital angular rate

Gimbaled solar array wings provide full power at any relative alignment of the Space Station and the sunline. Both solar voltaic and solar dynamic power generation systems were used in the study and are shown on the configuration to demonstrate a design option. Heat rejection is provided by a combination of body-mounted radiators on the modules and deployed radiators along the transverse boom.

The center of gravity is located at the center of the transverse boom and modules to provide the lowest possible microgravity environment within the modules. A low microgravity environment will be essential for many of the scientific experiments and commercial operations that will take place on the Station.

One of the principal advantages of this configuration is the excellent viewing afforded to all payloads, both externally-mounted and internally-mounted. The configuration also allows good accommodation of tether payloads and communication antennas. Excellent clearances are provided for Orbiter rendezvous and berthing, and for construction, servicing, and other operations activities. The truss-mounted subsystems and distribution equipment are mostly pre-integrated (prior to launch) to minimize on-orbit time, complexity, risk, and especially extravehicular activity (EVA) operations during buildup and assembly. The module-mounted subsystems and distribution equipment are also pre-integrated. Launch and assembly of the Dual Keel Space Station requires a minimum of sixteen Space Shuttle launches, including some but not all the payloads that will be mentioned in this report. Assembly is accomplished using the Orbiter Remote Manipulator System (RMS) and Space Station MRMS after it is installed.

GRAVITY GRADIENT TORQUE EQUATIONS

As was stated in the introduction, minimizing the cross products of inertia will also minimize the induced gravity gradient torques experienced by a Station. Reference 1 shows that gravity gradient torques on a body are governed by the following equation:

$$T_g = 3\omega^2 U_v \quad \{[\ I\ ]\ [U_{v_1},\ U_{v_2},\ U_{v_3}]\}$$

Or:

$$T_g = 3\omega^2 (U_{v_1}, U_{v_2}, U_{v_3}) \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} U_{v_1} \\ U_{v_2} \\ U_{v_3} \end{bmatrix}$$

Where: T_g = gravity gradient torque vector

ω = orbital angular rate

$[I]$ = inertia matrix

$[U_v]$ = unit vector, along the vertical, from the Earth's center to the spacecraft

The above torque equation can be reduced to a function of mass properties of the Station and an angle Θ, the angle which the Station is pitched off the local vertical, as shown below:

$$T_{g_x} = 3\omega^2 [(0.5 \sin 2\Theta)(I_{yy} - I_{zz}) - (I_{yz} \cos 2\Theta)]$$

$$T_{g_y} = 3\omega^2 [(0.5 \sin 2\Theta)(I_{zz} - I_{xx}) - (I_{xz} \cos 2\Theta)]$$

$$T_{g_z} = 3\omega^2 [(0.5 \sin 2\Theta)(I_{xx} - I_{yy}) - (I_{xy} \cos 2\Theta)]$$

Two limiting conditions apply to the above equations. First, for $\Theta=0$, the torques become solely a function of the cross products of inertia since $\sin(2\Theta)=0$ and $\cos(2\Theta)=1$. Secondly, for small angles of Θ, the principal moments of inertia are of the same magnitude, thus making their difference small. This combined with the small angle approximation that $\sin(2\Theta)$ is near zero and $\cos(2\Theta)$ is near one indicates that the torques are primarily a function of the cross products of inertia. Thus, by minimizing the cross products terms, there will be a minimal contribution to the torque components.

ASSIGNMENT OF VARIABLES AND COORDINATE DEFINITION

Even though standard payload elements have a variety of geometric shapes, only rectangular and square payloads will be considered in this analysis in order to simplify it. Fourteen values will be assigned with each element in each of two coordinate systems, eight in a local element system and six in a Station center of gravity (CG) centered system. The purpose of the two coordinate systems is to provide sufficient generality in specifying element dimensions and CGs as well as their location within the overall Station. In the local system, the element's mass and number are assigned. The x, y, and z coordinates of the element's CG are specified assuming (0,0,0) as a local origin, thus providing for any offset that the CG may have from the assumed geometric center at the center of the rectangle. The length, height, and depth values are assigned and are aligned along the local x, y, and z axes, respectively. In the global system, the x, y, and z coordinates of each element's geometric center are specified by the user to locate each payload within the overall Station. The global x, y, and z coordinates of each element's CG are derived by one of the programs from the sum of the local CG coordinate and the global geometric center coordinate. See Figure 2 for a description of the two coordinate systems.

The coordinate system used in this analysis is a standard cartesian coordinate system with the origin centered at the middle of the transverse (solar array) boom. The x axis is along the transverse boom, the y axis is along the keels, and the z axis completes the system by pointing along the direction of flight (See Figure 1.)

In the program formulation it is desirable to force all coordinates to be strictly nonnegative real numbers. This is done by biasing all of the coordinates by adding the point (1000,1000,1000) to all the global coordinates. This essentially locates the Station far out in quadrant I of an imaginary three-dimensional coordinate system. The inertias of the system are not changed due to the transformation of coordinates because the inertias are taken about the transformed system's center of gravity.

The representative payloads used in this analysis is described in Table I.

Table I. Description of payload elements

Number (i)	Description Name	Weight (kg)	Length (m)	Height (m)	Depth (m)
1	Transition Radiation & Ion Calorimeter (TRIC)	5,749	3.0	5.5	2.7
2	Pinhole Occulter Facility	3,602	3.6	4.6	2.7
3	Production Unit	4,500	4.3	6.4	4.3
4	Space Construction Experiment	4,001	29.0	24.4	20.1
5	Orbital Transfer Vehicle Servicing Experiment	8,001	8.5	19.2	8.5

MATHEMATICAL PROGRAM FORMULATION

I. INERTIAL CONSTRAINTS

In mathematical programming, one form an optimization program can take is to minimize an objective function of some set of variables subject to a certain number of constraint functions of these same variables. A program would attempt to solve the problem:

Minimize F(X)

Subject to: $G_j(X) \leq 0 \quad j = 1,n$

$$H_k(X) = 0 \quad k = 1,m$$

Where F(X) is the objective function, $G_j(X)$ is one of n inequality constraint functions, $H_k(X)$ is one of m equality constraint functions, and the vector X is the set of variables to be optimized. This analysis will derive an objective function, six equality constraints, and n inequality constraints where n will depend on the number of payloads.

The objective of the analysis is to minimize the three cross products of inertia for n elements about a given Station support structure. The structure itself has its own mass properties given by:

m_o = the Station mass

I_{xy_o}, I_{xz_o}, I_{yz_o} = the initial cross products of inertia without payloads

The final optimized location of the Station with payloads will be (x_{cg}, y_{cg}, z_{cg}). Each element will have the following characteristics:

m_i = mass of i^{th} element

x_i, y_i, z_i = x, y, and z global coordinates of each element's CG

The overall inertia of the Station with payloads is the objective function F which can be expressed as the sum of Station terms and payload terms:

F = Station terms + Payload terms

Where:

$$\text{Station terms} = I_{xy_o} + I_{xz_o} + I_{yz_o}$$

$$\text{Payload terms} = \left| \sum_{i=1}^{n} m_i (x_i)(y_i) \right| + \left| \sum_{i=1}^{n} m_i (x_i)(z_i) \right| + \left| \sum_{i=1}^{n} m_i (y_i)(z_i) \right|$$

Each of these terms changes if the Station CG is moved from $(x_{cg_o}, y_{cg_o}, z_{cg_o})$ to (x_{cg}, y_{cg}, z_{cg}). For the Station terms, this inertia change is added to the initial cross product of inertia due to the parallel axis theorem. For the payload terms, the inertial change is multiplied directly into the term.

Thus the objective function or overall Station inertia to be minimized becomes:

$$F = [\ I_{xy_o} + m_o(x_{cg_o} - x_{cg})(y_{cg_o} - y_{cg})] + [\ I_{xz_o} + m_o(x_{cg_o} - x_{cg})(z_{cg_o} - z_{cg})]$$

$$+ [\ I_{yz_o} + m_o(y_{cg_o} - y_{cg})(z_{cg_o} - z_{cg})] + |\ \sum_{i=1}^{n} m_i(x_i - x_{cg})(y_i - y_{cg})\ |$$

$$+ |\ \sum_{i=1}^{n} m_i(x_i - x_{cg})(z_i - z_{cg})\ | + |\ \sum_{i=1}^{n} m_i(y_i - y_{cg})(z_i - z_{cg})\ | \qquad (1)$$

Since the products of inertia can be negative, the absolute value of each of the payload product terms is taken so that any cancelling out of terms will not occur. The x_{cg}, y_{cg}, and z_{cg} of the Station and x_i, y_i, and z_i of each payload are the unknown variables to be found. The inertias about the final optimum CG once they are known are what are important to the gravity gradient control of the Station. When function F is minimized, all products of inertia are also minimized with respect to the optimum center of gravity.

The discontinuous absolute value terms in the objective function make the problem very difficult to solve in closed form. Using the techniques described in Hillier and Lieberman, Ref. 2, however, the three absolute value terms can be converted into three continuous terms in the objective function and three continuous terms that are included as three equality constraints in the set of constraint equations.

This is done using six auxiliary variables as shown in the following equations:

$$F = [\ I_{xy_o} + m_o(x_{cg_o} - x_{cg})(y_{cg_o} - y_{cg})] + [\ I_{xz_o} + m_o(x_{cg_o} - x_{cg})(z_{cg_o} - z_{cg})]$$

$$+ [I_{yz_o} + m_o(y_{cg_o} - y_{cg})(z_{cg_o} - z_{cg})] + (R_1 + S_1) + (R_2 + S_2) + (R_3 + S_3) \qquad (2)$$

Where the three equations in the constraint set are:

$$(R_1 - S_1) = \sum_{i=1}^{n} m_i(x_i - x_{cg})(y_i - y_{cg}) \qquad (3)$$

$$(R_2 - S_2) = \sum_{i=1}^{n} m_i(x_i - x_{cg})(z_i - z_{cg}) \qquad (4)$$

$$(R_3 - S_3) = \sum_{i=1}^{n} m_i(y_i - y_{cg})(z_i - z_{cg}) \qquad (5)$$

The differences in the auxiliary variables in the constraint set and their corresponding sums in the objective function are what convert the discontinuous absolute value terms in the objective function to six continuous terms in the overall problem.

The inertias are taken about the center of gravity of the structure, which is unknown until the optimization is complete. The next three constraints serve as the mathematical definition of the three coordinates of the overall center of gravity. These are given by x_{cg}, y_{cg}, z_{cg}:

$$x_{cg} = \frac{\sum_{i=1}^{n} m_i x_i}{M} \tag{6}$$

$$y_{cg} = \frac{\sum_{i=1}^{n} m_i y_i}{M} \tag{7}$$

$$z_{cg} = \frac{\sum_{i=1}^{n} m_i z_i}{M} \tag{8}$$

Where the total mass of the Station with payloads is $M = m_o + \sum_{i=1}^{n} m_i$.

In order to get equations (6)-(8) into the form of an equality constraint, H(X)=0 , a transposition is done. To get equation (6) into the form:

$$x_{cg} - \frac{\sum_{i=1}^{n} m_i x_i}{M} = 0$$

A similar transposition is done for equations (3)-(5). To transpose equation (3) into the form:

$$\sum_{i=1}^{n} m_i (x_i - x_{cg})(y_i - y_{cg}) + S_1 - R_1 = 0$$

Thus the six equality constraints consist of those defining the center of gravity, Equations (6)-(8), and those handling the auxiliary variables for the absolute value conversion, Equations (3)-(5). Only the CG coordinates of the payloads were used in the inertial constraints, since inertial constraints require inertial coordinates.

II. GEOMETRIC CONSTRAINTS

A. ELEMENT-ELEMENT INTERACTIONS

Inter-element interference criteria should be adhered to in order to preclude non-feasible solutions. Two means of modeling non-interference criteria are described below.

The interference problem is generally a three-dimensional problem. One must prohibit interference of one object with another object about the x, y, and z-axes. In this case, since no equipment can be placed on the front or back of the support truss, the problem is then reduced to a two-dimensional problem by eliminating the z-axis.

The problem is now to prohibit overlapping of any region defined by element i from interfering with a region defined by element j in the height and length dimensions. It is desirable to have a closed neighborhood around each payload that can be described by a continuous mathematical equation, such as a circle or an ellipse. A mathematically discontinuous closed neighborhood such as a square would not provide meaningful solutions and would be difficult to code into the optimization routine. One way this neighborhood can be achieved is to define a "constraint ellipse" such that the line of the ellipse passes through the points marking the maximum distance from the geometric centers of elements i and j when the elements just meet at a diagonal.(See Figure 3.) This methodology was derived from Reference 3.

The standard equation of an ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (9)$$

for the semi-major axis located about the x- axis, and

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \qquad (10)$$

for the semi-major axis located about the y- axis where a and b are given as the length of the semi-major and semi-minor axes, respectively. Since the elements i and j both have l and h values, the geometric center of element j on the constraint ellipse is defined as

$$[\ 1/2\ (l_i + l_j)\ ,\ 1/2\ (h_i + h_j)\ ] \qquad (11)$$

The geometric center of element i is defined as the origin. In order to minimize the maximum excursion of one element to another, the coordinate with the greatest value would be considered the direction of the semi-major axis. In order to determine the a and b values of the ellipse, the focal point of the ellipse is assumed to be located on the semi-major axis at one-half the sum of the length or height of the i^{th} and j^{th} elements, depending on the orientation of the semi-major axis. The distance from the center of the ellipse to its focal point is given as

$$d^2 = a^2 - b^2 \qquad (12)$$

Using equations (9) or (10), (11), and (12), the a and b values can be determined under the following conditions:

for $(l_i+l_j) \geq (h_i+h_j)$,

$$b^2 = \frac{4\,(h_i+h_j)^2 + \sqrt{16\,(h_i+h_j)^4 + 64\,(h_i+h_j)^2(l_i+l_j)^2}}{32} \qquad (13)$$

$$a^2 = b^2 + (l_i+l_j)^2/\,4 \qquad (14)$$

and for $(h_i+h_j) \geq (l_i+l_j)$,

$$b^2 = \frac{4\,(l_i+l_j)^2 + \sqrt{16\,(l_i+l_j)^4 + 64\,(l_i+l_j)^2(h_i+h_j)^2}}{32} \qquad (15)$$

$$a^2 = b^2 + (h_i+h_j)^2/\,4 \qquad (16)$$

The constraint is now imposed in a manner such that the center of gravity of element j can not enter the region defined by the constraint ellipse and is given by equations (17) and (18):

$$\frac{(x_j - x_i)^2}{a^2} + \frac{(y_j - y_i)^2}{b^2} \geq 1 \qquad (17)$$

or

$$\frac{(y_i - y_j)^2}{a^2} + \frac{(x_i - x_j)^2}{b^2} \geq 1 \qquad (18)$$

Thus two forms that the inequality constraints, $G(X) \leq 0$, can take are equations (17) and (18).

Another method of defining a constraint neighborhood around a payload is to circumscribe a "constraint circle" of fixed radius around it. The radius of this circle for element i is the distance from the geometric center to one of the corners and is denoted by r_i. In order to distinguish between long, thin payloads and short, square payloads, an aspect ratio criterion was used. If $(l_i+l_j)/(h_i+h_j) \geq 1.1$ or $(h_i+h_j)/(l_i+l_j) \geq 1.1$, ellipse interference paths were used between two payloads, where 1.1 is an arbitrary constant allowing for only 10% deviation from a square. In the case of short, square payloads where $(l_i+l_j)/(h_i+h_j) \leq 1.1$ or

$(h_i + h_j)/(l_i + l_j) \leq 1.1$, circular interference paths were used between two payloads. The use of circular interference paths for square payloads avoids the exclusive use of ellipses that could cause problems in the interference geometry of square payloads. Given two payloads that might interfere with one another, a circular constraint equation states that the distance between the geometric centers of the two payloads must be greater than or equal to the sum of the radii of their corresponding constraint circles. See Figure 3. The geometric center to geometric center distance is expressed using the standard cartesian distance formula:

$$(x_i - x_j)^2 + (y_i - y_j)^2 \geq (r_i + r_j) \tag{19}$$

This equation is transposed into an inequality constraint by the following:

$$(r_i + r_j) - (x_i - x_j)^2 + (y_i - y_j)^2 \leq 0 \tag{20}$$

Another form the inequality constraint, $G(X) \leq 0$, can take is equation (20). Thus the n number of inequality constraints will consist of G(1)-G(n) inequalities that preclude the interference of one payload with another, using either elliptical or circular interference equations. An equation is generated for all the possible non-redundant combinations of interaction between n payloads. Only the geometric center coordinates of the payloads are used in the geometric constraint equations, since geometric constraints require geometric coordinates.

B. STATION-ELEMENT INTERACTIONS

In the constraint formulation, the physical barriers of the Station must be considered in order to prevent payload-Station interference. This was accomplished by considering each rectangular region of the Station structure as a region to be optimized, with the exception that these regions had fixed explicitly defined coordinate locations. Non-redundant constraint equations are generated in G(1)-G(n) for Station-element interactions, but none are generated for Station-Station interactions since these would be meaningless because the Station elements are not allowed to move. The payloads must not interfere with the support trusses but be near them within allowable tolerances. The payload element can not be in free space in the interior of the Station solely for inertia optimizing purposes. An envelope well outside the perimeter of the Station outside of which the payloads could not move was also specified.

Viewing and contamination requirements of some missions can impose constraints on the elements. These constraints are included in the inequality constraint set G(1)-G(n) if a range of values is permitted or in the equality constraint set H(X) if an explicit fixed location is desired. These types of constraints were not used in this analysis, but it is obvious that payloads of the size of the Space Construction Experiment (Payload 4) should be placed outside of the center area of the Station because of their size.

ANALYSIS AND RESULTS

The computer code for Automated Design Synthesis (ADS) was selected to support the present analysis. ADS, described in References 4, is an automated optimization program capable of providing efficient optimization for a variety of problems. This particular problem, nonlinear in nature, was solved by the use of a combination of a strategy search and a one-dimensional search. Because the equations in both the objective function and the constraint set are at most quadratic in nature, relative minima must exist in their solutions. With this knowledge, Reference 4 suggests the use of sequential linear programming as a strategy and the modified method of feasible directions for the minimization. The one-dimensional search algorithm recommended was to find the minimum of an unconstrained function by first finding the bounds on the solution and then using polynomial interpolation. The solution of the problem is as follows along with the initial results of this type of analysis with a representative initial placement of the payload elements. The (1000,1000,1000) bias has been removed from all the coordinates.

Overall Center of Gravity (m)

	Initial	Optimized
x	0	-0.68
y	0	-0.85
z	0	0

Inertias $(kg\text{-}m^2)$

	Initial	Optimized	Δ %
xy	2.85600×10^6	8.31112×10^5	-71%
xz	1.04530×10^6	1.04530×10^6	0%
yz	4.00186×10^5	4.003019×10^5	+0.03%

Payload Locations (m)

Payload	X			Y			Z		
	Initial	Final	Δ	Initial	Final	Δ	Initial	Final	Δ
1	5	3	-2	35	33	-2	0	0	0
2	-5	-8	-3	-35	-45	-10	0	0	0
3	-5	-4	+1	18	21	3	0	[illegible]	[illegible]
4	5	8	3	-73	-90	-17	0	0	0
5	10	2	-8	-27	-37	-10	0	0	0

The payload initial positions and their displacements as a result of the optimization are shown in Figures 4 and 5, respectively. A comparison between the initial payload orientation and the final optimized orientation indicates that each payload shifted significantly, implying that the payloads were not placed in optimal initial positions. The two elements above the solar array boom shifted closer to the Station CG. The two elements below the solar array boom displaced downward as well as shifting more toward the Station CG. The Space Construction Experiment also shifted downward. This downward shifting would not be intuitively obvious in a non-automated procedure to minimize the inertias, since the natural inclination would be to move payloads below the solar array boom upwards toward the Station CG. This points out an advantage to a computer aided procedure. The final optimized positions of the payloads are not ideal for their attachment to the surrounding truss structures with short connecting trusses. This aspect of the problem could be contolled by adding constraints to restrict the movement of the payloads along the trusses of the support structure.

The overall I_{xy} of the Station was reduced by 71%, which would result in a small reduction in the gravity gradient induced torque which would in turn contribute to a small savings in the attitude control propellant over a certain mission length. This can be seen by referring to the gravity gradient torque equation for T_{g_z}. The increase of 0.03% in I_{yz} would have negligible effects. The small changes in the inertias involving the z-axis are due again to the fact that this is primarily a two-dimensional problem. The optimum inertias were derived from the following equations once the final payload and overall CG positions were known:

$$I_{xy} = [\, I_{xy_o} + m_o(x_{cg_o} - x_{cg})(y_{cg_o} - y_{cg})] + (R_1 - S_1) \qquad (21)$$

$$I_{xz} = [\, I_{xz_o} + m_o(x_{cg_o} - x_{cg})(z_{cg_o} - z_{cg})] + (R_2 - S_2) \qquad (22)$$

$$I_{yz} = [\, I_{yz_o} + m_o(y_{cg_o} - y_{cg})(z_{cg_o} - z_{cg})\,] + (\, R_3 - S_3) \qquad (23)$$

Even though this analysis does not show any startling results, it demonstrates the ability to rapidly optimize the placement of payloads on a Space Station.

CONCLUSIONS

The optimization methodologies used in operations research can be applied to the management of mass properties of a Dual Keel Space Station through the placement of discrete masses about the Station. The masses to be placed are the externally attached payloads. The payloads can be placed in such a manner that the inertia cross products can be reduced, thus minimizing the induced gravity-gradient torques on the Station. With the torques minimized, the momentum buildup over an orbital period will also be minimized.

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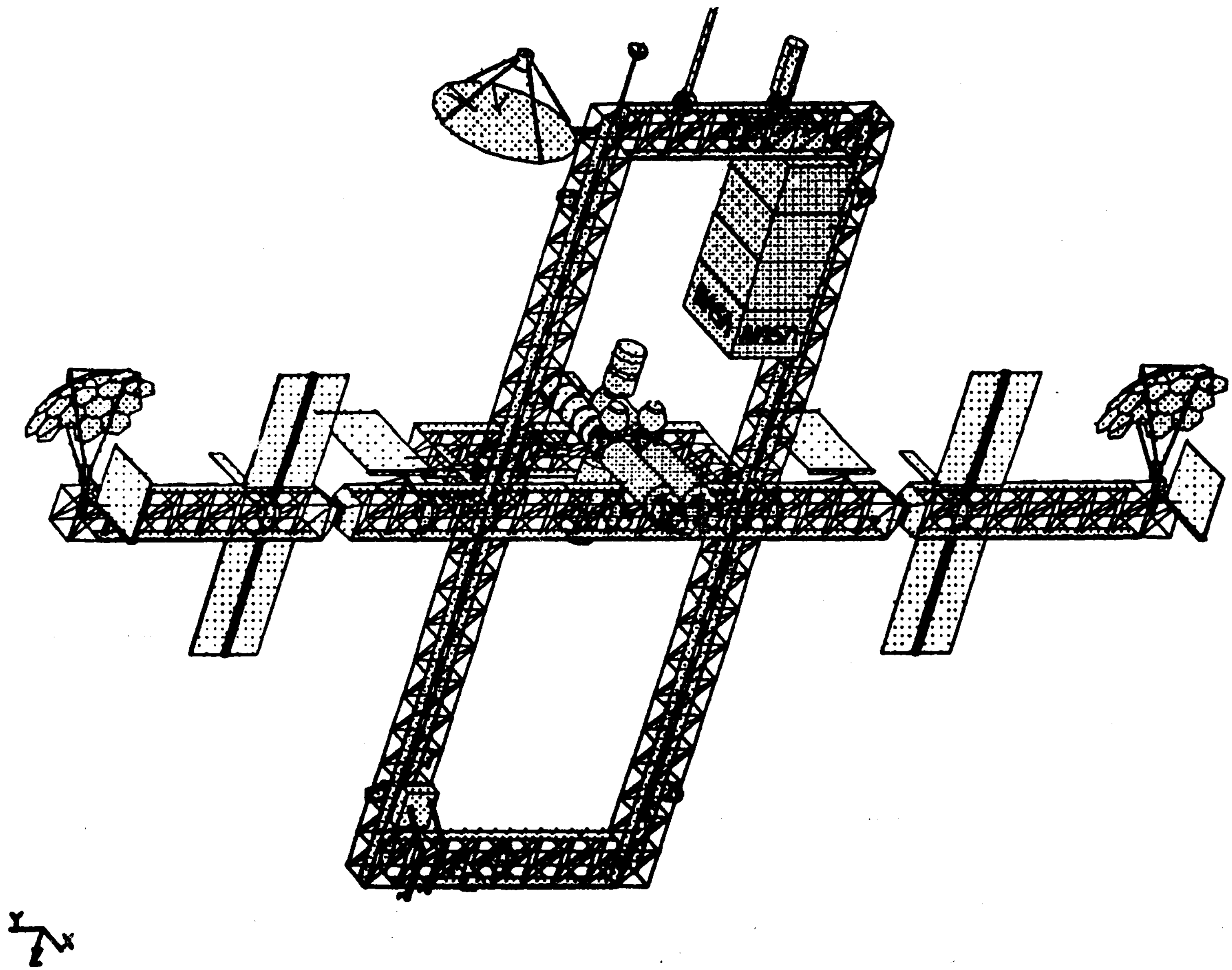

Figure 1. Dual Keel Space Station

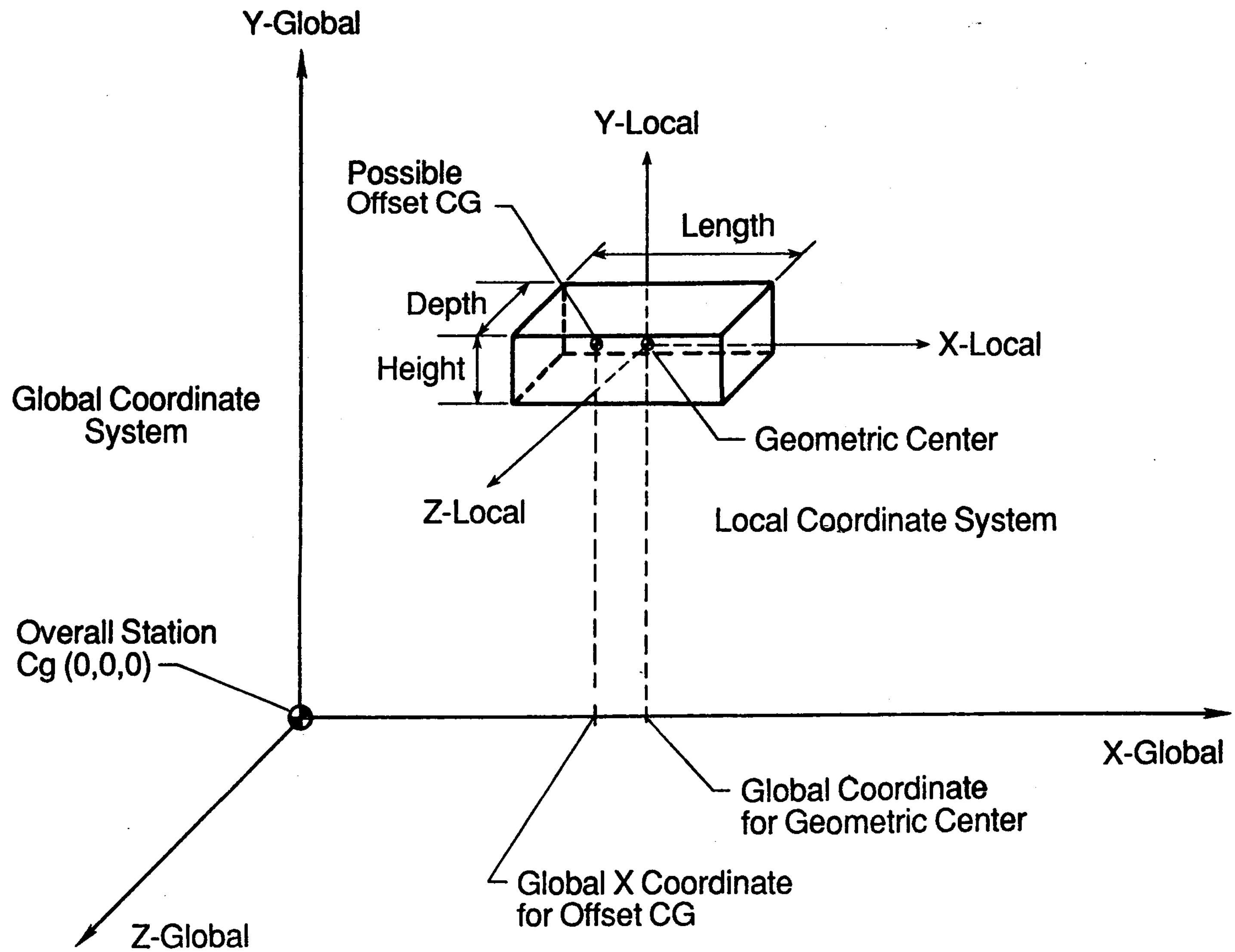

Figure 2. Coordinate Systems

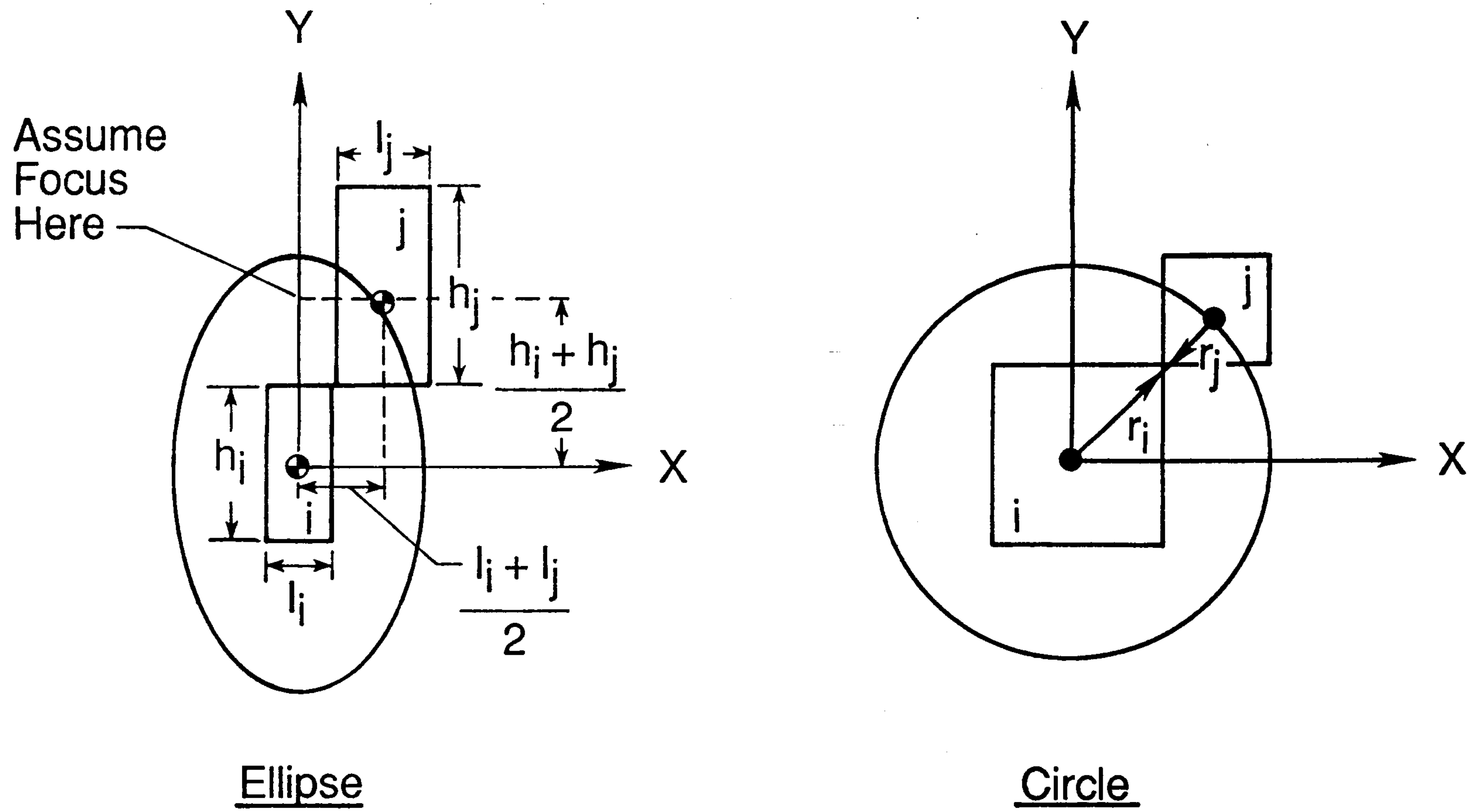

Figure 3. Geometric Constraint Definitions

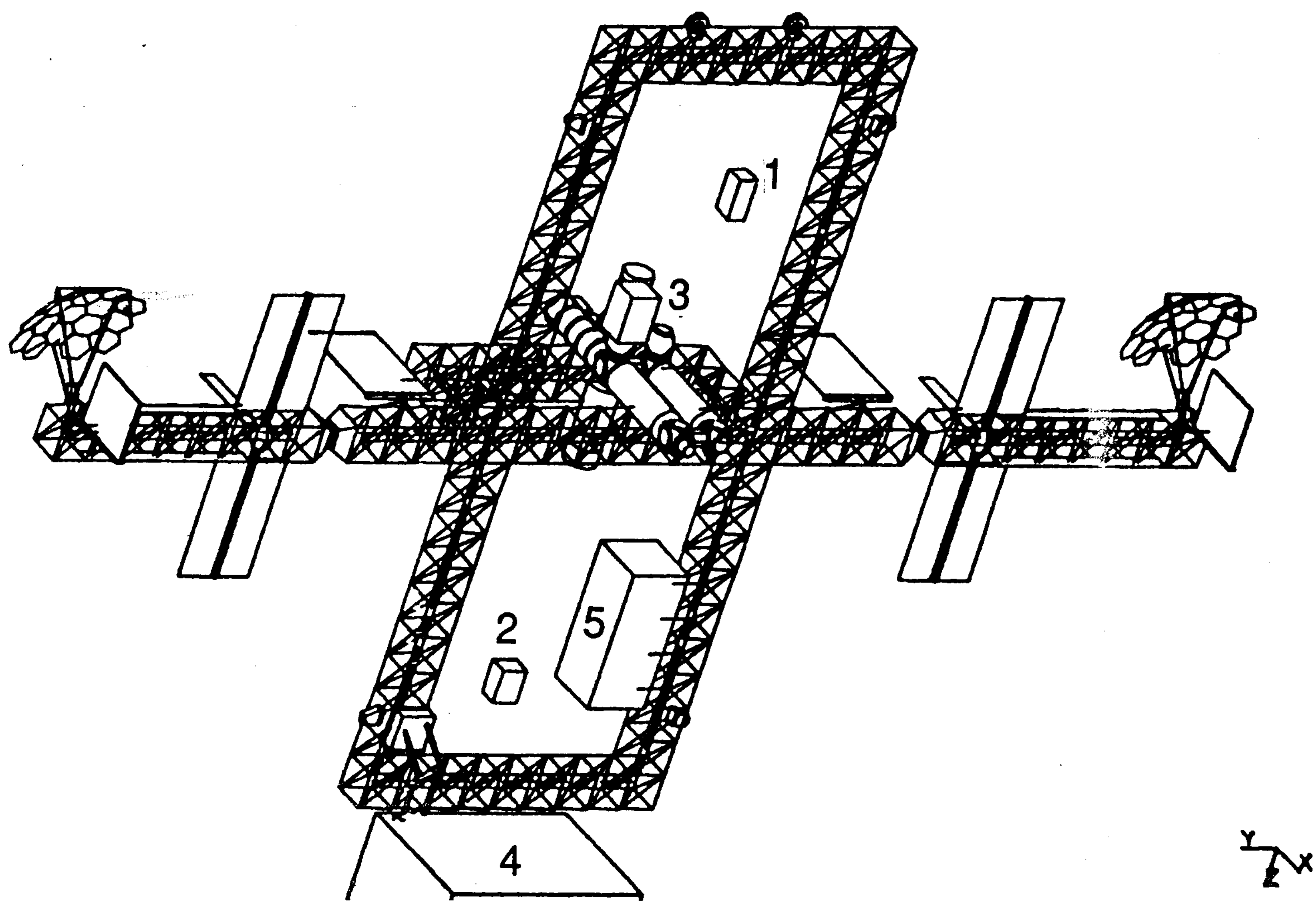

Figure 4. Initial Payload Positions

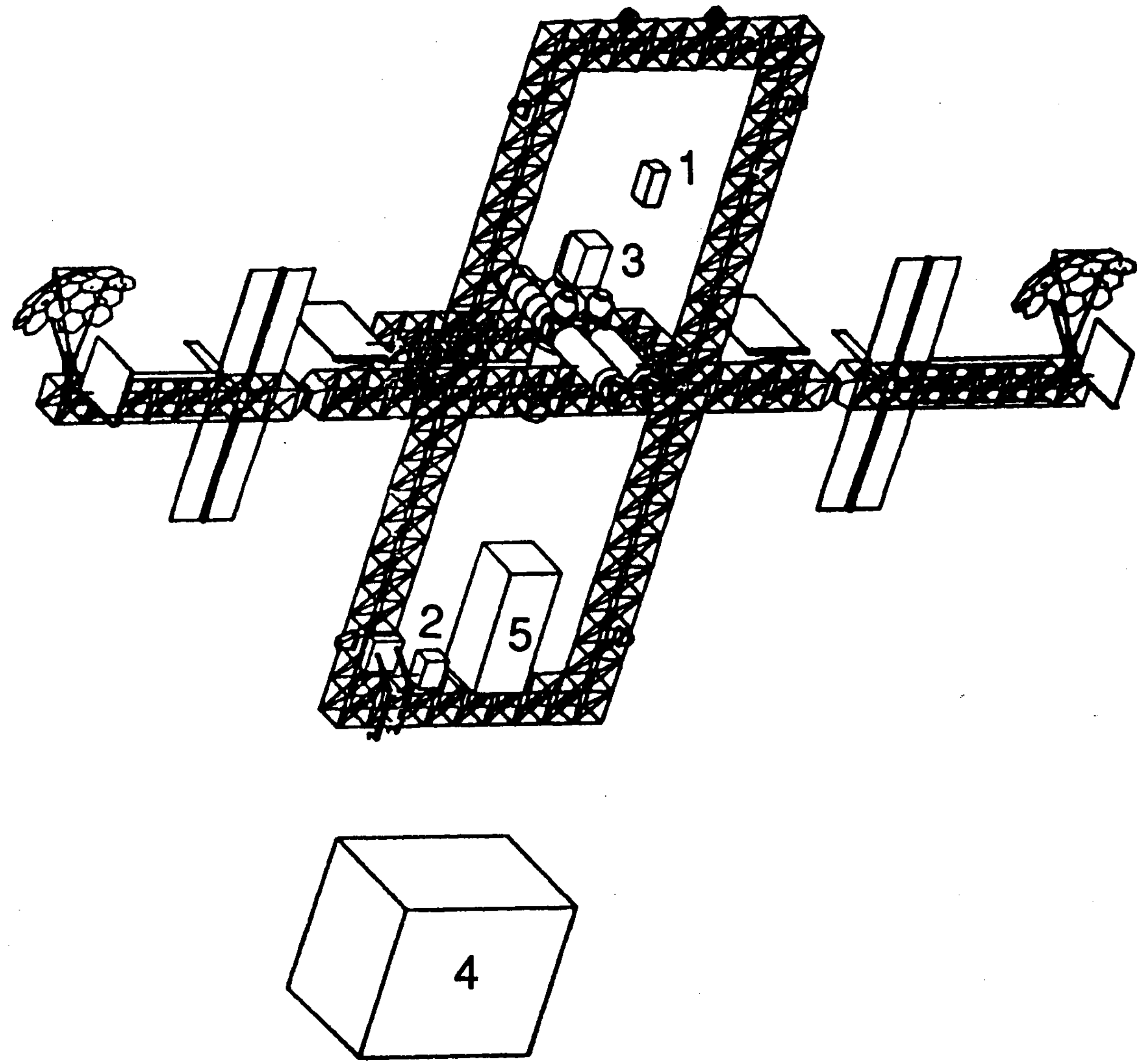

Figure 5. Optimized Payload Positions

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16. Abstract

In order to keep a Space Station in a stable low-Earth orbit, angular momentum storage and translational attitude control systems will have to be used. In order to minimize the size of these attitude control systems, the induced gravity gradient torque effects will have to be minimized. This can be done by minimizing the cross products of inertia of the Station through the management of payload placement with the Station geometry. A derived and automated methodology is presented in this paper which utilizes mathematical nonlinear programming techniques. An optimal arrangement of a set of five payloads on a Dual Keel Space Station was found that minimized the cross products of inertia and thus the required controllability resources.

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