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DESIGN SENSITIVITY ANALYSIS USING EAL:

Part I: Conventional Design Parameters

by

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ABSTRACT

A numerical implementation of design sensitivity analysis of built-up structures is presented, using the versatility and convenience of an existing finite element structural analysis code and its database management system. The finite element code used in the implementation presented is the Engineering Analysis Language (EAL), which is based on a hybrid method of analysis. It has been shown that design sensitivity computations can be carried out using the database management system of EAL, without writing a separate program and a separate database.

Conventional (sizing) design parameters such as cross-sectional area of beams or thickness of plates and plane elastic solid components are considered in this report. Compliance, displacement, and stress functionals are considered as performance criteria. The method presented in this paper is being extended to implement shape design sensitivity analysis using a domain method and a design component method. Results of shape design sensitivity analysis will be reported in Part II.

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CHAPTER I

INTRODUCTION

1.1. Purpose

To date there exist a wide variety of finite element structural analysis programs that are used as reliable tools for structural analysis. They give the designer pertinent information such as stresses, strains, and displacements of the structure being modeled. However, if this information reveals that the structure does not meet specified constraint requirements, the designer must make intuitive estimates on how to improve the design. If the structure is complex, it becomes very difficult to decide what step must be taken to improve the design. There is however, substantial literature [1] on the theory of design sensitivity analysis, which predicts the effect that structural design changes have on the performance of a structure.

The purpose of this work is to develop and implement structural design sensitivity analysis, using the adjoint variable method that takes advantage of the versatility and convenience of an existing finite element structural analysis code and its database management system and the theoretical foundation of structural design sensitivity analysis that is found in Ref. 1. The finite element code that will be used is the Engineering Analysis Language EAL [2].

Using the full capabilities of the EAL system, design sensitivities can be calculated within the program, without knowing the source code of the program. This has the advantage that the user deals with only one program, with only one database and no interfaces between different programs [3,4,5].

1.2. Continuum Approach of Design Sensitivity Analysis

A number of methods could be used to implement structural design sensitivity analysis, but the most powerful is the continuum approach [1]. This method can be implemented outside an existing finite element code [3,4,5], using only postprocessing data. This approach is convenient, because design sensitivity analysis software does not have to be embedded in an existing finite element code. The continuum approach to design sensitivity analysis calculation can also be implemented using a powerful database management system such as the Engineering Analysis Language (EAL). Using the database management system of EAL, only one database is necessary for computation of design sensitivity information. That is, it is not necessary to create interfaces and other datafiles to compute sensitivity information. Information on element shape functions used in the finite element model is, however, necessary for design sensitivity calculation.

The continuum approach to design sensitivity analysis can easily be extended to complex structural systems that have more than one structural component [6]. The design sensitivity vector is the derivative of a constraint functional with respect to design parameters. The magnitude of each component reflects how sensitive the constraint functional is to a change in the associated design parameter. If a component of the vector is negative, the corresponding design parameter should be decreased to increase the value of the constraint functional. If a component of the vector is positive, the corresponding design parameter should be increased to increase the value of the constraint functional. In addition, if the magnitude of a component of the vector is large, then the corresponding design parameter will have a more substantial effect on design improvement.

When a designer uses a finite element structural analysis code in design of a structure, it is most likely that a number of program runs are necessary before a substantially improved design is obtained. With the aid of a design sensitivity vector, the designer will know what direction to take to improve the design most efficiently.

CHAPTER II

DESIGN SENSITIVITY ANALYSIS METHOD

A detailed treatment of methods and procedures for calculating design sensitivities are given in Ref. 1, for constraint functionals such as compliance, displacement, stress, and eigenvalues. For compliance and eigenvalue functionals, the adjoint equation is the same as the state equation, thus no adjoint equation needs to be solved. Each displacement and stress functional requires an adjoint load computation and an adjoint equation must be solved. Note that the state equation and the adjoint equation differ only in their load terms. Using the reload or multiload option of an existing finite element code, the adjoint equation can be solved efficiently [3]. For the displacement functional, the adjoint load is a unit point load acting at the point where the displacement constraint is imposed. For the stress functional the shape function of the finite element code is necessary to calculate equivalent nodal forces of the adjoint load.

The equivalent nodal force computation of the adjoint load for the stress functional can be based on different principles. To be consistent with EAL, a hybrid formulation should be used, which requires computation of the applied loads in terms of stress coefficients. This is impractical, because EAL is formulated in terms of nodal displacement coefficients. Another method, which is consistent with the hybrid formulation of EAL, is based on the modified Hellinger-Reissner principle [10,11]. Here the equivalent nodal forces are expressed in terms of nodal displacement coefficients. To alleviate this difficulty, an equivalent nodal force computation is based on the formulation used in the displacement method of finite element analysis. That is, once the degrees of freedom (nodal displacements) of the element in EAL are known, a compatible displacement shape function that is defined on the same finite element and has the same degrees of freedom can be used for computation of equivalent nodal forces.

The flow chart of Fig. 2.1 shows the overall procedure. First, the model is defined by identifying the design variables, constraint functionals, finite element model, and loadings. In the next step, EAL is used to obtain structural response. With the structural response obtained, an adjoint load is calculated, external to EAL, using assumed displacement shape functions. The adjoint load is input to EAL, to obtain an adjoint response for each constraint using reload option. Using the original structural response and the adjoint response, design sensitivity information is computed for each constraint, by numerically integrating the design sensitivity expressions. The process is convenient, since it uses data that are available or easily computable in the database.

To give the basic idea of implementation of design sensitivity analysis and computation procedures, simple prototype structural components are investigated. Once design sensitivity analysis of structural components is completed, the design component method of Ref. 6 can be used for design sensitivity analysis of built-up structures. The procedures and equations necessary for analysis of

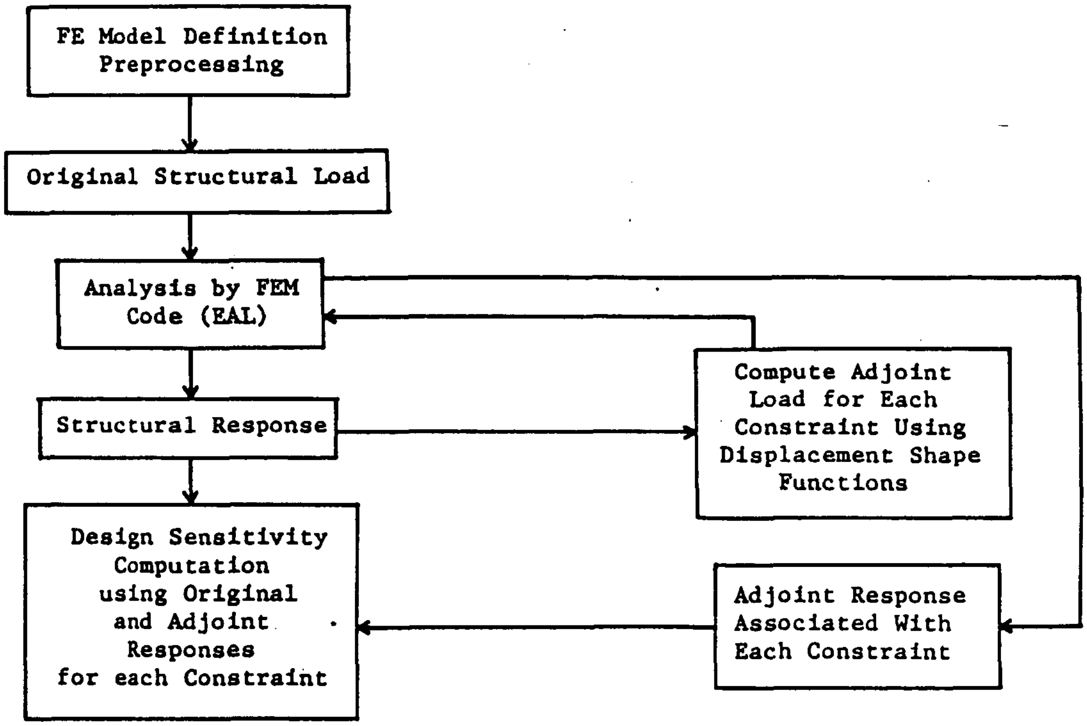

Figure 2.1. Flow Chart of Design Sensitivity Computation

structural components as membranes, plates, and beams are shown in the following sections.

2.1. Membranes

Consider a thin elastic clamped solid shown in Fig. 2.2, with thickness $u = h(x)$ of the membrane as a design parameter where $x = (x_1, x_2)$. The energy bilinear form and load linear form are [1],

$$a_u(z,\bar{z}) = \iint_\Omega h(x) \sum_{i,j=1}^{2} \sigma^{ij}(z)\, \varepsilon^{ij}(\bar{z}) d\Omega \tag{2.1.1}$$

and

$$\ell_u(\bar{z}) = \iint_\Omega \sum_{i=1}^{2} F^i \bar{z}^i d\Omega + \int_\Gamma \sum_{i=1}^{2} T^i \bar{z}^i d\Gamma \tag{2.1.2}$$

where $z = [z^1, z^2]^T$ is the displacement field, $F = [F^1, F^2]^T$ is the applied body force, $T = [T^1, T^2]^T$ is the boundary traction, and $\sigma^{ij}(z)$ and $\varepsilon^{ij}(\bar{z})$ are the stress and strain fields associated with the displacement z and the virtual displacement $\bar{z}$, respectively. The variational state equation is [1]

$$a_u(z,\bar{z}) = \ell_u(\bar{z}) \tag{2.1.3}$$

for all kinematically admissible virtual displacements $\bar{z}$.

First consider the functional that represents compliance of the structure,

$$\psi_1 = \iint_\Omega \sum_{i=1}^{2} F^i z^i d\Omega + \int_\Gamma \sum_{i=1}^{2} T^i z^i d\Gamma \tag{2.1.4}$$

The first variation of Eq. 2.1.4 is

$$\psi_1' = \iint_\Omega \sum_{i=1}^{2} F^i z^{i\prime} d\Omega + \int_\Gamma \sum_{i=1}^{2} T^i z^{i\prime} d\Gamma \tag{2.1.5}$$

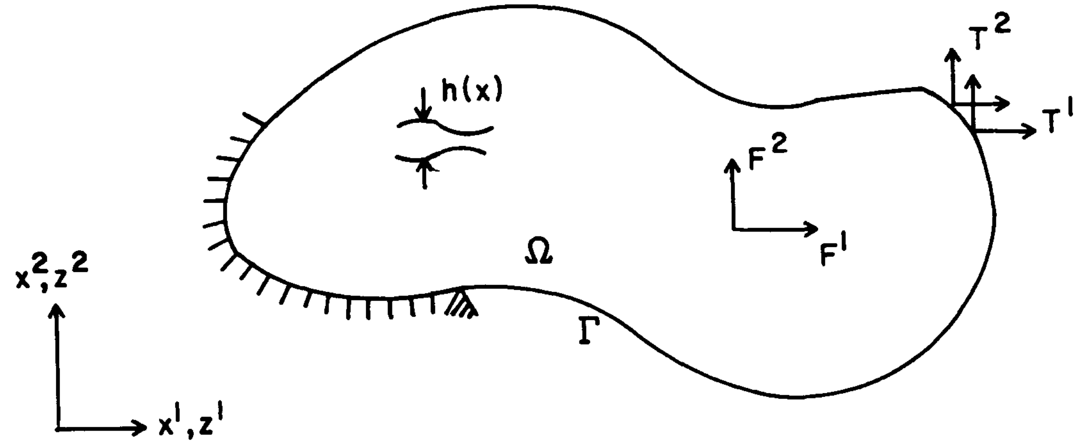

Figure 2.2. Clamped Plane Elastic Solid of Variable Thickness

In order to eliminate the dependence on variation of the state variable in Eq. 2.1.5, it is necessary to define the adjoint equation as [1]

$$a_u(\lambda,\bar{\lambda}) = \iint \sum_{i=1}^{2} F^i \bar{\lambda}^i d\Omega + \int_\Gamma \sum_{i=1}^{2} T^i \bar{\lambda}^i d\Gamma \qquad (2.1.6)$$

for all kinematically admissible virtual displacements $\bar{\lambda}$. Since the right side of Eqs. 2.1.3 and 2.1.6 are identical if $\bar{\lambda} = \bar{z}$, the adjoint equation does not need to be solved. Using the adjoint variable method, the design sensitivity is [1]

$$\psi_1' = -\iint_\Omega [\sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(z)]\delta h d\Omega \qquad (2.1.7)$$

since $z = \lambda$ for the compliance functional.

To numerically integrate Eqs. 2.1.4 and 2.1.7, a two-by-two Gauss-point integration procedure is used. The equations then become

$$\psi_1 = \sum_{k=1}^{N} \left[\sum_{\ell=1}^{4} \left[\sum_{h=1}^{2} (F^{\ell})(z^{\ell}) \right] W^{\ell} J^{\ell} \right]^{k} + \sum_{k} \left[\sum_{\ell=1}^{2} \left[\sum_{h=1}^{2} (T^{\ell})(z^{\ell}) \right] W^{\ell} J^{\ell} \right]^{k} \qquad (2.1.8)$$

and

$$\psi_1' = \sum_{k=1}^{N} \left[\sum_{\ell=1}^{4} \left[- \sum_{i,j=1}^{2} (\sigma^{ij}(z))^{\ell} (\varepsilon^{ij}(z))^{\ell} \right] W^{\ell} J^{\ell} \right]^{k} \delta h^{k} \qquad (2.1.9)$$

respectively, where J is the Jacobian, N is the total number of elements, superscript ℓ is the counter for the number of Gauss points, superscript k is the counter for the element number, W is the weighting constant for the ℓth Gauss point, and superscript h is the direction of the force and the displacement.

Since EAL gives only the boundary displacement shape function for the 4-noded membrane element E41, a bilinear shape function for displacement is adopted for integration in Eqs. 2.1.4 and 2.1.7. Using stress computation of membrane element E41 in EAL, stresses and strains can be expressed in matrix form as

$$\{\sigma^{11}(z),\ \sigma^{22}(z),\ \sigma^{12}(z)\}^T = [P]\{\beta\} \qquad (2.1.10)$$

$$\{\varepsilon^{11}(z),\ \varepsilon^{22}(z),\ \varepsilon^{12}(z)\}^T = [E]^{-1}\{\sigma\} \qquad (2.1.11)$$

where

$$[P] = \begin{bmatrix} 0 & 0 & 1 & x_2 & 0 \\ 0 & 1 & 0 & 0 & x_1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2.1.12)$$

$$\{\beta\} = \{b_1,\ b_2,\ b_3,\ b_4,\ b_5\}^T \tag{2.1.13}$$

$$[E] = \frac{E}{\left(1 - \nu^2\right)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \tag{2.1.14}$$

and [P] is the position coordinate matrix, which determines points where the stresses are obtained, $\{\beta\}$ is the stress coefficient vector, and [E] is the elasticity matrix for a plane stress problem [8,9].

Next consider the functional that represents the displacement z at a discrete point $\hat{x}$,

$$\psi_2 \equiv z(\hat{x}) = \iint_\Omega \hat{\delta}(x - \hat{x}) z(x)\ d\Omega \tag{2.1.15}$$

where $\hat{\delta}(x)$ is the Dirac delta. The first variation of Eq. 2.1.15 is

$$\psi_2' = \iint_\Omega \hat{\delta}(x - \hat{x}) z'(x) d\Omega \tag{2.1.16}$$

The adjoint equation in this case is [1,3]

$$a_u(\lambda, \bar{\lambda}) = \iint_\Omega \hat{\delta}(x - \hat{x}) \bar{\lambda}(x) d\Omega \tag{2.1.17}$$

for all kinematically admissible displacements $\bar{\lambda}$. This equation has a unique solution λ, which is the displacement field due to a unit point load acting at a point $\hat{x}$. Using the adjoint variable method, design sensitivity of the displacement functional is

$$\psi_2' = -\iint_\Omega \sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(\lambda)\ \delta h d\Omega \tag{2.1.18}$$

where λ is the solution of Eq. 2.1.17.

Using the same numerical techniques applied in the compliance constraint case, Eq. 2.1.18 can be evaluated as

$$\psi_2' = \sum_{k=1}^{N} \left[\sum_{\ell=1}^{4} [- \sum_{i,j=1}^{2} (\sigma^{ij}(z))^{\ell} (\varepsilon^{ij}(\lambda))^{\ell}] W^{\ell} J^{\ell} \right]^{k} \delta h^{k} \qquad (2.1.19)$$

where

$$\{\varepsilon(\lambda)\} = [E]^{-1}\{\sigma(\lambda)\}$$

$$= [E]^{-1}[P]\{\beta\} \qquad (2.1.20)$$

Finally consider the general functional that represents a locally averaged stress on an element as

$$\psi_3 = \iint_{\Omega} g(\sigma(z)) m_p d\Omega \qquad (2.1.21)$$

where m_p is a characteristic function, defined on a finite element Ω_p as

$$m_p = \begin{cases} \dfrac{1}{\iint_{\Omega_p} d\Omega} & x \in \Omega_p \\ 0 & x \notin \Omega_p \end{cases} \qquad (2.1.22)$$

and g is the stress function. The first variation of Eq. 2.1.21 is

$$\psi_3' = \iint_{\Omega} [\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z)\ \sigma^{ij}(z')] m_p d\Omega \qquad (2.1.23)$$

Replacing the variation in state z' by a virtual displacement $\bar{\lambda}$, the adjoint equation is obtained as [1,3]

$$a_u(\lambda,\bar{\lambda}) = \iint_{\Omega} [\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z)\ \sigma^{ij}(\bar{\lambda})] m_p d\Omega \qquad (2.1.24)$$

for all kinematically admissible displacements $\bar{\lambda}$. Equation 2.1.24 has a unique solution for a displacement field λ. Using the adjoint variable method, design sensitivity of the stress functional is

$$\psi_3' = -\iint_{\Omega}[\sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(\lambda)]\delta h d\Omega \tag{2.1.25}$$

where λ is the solution of Eq. 2.1.24.

Using the same numerical techniques applied in the compliance constraint case, Eqs. 2.1.21 and 2.1.25 become

$$\psi_3 = \sum_{\ell=1}^{4} [g^{\ell}(\sigma^{\ell}(z))m_p W^{\ell}]J^{\ell} \tag{2.1.26}$$

and

$$\psi_3' = \sum_{k=1}^{N} \left[\sum_{\ell=1}^{4} [-\sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(\lambda)]^{\ell} W^{\ell} J^{\ell}\right]^{k} \delta h^{k} \tag{2.1.27}$$

The right side of Eq. 2.1.24 can be evaluated as

$$\sum_{\ell=1}^{4} [\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z)\ \sigma^{ij}(\bar{\lambda})]^{\ell} m_p W^{\ell} J^{\ell} \tag{2.1.28}$$

After the adjoint load of Eq. 2.1.17 is calculated, the adjoint displacement field and resulting adjoint stress and strain fields are evaluated. The adjoint strains are then used in evaluating Eq. 2.1.27 for sensitivity of the constraint functional.

If von Mises' stress criteria is selected in the constraint functional, then

$$g = [(\sigma^{11})^2 + (\sigma^{22})^2 + 3(\sigma^{12})^2 - \sigma^{11}\sigma^{22}]^{1/2} \tag{2.1.29}$$

and

$$\frac{\partial g}{\partial \sigma^{11}} = (2\sigma^{11} - \sigma^{22})/2g$$

$$\frac{\partial g}{\partial \sigma^{22}} = (2\sigma^{22} - \sigma^{11})/2g \qquad (2.1.30)$$

$$\frac{\partial g}{\partial \sigma^{12}} = 3\sigma^{12}/g$$

which can be written in vector form as

$$\{\frac{\partial g}{\partial \sigma^{ij}}\} = \{\frac{\partial g}{\partial \sigma^{11}}, \frac{\partial g}{\partial \sigma^{22}}, \frac{\partial g}{\partial \sigma^{12}}\}^T \qquad (2.1.31)$$

The equivalent nodal force is computed, based on the modified Hellinger-Reissner principle. The stress can be written as

$$\{\sigma\} = [P]\{\beta\} \qquad (2.1.32)$$

(see also Eq. 2.1.10). The stress coefficients $\{\beta\}$ can be expressed in terms of the nodal displacement coefficients $\{q\}$ as [7]

$$\{\beta\} = [H^{-1}][T]\{q\} \qquad (2.1.33)$$

if $\{\nabla\sigma\} = \{0\}$, where

$$\{\nabla\sigma\} = \{\frac{\partial \sigma^{11}}{\partial x_1}, \frac{\partial \sigma^{22}}{\partial x_2}, \frac{\partial \sigma^{12}}{\partial x_i}\}^T \qquad i=1,2 \qquad (2.1.34)$$

$\{\nabla\sigma\}$ can be written as

$$\{\nabla\sigma\} = [\nabla P]\{\beta\} \qquad (2.1.35)$$

where

$$[\nabla P] = \begin{bmatrix} \frac{\partial}{\partial x_1}(0 & 0 & 1 & x_2 & 0) \\ \frac{\partial}{\partial y_2}(0 & 1 & 0 & 0 & x_1) \\ \frac{\partial}{\partial x_i}(1 & 0 & 0 & 0 & 0) \end{bmatrix} \qquad i=1,2$$

Using Eq. 2.1.34, $[\nabla P]$ in Eq. 2.1.35 becomes zero. Note that, if $[\nabla P]$ is not zero, Eq. 2.1.33 is not valid. Using Eq. 2.1.33, Eq. 2.1.32 can be written as

$$\{\sigma\} = [P][H^{-1}][T]\{q\} \tag{2.1.36}$$

Using Eqs. 2.1.27, 2.1.31, and 2.1.36, the equivalent nodal forces $\{F\}$ can be written as

$$\{F\} = \sum_{\ell=1}^{4} \{\frac{\partial g}{\partial \sigma^{ij}}\}^T [P][H^{-1}][T] W^{\ell} J^{\ell} \tag{2.1.37}$$

which is valid only if $[\nabla P]$ is zero.

2.2. Bending of Plates

Consider the clamped plate in Fig. 2.3 of variable thickness $u = t(x)$, with a distributed load $f(x)$. For this design independent

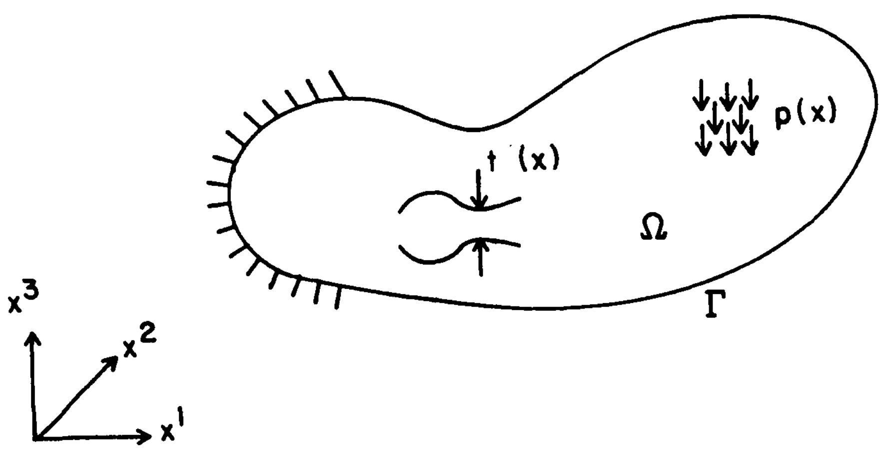

Figure 2.3. Clamped Plate of Variable Thickness

loading, the energy bilinear form and the load linear form for the plate are given as [1]

$$a_u(z,\bar{z}) = \iint_\Omega \hat{D}(u)[z_{11}\bar{z}_{11} + z_{22}\bar{z}_{22} + \nu(z_{22}\bar{z}_{11} + z_{11}\bar{z}_{22})$$

$$+ 2(1 - \nu)z_{12}\bar{z}_{12}]d\Omega \tag{2.2.1}$$

and

$$\ell(\bar{z}) = \iint_\Omega f\bar{z}d\Omega \tag{2.2.2}$$

where $\hat{D}(u) = Et/[12(1 - \nu^2)]$ is flexural rigidity, E is Young's modulus, ν is the Poisson's ratio, and f is externally applied pressure. The governing state equation is [1]

$$a_u(z,\bar{z}) = \ell(\bar{z}) \tag{2.2.3}$$

for all kinematically admissible displacements $\bar{z}$.

First consider the functional that represents compliance of the structure,

$$\psi_4 = \iint_\Omega fzd\Omega \tag{2.2.4}$$

The first variation of Eq. 2.2.4 is

$$\psi_4' = \iint_\Omega fz'd\Omega \tag{2.2.5}$$

The adjoint equation is defined as [1]

$$a_u(\lambda,\bar{\lambda}) = \iint_\Omega f\bar{\lambda}d\Omega \tag{2.2.6}$$

for all kinematically admissible displacements $\bar{\lambda}$. As in the membrane case, the right sides of Eqs. 2.2.3 and 2.2.6 is identical, so $\bar{\lambda} = \bar{z}$. Using the adjoint variable method, design sensitivity of the compliance functional is [1,3,4]

$$\psi_4' = \iint_\Omega \{-Et^2[z_{11}^2 + z_{22}^2 + 2\nu z_{11}z_{22} + 2(1 - \nu)z_{12}^2]/4(1 - \nu^2)\}\ \delta t d\Omega$$

$$= - \iint_\Omega [\sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(z)]\ \delta t d\Omega \qquad (2.2.7)$$

where $\sigma^{ij}(z)$ and $\varepsilon^{ij}(z)$ are stress and strain of the extreme fiber, given as

$$\varepsilon^{ij} = - \frac{tz_{ij}}{2}, \qquad i,j = 1,\ 2 \qquad (2.2.8)$$

and

$$\sigma^{11} = - \frac{Et}{2(1 - \nu^2)}(z_{11} + \nu z_{22})$$

$$\sigma^{22} = - \frac{Et}{2(1 - \nu^2)}(z_{22} + \nu z_{11}) \qquad (2.2.9)$$

$$\sigma^{12} = - \frac{Et}{2(1 - \nu^2)} z_{12}$$

To evaluate Eqs. 2.2.4 and 2.2.7 numerically, a two-by-two Gauss-point integration procedure is used. Equations 2.2.4 and 2.2.7 become

$$\psi_4 = \sum_{k=1}^{N} \{ \sum_{\ell=1}^{4} [f^\ell z^\ell]\}^k W^\ell J^\ell \qquad (2.2.10)$$

and

$$\psi_4' = - \sum_{k=1}^{N} \{ \sum_{\ell=1}^{4} [\sum_{i,j=1}^{2} (\sigma^{ij}(z))^\ell (\varepsilon^{ij}(z))^\ell] W^\ell J^\ell\}^k \delta t^k \qquad (2.2.11)$$

For the Gauss integration, $\sigma^{ij}(z)$ and $\varepsilon^{ij}(z)$ are obtained at each Gauss point, using the same moment formulation as in EAL. The moment vector

[M] is defined as

$$\{M^{11}(z),\ M^{22}(z),\ M^{12}(z)\}^T = [P]\{\beta\} \tag{2.2.12}$$

where

$$[P] = \begin{bmatrix} 1 & 0 & 0 & x_2 & 0 & x_1 & 0 & 0 & 0 & x_1x_2 & 0 \\ 0 & 1 & 0 & 0 & x_1 & 0 & x_2 & 0 & 0 & 0 & x_1x_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & x_2 & x_1 & 0 & 0 \end{bmatrix} \tag{2.2.13}$$

$$\{\beta\} = \{b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6,\ b_7,\ b_8,\ b_9,\ b_{10},\ b_{11}\}^T \tag{2.2.14}$$

Thus,

$$\{\sigma^{11}(z),\ \sigma^{22}(z),\ \sigma^{12}(z)\}^T = \frac{6}{t^2}\{M^{11}(z),\ M^{22}(z),\ -M^{12}(z)\}^T \tag{2.2.15}$$

$$\{\varepsilon^{11}(z),\ \varepsilon^{22}(z),\ \varepsilon^{12}(z)\}^T = [E]^{-1}\{\sigma^{11}(z),\ \sigma^{22}(z),\ \sigma^{12}(z)\}^T \tag{2.2.16}$$

where $[E]^{-1}$ is given in Eq. 2.1.14.

Next consider the functional that represents displacement z at a discrete point $\hat{x}$,

$$\psi_5 \equiv z(\hat{x}) = \iint_\Omega \hat{\delta}(x - \hat{x})z(x)d\Omega \tag{2.2.17}$$

The first variation of Eq. 2.2.17 is

$$\psi_5' = \iint_\Omega \hat{\delta}(x - \hat{x})z'(x)d\Omega \tag{2.2.18}$$

The adjoint equation is defined as [1]

$$a_u(\lambda,\bar{\lambda}) = \iint_\Omega \hat{\delta}(x - \hat{x})\bar{\lambda}(x)d\Omega \tag{2.2.19}$$

for all kinematically admissible displacements $\bar{\lambda}$. This equation has a unique solution λ, which is the plate displacement field due to a unit vertical load at point $\hat{x}$. Using the adjoint variable method [1], design sensitivity of the displacement functional is

$$\psi_5' = \iint_\Omega \{- Et^2[z_{11}\lambda_{11} + z_{22}\lambda_{22} + \nu(z_{11}\lambda_{11} + z_{22}\lambda_{22})$$

$$+ 2(1 - \nu)z_{12}\lambda_{12}]/4(1 - \nu^2)\} \delta t d\Omega \qquad (2.2.20)$$

Using Eqs. 2.2.8 and 2.2.9, Eq. 2.2.20 can be rewritten as

$$\psi_5' = - \iint_\Omega \sum_{i,j=1}^{2} \sigma^{ij}(z)\, \varepsilon^{ij}(\lambda)\delta t d\Omega \qquad (2.2.21)$$

Using the same numerical integration as in the compliance case, Eq. 2.2.21 becomes

$$\psi_5' = \sum_{k=1}^{N} \{- \sum_{\ell=1}^{4} [\sum_{i,j=1}^{2} (\sigma^{ij}(z))^\ell (\varepsilon^{ij}(\lambda))^\ell W^\ell J^\ell\}^k \delta t^k \qquad (2.2.22)$$

where $\varepsilon^{ij}(\lambda)$ are strains obtained by applying the adjoint load. The procedure is the same as in Eqs. 2.2.12 thru 2.2.16.

Finally, consider the functional that represents a locally averaged stress at the extreme fiber of the plate,

$$\psi_6 = \iint_\Omega g(\sigma(z))m_P d\Omega \qquad (2.2.23)$$

where $g(\sigma(z))$ is chosen as the von Mises' stress criteria and m_P is a characteristic function on finite element Ω_P, defined as

$$m_P = \begin{cases} \dfrac{1}{\iint_{\Omega_P} d\Omega} & x \in \Omega_P \\ 0 & x \notin \Omega_P \end{cases} \qquad (2.2.24)$$

The first variation of Eq. 2.2.23 is

$$\psi_6' = \iint_\Omega \Big[\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}(z)} \sigma^{ij}(z') \Big] m_p d\Omega \qquad (2.2.25)$$

The adjoint equation is defined as [1]

$$a_u(\lambda,\bar{\lambda}) = \iint_\Omega \Big[\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z)\ \sigma^{ij}(\bar{\lambda}) \Big] m_p d\Omega \qquad (2.2.26)$$

for all kinematically admissible displacement $\bar{\lambda}$. Using the adjoint variable method, design sensitivity of the stress functional is

$$\psi_6' = \iint_\Omega - \Big[\sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(\lambda) \Big] \delta t d\Omega + \iint_\Omega \frac{\partial g}{\partial t}(\sigma(z)) m_p d\Omega \qquad (2.2.27)$$

where λ is the solution of Eq. 2.2.26. Using two-by-two Gauss-point integration, Eqs 2.2.23 and 2.2.27 can be expressed as

$$\psi_6 = \sum_{\ell=1}^{4} [g(\sigma^\ell(z)) m_p] W^\ell J^\ell \qquad (2.2.28)$$

and

$$\psi_6' = \sum_{k=1}^{N} \Big\{ \sum_{\ell=1}^{4} \Big[- \sum_{i,j=1}^{2} (\sigma^{ij}(z))^\ell (\varepsilon^{ij}(\lambda))^\ell \Big] W^\ell J^\ell + \sum_{\ell=1}^{4} \Big[\frac{\partial g(\sigma(z))}{\partial t} \Big] W^\ell J^\ell \Big\}^k \delta t^k$$

(2.2.29)

The right side of Eq. 2.2.26 can be written as

$$\sum_{\ell=1}^{4} \Big[\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z)\ \sigma^{ij}(\bar{\lambda}) \Big]^\ell m_p W^\ell J^\ell \qquad (2.2.30)$$

For the von Mises' stress criteria, the last term on side right side of Eq. 2.2.29 becomes

$$\sum_{\ell=1}^{4} \Big\{ \frac{1}{t} \Big[(\sigma^{11})^2 + (\sigma^{22})^2 + 3(\sigma^{12})^2 - \sigma^{11}\sigma^{22} \Big]^{1/2} \Big\} W^\ell J^\ell m_p \qquad (2.2.31)$$

For the displacement method, the equivalent nodal force for the adjoint load can be computed in a consistent way, by using the displacement shape functions of the code and then using the same procedure as in Refs. 3,4 and 5. For the hybrid method, Reissner's principle can not be used, because $[\nabla P]$ is not necessarily zero. To alleviate this difficulty, it is proposed to select an acceptable displacement shape function for the adjoint load calculation. That is, once degrees of freedom of the element in EAL are known, a compatible shape function that has the same degrees of freedom can be selected. The term $\frac{\partial g}{\partial \sigma^{ij}}$ is the same as for membrane elements (Eq. 2.1.31). Stresses in a displacement formulation are given as [8,9]

$$\{\sigma\} = [E][B]\{q\} \tag{2.2.32}$$

Using Eqs. 2.1.31, 2.2.29, and 2.2.32, the equivalent nodal forces $\{F\}$ can be evaluated as

$$\{F\} = \sum_{\ell=1}^{4} \{\frac{\partial g}{\partial \sigma}\}^{\ell T}[E][B]m_p W^{\ell} J^{\ell} \tag{2.2.33}$$

2.3. Beams

Consider a beam with variable width and height, as shown in Fig. 2.4. Width and height are the design parameters; i.e., $u = [b(x_3),h(x_3)]^T$. The energy bilinear form and the load linear form of the beam are

$$a_m(z,\bar{z}) = \int_0^{\ell} E[bh(w_3\bar{w}_3 + \frac{hb^3}{12}(v_{33}\bar{v}_{33})) + \frac{bh^3}{12}(\tilde{u}_{33}\tilde{\bar{u}}_{33})]dx_3 \tag{2.3.1}$$

and

$$\ell(\bar{z}) = \int_0^{\ell} F^T\bar{z}dx_3 \tag{2.3.2}$$

where $F = [f^1, f^2, f^3]$ is the distributed load, E is Young's modulus, $hb^3/12$ is the moment of inertia with respect to x_1-axis [2], $bh^3/12$ is the moment of inertia with respect to x_2-axis, $z = [\tilde{u},v,w]^T$ is the

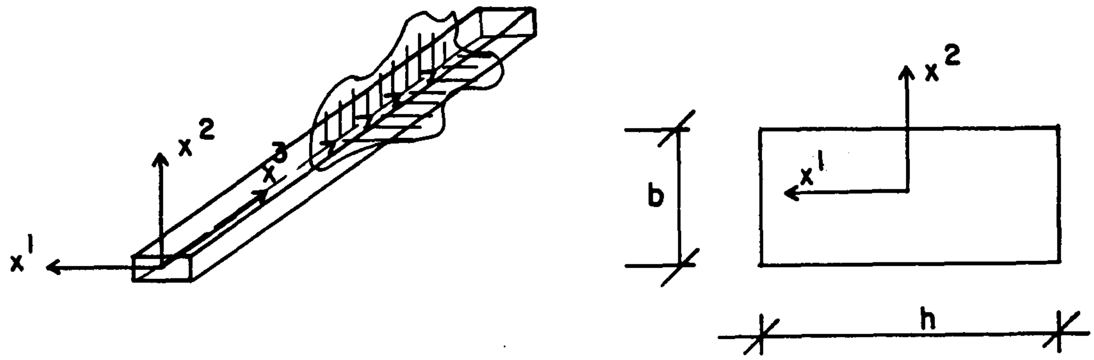

Figure 2.4. Beam of Variable Thickness and Width

displacement field, in x_1-, x_2-, and x_3-directions, respectively, $\tilde{u}_{33}$ and v_{33} are curvatures of the displacement field, and $\bar{v}_{33}$ and $\bar{\tilde{u}}_{33}$ are curvatures of the virtual displacement field $\bar{z} = [\bar{\tilde{u}},\bar{v},\bar{w}]^T$.

In EAL [2], the beam element E21 is based on Timoshenko beam theory and includes torsion. The derivation here is based on technical beam theory without torsion. Thus, if the energy bilinear form of Eq. 2.3.1 is used, results of sensitivity analysis are applicable only if shear and torsion are negligible. However, it can be easily extended to include torsional effects.

Note that the load is applied in the positive direction of the coordinate axis. The state equation is [1]

$$a_u(z,\bar{z}) = \ell(\bar{z}) \tag{2.3.3}$$

for all kinematically admissible displacements $\bar{z}$.

First consider the functional that represents compliance of the structure,

$$\psi_7 = \int_0^\ell F^T z dx_3 \tag{2.3.4}$$

The first variation of Eq. 2.3.4 is

$$\psi_7' = \int_0^\ell F^T z' dx_3 \tag{2.3.5}$$

By replacing the variation of z' in Eq. 2.3.5 by a virtual displacement $\bar{\lambda}$, an adjoint equation is defined as [1]

$$a_u(\lambda,\bar{\lambda}) = \int_0^\ell F^T\bar{\lambda} dx \tag{2.3.6}$$

for all kinematically admissible displacements $\bar{\lambda}$. Since Eq. 2.3.2 is identical to Eq. 2.3.6, $\lambda = z$. Using the adjoint variable method of design sensitivity analysis,

$$\begin{aligned}\psi_7' = &- \int_0^\ell \{Ehw_3^2 + \frac{3Eb^2h}{12} v_{33}^2 + \frac{Eh^3}{12} \tilde{u}_{33}^2\}\delta b dx_3 \\ &- \int_0^\ell \{Ebw_3^2 + \frac{Eb^3}{12} v_{33}^2 + \frac{3Ebh^2}{12} \tilde{u}_{33}^2\}\delta h dx_3\end{aligned} \tag{2.3.7}$$

A cubic displacement shape function is assumed for beam bending and a linear displacement shape function is assumed for axial displacement.

To numerically integrate Eqs. 2.3.4 and 2.3.7, a two-point Gaussian integration procedure is used. Equations 2.3.4 and 2.3.7 can be evaluated as

$$\psi_7 = \sum_{k=1}^{N} \Big[\sum_{\ell=1}^{2} F^{\ell T} z^{\ell} J^{\ell} W^{\ell} \Big]^k \qquad (2.3.8)$$

and

$$\psi_7' = - \sum_{k=1}^{N} \Big\{ \sum_{\ell=1}^{2} E\Big[h(w_3^{\ell})^2 + \frac{hb^2}{4}(v_{33}^{\ell})^2 + \frac{h^3}{12}(\tilde{u}_{33}^{\ell})^2\Big]W^{\ell}J^{\ell}\Big\}^k \delta b^k$$
$$- \sum_{k=1}^{N} \Big\{ \sum_{\ell=1}^{2} E\Big[b(w_3^{\ell})^2 + \frac{b^3}{12}(v_{33}^{\ell})^2 + \frac{bh^2}{4}(\tilde{u}_{33}^{\ell})^2\Big]W^{\ell}J^{\ell}\Big\}^k \delta h^k \qquad (2.3.9)$$

where N is the total number of elements, ℓ is the Gauss point counter, W is the weighting constant for the Gauss point, and J is the Jacobian.

Next, consider the functional that represents displacement z at a discrete point $\hat{x}_3$,

$$\psi_8 \equiv z(\hat{x}_3) = \int_0^{\ell} \hat{\delta}(x_3 - \hat{x}_3)z(x_3)dx_3 \qquad (2.3.10)$$

where $\hat{\delta}$ is the Dirac delta. The first variation of Eq. 2.3.10 is

$$\psi_8' = \int_0^{\ell} \hat{\delta}(x_3 - \hat{x}_3)z'(x_3)dx_3 \qquad (2.3.11)$$

The adjoint equation is defined as [1]

$$a(\lambda,\bar{\lambda}) = \int_0^{\ell} \hat{\delta}(x_3 - \hat{x}_3)\bar{\lambda}(x_3)dx_3 \qquad (2.3.12)$$

for all kinematically admissible displacements $\bar{\lambda}$. Equation 2.3.12 has a unique solution λ, where $\lambda = [\tilde{\lambda}^u, \lambda^v, \lambda^w]^T$ is beam displacement due to a unit load acting at a point $\hat{x}_3$. Using the adjoint variable method the design sensitivity analysis is

$$\psi_8' = -\int_0^{\ell}\{Ehw_3\lambda_3^w + \frac{Ehb^2}{4}v_{33}\lambda_{33}^v + \frac{h^3}{12}\tilde{u}_{33}\lambda_{33}^{\tilde{u}}\}\delta b dx_3$$

$$-\int_0^{\ell}\{Ebw_3\lambda_3^w + \frac{Eb^3}{12}v_{33}\lambda_{33}^v + \frac{Eh^2b}{4}\tilde{u}_{33}\lambda_{33}^{\tilde{u}}\}\delta h dx_3 \qquad (2.3.13)$$

Using the two-point Gaussian integration procedure, Eq. 2.3.13 becomes

$$\psi_8' = -\sum_{k=1}^{N}\{\sum_{\ell=1}^{2}E[hw_3^{\ell}\lambda_3^{w\ell} + \frac{hb^2}{4}v_{33}^{\ell}\lambda_{33}^{v\ell} + \frac{h^3}{12}\tilde{u}_{33}^{\ell}\lambda_{33}^{\tilde{u}\ell}]W^{\ell}J^{\ell}\}^k\delta b^k$$

$$-\sum_{k=1}^{N}\{\sum_{\ell=1}^{2}E[bw_3^{\ell}\lambda_3^{w\ell} + \frac{b^3}{12}v_{33}^{\ell}\lambda_{33}^{v\ell} + \frac{bh^2}{12}\tilde{u}_{33}^{\ell}\lambda_{33}^{\tilde{u}\ell}]W^{\ell}J^{\ell}\}^k\delta h^k \quad (2.3.14)$$

Finally, consider the functional that represents extreme fiber stresses in the beam,

$$\psi_9 = \int_0^{\ell}\{w_3 - \text{sign}(b)\,\frac{b}{2}\,v_{33} - \text{sign}(h)\,\frac{h}{2}\,\tilde{u}_{33}\}Em_p dx_3 \qquad (2.3.15)$$

where h/2 is the half-depth of the beam, b/2 is the half width of the beam, sign(b) and sign(h) are +1 or −1 and indicate at what extreme fiber the sensitivity is computed, and m_p is a characteristic function that is defined on a finite element dx_p as

$$m_p = \begin{cases} \dfrac{1}{\int_{dx_p} dx} & x \in dx_p \\ 0 & x \notin dx_p \end{cases} \qquad (2.3.16)$$

The first variation of Eq 2.3.15 is

$$\psi_9' = \int_0^{\ell} Em_p\{w_3' - \text{sign}(b)\,\frac{b}{2}\,v_{33}' - \text{sign}(h)\,\frac{h}{2}\,\tilde{u}_{33}'\}dx_3$$
$$- \int_0^{\ell} Em_p(\text{sign}(b)\,\frac{v_{33}}{2}\,\delta b + \text{sign}(h)\,\frac{\tilde{u}_{33}}{2}\,\delta h)dx_3 \qquad (2.3.17)$$

Replacing the first variation z' in Eq. 2.3.17 by a virtual displacement $\bar{\lambda} = [\bar{\lambda}^u, \bar{\lambda}^v, \bar{\lambda}^w]^T$, the adjoint equation is defined as

$$a(\lambda,\bar{\lambda}) = \int_0^{\ell} Em_p\{\bar{\lambda}_3^w - \text{sign}(b)\,\frac{b}{2}\,\bar{\lambda}_{33}^v - \text{sign}(h)\,\frac{h}{2}\,\bar{\lambda}_{33}^{\tilde{u}}\}dx_3 \quad (2.3.18)$$

for all kinematically admissible displacements $\bar{\lambda}$. Equation 2.3.18 has a unique solution for a displacement field λ. Using the adjoint variable method of design sensitivity analysis,

$$\psi_9' = - \int_0^{\ell}\{Ehw_3\lambda_3^w + \frac{Ehb^2}{4}\,v_{33}\lambda_{33}^v + \frac{Eh^3}{12}\,\tilde{u}_{33}\lambda_{33}^{\tilde{u}}\}\delta b dx_3$$
$$- \int_0^{\ell} Em_p \text{sign}(b)\,\frac{v_{33}}{2}\,\delta b dx_3 - \int_0^{\ell}\{Ebw^3\lambda_3^w + \frac{Eb^3}{12}\,v_{33}\lambda_{33}^v$$
$$+ \frac{Ebh^2}{12}\,\tilde{u}_{33}\lambda_{33}^{\tilde{u}}\}\delta h dx_3 - \int_0^{\ell} Em_p \text{sign}(h)\,\frac{\tilde{u}_{33}}{2}\,\delta h dx_3 \qquad (2.3.19)$$

where λ is the solution of Eq. 2.3.18. With the two-point Gauss integration procedure, integrals in Eqs. 2.3.15 and 2.3.19 are evaluated as

$$\psi_9 = \sum_{\ell=1}^{2} \frac{E}{dx_p}(w_3^{\ell} - \text{sign}(b)\,\frac{b}{2}\,v_{33}^{\ell} - \text{sign}(h)\,\frac{h}{2}\,\tilde{u}_{33}^{\ell})W^{\ell}J^{\ell} \qquad (2.3.20)$$

$$\psi_9' = - \sum_{k=1}^{N}\{\sum_{\ell=1}^{2} E[hw_3^{\ell}\lambda_3^{w\ell} + \frac{hb^2}{4}\,v_{33}^{\ell}\lambda_{33}^{v\ell} + \frac{h^3}{12}\,\tilde{u}_{33}^{\ell}\lambda_{33}^{\tilde{u}\ell}]W^{\ell}J^{\ell}\}^k\,\delta b^k$$
$$- \sum_{k=1}^{N}\{\sum_{\ell=1}^{2} E[bw_3^{\ell}\lambda_3^{w\ell} + \frac{b^3}{12}\,v_{33}^{\ell}\lambda_{33}^{v\ell} + \frac{bh^2}{4}\,\tilde{u}_{33}^{\ell}\lambda_{33}^{\tilde{u}\ell}]W^{\ell}J^{\ell}\}^k\,\delta h^k$$
$$- \sum_{k=1}^{N}\{\sum_{\ell=1}^{2} Em_p\,\frac{v_{33}^{\ell}}{2}\,\text{sign}(b)\,W^{\ell}J^{\ell}\}^k\delta b^k - \sum_{k=1}^{N}\{\sum_{\ell=1}^{2} Em_p\,\frac{\tilde{u}_{33}^{\ell}}{2}\,\text{sign}(h)W^{\ell}J^{\ell}\}^k\,\delta h^k$$
$$(2.3.21)$$

The right hand side of Eq. 2.3.18 can be written as

$$\sum_{k=1}^{N} m_p \{ \sum_{\ell=1}^{2} E(\lambda_{33}^{w_\ell} - \text{sign}(b)\,\frac{b}{2}\,\lambda_{33}^{v_\ell} - \text{sign}(h)\,\frac{h}{2}\,\lambda_{33}^{\tilde{u}_\ell}) W^\ell J^\ell \}^k \qquad (2.3.22)$$

2.4. Built-Up Structures

A general structure is a collection of structural components that are interconnected by kinematic constraints at their boundaries. Results stated in this section are from Refs. 1 and 6. The energy bilinear form of a general system, consisting of beams, membranes, and plates can be written as

$$a_u(z,\bar{z}) = \sum\, [a_u(z,\bar{z})]^M + \sum\, [a_u(z,\bar{z})]^P + \sum\, [a_u(z,\bar{z})]^B \qquad (2.4.1)$$

where $[a_u(z,\bar{z})]^M$, $[a_u(z,\bar{z})]^P$, and $[a_u(z,\bar{z})]^B$ are given in Eqs. 2.1.1, 2.2.1, and 2.3.1, respectively. The load linear form of a general system can be written as

$$\ell_u(\bar{z}) = \sum\, [\ell_u(\bar{z})]^M + \sum\, [\ell_u(\bar{z})]^P + \sum\, [\ell_u(\bar{z})]^B \qquad (2.4.2)$$

where $[\ell_u(\bar{z})]^M$, $[\ell_u(\bar{z})]^P$, and $[\ell_u(\bar{z})]^B$ are given in Eqs. 2.1.2, 2.2.2, and 2.3.2, respectively. The state equation is [1]

$$a_u(z,\bar{z}) = \ell_u(\bar{z}) \qquad (2.4.3.)$$

for all kinematically admissible virtual displacements $\bar{z}$. Since the energy bilinear and load linear forms of the state equation are just the sum of energy bilinear and load linear forms of each structural component, the design sensitivity equation of the system is a simple additive process [1,6]. The generalized design sensitivity of a built-up structure is

$$\psi' = \psi'_b \delta b + \psi'_h \delta h + \psi'_t \delta t \tag{2.4.4}$$

which is the sum of the sensitivities of each structural component, given in Sections 2.1, 2.2, and 2.3, respectively.

2.5. Coupling of Bending and Membrane Effect

2.5.1. Formulation of Membrane Plus Bending

A clamped plate combining bending and stretching is shown in Fig. 2.5 with the laterally distributed load f(x) and in-plane traction load $T = [T^1, T^2]^T$.

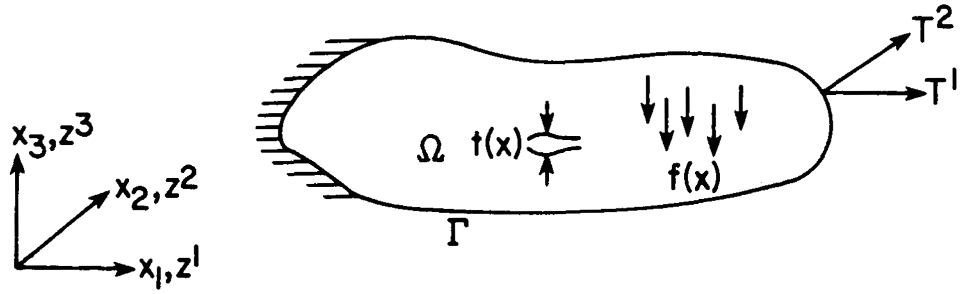

Figure 2.5. Clamped Plate of Variable Thickness

Assuming that bending and stretching are decoupled, one can obtain the energy bilinear form by adding the plate and membrane energy bilinear forms as in the case of the built-up structure.

$$a_u(z,\bar{z}) = [a_u(z,\bar{z})]^M + [a_u(z,\bar{z})]^P \tag{2.5.1}$$

where $[a_u(z,\bar{z})]^M$ and $[a_u(z,\bar{z})]^P$ are given in Eqs. 2.1.1 and 2.2.1, respectively. The membrane thickness h(x) has to be replaced by t(x) in this case. Likewise, the load linear form can be expressed as

$$\ell_u(\bar{z}) = [\ell_u(\bar{z})]^M + [\ell_u(\bar{z})]^P \tag{2.5.2}$$

where $[\ell_u(\bar{z})]^M$ and $[\ell_u(\bar{z})]^P$ are given in Eqs. 2.1.2 and 2.2.2, respectively. The state equation is

$$a_u(z,\bar{z}) = \ell_u(\bar{z}) \tag{2.5.3}$$

for all kinematically admissible virtual displacements $\bar{z}$.

Consider the functional representing the allowable stresses in the middle plane due to stretching and on the surface due to bending as

$$\psi_{10} = \psi_3 + \psi_6 \tag{2.5.4}$$

where ψ_3 and ψ_6 are given in Eqs. 2.1.21 and 2.2.23, respectively.

Forces and moments of the component are [8]

$$\{N_{x_1} N_{x_2} N_{x_1 x_2} M_{x_1} M_{x_2} M_{x_1 x_2}\}^T = \int_{-t/2}^{t/2} \begin{Bmatrix} [\sigma_M] \\ [\sigma_B] \end{Bmatrix} dx_3 \tag{2.5.5}$$

where $\{\sigma_M\} = [E]\{\varepsilon_M\}$ are the membrane stresses and $\{\sigma_B\} = x_3[E]\{\kappa\}$ are the bending stresses. The curvatures $\{\kappa\}$ are

$$\{\kappa\} = \{z^3_{11} \;\; z^3_{22} \;\; 2z^3_{12}\} \tag{2.5.6}$$

Membrane and bending stress resultants can be decoupled if the plate is symmetric with respect to the x_1-x_2 plane. In EAL, moments and forces are related as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = [C]\begin{Bmatrix} \varepsilon_M \\ \kappa \end{Bmatrix} = \begin{bmatrix} C_{mm} & C_{mb} \\ C_{bm} & C_{bb} \end{bmatrix}\begin{Bmatrix} \varepsilon_M \\ \kappa \end{Bmatrix} \tag{2.5.7}$$

where [C] is the coupling coefficient matrix [2].

Design sensitivity analysis of the stress functional is chosen to illustrate the procedure. The sensitivity is the summation of membrane and plate sensitivity as

$$\psi'_{10} = \psi'_3 + \psi'_6$$

$$= \iint_\Omega [-\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)]\delta t \, d\Omega + \iint_\Omega [-\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)]\delta t \, d\Omega$$

$$+ \iint_\Omega \frac{\partial g}{\partial t} [\sigma(z)]M_p \, \delta t \, d\Omega \qquad (2.5.8)$$

which are given in Eqs. 2.1.25 and 2.2.27. Membrane stresses and strains are expressed in Eqs. 2.1.10 and 2.1.11, respectively and bending stresses and strains on the surface are expressed in Eqs. 2.2.15 and 2.2.16, respectively.

For the design sensitivity of the displacement functional, Eq. 2.1.18 can be used for the in-plane displacements due to stretching and Eq. 2.2.22 for the displacement z^3 due to bending.

2.5.2. Numerical Examples

To demonstrate the numerical accuracy of the approach, a numerical example is tested. The finite element plate model is given in Fig. 2.6 which is restrained at one side and loaded with a distributed tensile load in the positive x_1 direction and nodal loads of 50lb at nodes 21, 42, 63, 84 and 105 in the negative x_3 direction as shown in Fig. 2.6. It contains 80 E43 elements (EAL bending plus membrane element type), 105 nodal points, and 500 degrees of freedom with the design variable thickness $u = t(x_1, x_2)$. The material properties are the same as the one in Section 4.1.

Design sensitivity results of the von Mises' stress functional are given in Table 1 for Membrane, Bending and Total sensitivities with the perturbation of 0.01t. Since the stress resultants can be decoupled, the membrane sensitivity is expected to be the same as that of the original membrane model given in Table 7. However slightly different numerical values are obtained because numerical difference can occur in the decomposition process of membrane plus bending stiffness matrix when the process is compared to the membrane stiffness matrix alone.

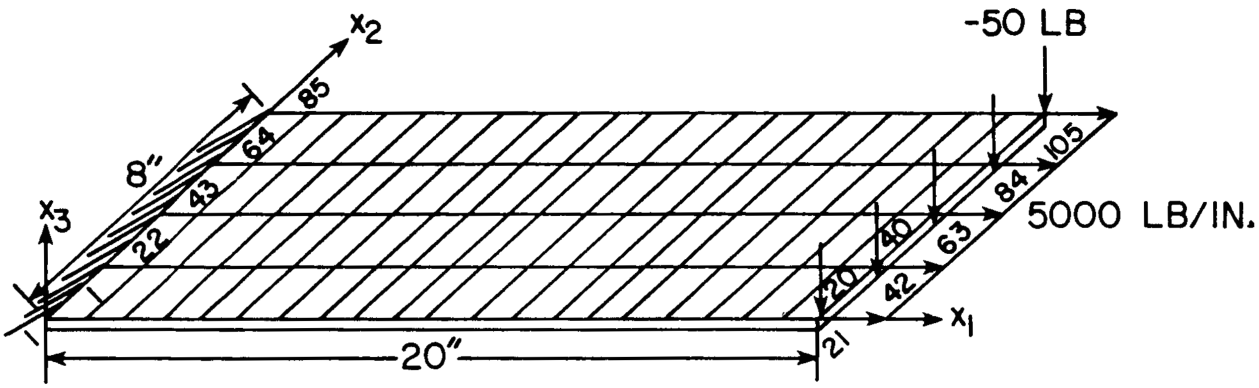

Figure 2.6. Membrane plus Bending Plate Finite Element Model

The same nodal points as in Table 6 are selected to check the accuracy of the design sensitivity of the displacement functional in the x_1, x_2 and negative x_3 directions. Design sensitivity predictions and finite differences, with $\delta t = 0.05t$ are given in Table 2.

Table 1. Design Sensitivity Check for von Mises' Stress of Membrane plus Bending Plate with $\delta t = 0.01t$

El. No.		$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
	M	10053.27	9953.71	-99.56	-100.53	101.0
1	B	12459.37	12194.25	-265.12	-249.86	94.3
	T	22512.63	22147.96	-364.67	-350.93	96.2
	M	9995.58	9896.59	-98.99	-99.96	101.0
10	B	7897.13	7732.11	-165.01	-158.86	96.3
	T	17892.71	17628.71	-264.00	-258.81	98.0
	M	9999.86	9900.97	-98.89	-100.00	101.1
20	B	748.51	731.58	-16.93	-16.72	98.9
	T	10748.38	10632.55	-115.82	-116.72	100.8
	M	8570.37	8485.51	-84.86	-85.7	101.0
21	B	14344.51	14040.40	-304.05	-289.13	95.1
	T	22914.89	22525.91	-388.89	-374.84	96.4
	M	10019.71	9920.49	-99.21	-100.19	101.0
30	B	7727.56	7565.95	-161.61	-155.44	96.2
	T	17747.27	17486.45	-260.82	-255.64	98.0
	M	9999.20	9900.91	-98.30	-99.99	101.7
40	B	675.18	642.77	-14.41	-13.94	96.7
	T	10657.08	10543.67	-113.41	-113.93	100.5

M: membrane, B: bending, and T: total

Table 2. Design Sensitivity Check for Displacement of Membrane plus Bending Plate, $\delta t = 0.05t$

Node No.	Dir.	$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
63	x_1	6.631E-3	6.315E-3	-3.157E-4	-3.316E-4	105.0
63	x_3	-2.585E-1	-2.214E-1	3.710E-2	3.895E-2	105.0
74	x_1	3.297E-3	3.141E-3	-1.570E-4	-1.649E-4	105.0
74	x_2	-2.011E-4	-1.915E-4	0.961E-5	1.004E-5	105.0
74	x_3	-7.995E-2	-6.842E-2	1.153E-2	1.205E-2	104.5
105	x_1	6.631E-3	6.315E-3	-3.157E-4	-3.316E-4	105.0
105	x_2	-3.996E-4	-3.804E-4	1.914E-5	1.995E-5	104.2
105	x_3	-2.582E-1	-2.211E-1	3.707E-2	3.895E-2	105.1

Since the displacements can be decoupled in membrane plus bending elements, the sensitivities for x_1 and x_2 directions are the same as that of Table 6. The sensitivity for x_2 direction displacement at node 63 is not considered since the displacement is zero due to the symmetry of loads and structure.

2.6. Design Sensitivity Analysis of Pointwise Stress Functional

2.6.1. Membranes

Consider the general functional that represents a locally averaged stress on an element as in Eq. 2.1.21.

$$\psi_3 = \iint_\Omega g(\sigma(z))m_p \, d\Omega \tag{2.6.1}$$

where m_p is defined on a finite element Ω_p in Eq. 2.1.22. If we have a smooth problem so the stress is continuous, we can consider pointwise constraint. In this case, by letting the test area shrink to a point $\hat{x}$, m_p becomes a Dirac delta measure. Thus

$$\begin{aligned}\psi_{11} &= g\big(\sigma(z(\hat{x}))\big) \\ &= \iint_\Omega g\big(\sigma(z\ \ )\big)\delta(x - \hat{x})d\Omega\end{aligned} \tag{2.6.2}$$

The first variation of Eq. 2.6.2 is

$$\begin{aligned}\psi'_{11} &= \sum_{i,j=1}^{2} \frac{\partial g\big(z(\hat{x})\big)}{\partial\sigma^{ij}}\,\sigma^{ij}\big(z'(\hat{x})\big) \\ &= \iint_\Omega \sum_{i,j=1}^{2} \frac{\partial g(z)}{\partial\sigma^{ij}}\,\sigma^{ij}(z')\delta(x - \hat{x})d\Omega\end{aligned} \tag{2.6.3}$$

Replacing the variation in state z' by a virtual displacement $\bar{\lambda}$, the adjoint equation is obtained as

$$a_u(\lambda,\bar{\lambda}) = \iint_\Omega \sum_{i,j=1}^{2} \frac{\partial g(z)}{\partial\sigma^{ij}}\,\sigma^{ij}(\bar{\lambda})\delta(x - \hat{x})d\Omega \tag{2.6.4}$$

for all kinematically admissible virtual displacements $\bar{\lambda}$. Using the adjoint variable method, the design sensitivity of the pointwise stress functional is

$$\psi'_{11} = -\iint_\Omega \Big[\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)\Big]\delta h\ d\Omega \tag{2.6.5}$$

which can be computed using information at Gauss points as

$$\psi'_{11} = \sum_{k=1}^{N} \left[\sum_{\ell=1}^{4}\Big[- \sum_{i,j=1}^{2} \big(\sigma^{ij}(z)^{\ell}\big(\varepsilon^{ij}(\lambda)\big)^{\ell}\Big]W^{\ell}J^{\ell}\right]^{k} \delta h^{k} \tag{2.6.6}$$

where N is the total number of elements, ℓ is the counter for the number of Gauss points, k is the counter for the element number, J is the Jacobian, and W is the weighting constant for the ℓth Gauss point.

If von Mises' stress functional is selected as the constraint functional, the formulations for stresses are given in Section 2.1.

2.6.2 Plates

Consider the functional that represents a locally averaged stress on the surface of the plate as in Eq. 2.2.23. If we consider the pointwise stress, the stress functional and its first variation will become

$$\begin{aligned}\psi_{12} &= g\big(\sigma(z(\hat{x}))\big) \\ &= \iint_\Omega g\big(\sigma(z)\big)\delta(x-\hat{x})d\Omega\end{aligned} \tag{2.6.7}$$

and

$$\begin{aligned}\psi'_{12} &= \sum_{i,j=1}^{2} \frac{\partial g\big(z(\hat{x})\big)}{\partial\sigma^{ij}}\,\sigma^{ij}\big(z'(\hat{x})\big) \\ &= \iint_\Omega \sum_{i,j=1}^{2} \frac{\partial g(z)}{\partial\sigma^{ij}}\,\sigma^{ij}(z')\delta(x-\hat{x})d\Omega\end{aligned} \tag{2.6.8}$$

The adjoint equation can be defined as

$$\begin{aligned}a_u(\lambda,\bar{\lambda}) &= \sum_{i,j=1}^{2} \frac{\partial g\big(z(\hat{x})\big)}{\partial\sigma^{ij}}\,\sigma^{ij}\big(\bar{\lambda}(\hat{x})\big) \\ &= \iint_\Omega \sum_{i,j=1}^{2} \frac{\partial g(z)}{\partial\sigma^{ij}}\,\sigma^{ij}(\bar{\lambda})\delta(x-\hat{x})d\Omega\end{aligned} \tag{2.6.9}$$

Then the design sensitivity of the pointwise stress functional can be obtained using the adjoint variable method as

$$\psi_{12}' = \iint_{\Omega} - \left[\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)\right]\delta t \; d\Omega + \iint_{\Omega} \frac{\partial g}{\partial t} \left(\sigma(z)\right)\delta(x - \hat{x})d\Omega \; \ldots \tag{2.6.10}$$

which can be computed as

$$\psi_{12}' = \sum_{k=1}^{N} \left[\sum_{\ell=1}^{4} \left[- \sum_{i,j=1}^{2} \left(\sigma^{ij}(z)\right)^{\ell} \left(\varepsilon^{ij}(\lambda)\right)^{\ell}\right] W^{\ell} J^{\ell} + \sum_{i,j=1}^{2} \frac{\partial g\left(\sigma^{ij}(z)\right)}{\partial t} \delta(x - \hat{x}) \right]^{k} \delta t^{k} \tag{2.6.11}$$

where N, ℓ, and k are explained in Section 2.6.1. In case of von Mises' stress constraint, the formulations of stresses for plate are given in Section 2.2.

2.6.3. Numerical Examples

The examples given in Sections 4.1 and 4.2 are used to test the accuracy of design sensitivity of the pointwise stress constraint. For the pointwise stress sensitivity, the first Gauss point shown in Fig. 2.7 is selected as $\hat{x}$.

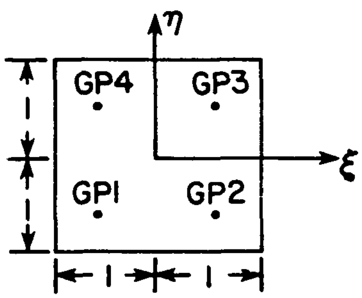

Figure 2.7. 2-by-2 Gauss Points

To check sensitivity of the stress constraint of Eq. 2.6.2, the equivalent nodal forces of the adjoint load are computed using right side of Eq. 2.6.4. Design sensitivity analysis results for the von Mises' pointwise stress are given in Table 3 for several finite elements with the perturbation of $\delta t = 0.01t$.

Accuracy of the sensitivity analysis results of the pointwise stresses at the first Gauss point of elements are almost equal to that of averaged stresses except at the first element. The design sensitivity results compared to the finite difference approximations are excellent. Especially the sensitivity of pointwise stress of the first element is better than that of the averaged stress in Table 7.

The design sensitivities of the pointwise stress of plate problem are computed using Eq. 2.6.11. The adjoint load of Eq. 2.6.9 is used to get the equivalent nodal forces. Design sensitivity results of the von Mises' functional at the first Gauss point are given in Table 4 for several finite elements. The design perturbation for the finite difference computation is $\delta t = 0.001t$. The sensitivity results of pointwise stress are not quite excellent when compared to Table 10. It is found that the bending stresses are not evenly distributed on the plate surface for a given finite element element. That is, values of stress at four Gauss points are rather different from the average value.

Table 3. Membrane Design Sensitivity Check for Pointwise Stress at the first Gauss point, $\delta t = 0.01t$

El. No.	$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
1	10496.95	10392.99	-103.95	-104.97	101.0
10	9990.33	9891.39	-98.94	-99.90	101.0
20	9999.99	9900.96	-99.03	-100.0	101.0
21	8453.86	8370.15	-83.71	-84.54	101.0
30	10015.01	9915.84	-99.17	-100.15	101.0
40	9999.93	9900.94	-98.99	-99.99	101.0

Table 4. Plate Design Sensitivity Check for Pointwise Stress at the first Gauss point, $\delta t = 0.001t$

El. No.	$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
1	35.64	35.57	-0.071	-0.074	104.7
2	830.29	828.62	-1.661	-1.665	100.2
3	1841.86	1838.17	-3.691	-3.609	97.8
4	2716.80	2711.37	-5.432	-5.331	98.1
5	3252.14	3245.63	-6.510	-6.374	97.9
7	1157.15	1154.83	-2.314	-2.376	102.6
8	1273.81	1271.26	-2.553	-2.473	95.7
9	1169.60	1167.25	-2.341	-2.112	90.2
10	1038.10	1036.02	-2.078	-1.735	83.5
13	1310.30	1307.68	-2.620	-2.670	101.9
14	1151.13	1148.83	-2.307	-2.432	105.4
15	901.55	899.75	-1.805	-1.985	109.9
19	1402.82	1400.01	-2.813	-3.057	108.7
20	1533.27	1530.20	-3.069	-3.291	107.0
25	1883.25	1879.47	-3.77	-3.901	103.5

CHAPTER III

PROGRAMMING ASPECTS

So far analytical results and numerical algorithms for design sensitivity analysis have been stated. This Chapter outlines the basic organization of the EAL database management system and a structural design sensitivity analysis program that has been implemented using EAL.

3.1. The EAL Database Management System

Design sensitivity analysis of structural components and built-up structures can be implemented using a database management system. A survey of database management systems can be found in Ref.12. EAL can be considered as a database management system [12], as well as a higher order programming language with advanced programming concepts [13].

EAL is a set of independent processors that communicate with each other through a random access database (Fig. 3.1). The database is manipulated according to user commands. The commands can be combined in a runstream, which can be stored in the database as a runstream dataset. Runstream datasets are driven by the Execution Control System (ECS), which also allows branching and looping within a runstream dataset. The ECS allows the user to call a runstream dataset from the database within a called runstream dataset. The ECS also initiates the execution of a new processor.

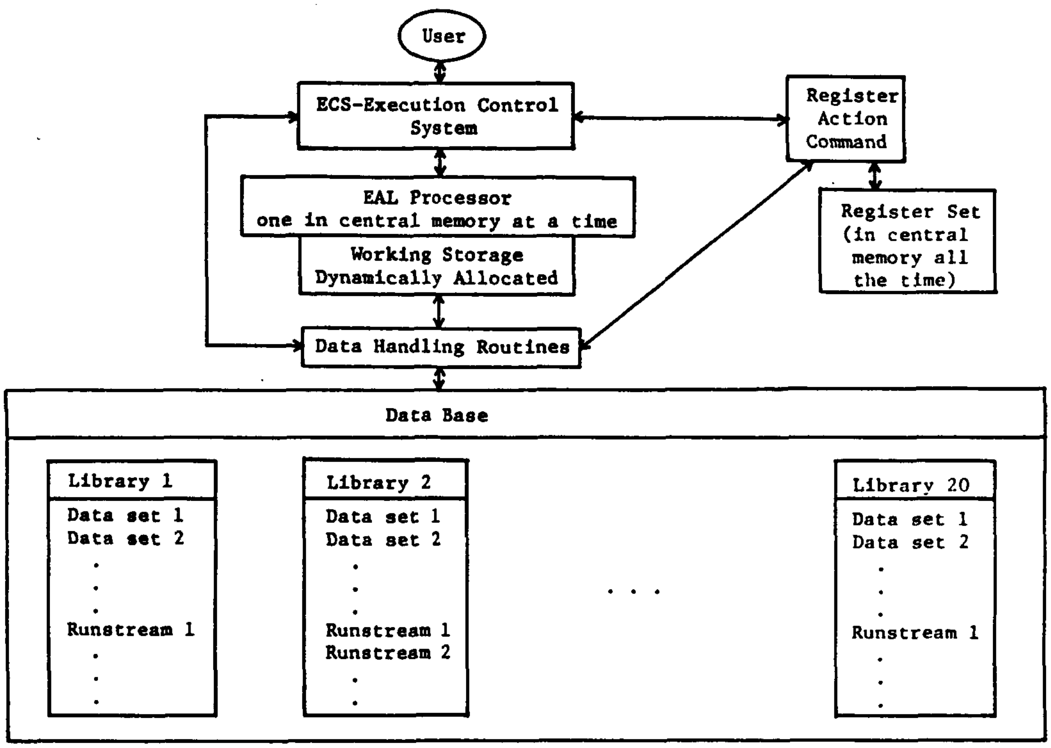

Figure 3.1 Data Flow in EAL

Besides the regular database, EAL has a set of registers, which are stored in core. The user can assign a name, a type code (real, integer, or alphanumeric), and a value to each register. The contents of a register can be manipulated by register action commands. The registers, together with the ECS, enable controlled branching to be performed in runstream datasets. Register action commands have a higher priority than other commands, so whenever a register action command is given, the regular command procedure will be interrupted and the register command will be executed first.

To manage the database, a set of database utility functions are provided. The database consists of one or more libraries, where each library contains a set of named datasets. The contents of each library are stored in a separate table, which for each dataset stores the name, data type, block size, number of columns, total number of words in a dataset, and location in the library. If many datasets are stored in one library, database overhead disk I/O can be very large. Database utility functions allow the user to change information in the table of contents, copy datasets from one library to another, and erase information in libraries.

To manipulate information within each library, EAL provides the Arithmetic Utility System (AUS). AUS allows a wide range of vector and matrix manipulations as well as storing new datasets that are specified by the user. Matrices can be stored as full matrices or in a sparse storage forum.

Besides the database management system, EAL contains a variety of processors common to finite element analysis programs. Although EAL's element library is limited, it contains one, two, and three dimensional elements. The program is able to solve static as well as buckling problems. The dynamic analysis part can handle eigenvalue and eigenvector solution and forced dynamic response analysis. Substructuring and graphical pre- and post-processing are also available. Because the global system matrices (stiffness, mass, and geometrical stiffness matrices) are usually very large and sparse, a sparse storage technique for hypermatrices is used for global matrices. Thus, the user does not have to worry about bandwidth (as in SAP IV [14]) or wave front length (as in ANSYS [5]). For the solution of a large system of equations, the global stiffness matrix is factored according to Ref. 15.

EAL also allows the user to write his own program and combine it with the EAL database system. Because the user can use Fortran callable data handling routines, it is relatively easy to create new processors for the EAL database management system [16]. Writing a separate processor for a specific task is time consuming, but the result will be a more efficient system than just using the registers and the arithmetic utility processor. No separate processor is written in this work, because the goal is to show the feasibility of combining the design sensitivity analysis method with the EAL database management system.

3.2. Program Organization

Using the Engineering Analysis Language EAL, a general purpose design sensitivity analysis program has been written. The program can handle three types of constraint functionals; compliance, displacement, and stress. Three types of elements (element types E21, E41, and E42) can be used to model a structure and evaluate design sensitivities. A flow chart of the program is given in Figure 3.2. To use the program, the user sets the system control parameter, gives information about the design variables, specifies the constraints, sets up his finite element model. The finite element model is described in the runstream dataset INIT MODL 0 0, where all other information is given in the runstream dataset PARA SET 0 0. After that, the program automatically computes sensitivities for the given constraints and design variables. System control parameters are as follows:

NLST - Number of load cases
LCAS - Actual load case

NBTD - Number of independent beam/truss design variables
NMDV - Number of independent membrane design variables
NPDV - Number of independent plate design variables
NDV - Total number of design variables

DE21 - Number of beam/truss elements
DE41 - Number of membrane elements
DE42 - Number of plate elements
DETO - Total number of elements

For simplification of the checking process only, the program allows only one control parameter per finite element. For beam elements, the

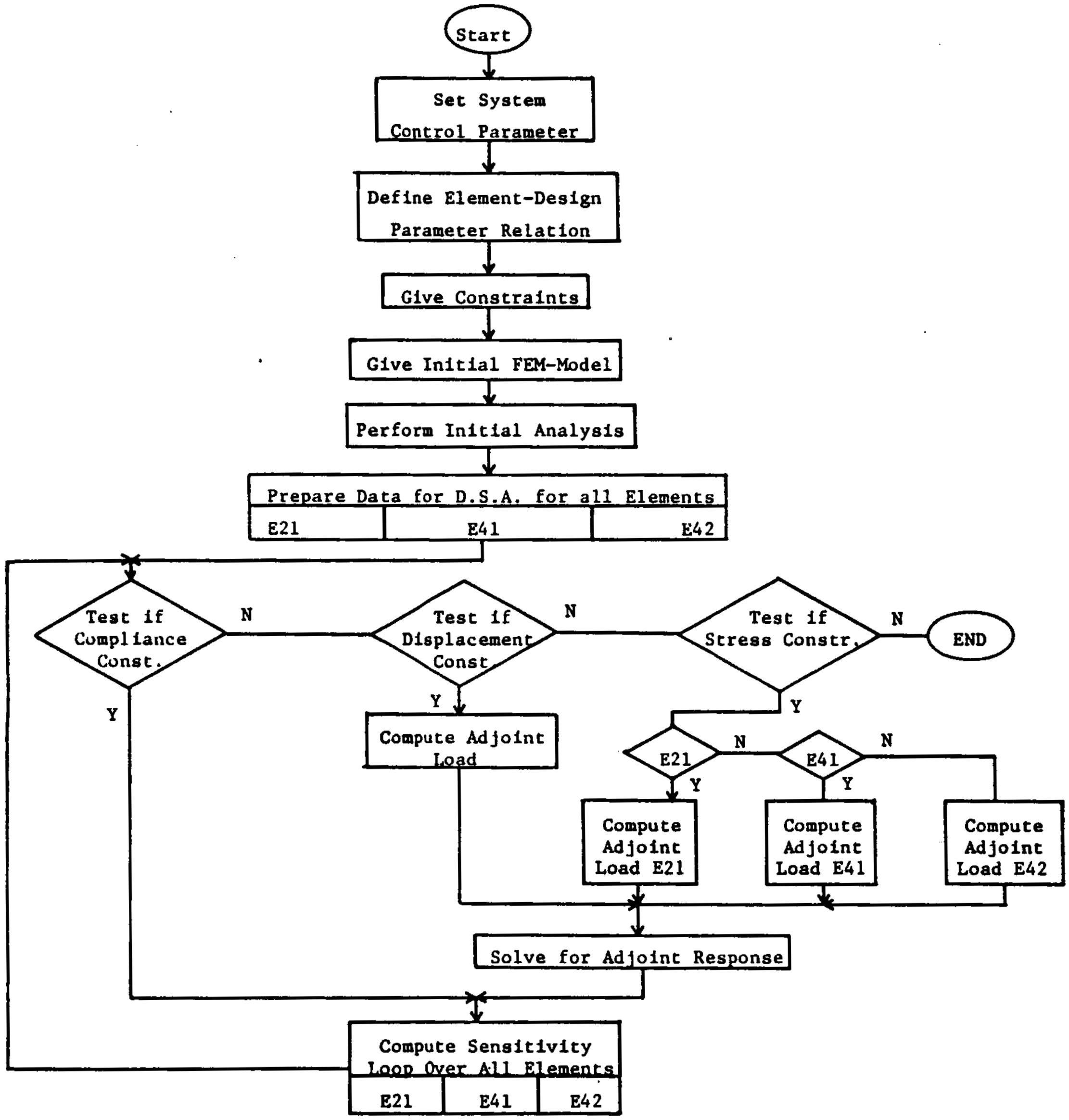

Figure 3.2 Program Organization

user can chose the height, the width, or a combination of both as independent design parameter. Weighting values for beam design parameter are introduced for selecting the height or width or any combination as design variables. The weighting values for beam design parameters are given in the table DESV VALU 0 0.

The relationship between elements and design parameters are defined in the lists given below. For each element, there is one entry in the appropriate table that gives the design variable group.

ED21 REL 0 0 Relationship for beam/truss elements

ED41 REL 0 0 Relationship for membrane elements

ED42 REL 0 0 Relationship for plate elements

The constraint control parameters are

CCOM - Compliance constraint

CDIS - Number of displacement constraints

CS21 - Number of stress constraints for element typ E21

CS41 - Number of stress constraints for element typ E41

CS42 - Number of stress constraints for element typ E42

CTOT - Total number of constraints

For every constraint group, with the exception of the compliance constraint, a table is required to describe the location of the constraint. For displacement constraints, two entries are needed for each constraint. The first entry is the node number on which the displacement sensitivity is evaluated and the second entry is the direction of the displacement constraint. For each stress constraint group (E21, E41, and E42) a separate table is needed that gives the element numbers on which the stress constraint functionals are evaluated. The tables are

ST21 LIST 0 0 - Gives the constraints for beam/truss elements

ST41 LIST 0 0 - Gives the constraints for membrane elements

ST43 LIST 0 0 - Gives the constraints for plate elements

For the stress constraint in a beam element (E21), the maximal stress is at one of the four corners of the beam cross section. The program computes stresses at four corners, finds the corner with the largest absolute value of stress, and computes sensitivity of the maximum stress.

Additional parameters are;

IDGP=1 - Pointwise stress at Gauss point case

IDGP=0 - Averaged stress case

(1) For E43 elements,

DE43 - Number of E43 plate elements

ED43 REL 0 0 Relationship for E43 plate elements

CS43 - Number of stress constraints for element type E43

For results,

CIND has an additional number between 60,000 and 70,000 indicating the stress constraint of E43 element.

(2) For pointwise stress constraint,

GPNO LIST 0 0 Indicates the Gauss point at which pointwise stress will be evaluated

For results,

DVAL E42 1 1	The value of pointwise stress constraint functional at Gauss point listed in input data set of E42
DVAL E41 1 1	The value of pointwise stress constraint functional at Gauss point listed in input data set of E41

CHAPTER IV

NUMERICAL EXAMPLES

In order to check whether the design sensitivity information obtained is accurate, a comparison is made with the finite difference $\Delta\psi$. An appropriate design perturbation Δu must be selected, in order to obtain a meaningful finite difference of the constraint functional. That is, if Δu is too small, $\Delta\psi = \psi(u + \Delta u) - \psi(u)$ may be inaccurate, due to loss of significant digits in the difference. On the other hand, if Δu is too large, $\Delta\psi$ will contain nonlinear terms and the comparison with ψ' will be meaningless. The design sensitivity ψ' of a constraint functional is the scalar product of the design sensitivity vector $\frac{\partial\psi}{\partial u}$ and the design variable perturbation vector δu. That is, the perturbation of an element design parameter is multiplied by the corresponding sensitivity component and the sum of all products is the design sensitivity ψ'.

4.1. Membrane

The finite element membrane model in Fig.4.1 is a simple plane elastic solid that is restrained at one end and loaded with a distributed tensile load at the other end. It contains 80 isoparametric elements (EAL plane stress element, type E41), 105 nodal points, and 200 degrees of freedom, with the design variable thickness $u = h(x)$. Young's modulus and Poisson's ratio are given as $E = 3x10^7$ psi and $\gamma = 0.3$, respectively. Each finite element has uniform thickness,

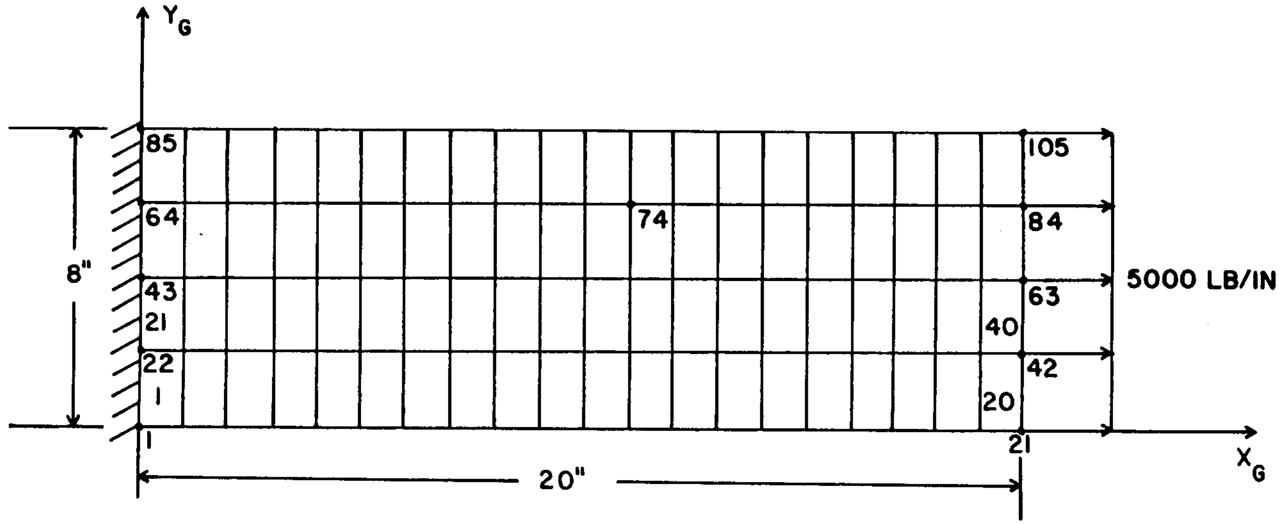

Figure 4.1 Plane Elastic Solid Finite Element Mode

so that a maximum possible number of design variables is 80. For simplicity, a uniform thickness of h = 0.5 in. is used for the sensitivity check.

Compliance sensitivity results are shown in Table 5, where $\Delta\psi = \psi(h + \Delta h) - \psi(h)$ and ψ' is the predicted value computed from Eq. 2.1.9, with design perturbations of $\delta h = 0.01h$ and $\delta h = 0.05h$. The percent accuracy of the sensitivity prediction is computed using the ratio $\psi' \times 100/\Delta\psi$.

Table 5. Membrane Design Sensitivity Check for Compliance

δh	$\psi(h)$	$\psi(h+\Delta h)$	$\Delta\psi$	ψ'	Ratio %
0.01h	265.24	262.62	-2.63	-2.65	101.0%
0.05h	265.24	252.62	-12.63	-13.26	105.0%

Several discrete nodal points shown in Fig. 4.1 are selected to check accuracy of design sensitivity of the displacement functional of Eq. 2.1.19. In order to compute this equation, the adjoint strain $\varepsilon^{ij}(\lambda)$ due to the adjoint load is needed. For each direction of displacement on a node, there is a separate sensitivity calculation that produces an adjoint strain $\varepsilon^{ij}(\lambda)$. Design sensitivity predictions and differences, with $\delta h = 0.05h$, are given in Table 6.

Table 6. Membrane Design Sensitivity Check for Displacement

Node No.	Dir.	$\psi(h)$	$\psi(h+\Delta h)$	$\Delta\psi$	ψ'	Ratio %
74	X	3.297E-3	3.141E-3	-1.570E-4	-1.649E-4	105.0
74	Y	-2.011E-4	-1.915E-4	0.961E-5	1.004E-5	105.0
105	X	6.631E-3	6.315E-3	-3.157E-4	-3.315E-4	105.0
105	Y	-3.996E-4	-3.804E-4	1.914E-5	1.995E-5	104.2
63	X	6.631E-3	6.315E-3	-3.157E-4	-3.316E-4	105.0

To check the stress constraint sensitivity of Eq. 2.1.27, the equivalent nodal force of the adjoint load on the right of Eq. 2.1.37 is computed for the finite element adjoint analysis and $\varepsilon^{ij}(\lambda)$ is obtained for each constrained element. Design sensitivity results for von Mises' stress functionals are given in Table 7, for several finite elements. Perturbations are $\delta h = 0.01h$ and $\delta h = 0.05h$, for the von Mises' stress criteria.

With all three constraint functionals, design sensitivity results compared to the finite difference approximation are excellent. It is interesting to note that in Tables 5, 6, and 7, the finite difference approximation is nearly 1% of the constraint functional when $\delta h = 0.01h$ and nearly 5% of the constraint functional when $\delta h = 0.05h$. These results also show that as δh approaches zero, the ratio $\psi'/\Delta\psi$ approaches one.

Table 7. Membrane Design Sensitivity Check for Stress

(a) von Mises' Stress with δh = 0.01h

El. No.	$\psi(h)$	$\psi(h+\Delta h)$	$\Delta\psi$	ψ'	Ratio %
1	10053.402	9953.863	-99.539	-106.804	107.3
10	9995.646	9896.680	-98.966	-99.957	101.0
20	10000.141	9901.130	-99.011	-99.999	101.0
21	8570.358	8485.503	-84.855	-86.209	101.0
30	10019.743	9920.537	-99.206	-100.197	101.0
40	10000.065	9901.054	-99.011	-99.996	100.9

(b) von Mises' Stress with δh = 0.05h

El. No.	$\psi(h)$	$\psi(h+\Delta h)$	$\Delta\psi$	ψ'	Ratio %
1	10053.402	9574.668	-478.734	-534.021	111.5
10	9995.646	9519.663	-475.983	-499.958	105.0
20	10000.141	9523.944	-476.147	-499.996	105.0
21	8570.358	8162.246	-408.112	-431.046	105.6
30	10019.743	9542.612	-477.131	-500.983	105.0
40	10000.065	9523.871	-476.194	-499.982	105.0

4.2 Bending of Plates

The clamped plate element model shown in Fig. 4.2 is uniformly loaded with a pressure $f(x) = -1.5$ lb/in in the z direction. Since the model is symmetric with respect to the center and symmetric boundary conditions are applied, only one quarter of the plate is analyzed. The quarter model contains 25 4-node quadrilateral thin plate elements of type E42. It has 36 nodal points and 85 degrees of freedom.

The design variable is the plate thickness $u = t(x)$. Young's Modulus and Poisson's ratio are $E = 30.5 \times 10^7$ psi and $\gamma = 0.3$, respectively. The constant plate thickness is $t = 0.4$ in. Self-weight of the plate is neglected.

Compliance sensitivity results are shown in Table 8, where $\Delta\psi$ $\psi(t + \Delta t) - \psi(t)$ and ψ' is the predicted value that is computed from Eq. 2.2.11, with design perturbations $\delta t = 0.01t$ and $\delta t = 0.05t$. The 1% design perturbation for the compliance constraint functional gives good correlation between design sensitivity and the finite difference approximation. However the 5% perturbation shows nonlinearity in the compliance of the plate element.

Table 8. Plate Design Sensitivity Check for Compliance

Δt	$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
0.01t	5.0016	4.8545	-0.1471	-0.1501	102.0
0.05t	5.0016	4.3205	-0.6811	-0.7502	110.2

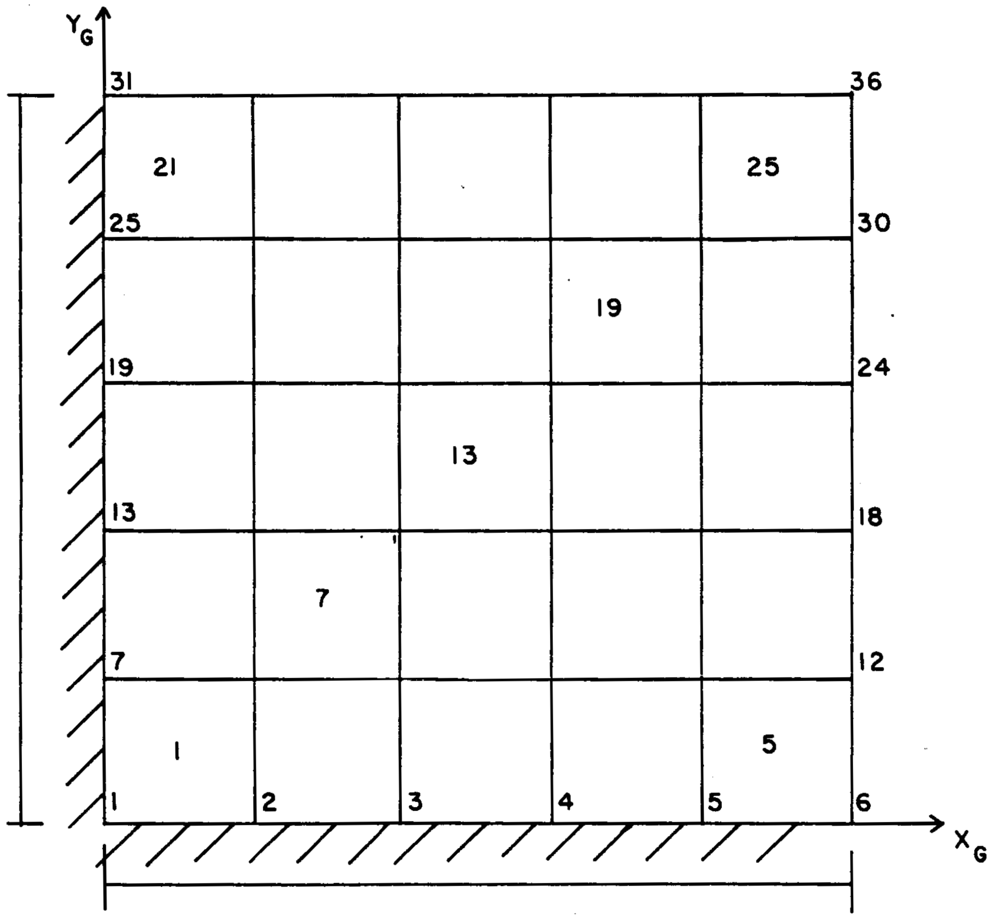

Figure 4.2 Bending Plate Finite Element Model

Several discrete points in Fig. 4.2 are selected to check design sensitivity accuracy for the displacement functional in Eq. 2.2.22. In order to compute this equation, as in the membrane case, the adjoint strain $\varepsilon^{ij}(\lambda)$ due to the adjoint load is needed. Some displacement results are shown in Table 9 for a design perturbation of $\Delta t = 0.01t$. The design sensitivity results of Table 9 agree very well with the finite difference approximation.

Table 9. Plate Design Sensitivity Check for Displacement

Node No.	$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
9	1.9287E-3	1.8720E-3	-5.6720E-5	-5.7861E-5	102.0
10	3.0860E-3	2.9952E-3	-9.0760E-5	-9.2579E-5	102.0
14	1.9287E-3	1.8720E-3	-5.6720E-5	-5.7861E-5	102.0
17	1.1349E-2	1.1015E-2	-3.3380E-4	-3.4046E-4	102.0
20	3.0860E-3	2.9952E-3	-9.0760E-5	-9.2579E-5	102.0
23	1.8585E-2	1.8038E-2	-5.4660E-4	-5.5753E-4	102.0
27	1.1349E-2	1.1015E-2	-3.3380E-4	-3.4046E-4	102.0
28	1.8585E-2	1.8038E-2	-5.4660E-4	-5.5753E-4	102.0
32	4.1105E-3	3.9896E-3	-1.2088E-4	-1.2331E-4	102.0
35	2.5256E-2	2.4514E-2	-7.4280E-4	-7.5767E-4	102.0
36	2.7124E-2	2.6327E-2	-7.9780E-4	-8.1372E-4	102.0

To check sensitivity of the constraint functional of Eq. 2.2.29, the equivalent nodal forces of the adjoint load are computed with Eq. 2.2.33. Design sensitivity results for the von Mises' stress functional are given in Table 10, for several different elements. The perturbation for the finite difference calculation is $\delta t = 0.001t$. Note that even though the equivalent nodal force calculation for the adjoint load is not consistent with the hybrid method, since a displacement shape function is used, design sensitivity accuracy is excellent.

Table 10. Design Sensitivity Check for Stress Von Mises' Stress with $\delta t = 0.001t$

El. No.	$\psi(t)$	$\psi(t+\Delta t)$	$\Delta\psi$	ψ'	Ratio %
1	451.13	450.23	-0.90	-0.889	98.7
2	1128.39	1126.14	-2.25	-2.185	97.1
3	1762.13	1758.61	-3.52	-3.476	98.7
4	2268.37	2263.84	-4.53	-4.518	99.7
5	2549.64	2544.54	-5.10	-5.097	99.9
7	1253.26	1250.76	-2.50	-2.543	101.7
8	1248.91	1246.42	-2.49	-2.471	99.2
9	1034.00	1031.93	-2.07	-1.967	95.0
10	809.66	808.04	-1.62	-1.460	90.5
13	1288.46	1285.89	-2.57	-2.548	99.1
14	1210.50	1208.09	-2.41	-2.380	98.8
15	1084.12	1081.96	-2.16	-2.134	98.8

Table 10--continued

19	1534.12	1531.06	-3.06	-3.057	99.9
20	1680.28	1676.92	-3.36	-3.358	99.9
25	1963.52	1959.61	-3.91	-3.927	100.4

4.3 Beams

A cantilever beam finite element model shown in Fig. 4.3 is loaded with a force F = [0.0 10.0 -10.0] lb. at the tip. It contains 20 2-node beam elements of type E21 and 21 nodal points with six degrees of freedom each. The beam has a rectangular cross-section with constant width and height, b = 0.5 in. and h = 0.25 in., respectively. Young's modulus and Poisson's ratio are $E = 30.5\times10^{7}$ psi and $\Upsilon = 0.3$, respectively. Self weight is excluded in the analysis.

Compliance sensitivity results are shown in Table 11, where $\Delta\psi = \psi(u + \Delta u) - \psi(u)$ and ψ' is the predicted value calculated from Eq. 2.3.9, with design perturbations $\delta b = 0.01b$ and $\delta h = 0.01h$.

Table 11. Beam Design Sensitivity Check for Compliance

δh	δb	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi$	ψ'	Ratio %
0.01h	0.01b	22.524	21.646	-0.8789	-0.9010	102.5

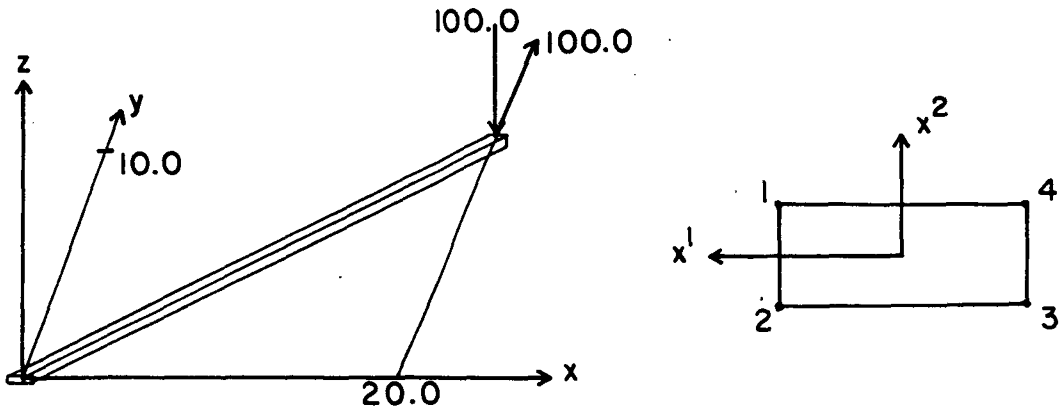

Figure 4.3. Beam Finite Element Model

Several discrete points along the beam are selected to check accuracy of design sensitivity of the displacement functional of Eq. 2.3.14. In order to compute Eq. 2.3.14, the beam curvature of the adjoint displacement field is needed. Displacement results are shown in Table 12 for design perturbations of $\delta b = 0.01b$ and $\delta h = 0.01h$. Results show that design sensitivity predictions are close to the finite difference approximation.

Table 12. Beam Design Sensitivity Check for Displacement

Node No.	Direction	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi$	ψ'	Ratio %
3	3	-0.0272	-0.0261	0.00106	0.00109	102.6
6	3	-0.1613	-0.1550	0.00629	0.00645	102.6
9	3	-0.3904	-0.3752	0.01523	0.01562	102.5
12	3	-0.6955	-0.6684	0.02713	0.02783	102.5
15	3	-1.0577	-1.0164	0.04130	0.04231	102.4
18	3	-1.4578	-1.4009	0.05690	0.05831	102.5
21	3	-1.8769	-1.8037	0.07320	0.07508	102.6
3	2	0.00546	0.00525	-0.2126E-3	-0.2192E-3	103.1
6	2	0.03229	0.03104	-0.001259	-0.001294	102.8
9	2	0.07814	0.07509	-0.003046	-0.003129	102.7
12	2	0.13918	0.13375	-0.005430	-0.005572	102.6
15	2	0.21162	0.20337	-0.008250	-0.008470	102.6
18	2	0.29167	0.28030	-0.011370	-0.011670	102.6
21	2	0.37551	0.36087	-0.014640	-0.015028	102.6

To check stress constraint sensitivity of Eq. 2.3.21, the equivalent nodal force for the adjoint load and the right side of Eq. 2.3.18 must be computed for finite element adjoint analysis and the adjoint displacement field λ must be obtained for each constraint element. Stress results for several finite elements are shown in Table 13 for design perturbations $\delta h = 0.01h$ and $\delta b = 0.01b$.

The reduced accuracy in correlation between the finite difference calculation and the first variation for the element number 20 results from an inaccurate finite difference calculation.

Table 13. Beam Design Sensitivity Check for Stress

El. No.	Fiber	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi$	ψ'	Ratio %
1	4	60613.5	58831.9	-1781.6	-1818.0	102.0
5	4	48187.0	46770.9	-1416.1	-1445.2	102.0
10	4	32654.9	31695.6	-959.3	-979.1	102.1
15	4	17121.3	16621.7	-499.6	-512.8	102.6
20	4	1592.9	1541.7	-51.2	-62.3	121.8

4.4 Built-Up Structures

A built-up structure that uses beams and plates is shown in Fig. 4.4. The structure is clamped on two edges, with symmetric boundary conditions applied along the other two edges. A uniform pressure $f(x) = 1.5$ lb/in is applied on the top surface of the plate. The model contains 20 beam elements (E21) and 25 plate elements (E42). Beam width and beam height are two dependent design variables of the first independent design parameter, and the plate thickness is the second independent design parameter. The initial values are $b = 0.05$ in., $h = 0.40$ in., and $t = 0.1$ in. Young's modulus and Poisson's ratio for the beams and plates are $E = 30.5 \times 10^7$ psi and $\gamma = 0.3$, respectively.

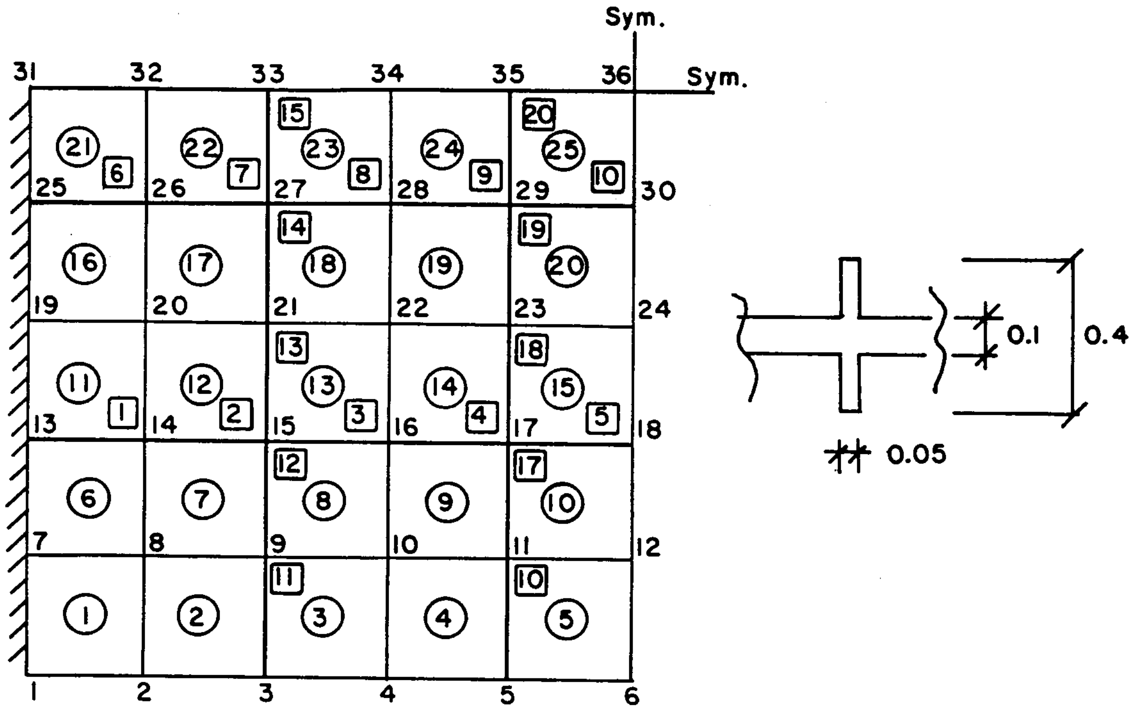

Figure 4.4 Built-Up Structure Finite Element Model

Self weight is neglected. An example of the design sensitivity vector $\frac{\partial \psi}{\partial u}$ is given in Appendix A1.

Compliance sensitivity results are shown in Table 14, where $\Delta\psi = \psi(u+\Delta u) - \psi(u)$ and ψ' is the predicted value computed from Eq. 2.4.4, with the design perturbations $\delta b = 0.01b$ and $\delta h = 0.01h$ for the first control parameter and perturbation of $\delta t = 0.01t$ for the second control parameter.

Table 14. Built-Up Structure Design Sensitivity Check for Compliance

Control Parameter	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi$	ψ'	Ratio %
1	110.084	109.086	-0.998	-0.9893	99.1
2	110.084	107.560	-2.524	-2.5577	101.3

Several discrete points in Fig. 4.4 are selected to check design sensitivity accuracy for the displacement functional in Eq. 2.4.4. In order to compute this equation, just as in the case of single components, the adjoint strain $\varepsilon^{ij}(\lambda)$ is needed. Some displacement results are shown in Table 15 for the design perturbations $\delta b = 0.01b$ and $\delta h = 0.01h$ for the first control parameter and a perturbation of $\delta t = 0.01t$ for the second control parameter.

Table 15. Built-Up Structure Design Sensitivity for Displacement

Node No.	Control Parameter	ψ(u)	ψ(u+Δu)	Δψ	ψ'	Ratio %
8	1	-0.02115	-0.02131	0.000197	0.000196	99.6
8	2	-0.02115	-0.02102	0.000492	0.000498	101.2
9	1	-0.06217	-0.06157	0.000605	0.000606	100.1
9	2	-0.06217	-0.06078	0.001394	0.001420	101.3
10	1	-0.10086	-0.09992	0.000941	0.000937	99.6
10	2	-0.10086	-0.09857	0.002295	0.002324	101.3
11	1	-0.12491	-0.12372	0.001190	0.001192	100.2
11	2	-0.12491	-0.12209	0.002820	0.002855	101.2
12	1	-0.13473	-0.13348	0.001250	0.001242	99.3
12	2	-0.13473	-0.13166	0.003070	0.003111	101.3
15	1	-0.18204	-0.18032	0.001720	0.001715	99.7
15	2	-0.18204	-0.17792	0.004120	0.004174	101.3
16	1	-0.29650	-0.29375	0.002750	0.002727	99.1
16	2	-0.29650	-0.28974	0.006760	0.006845	101.3
17	1	-0.37298	-0.36955	0.003430	0.003407	99.3
17	2	-0.37298	-0.36447	0.008510	0.008626	101.4
18	1	-0.40049	-0.39683	0.003660	0.003628	99.1
18	2	-0.40049	-0.39133	0.009160	0.009285	101.4
22	1	-0.48626	-0.48186	0.004400	0.004365	99.2
22	2	-0.48626	-0.47511	0.011150	0.011300	101.3
23	1	-0.61524	-0.60970	0.005540	0.005478	98.9

Table 15--continued

23	2	-0.61524	-0.60110	0.014140	0.014329	101.3
24	1	-0.66123	-0.65533	0.005900	0.005840	99.0
24	2	-0.66123	-0.64600	0.015230	0.015434	101.3
29	1	-0.78180	-0.77486	0.006940	0.006861	98.9
29	2	-0.78180	-0.76377	0.018030	0.018279	101.4
30	1	-0.84074	-0.83332	0.007420	0.007334	98.9
30	2	-0.84074	-0.82131	0.019430	0.019688	101.3
36	1	-0.90440	-0.89647	0.007930	0.007836	98.8
36	2	-0.90440	-0.88347	0.020930	0.021217	101.3

To check design sensitivity of the stress constraint functional for plate elements in the built-up structure, the equivalent nodal force for the adjoint load of each constraint must be computed. Design sensitivity results for the von Mises' stress functional are given in Table 16, for several different elements. The perturbations $\delta h = 0.01h$ and $\delta b = 0.01b$ are for finite difference calculation of the first control parameter and $\delta t = 0.01t$ is the perturbation of the second control parameter. Note that the equivalent nodal force calculation for the adjoint load is done with shape functions that are inconsistent with the hybrid method, but this has no effect on accuracy of design sensitivity calculations.

Table 16. Design Sensitivity Check for Plate Stress Von Mises' Stress with $\delta t = 0.01t$

El. No.	Control Parameter	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi$	ψ'	Ratio %
1	1	3669.32	3635.34	-33.98	-34.11	100.4
1	2	3669.32	3621.67	-47.65	-47.95	100.6
2	1	9094.17	9005.53	-89.17	-86.17	96.6
2	2	9094.17	8979.13	-115.04	-111.05	96.5
3	1	14410.77	14276.46	-134.31	-132.65	98.8
3	2	14410.77	14223.77	-187.00	-184.91	98.9
4	1	18484.13	18309.74	-174.39	-172.25	98.8
4	2	18484.13	18245.95	-238.18	-238.23	100.0
5	1	20882.59	20688.57	-194.02	-192.30	99.1
5	2	20882.59	20611.15	-271.44	-272.81	100.5
7	1	10370.80	10277.77	-93.03	-92.73	99.4
7	2	10370.80	10233.58	-137.22	-140.06	102.1
8	1	10381.71	10288.66	-93.05	-91.21	98.0
8	2	10381.71	10244.29	-137.42	-137.10	99.8
9	1	8802.67	8730.74	-71.93	-68.44	95.1
9	2	8802.67	8681.11	-121.56	-115.74	95.2
10	1	6956.20	6900.33	-55.87	-51.49	92.2
10	2	6956.20	6859.40	-96.80	-87.96	90.9
13	1	10757.59	10664.65	-92.94	-90.63	97.5
13	2	10757.59	10612.64	-144.95	-143.94	99.3

Table 16--continued

14	1	10079.36	9990.00	-89.36	-87.01	97.4
14	2	10079.36	9945.17	-134.19	-131.87	98.3
15	1	9011.52	8931.79	-79.73	-77.53	97.2
15	2	9011.52	8891.51	-120.01	-117.59	98.0
19	1	12794.24	12682.81	-111.43	-109.85	98.6
19	2	12794.24	12622.52	-171.71	-172.01	100.2
20	1	14073.98	13952.86	-121.12	-119.39	98.6
20	2	14073.98	13884.00	-189.98	-190.94	100.5
25	1	16610.25	16472.22	-138.03	-135.45	98.1
25	2	16610.25	16382.40	-227.85	-229.33	100.6

To check stress constraint sensitivity for beam elements in the built-up structure, the equivalent nodal force for the adjoint load on beam elements must be computed, so that the adjoint displacement field λ can be calculated. Stress results for several beam elements are shown in Table 17.

The forward finite difference method has been used so far to check the accuracy of the design sensitivity prediction. The prediction of the gradient for the first design parameter is very small for the beam element, which implies that the function has a nearly zero slope. For better finite difference approximation, the central finite difference method is used, to compare the accuracy of the prediction. The central finite difference method is defined as

$$\Delta\psi = \frac{\psi(u + \Delta u) - \psi(u - \Delta u)}{2}$$

The perturbations are $\delta b = 0.01b$ and $\delta h = 0.01h$ for the first control parameter Δu, and $\delta t = 0.01t$ is the perturbation for second control parameter.

Table 17. Design Sensitivity Check for Beam Stress

El. No.	Fiber	Control Parameter	$\psi(u)$	$\psi(u+\Delta u)$	$\psi(u-\Delta u)$	$\Delta\psi$	ψ'	Ratio %
1	4	1	40182.0	40191.2	40161.1	15.1	17.8	118.7
1	4	2	40182.0	39277.9	41109.3	-915.7	-915.7	100.0
2	4	1	7430.8	7457.8	7401.8	28.0	27.9	99.9
2	4	2	7430.8	7245.7	7621.7	-188.0	-188.0	100.0
3	4	1	-9945.1	-9947.6	-9939.7	-3.9	-4.6	117.4
3	4	2	-9945.1	-9721.0	-10174.6	226.8	226.8	100.0
4	4	1	-17328.3	-17337.9	-17313.5	-12.2	-13.1	107.4
4	4	2	-17328.3	-16934.5	-17732.3	398.9	399.0	100.0
5	4	1	-20339.1	-20363.2	-20309.4	-26.9	-28.1	104.5
5	4	2	-20339.1	-19867.9	-20823.6	477.9	477.8	100.0
6	4	1	82011.8	82049.4	81950.6	49.4	56.0	113.4
6	4	2	82011.8	80152.9	83919.1	-1883.1	-1883.1	100.0
7	4	1	17977.1	18035.4	17914.1	60.6	62.9	103.8
7	4	2	17977.1	17534.3	18433.3	-449.5	-449.6	100.0
8	4	1	-18646.6	-18639.5	-18648.1	4.3	1.7	40.0
8	4	2	-18646.6	-18234.8	-19067.7	416.5	416.5	100.0

Table 17--continued

9	4	1	-36874.1	-36910.3	-36648.1	-41.5	-43.5	104.7
9	4	2	-36874.1	-36024.9	-37746.3	860.7	860.8	100.0
10	4	1	-44467.5	-44534.1	-44388.5	-72.8	-77.2	106.1
10	4	2	-44467.5	-43426.8	-45537.6	1055.4	1055.3	100.0

CHAPTER V

CONCLUSIONS

Results of this study show that it is possible to combine the design sensitivity algorithms of Ref. 1 with the database management system of EAL. For stress constraints as performance criteria, it is necessary to compute equivalent nodal forces for the adjoint load. For plate elements, the EAL finite element analysis is based on a hybrid formulation, but a displacement finite element formulation is used for evaluating the equivalent adjoint nodal forces. Nevertheless, results of the design sensitivity analysis are very accurate, which indicates that it is not necessary to compute equivalent nodal forces for the adjoint load using exactly the same shape functions that are employed in finite element analysis.

A database management system with a finite element capability and the adjoint variable method of design sensitivity analysis, permit implementation of a design sensitivity analysis method that does not require differentiation of element stiffness and mass matrices. It is shown that a database management system can be used to implement design sensitivity analysis, so only one program with one database is necessary.

Work is progressing to extend the methods presented in this report to include shape (geometric) design parameters. A domain method [17] for shape design sensitivity analysis and a design component method [6] for sensitivity analysis of built-up structures are used for software implementation. Numerical implementation and results of shape design sensitivity analysis will be reported in Part II: Shape Design Parameters.

REFERENCES

1. Haug, E.J., Choi, K.K., and Komkov, V., Structural Design Sensitivity Analysis, Academic Press, New York, N.Y., 1985.

2. Whetstone, W.D., EISI-EAL Engineering Analysis Language, Reference Manual, EIS Inc., July 1983.

3. Choi, K.K., Santos, L.T., Frederick, M.C., "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes," Journal of Mechanics, Transmissions, and Automation in Design, 85-DET-77.

4. Frederick, M.C., and Choi, K.K., "Design Sensitivity Analysis with APPLICON IFAD Using the Adjoint Variable Method," Technical Report 84-17, Center for Computer Aided Design, University of Iowa, 1984.

5. Santos, J.L.T., and Choi, K.K., "Implementation of Design Sensitivity Analysis with ANSYS," Technical Report 85- , Center for Computer Aided Design, University of Iowa, 1985.

6. Choi, K.K., and Seong, H.G., "Design Component Method for Sensitivity Analysis of Built-up Structures," Journal of Structural Mechanics, to appear, 1986.

7. Pian, T.H.H., "Derivation of Element Stiffness Matrices by Assumed Stress Distribution," AIAA, July 1964.

8. Cook, R.D., Concept and Application of the Finite Element Analysis, John Wiley and Sons Inc., New York, 1981.

9. Zienswicz, O.O., The Finite Element Method, McGraw-Hill, London 1977.

10. Washizu, K., Variational Methods in Elasticity and Plasticity, Pergamon Press, 1982

11. Pian, T.H.H., "Finite Element Methods by Variational Principles with Relaxed Continuity Requirement,", in Variational Methods in Engineering, edited by Brebbia, C.A., Tottenham, H., Southampton University Press, 1973.

12. Sreekanta Murthy, T., Arora, J.S., "A Survey of Database Management in Engineering," Advanced Engineering Software, Vol 7 (3), 1985.

13. Prasad, B., "An Integrated System for Optimal Structural Synthesis and Remodelling," Computers and Structures, Vol 20 (5), 1985.

14. Bathe, K.J., Wilson, E.L., Peterson, F.E., SAP IV: A Structural Analysis Program for Static and Dynamic Response of a System, College of Engineering, University of California, Berkley, 1974.

15. Whetstone, W.D., "Computer Analysis of Large Linear Frames,", ASCE, Journal of the Structural Division, Vol 95, November 1969.

16. Giles, G.L., Haftka, R.T., "SPAR Data Handling Routines," NASA Technical Memorandum 78701, September 1978.

17. Choi, K.K. and Seong, H.G., "Domain Method For Shape Design Sensitivity Analysis of Built-Up Structures", Computer Method in Applied Mechanics and Engineering, to appear, 1986.

APPENDIX A1 - DESIGN SENSITIVITY VECTORS

This appendix lists the design sensitivity vectors $\frac{\partial \psi}{\partial u}$ for the built-up structure for the compliance constraint, for the displacement constraint at node 36, for the stress constraint in beam element 1, and for the stress constraint in plate element 25.

Table 18. Sensitivity Vectors for the Compliance Constraint

Beam element	$\frac{\partial \psi}{\partial b_i}$	$\frac{\partial \psi}{\partial h_i}$	Plate element	$\frac{\partial \psi}{\partial t_i}$
1	-11.62	-30.97	1	-9.83
2	-0.62	-1.65	2	-47.94
3	-0.73	-1.94	3	-127.21
4	-1.97	-5.26	4	-219.05
5	-2.72	-7.26	5	-282.67
6	-47.55	-126.80	6	-47.94
7	-3.13	-8.34	7	-50.04
8	-2.51	-6.68	8	-50.25
9	-8.94	-23.85	9	-38.58
10	-12.97	-34.58	10	-27.41
11	-11.62	-30.97	11	-127.20
12	-0.62	-1.65	12	-50.25
13	-0.73	-1.94	13	-59.09
14	-1.97	-5.26	14	-62.54

Table 18--continued

15	-2.72	-7.26	15	-59.77
16	-47.55	-126.80	16	-219.03
17	-3.13	-8.34	17	-38.58
18	-2.51	-6.68	18	-62.54
19	-8.94	-23.85	19	-114.17
20	-12.97	-34.58	20	-145.47
			21	-282.65
			22	-27.41
			23	-59.77
			24	-145.47
			25	-202.79
$\sum$	-185.52	-494.66	$\sum$	-2557.65
δu	0.004	0.0005	δu	0.001
ψ'	-0.7421	-0.2473	ψ'	-2.557

In Table 18, the sum of the sensitivity components $\frac{\partial\psi}{\partial u_i}$ $i=1,\cdots,n$ is given in the third row from the bottom. When multiplying the sum of the sensitivity components with the perturbation δu of the design variable u, one gets the first variation, which is given in the last row of Table 18. Note that this result coincides with results given in Table 14. Results in Tables 19, 20, and 21 coincide with results given in Tables 15, 16, and 17, respectively.

Table 19. Sensitivity Vectors for the Displacement Constraint

Beam element	$\frac{\partial\psi}{\partial b}$	$\frac{\partial\psi}{\partial h}$	Plate element	$\frac{\partial\psi}{\partial t}$
1	0.0604	0.1610	1	0.040
2	0.0074	0.0197	2	0.251
2	0.0010	0.0028	3	0.755
4	0.0144	0.0386	4	1.426
5	0.0286	0.0763	5	1.934
6	0.3205	0.8547	6	0.251
7	0.0405	0.1079	7	0.355
8	-0.0068	-0.0181	8	0.429
9	0.0587	0.1564	9	0.394
10	0.2098	0.5595	10	0.311
11	0.0604	0.1610	11	0.755
12	0.0074	0.0197	12	0.429
13	0.0010	0.0028	13	0.448
14	0.0144	0.0386	14	0.457
15	0.0286	0.0763	15	0.389
16	0.3205	0.8547	16	1.426
17	0.0405	0.1079	17	0.394
18	-0.0068	-0.0181	18	0.457
19	0.0589	0.1564	19	0.958
20	0.2098	0.5594	20	1.490

Table 19--continued

			21	1.934
			22	0.311
			23	0.389
			24	1.490
			25	3.744
$\sum$	1.469	3.9176	$\sum$	21.217
δ_u	0.004	0.0005	δ_u	0.001
ψ'	0.005876	0.0001958	ψ'	0.021217

Table 20. Sensitivity Vectors for the Beam Stress Constraint

Beam element	$\frac{\partial\psi}{\partial b}$	$\frac{\partial\psi}{\partial h}$	Plate element	$\frac{\partial\psi}{\partial t}$
1	77168	-62100	1	-15002
2	9230	24614	2	-52185
2	-652	-1739	3	-80766
4	603	1610	4	-87753
5	1108	2955	5	-83260
6	-18728	-49942	6	-88775
7	-97	-258	7	-13790
8	-1728	-4608	8	4890
9	-2162	-5765	9	3402
10	-1163	-3102	10	-1870
11	-13565	-36174	11	-111076
12	618	1648	12	23378
13	-2072	-5524	13	-45076
14	587	1565	14	-21198
15	1405	3746	15	-4547
16	-15697	-41858	16	-104176
17	-709	-1889	17	1248
18	-1428	-3807	18	-29999
19	-2440	-6506	19	-30033
20	-1815	-4841	20	-20950

Table 20--continued

			21	-97175
			22	137
			23	-7950
			24	-23568
			25	-29563
$\sum$	28463	-191975	$\sum$	-915657
δ_u	0.004	0.0005	δ_u	0.001
ψ'	113.85	-95.99	ψ'	-915.657

Table 21. Sensitivity Vectors for the Plate Stress Constraint

Beam element	$\frac{\partial\psi}{\partial b}$	$\frac{\partial\psi}{\partial h}$	Plate element	$\frac{\partial\psi}{\partial t}$
1	-641	-1711	1	-361
2	-117	-312	2	-2660
2	19	51	3	-8478
4	-201	-538	4	-16560
5	-444	-1185	5	-22898
6	-3784	-10090	6	-2660
7	-865	-2307	7	-4650
8	1316	3509	8	-6543
9	2632	7018	9	-6805
10	-10611	-28298	10	-5715
11	-641	-1711	11	-8477
12	-117	-312	12	-6543
13	19	51	13	-7439
14	-201	-538	14	-6114
15	-444	-1185	15	4866
16	-3784	-10090	16	-16560
17	-865	-2307	17	-6805
18	1316	3509	18	-6115
19	2632	7018	19	-803
20	-10612	-28299	20	2472

Table 21--continued

			21	-22897
			22	-5714
			23	4864
			24	2472
			25	-79195
$\sum$	-25392	-67726	$\sum$	-229328
δ_u	0.004	0.0005	δ_u	0.001
ψ'	-101.57	-33.86	ψ'	-229.328

APPENDIX A2 - SOURCE PROGRAM

```
*XQT U1
$
$------------------------------------------------------------------------
$
$   Program for computing the first variation of stress, displacement
$   and compliance constraints
$
$   Written by  B. DOPKER
$
$------------------------------------------------------------------------
$
*(29 DRIV GLOB)                          ENDDRIV
$
*DCALL,(29 PARA SET  0 0)
*DCALL,(29 INIT MODL 0 0)
*DCALL,(29 DSA  COM  0 0)
$
*RETURN
$
*ENDDRIV
$
$------------------------------------------------------------------------
$
*(29 PARA SET  0 0)               ENDPARA
$
!NLST=1    $ NUMBER OF LOAD CASES
$
!LCAS=1     $  NORMAL LOAD CASE
$
$
$
!NBTD=1    $ NUMBER OF INDEPENDENT BEAM/TRUSS DESIGN VARIABLES
!NMDV=0    $ NUMBER OF INDEPENDENT MEMBRANE DESIGN VARIABLES
!NPDV=1    $ NUMBER OF INDEPENDENT PLATE DESIGN VARIABLES
$
!NDV=NBTD+NMDV+NPDV
$
!DE21=20   $ NUMBER OF BEAM/TRUSS ELEMENTS
!DE41=0    $ NUMBER OF MEMBRANE ELEMENTS
!DE42=25   $ NUMBER OF PLATE ELEMENTS
$
!DETO=DE21+DE41+DE42
$
$   DEFINE INITIAL VALUE FOR ALL DESIGN VARIABLES
$
*XQT AUS
TABLE(NI=2,NJ="NBTD"): DESV VALU 0 0
J=1: 1.0      1.0
$
$
$ DEFINE RELATION BETWEEN ELEMENTS AND DESIGN VARIABLES (SIZING)
$
$  BEAM/TRUSS
*JZ(DE21,10)
TABLE(NJ="DE21",TYPE=0): ED21 REL 0 0
```

```
    J=1 :  1
    J=2 :  1
    J=3 :  1
    J=4 :  1
    J=5 :  1
    J=6 :  1
    J=7 :  1
    J=8 :  1
    J=9 :  1
    J=10:  1
    J=11:  1
    J=12:  1
    J=13:  1
    J=14:  1
    J=15:  1
    J=16:  1
    J=17:  1
    J=18:  1
    J=19:  1
    J=20:  1
*LABEL 10
$  MEMBRANE
*JZ(DE41,30)
 TABLE(NJ="DE41",TYPE=0): 1 ED41 REL 0  0
    J=1 :  1
    J=2 :  1
    J=3 :  1
    J=4 :  1
    J=5 :  1
    J=6 :  1
    J=7 :  1
    J=8 :  1
    J=9 :  1
    J=10:  1
    J=11:  1
    J=12:  1
    J=13:  1
    J=14:  1
    J=15:  1
    J=16:  1
    J=17:  1
    J=18:  1
    J=19:  1
    J=20:  1
    J=21:  1
    J=22:  1
    J=23:  1
    J=24:  1
    J=25:  1
*LABEL 30
$  PLATE
*JZ(DE42,40)
 TABLE(NJ="DE42",TYPE=0):  ED42 REL 0 0
    J=1 :  2
    J=2 :  2
    J=3 :  2
    J=4 :  2
    J=5 :  2
    J=6 :  2
    J=7 :  2
```

```
     J=8 :  2
     J=9 :  2
     J=10:  2
     J=11:  2
     J=12:  2
     J=13:  2
     J=14:  2
     J=15:  2
     J=16:  2
     J=17:  2
     J=18:  2
     J=19:  2
     J=20:  2
     J=21:  2
     J=22:  2
     J=23:  2
     J=24:  2
     J=25:  2
*LABEL 40
$
$CONTRAINT DEFINITION
$
!CCOM=0            $   COMPLIANCE CONTRAINTS
!CDIS=0            $   NUMBER OF DISPLACEMENT CONSTRAINTS
!CS21=0            $   NUMBER OF STRESS CONSTRAINTS IN BEAM/TRUSS ELEMENTS
!CS41=0            $   NUMBER OF STRESS CONSTRAINTS IN MEMBRANE ELEMENTS
!CS42=1            $   NUMBER OF STRESS CONSTRAINTS IN PLATE ELEMENTS
!CSTR=CS21+CS41+CS42
$
!CTOT=CCOM+CDIS+CSTR
$
$
$TABLES FOR CONSTRAINT NODES/ELEMENTS
*JZ(CDIS,100)
 TABLE(NI=2,NJ="CDIS",TYPE=0):  DICO LIST 1 1
      J=1 :36     3           $  FIRST NUMBER IS THE NODE, SECOND IS DIRECTION
$     J=2 : 9     3
$     J=3 :10     3
$     J=4 :11     3
$     J=5 :12     3
*LABEL 100
*JZ(CS21,110)
   TABLE(NJ="CS21",TYPE=0):  ST21 LIST
      J=1 :  1
*LABEL 110
$
*JZ(CS41,130)
   TABLE(NJ="CS41",TYPE=0):  ST41 LIST
      J=1 :  1
      J=2 :  25
$     J=3 :  10
$     J=4 :  15
$     J=5 :  21
$     J=6 :  31
$     J=7 :  61
$     J=8 :  64
*LABEL 130
$
*JZ(CS42,140)
   TABLE(NJ="CS42",TYPE=0):  ST42 LIST
```

```
      J=1 : 25
*LABEL 140
$
*RETURN
*                                       ENDPARA
$
$-----------------------------------------------------------------------
$
*(29 INIT MODL 0 0)                 ENDINIT
$
*XQT TAB
START 36
TITLE 'TESTING OF MEMBRANE OF 80 ELEMENTES
MATERIAL CONSTANTS
1  30500000. 0.3  300.0
JOINT LOCATIONS
 1 -20.0 -20.0    0.0         0.0 -20.0   0.0   6 1 6
 6 -20.0   0.0    0.0         0.0   0.0   0.0
MREF
FORMAT=2
1,1,30.0,30.0,0.0
SA
1 0.10
BA
RECT 1,0.40   0.05
CONSTRAINT DEFINITION 1
ZERO 1 2 6
1,36
SYMMETRY PLANE=1
SYMMETRY PLANE=2
 ZERO 1 2 3 4 5 6
  1,6,1
  7,31,6
*XQT ELD
 E21
   13 14 1 5
   25 26 1 5
   3 9 1 5
   5 11 1 5
 E42
  1 2 8 7   1   5 5
*XQT E
*XQT EKS
*XQT TAN
*XQT K
*XQT DRSI
*XQT AUS
ALPHA: CASE TITLE
 1'UNIFORM ELEMENT PRESSURE
TABLE: NODAL PRESSURE
J=1,36:   -1.0
*XQT EQNF
RESET SET="LCAS"
*XQT SSOL
RESET SET="LCAS"
*XQT VPRT
PRINT 1 STAT DISP "LCAS" 1
$
!TEMP=DETO-DE21
*JZ(TEMP,200)
```

```
*JZ(DE41,156)
*XQT ES
PMODE= 2
OUTLIB=1
OUTPUT TYPE=ESB
U= STAT DISP "LCAS" 1
E41 1
OUTPUT TYPE=ESD
U= STAT DISP "LCAS" 1
E41 1
*LABEL 156
*JZ(DE42,157)
*XQT ES
PMODE=2
OUTLIB=1
OUTPUT TYPE=ESB
U= STAT DISP "LCAS" 1
E42 1
OUTPUT TYPE=ESD
U= STAT DISP "LCAS" 1
E42 1
*LABEL 157
*LABEL 200
!TEMP=FREE()
$
*RETURN
$
*ENDINIT
$
$-----------------------------------------------------------------------------------
$
*(29 DSA  COM  0 0)                    ENDDSAC
$
$  PREPARE DATA FOR LATER USAGE IN SENSITIVITY CALCULATION
$
*JZ(DE21,40)
*DCALL,(29 PREP E21 0 0)
*LABEL 40
*JZ(DE41,50)
*DCALL,(29 PREP E41 0 0)
*LABEL 50
*JZ(DE42,60)
*DCALL,(29 PREP E42 0 0)
*LABEL 60
$
$   LOOP OVER ALL CONSTRAINTS
$
!CLC=0              $  REGISTER FOR CONSTRAINT COUNTING
*LABEL 500
!CLC=CLC+1
!ADLC=1000+CLC
!PSI=0.0            $  REGISTER FOR VARIATION SUMMATION
!VMST=0.0
!POIN=1
!CLC
$
*DCALL,(29 ADJO COMP 0 0)
*DCALL,(29 ADJO SOLU 0 0)
*DCALL,(29 SENS CALC 0 0)
$
```

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```
!CLCC=CLC-CTOT
*JNZ(CLCC,500)
$
*RETURN
$
*ENDDSAC
$
$--------------------------------------------------------------------
$
*(29 ADJO COMP 0 0)                                   ENDADCO
$
!TEMP=CLC-1
!JENT=0
*JGZ(TEMP,870)
*JZ(CCOM,870)
$
$  COMPUTE COMPLIANCE
$
*XQT DCU
TOCC 1 STAT DISP "LCAS" 1: N3="ADLC"
$
*JUMP 1000
$
*LABEL 870
$
!TEMP=CLC-CCOM-CDIS
*JGZ(TEMP,880)
$
$  COMPUTE ADJOINT FOR DISPLACEMENTS
$
!TEMP=CLC-CCOM
$GET TEMP'S ENTRY OF DICO LIST
!JENT=DS 1,"TEMP",1   (1  DICO LIST 1 1)
!IDIR=DS 2,"TEMP",1   (1  DICO LIST 1 1)
$
*XQT AUS
SYSVEC: APPL FORC "ADLC" 1
I="IDIR"
J="JENT"
1.0
$
!IDIR=FREE()
$
*JUMP 1000
$
*LABEL 880
$
$  COMPUTE STRESS ADJOINTS
$  ------------------------
$
$  FOR E21 STRSS CONSTRAINTS
!TEMP=CLC-CCOM-CDIS-CS21
!TEMP
*JGZ(TEMP,890)
$
!TEMP=CLC-CCOM-CDIS
!TEMP
!DE21
$GET TEMP'S ENTRY OF ST21 LIST
!JENT=DS "TEMP",1 ,1   (1  ST21 LIST 1 1)
```

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```
$
$ COMPUTE STRESS CONSTR FOR ELEMENT: JENT E21
$
*DCALL, (29 ADJO E21 0 0)
$
*JUMP 1000
$
*LABEL 890
$
$                          FOR E41 STRSS CONSTRAINTS
$
!TEMP=CLC-CCOM-CDIS-CS21-CS41
!TEMP
*JGZ(TEMP,910)
$
!TEMP=CLC-CCOM-CDIS-CS21
!TEMP
$GET TEMP'S ENTRY OF ST41 LIST
!DE41
!JENT=DS "TEMP",1 ,1  (1 ST41 LIST 1 1)
$
$ COMPUTE STRESS CONSTR FOR ELEMENT: JENT E41
$ .
*DCALL, (29 ADJO E41 0 0)
$
*JUMP 1000
$
*LABEL 910
!TEMP=FREE()
$
$
$  FOR E42 STRSS CONSTRAINTS
!TEMP=CLC-CCOM-CDIS-CS21-CS41-CS42
!TEMP
*JGZ(TEMP,1000)
$
!TEMP=CLC-CCOM-CDIS-CS21-CS41
!TEMP
!DE42
$GET TEMP'S ENTRY OF ST42 LIST
!JENT=DS "TEMP",1 ,1  (1  ST42 LIST 1 1)
$
$ COMPUTE STRESS CONSTR FOR ELEMENT: JENT E42
$
*DCALL, (29 ADJO E42 0 0)
$
*LABEL 1000
!TEMP=FREE()
!JENT=FREE()
$
*RETURN
$
*ENDADCO
$
$-----------------------------------------------------------------
$
*(29 ADJO SOLU 0 0)                    ENDADSO
$
!TEMP=CLC-1
*JGZ(TEMP,885)
```

```
*JZ(CCOM,885)
$
$     THIS JUMP IS ONLY FOR COMPLIANCE CONSTRAINTS
$
*JUMP 886
$
*LABEL 885
*XQT SSOL
RESET SET="ADLC"
$
*XQT VPRT
$
PRINT 1 STAT DISP "ADLC"
$
*LABEL 886
$
!TEMP=DETO-DE21
*JZ(TEMP,158)
*JZ(DE41,156)
*XQT ES
PMODE= 2
OUTLIB=1
OUTPUT TYPE=ESB
U= STAT DISP "ADLC" 1
E41 1
*LABEL 156
*JZ(DE42,157)
*XQT ES
PMODE=2
OUTLIB=1
OUTPUT TYPE=ESB
U= STAT DISP "ADLC" 1
E42 1
*LABEL 157
*LABEL 158
!TEMP=FREE()
$
*RETURN
*                                    ENDADSO
$
$-----------------------------------------------------------------------
$
*(29 SENS CALC 0 0)                  ENDSECA
$
!ICDV=1
!TEMP=0
*LABEL 300
!PSI=0.0
$
*JZ(DE21,140)
*DCALL,(29 SECA E21 0 0)
*LABEL 140
$
*JZ(DE41,150)
*DCALL,(29 SECA E41 0 0)
*LABEL 150
$
*JZ(DE42,160)
*DCALL,(29 SECA E42 0 0)
*LABEL 160
```

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```
$
!ICDV
!NDV
!ICDV=ICDV+1
!TEMP=NDV-ICDV+1
!ICDV
!TEMP
$
*JGZ(TEMP,300)
$
!TEMP=FREE()
!ICDV=FREE()
$
*RETURN
$
*ENDSECA
$
$------------------------------------------------------------------
*(29 PREP E21 0 0)                              ENDPE21
$
$  THIS SUBROUTINE IS FOR ELEMENT TYP E21
$
$   THE PRINCIPAL LIBRARY IS 13
$   TEMPORY LIBRARY IS 17. WILL BE DELETED AT THE END OF THIS ROUTINE
$
*XQT E11
$
EXTRACT:  SOURCE=E21:   CONTENT SPEC:  GEOM  1 $ LENGTH OF ELEMENTS
CREATE 13 LENG E21 0 0
$
EXTRACT:  SOURCE=E21:   CONTENT SPEC:  GEOM  5,13 $ DIRECTION COSINES
CREATE 13 DIRC E21 0 0
$
EXTRACT:  SOURCE=E21:   CONTENT SPEC:  GEOM  14,31 $ ELEMENT-NODAL RELATION
CREATE 13 ELNO E21 0 0
$
EXTRACT:  SOURCE=E21:   CONTENT SPEC:  MATE 1,2 $ MATERIAL PROPERTIES
CREATE 13 MATP E21 0 0
$
EXTRACT.  SOURCE=E21:   CONTENT SPEC:  INTE 1,18  $ CONNECTIVITY
CREATE 13 CONN E21 0 0
$
EXTRACT:  SOURCE=E21:   CONTENT SPEC:  SECTION 16,23  $ STRESS POINT LOCATION
CREATE 13 SPLO E21 0 0
$
*XQT DCU
TOCC 13 CONN E21 0 0 :  TYPE=0
$  IF NOT ALL NODAL INFORMATION IS IN GLOBAL COORDINATES USE LTOG
$
*XQT AUS
!ICOU=0
*LABEL 100
!ICOU=ICOU+1
!P1=DS,13,"ICOU",1 (13 CONN E21 0 0)
!P2=DS,14,"ICOU",1 (13 CONN E21 0 0)
!UX1=DS,1,"P1",1  (1 STAT DISP "LCAS" 1)
!UY1=DS,2,"P1",1  (1 STAT DISP "LCAS" 1)
!UZ1=DS,3,"P1",1  (1 STAT DISP "LCAS" 1)
!RX1=DS,4,"P1",1  (1 STAT DISP "LCAS" 1)
!RY1=DS,5,"P1",1  (1 STAT DISP "LCAS" 1)
```

```
!RZ1=DS,6,"P1",1   (1 STAT DISP "LCAS" 1)
!UX2=DS,1,"P2",1   (1 STAT DISP "LCAS" 1)
!UY2=DS,2,"P2",1   (1 STAT DISP "LCAS" 1)
!UZ2=DS,3,"P2",1   (1 STAT DISP "LCAS" 1)
!RX2=DS,4,"P2",1   (1 STAT DISP "LCAS" 1)
!RY2=DS,5,"P2",1   (1 STAT DISP "LCAS" 1)
!RZ2=DS,6,"P2",1   (1 STAT DISP "LCAS" 1)
!P2=FREE()
$
TABLE(NI=3,NJ=4): 17 EL21 DISP "ICOU" O
J=1: "UX1"  "UY1"  "UZ1"
J=2: "RX1"  "RY1"  "RZ1"
J=3: "UX2"  "UY2"  "UZ2"
J=4: "RX2"  "RY2"  "RZ2"
DEFINE B11=13 DIRC E21 O O
!P1=ICOU-1*9
!P1
TABLE(NI=3,NJ=3): 13 EL21 ROTA "ICOU" O
TRANSFER(SOURCE=B11,ILIM=9,SBASE="P1")
!P1=FREE()
DEFINE B11=13 EL21 ROTA "ICOU" O
DEFINE B12=17 EL21 DISP "ICOU" O
13 DI21 ELRF "ICOU" O=RPROD(B11,B12)
!UX1=DS, 1,1,1  (13 DI21 ELRF "ICOU" O)
!UY1=DS, 2,1,1  (13 DI21 ELRF "ICOU" O)
!UZ1=DS, 3,1,1  (13 DI21 ELRF "ICOU" O)
!RX1=DS, 1,2,1  (13 DI21 ELRF "ICOU" O)
!RY1=DS, 2,2,1  (13 DI21 ELRF "ICOU" O)
!RZ1=DS, 3,2,1  (13 DI21 ELRF "ICOU" O)
!UX2=DS, 1,3,1  (13 DI21 ELRF "ICOU" O)
!UY2=DS, 2,3,1  (13 DI21 ELRF "ICOU" O)
!UZ2=DS, 3,3,1  (13 DI21 ELRF "ICOU" O)
!RX2=DS, 1,4,1  (13 DI21 ELRF "ICOU" O)
!RY2=DS, 2,4,1  (13 DI21 ELRF "ICOU" O)
!RZ2=DS, 3,4,1  (13 DI21 ELRF "ICOU" O)
$
!L=DS,"ICOU",1,1  (13 LENG E21 O O)
!ICOU
!XGP1=1.0-0.57735*L*0.5
!XGP2=1.0+0.57735*L*0.5
!A1=-1.0/L*UZ1
!A2=1.0/L*UZ2
!UZG1=A1+A2
!UZG2=A1+A2
!A1=1.0/L
!A2=A1*A1
!A3=A2*A1
!E1=-6.0*A2
!E2=12.0*XGP1*A3
!E3=-4.0*A1
!E4=6.0*XGP1*A2
!E5=E3*0.5
!WXG1=E1+E2*UY1
!WXG1=-E3-E4*RX1+WXG1
!WXG1=-E1-E2*UY2+WXG1
!WXG1=-E5-E4*RX2+WXG1
!WYG1=E1+E2*UX1
!WYG1=E3+E4*RY1+WYG1
!WYG1=-E1-E2*UX2+WYG1
!WYG1=E5+E4*RY2+WYG1
```

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```
!E2=12.0*XGP2*A3
!E4=6.0*XGP2*A2
!WXG2=E1+E2*UY1
!WXG2=-E3-E4*RX1+WXG2
!WXG2=-E1-E2*UY2+WXG2
!WXG2=-E5-E4*RX2+WXG2
!WYG2=E1+E2*UX1
!WYG2=E3+E4*RY1+WYG2
!WYG2=-E1-E2*UX2+WYG2
!WYG2=E5+E4*RY2+WYG2
!XGP1=FREE()
!XGP2=FREE()
!A1=FREE()
!A2=FREE()
!A3=FREE()
!L=FREE()
!E1=FREE()
!E2=FREE()
!E3=FREE()
!E4=FREE()
!E5=FREE()
TABLE(NI=2,NJ=3): 13 ELST E21 "ICOU" 0
J=1:  "UZG1"   "UZG2"
J=2:  "WXG1"   "WXG2"
J=3:  "WYG1"   "WYG2"
$
!TEMP=ICOU-DE21
*JNZ(TEMP,100)
!TEMP=FREE()
!ICOU=FREE()
!UZG1=FREE()
!UZG2=FREE()
!WXG1=FREE()
!WXG2=FREE()
!WYG1=FREE()
!WYG2=FREE()
!UX1=FREE()
!UY1=FREE()
!UZ1=FREE()
!RX1=FREE()
!RY1=FREE()
!RZ1=FREE()
!UX2=FREE()
!UY2=FREE()
!UZ2=FREE()
!RX2=FREE()
!RY2=FREE()
!RZ2=FREE()
*XQT DCU
ERASE 17
*RETURN
*                              ENDPE21
$
$-----------------------------------------------------------------------
*(29 SECA E21 0 0)                        ENDSC21
$
!TEMP=0
$
*XQT AUS
!ICOU=0
```

```
*LABEL 100
!ICOU=ICOU+1
!TEMP=DS 1,"ICOU",1 (1 ED21 REL 0 0)
!ICDV
!TEMP
!TEMP=TEMP-ICDV
*JNZ(TEMP, 200)
!P1=DS,13,"ICOU",1 (13 CONN E21 0 0)
!P2=DS,14,"ICOU",1 (13 CONN E21 0 0)
!UX1=DS,1,"P1",1  (1 STAT DISP "ADLC" 1)
!UY1=DS,2,"P1",1  (1 STAT DISP "ADLC" 1)
!UZ1=DS,3,"P1",1  (1 STAT DISP "ADLC" 1)
!RX1=DS,4,"P1",1  (1 STAT DISP "ADLC" 1)
!RY1=DS,5,"P1",1  (1 STAT DISP "ADLC" 1)
!RZ1=DS,6,"P1",1  (1 STAT DISP "ADLC" 1)
!UX2=DS,1,"P2",1  (1 STAT DISP "ADLC" 1)
!UY2=DS,2,"P2",1  (1 STAT DISP "ADLC" 1)
!UZ2=DS,3,"P2",1  (1 STAT DISP "ADLC" 1)
!RX2=DS,4,"P2",1  (1 STAT DISP "ADLC" 1)
!RY2=DS,5,"P2",1  (1 STAT DISP "ADLC" 1)
!RZ2=DS,6,"P2",1  (1 STAT DISP "ADLC" 1)
!P1=FREE()
!P2=FREE()
$
TABLE(NI=3,NJ=4): 17 EL21 DISP "ICOU" "ADLC"
J=1. "UX1"  "UY1"  "UZ1"
J=2: "RX1"  "RY1"  "RZ1"
J=3: "UX2"  "UY2"  "UZ2"
J=4: "RX2"  "RY2"  "RZ2"
DEFINE B11=13 EL21 ROTA "ICOU" 0
DEFINE B12=17 EL21 DISP "ICOU" "ADLC"
17 DI21 ELRF "ICOU" "ADLC"=RPROD(B11,B12)
!UX1=DS, 1,1,1  (17 DI21 ELRF "ICOU" "ADLC")
!UY1=DS, 2,1,1  (17 DI21 ELRF "ICOU" "ADLC")
!UZ1=DS, 3,1,1  (17 DI21 ELRF "ICOU" "ADLC")
!RX1=DS, 1,2,1  (17 DI21 ELRF "ICOU" "ADLC")
!RY1=DS, 2,2,1  (17 DI21 ELRF "ICOU" "ADLC")
!RZ1=DS, 3,2,1  (17 DI21 ELRF "ICOU" "ADLC")
!UX2=DS, 1,3,1  (17 DI21 ELRF "ICOU" "ADLC")
!UY2=DS, 2,3,1  (17 DI21 ELRF "ICOU" "ADLC")
!UZ2=DS, 3,3,1  (17 DI21 ELRF "ICOU" "ADLC")
!RX2=DS, 1,4,1  (17 DI21 ELRF "ICOU" "ADLC")
!RY2=DS, 2,4,1  (17 DI21 ELRF "ICOU" "ADLC")
!RZ2=DS, 3,4,1  (17 DI21 ELRF "ICOU" "ADLC")
!ICOU
$
!L=DS,"ICOU",1,1  (13 LENG E21 0 0)
!XGP1=1.0-0.57735*L*0.5
!XGP2=1.0+0.57735*L*0.5
!A1=-1.0/L*UZ1
!A2=1.0/L*UZ2
!LUZ1=A1+A2
!LUZ2=A1+A2
!A1=1.0/L
!A2=A1*A1
!A3=A2*A1
!E1=-6.0*A2
!E2=12.0*XGP1*A3
!E3=-4.0*A1
!E4=6.0*XGP1*A2
```

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```
!E5=E3*0.5
!WXG1=E1+E2*UY1
!WXG1=-E3-E4*RX1+WXG1
!WXG1=-E1-E2*UY2+WXG1
!LWX1=-E5-E4*RX2+WXG1
!WYG1=E1+E2*UX1
!WYG1=E3+E4*RY1+WYG1
!WYG1=-E1-E2*UX2+WYG1
!LWY1=E5+E4*RY2+WYG1
!E2=12.0*XGP2*A3
!E4=6.0*XGP2*A2
!WXG2=E1+E2*UY1
!WXG2=-E3-E4*RX1+WXG2
!WXG2=-E1-E2*UY2+WXG2
!LWX2=-E5-E4*RX2+WXG2
!WYG2=E1+E2*UX1
!WYG2=E3+E4*RY1+WYG2
!WYG2=-E1-E2*UX2+WYG2
!LWY2=E5+E4*RY2+WYG2
!XGP1=FREE()
!XGP2=FREE()
!A3=FREE()
!E3=FREE()
!E4=FREE()
!E5=FREE()
!UZG1=DS,1,1,1   (13 ELST E21 "ICOU" O)
!UZG2=DS,2,1,1   (13 ELST E21 "ICOU" O)
!WXG1=DS,1,2,1   (13 ELST E21 "ICOU" O)
!WXG2=DS,2,2,1   (13 ELST E21 "ICOU" O)
!WYG1=DS,1,3,1   (13 ELST E21 "ICOU" O)
!WYG2=DS,2,3,1   (13 ELST E21 "ICOU" O)
$
$  COMPUTE DESIGNS SENSITIVITY
$
!E=DS,1,"ICOU",1   (13 MATP E21 O O)
!B=DS,2,"ICOU",1   (13 SPLO E21 O O)
!H=DS,1,"ICOU",1   (13 SPLO E21 O O)
!B=B*2.0
!H=H*2.0
!WEIB=DS 1,"ICDV",1  (1 DESV VALU O O)
!WEIH=DS 2,"ICDV",1  (1 DESV VALU O O)
!DELB=1.0*WEIB*B
!DELH=1.0*WEIH*H
!ICDV
!A1=UZG1*LUZ1
!A2=UZG2*LUZ2
!E1=A1+A2*E*H*DELB*L/2
!E2=A1+A2*E*B*DELH*L/2
!PSI=PSI-E1-E2
!E1=A1+A2*E*H*WEIB*L/2
!E2=A1+A2*E*B*WEIH*L/2
!BDEL=-E1*1.0
!HDEL=-E2*1.0
!A1=WXG1*LWX1
!A2=WXG2*LWX2
!E1=A1+A2*E*B*B*B*DELH*L/24.0
!E2=A1+A2*3.0*E*H*B*B*DELB*L/24.0
!PSI=PSI-E1-E2
!E1=A1+A2*E*B*B*B*WEIH*L/24.0
!E2=A1+A2*3.0*E*H*B*B*WEIB*L/24.0
```

```
!BDEL=BDEL-E2
!HDEL=HDEL-E1
!A1=WYG1*LWY1
!A2=WYG2*LWY2
!E1=A1+A2*E*H*H*H*DELB*L/24.0
!E2=A1+A2*3.0*E*B*H*H*DELH*L/24.0
!PSI=PSI-E1-E2
!E1=A1+A2*E*H*H*H*WEIB*L/24.0
!E2=A1+A2*3.0*E*B*H*H*WEIH*L/24.0
!BDEL=BDEL-E1
!HDEL=HDEL-E2
!PSI
$
!TEMP=CLC-CCOM-CDIS-CS21-CS41-1
!TEMP=CLC-CCOM-CDIS-1
*JLZ(TEMP,155)
!TEMP=TEMP+1-CS21
*JGZ(TEMP,155)
!TEMP=CLC-CCOM-CDIS
!JENT=DS "TEMP",1 ,1   (1   ST21 LIST 1 1)
!JENT
!TEMP=ICOU-JENT
!JENT=FREE()
*JNZ(TEMP,155)
!SIB=1.0
!SIH=1.0
!TEMP=POIN-2
*JNZ(TEMP,444)
!SIB=-1.0
!SIH=1.0
*JUMP(448)
*LABEL 444
!TEMP=POIN-3
*JNZ(TEMP,446)
!SIB=-1.0
!SIH=-1.0
*JUMP(448)
*LABEL 446
!TEMP=POIN-4
*JNZ(TEMP,448)
!SIB=1.0
!SIH=-1.0
*LABEL 448
!SIB
!SIH
$
!INT1=WXG1+WXG2*E*0.25*DELB*SIB
!INT2=WYG1+WYG2*E*0.25*DELH*SIH
!PSI
!PSI=PSI-INT1-INT2
!PSI
!INT1=WXG1+WXG2*E*0.25*WEIB*SIB
!INT2=WYG1+WYG2*E*0.25*WEIH*SIH
!HDEL=HDEL-INT2
!BDEL=BDEL-INT1
!INT1=FREE()
!INT2=FREE()
!SIB=FREE()
!SIH=FREE()
$
```

```
*LABEL 135
!TEMP=FREE()
!DELB=FREE()
!DELH=FREE()
$
!ICOU
!PSI
!BDEL
!HDEL
!A1=FREE()
!A2=FREE()
!E1=FREE()
!E2=FREE()
!L=FREE()
!SIG1=FREE()
!SIG2=FREE()
$
*LABEL 200
!TEMP=ICOU-DE21
*JNZ(TEMP,100)
!ICOU=FREE()
!TEMP=FREE()
!UZG1=FREE()
!UZG2=FREE()
!WXG1=FREE()
!WXG2=FREE()
!WYG1=FREE()
!WYG2=FREE()
!LUZ1=FREE()
!LUZ2=FREE()
!LWX1=FREE()
!LWX2=FREE()
!LWY1=FREE()
!LWY2=FREE()
!E=FREE()
!B=FREE()
!H=FREE()
!UX1=FREE()
!UY1=FREE()
!UZ1=FREE()
!RX1=FREE()
!RY1=FREE()
!RZ1=FREE()
!UX2=FREE()
!UY2=FREE()
!UZ2=FREE()
!RX2=FREE()
!RY2=FREE()
!RZ2=FREE()
*RETURN                                       ENDSC21
*
$
$-----------------------------------------------------------------
$                                             ENDAD21
*(29 ADJO E21 0 0)
$
*XQT AUS
$
$  COMPUTE AVERAGE ELEMENT STRESS
$
```

```
!UZG1=DS,1,1,1  (13 ELST E21 "JENT" 0)
!UZG2=DS,2,1,1  (13 ELST E21 "JENT" 0)
!WXG1=DS,1,2,1  (13 ELST E21 "JENT" 0)
!WXG2=DS,2,2,1  (13 ELST E21 "JENT" 0)
!WYG1=DS,1,3,1  (13 ELST E21 "JENT" 0)
!WYG2=DS,2,3,1  (13 ELST E21 "JENT" 0)
!L=DS,"JENT",1,1   (13 LENG E21 0 0)
!E=DS,1,"JENT",1   (13 MATP E21 0 0)
!B=DS,2,"JENT",1   (13 SPLO E21 0 0)
!H=DS,1,"JENT",1   (13 SPLO E21 0 0)
!B=B*2.0
!H=H*2.0
!H
!B
!SIG1=E*UZG1
!SIG1=-E*0.5*B*WXG1+SIG1
!SIG1=-E*0.5*H*WYG1+SIG1
!SIG2=E*UZG2
!SIG2=-E*0.5*B*WXG2+SIG2
!SIG2=-E*0.5*H*WYG2+SIG2
!VMS1=SIG1+SIG2*0.5
!SIG1=E*UZG1
!SIG1=+E*0.5*B*WXG1+SIG1
!SIG1=-E*0.5*H*WYG1+SIG1
!SIG2=E*UZG2
!SIG2=+E*0.5*B*WXG2+SIG2
!SIG2=-E*0.5*H*WYG2+SIG2
!VMS2=SIG1+SIG2*0.5
!SIG1=E*UZG1
!SIG1=+E*0.5*B*WXG1+SIG1
!SIG1=+E*0.5*H*WYG1+SIG1
!SIG2=E*UZG2
!SIG2=+E*0.5*B*WXG2+SIG2
!SIG2=+E*0.5*H*WYG2+SIG2
!VMS3=SIG1+SIG2*0.5
!SIG1=E*UZG1
!SIG1=-E*0.5*B*WXG1+SIG1
!SIG1=+E*0.5*H*WYG1+SIG1
!SIG2=E*UZG2
!SIG2=-E*0.5*B*WXG2+SIG2
!SIG2=+E*0.5*H*WYG2+SIG2
!VMS4=SIG1+SIG2*0.5
!UZG1=FREE()
!UZG2=FREE()
!WXG1=FREE()
!WXG2=FREE()
!WYG1=FREE()
!WYG2=FREE()
!VMS1
!VMS2
!VMS3
!VMS4
!VMC1=ABS(VMS1)
!VMC2=ABS(VMS2)
!VMC3=ABS(VMS3)
!VMC4=ABS(VMS4)
!VMST=VMS1
!VMCS=VMC1
!POIN=1
!SIB=1.0
```

```
!SIH=1.0
!COMP=VMCS-VMC2
*JGZ(COMP,444)
!VMST=VMS2
!POIN=2
!SIB=-1.0
!SIH=1.0
*LABEL 444
!COMP=VMCS-VMC3
*JGZ(COMP,445)
!VMST=VMS3
!POIN=3
!SIB=-1.0
!SIH=-1.0
*LABEL 445
!COMP=VMCS-VMC4
*JGZ(COMP,446)
!VMST=VMS4
!POIN=4
!SIB=1.0
!SIH=-1.0
*LABEL 446
!VMST
!POIN
!COMP=FREE()
!VMS1=FREE()
!VMS2=FREE()
!VMS3=FREE()
!VMS4=FREE()
!VMC1=FREE()
!VMC2=FREE()
!VMC3=FREE()
!VMC4=FREE()
!VMCS=FREE()
!SIG1=FREE()
!SIG2=FREE()
!XGP1=1.0-0.57735*L*0.5
!XGP2=1.0+0.57735*L*0.5
!AD3=-1.0*E/L
!AD9=1.0*E/L
!A1=1.0/L
!A2=A1*A1
!A3=A2*A1
!E1=-6.0*A2
!E2=12.0*XGP1*A3
!E3=-4.0*A1
!E4=6.0*XGP1*A2
!E5=E3*0.5
!E6=12.0*XGP2*A3
!E7=6.0*XGP2*A2
!AD=2.0*E1+E2+E6/2.0
!AD2=-AD*E*SIB*B/2.0
!AD1=-AD*E*SIH*H/2.0
!AD=-2.0*E1-E2-E6/2.0
!AD8=-AD*E*SIB*B/2.0
!AD7=-AD*E*SIH*H/2.0
!AD=2.0*E3+E4+E7/2.0
!AD4=AD*E*SIB*B/2.0
!AD5=-AD*E*SIH*H/2.0
!AD=2.0*E5+E4+E7/2.0
```

```
!AD10=AD*E*SIB*B/2.0
!AD11=-AD*E*SIH*H/2.0
!AD=FREE()
!XGP1=FREE()
!XGP2=FREE()
!E=FREE()
!B=FREE()
!H=FREE()
!L=FREE()
!A1=FREE()
!A2=FREE()
!A3=FREE()
!E1=FREE()
!E2=FREE()
!E3=FREE()
!E4=FREE()
!E5=FREE()
!E6=FREE()
!E7=FREE()
!SIB=FREE()
!SIH=FREE()
TABLE(NI=3,NJ=4):  17 ROTA AD21 "ADLC" O
J=1:  "AD1"   "AD2"   "AD3"
J=2:  "AD4"   "AD5"    0.0
J=3:  "AD7"   "AD8"   "AD9"
J=4:  "AD10"  "AD11"   0.0
DEFINE B11=13 EL21 ROTA "JENT" O
17 EL21 ROTT "JENT" O=RTRAN(B11)
DEFINE B12=17 EL21 ROTT "JENT" O
DEFINE B11=17 ROTA AD21 "ADLC" O
17 GLAD E21 "ADLC" O=RPROD(B12,B11)
!AD1=DS, 1,1,1  (17 GLAD E21 "ADLC" O)
!AD2=DS, 2,1,1  (17 GLAD E21 "ADLC" O)
!AD3=DS, 3,1,1  (17 GLAD E21 "ADLC" O)
!AD4=DS, 1,2,1  (17 GLAD E21 "ADLC" O)
!AD5=DS, 2,2,1  (17 GLAD E21 "ADLC" O)
!AD6=DS, 3,2,1  (17 GLAD E21 "ADLC" O)
!AD7=DS, 1,3,1  (17 GLAD E21 "ADLC" O)
!AD8=DS, 2,3,1  (17 GLAD E21 "ADLC" O)
!AD9=DS, 3,3,1  (17 GLAD E21 "ADLC" O)
!AD10=DS, 1,4,1  (17 GLAD E21 "ADLC" O)
!AD11=DS, 2,4,1  (17 GLAD E21 "ADLC" O)
!AD12=DS, 3,4,1  (17 GLAD E21 "ADLC" O)
!P1=DS,13,"JENT",1 (13 CONN E21 O O)
!P2=DS,14,"JENT",1 (13 CONN E21 O O)
SYSVEC: APPL FORC "ADLC" O
I=1
J="P1"   :  "AD1"
J="P2"   :  "AD7"
I=2
J="P1"   :  "AD2"
J="P2"   :  "AD8"
I=3
J="P1"   :  "AD3"
J="P2"   :  "AD9"
I=4
J="P1"   :  "AD4"
J="P2"   :  "AD10"
I=5
J="P1"   :  "AD5"
```

```
J="P2"   :  "AD11"
I=6
J="P1"   :  "AD6"
J="P2"   :  "AD12"
!P1=FREE()
!P2=FREE()
!AD1=FREE()
!AD2=FREE()
!AD3=FREE()
!AD4=FREE()
!AD5=FREE()
!AD6=FREE()
!AD7=FREE()
!AD8=FREE()
!AD9=FREE()
!AD10=FREE()
!AD11=FREE()
!AD12=FREE()
*XQT U1
*SHOW
*XQT DCU
ERASE 17
*RETURN
*                                        ENDAD21
$------------------------------------------------------------------------
$
*(29 PREP E41 0 0)                                   ENDPE41
$
$  THIS SUBROUTINE IS FOR ELEMENT TYP E41
$
$   THE PRINCIPAL LIBRARY IS 14
$   TEMPORY LIBRARY IS 15    CAN BE DELETED AT THE END OF THIS ROUTINE
$
*XQT EI1
$
EXTRACT:  SOURCE=E41:   CONTENT SPEC:  GEOM  1 $  AREA OF ELEMENTS
CREATE 14 AREA E41 0 0
$
EXTRACT:  SOURCE=E41:   CONTENT SPEC:  GEOM  5,12 $ LOCAL ELEM. REF. FRAME
CREATE 14 ELRF E41 0 0
$
EXTRACT:  SOURCE=E41:   CONTENT SPEC:  GEOM  22,57 $ ELEMENT-NODAL RELATION
CREATE 14 ELNO E41 0 0
$
EXTRACT:  SOURCE=E41:   CONTENT SPEC:  MATE 1,2 $ MATERIAL PROPERTIES
CREATE 14 MATP E41 0 0
$
EXTRACT:  SOURCE=E41:   CONTENT SPEC:  INTE 1,16  $ CONNECTIVITY
CREATE 14 CONN E41 0 0
$
EXTRACT:  SOURCE=E41:   CONTENT SPEC:  S 1,25    $  HINV T
CREATE 14 HIT E41 0 0
$
*XQT DCU
TOCC 14 CONN E41 0 0 :  TYPE=0
*XQT U1
!ENT1=0.6220084
!ENT2=1.0/6.0
!ENT3=0.0446582
*TI(14 SHAP FUNC 1 1)
```

```
"ENT1"   "ENT2"   "ENT3"   "ENT2"
"ENT2"   "ENT1"   "ENT2"   "ENT3"
"ENT3"   "ENT2"   "ENT1"   "ENT2"
"ENT2"   "ENT3"   "ENT2"   "ENT1"
!ENT1=FREE()
!ENT2=FREE()
!ENT3=FREE()
$
*XQT AUS
!ICOU=1
!TEMP=0
*LABEL 200
!TEMP=FREE()
!B1=ICOU-1*8
DEFINE A1=14 ELRF E41 0 0
TABLE(NI=2,NJ=4): 15 ELEM REFT "ICOU" 99
TRANSFER (SOURCE=A1,ILIM=8,SBASE="B1")
!B1=FREE()
DEFINE BQ1=15 ELEM REFT "ICOU" 99
14 ELEM REFE "ICOU" 0=RTRAN(BQ1)
$
$     COMPUTE DETERMINATES FOR INTEGRATION
!IL=1
!GP1=0.57735
*LABEL 55
!PXI=-1.0*GP1
!PET=-1.0*GP1
!TEMP=IL-2
*JNZ(TEMP,35)
!PXI=+1.0*GP1
!PET=-1.0*GP1
*LABEL 35
!TEMP=IL-3
*JNZ(TEMP,37)
!PXI=+1.0*GP1
!PET=+1.0*GP1
*LABEL 37
!TEMP=IL-4
*JNZ(TEMP,39)
!PXI=-1.0*GP1
!PET=+1.0*GP1
*LABEL 39
!DUM1=-1.0+PET*0.25
!DUM2=-1.0+PXI*0.25
!DUM3=+1.0-PET*0.25
!DUM4=-1.0-PXI*0.25
!DUM5=+1.0+PET*0.25
!DUM6=+1.0+PXI*0.25
!DUM7=-1.0-PET*0.25
!DUM8=+1.0-PXI*0.25
TABLE(NI=2,NJ=4):  15 HELP E41 "IL" 99
J=1 :   "DUM1"   "DUM2"
J=2 :   "DUM3"   "DUM4"
J=3 :   "DUM5"   "DUM6"
J=4 :   "DUM7"   "DUM8"
DEFINE E1=15 HELP E41 "IL" 99
DEFINE BB2=14 ELEM REFE "ICOU" 0
15 JAC E41 "ICOU" "IL"=RPROD (E1,BB2)
!DUM1=DS 1,1,1 (15 JAC E41 "ICOU" "IL")
!DUM2=DS 1,2,1 (15 JAC E41 "ICOU" "IL")
```

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```
!DUM3=DS 2,1,1 (15 JAC E41 "ICOU" "IL")
!DUM4=DS 2,2,1 (15 JAC E41 "ICOU" "IL")
!DUM5=DUM1*DUM4
!DUM6=DUM2*DUM3
!DUM8=DUM5-DUM6
!DUM8
!TEMP=IL-1
*JNZ(TEMP,45)
!DET1=DUM8
*JUMP 53
*LABEL 45
!TEMP=IL-2
*JNZ(TEMP,47)
!DET2=DUM8
*JUMP 53
*LABEL 47
!TEMP=IL-3
*JNZ(TEMP,49)
!DET3=DUM8
*JUMP 53
*LABEL 49
!TEMP=IL-4
*JNZ(TEMP,53)
!DET4=DUM8
*LABEL 53
!IL=IL+1
!TEMP=IL-5
*JNZ(TEMP,55)
TABLE(NI=4,NJ=1):14 DETE E41 "ICOU" O
J=1. "DET1" "DET2" "DET3" "DET4"
!DET1=FREE()
!DET2=FREE()
!DET3=FREE()
!DET4=FREE()
!DUM1=FREE()
!DUM2=FREE()
!DUM3=FREE()
!DUM4=FREE()
!DUM5=FREE()
!DUM6=FREE()
!DUM7=FREE()
!DUM8=FREE()
!IL=FREE()
!PXI=FREE()
!PET=FREE()
!GP1=FREE()
$
DEFINE BB1=14 SHAP FUNC 1 1
DEFINE BB2=14 ELEM REFE "ICOU" O
15 GAUS POIN "ICOU" O= RPROD (BB1,BB2)
$
!XGP1=DS 1,1,1 (15 GAUS POIN "ICOU" O)
!YGP1=DS 1,2,1 (15 GAUS POIN "ICOU" O)
!XGP2=DS 2,1,1 (15 GAUS POIN "ICOU" O)
!YGP2=DS 2,2,1 (15 GAUS POIN "ICOU" O)
!XGP3=DS 3,1,1 (15 GAUS POIN "ICOU" O)
!YGP3=DS 3,2,1 (15 GAUS POIN "ICOU" O)
!XGP4=DS 4,1,1 (15 GAUS POIN "ICOU" O)
!YGP4=DS 4,2,1 (15 GAUS POIN "ICOU" O)
$
```

```
TABLE(NI=12,NJ=5):14 PMAT E41 "ICOU" 0
J=1: 0.0   0.0   1.0  0.0   0.0   1.0   0.0   0.0  1.0   0.0  0.0  1.0
J=2: 0.0   1.0   0.0  0.0   1.0   0.0   0.0   1.0  0.0   0.0  1.0  0.0
J=3: 1.0   0.0   0.0  1.0   0.0   0.0   1.0   0.0  0.0   1.0  0.0  0.0
J=4:  "YGP1" 0.0  0.0 "YGP2" 0.0  0.0   "YGP3" 0.0  0.0 "YGP4" 0.0  0.0
J=5: 0.0 "XGP1" 0.0  0.0  "XGP2" 0.0   0.0  "XGP3" 0.0  0.0 "XGP4" 0.0
!XGP1=FREE()
!YGP1=FREE()
!XGP2=FREE()
!YGP2=FREE()
!XGP3=FREE()
!YGP3=FREE()
!XGP4=FREE()
!YGP4=FREE()
$
!NU=DS 2,"ICOU",1  (14 MATP E41  0 0)
!NUP1=NU+1.0
!NU=-1.0*NU
$
TABLE(NI=12,NJ=12): 14 SIEP E41 "ICOU" 0
J=1: 1.0  "NU"  0.0   0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0
J=2: "NU" 1.0   0.0   0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0
J=3: 0.0  0.0 "NUP1"  0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0
J=4: 0.0 0.0 0.0   1.0 "NU"   0.0   0.0 0.0 0.0  0.0 0.0 0.0
J=5: 0.0 0.0 0.0  "NU" 1.0    0.0   0.0 0.0 0.0  0.0 0.0 0.0
J=6: 0.0 0.0 0.0   0.0 0.0 "NUP1"   0.0 0.0 0.0  0.0 0.0 0.0
J=7: 0.0 0.0 0.0   0.0 0.0 0.0  1.0 "NU"    0.0  0.0 0.0 0.0
J=8: 0.0 0.0 0.0   0.0 0.0 0.0  "NU" 1.0    0.0  0.0 0.0 0.0
J=9: 0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 "NUP1"   0.0 0.0 0.0
J=10: 0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0   1.0 "NU" 0.0
J=11: 0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0   "NU" 1.0 0.0
J=12: 0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0   0.0 0.0 "NUP1"
!NU=FREE()
!NUP1=FREE()
$
!TEMP=FREE()
!B1=ICOU-1*5
DEFINE A2=ESB E41 1 1
TABLE(NI=5, NJ=1):15 BBB E41 "ICOU" 0
TRANSFERR(SOURCE=A2, ILIM=5, SBASE="B1")
!B1=FREE()
$
DEFINE BB1=14 PMAT E41 "ICOU" 0
DEFINE BB2=15 BBB E41 "ICOU" 0
!SAGR=DS 8,"ICOU", 1 (14 CONN E41 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
14 GPST E41 "ICOU" 0= RPROD ("OOTH" BB1,BB2)
!OOTH=FREE()
!SAGR=FREE()
$
!ICOU=ICOU+1
!TEMP=DE41-ICOU+1
*JCZ(TEMP,200)
!B1=FREE()
!ICOU=FREE()
!TEMP=FREE()
$
*XQT DCU
ERASE 15
*RETURN
```

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```
*                   ENDPE41
$
$-----------------------------------------------------------------------
*(29 SECA E41 0 0)                    ENDSC41
$
*XQT AUS
$
!ICDU=1
!TEMP=0
*LABEL 200
$
!ELGR=DS 1,"ICOU",1  (1 ED41 REL 0 0 )
!TEMP=ELGR-ICDV
*JNZ(TEMP,195)
$
!B1=ICOU-1*5
DEFINE A2=1 ESB E41 "ADLC"  1
TABLE(NI=5, NJ=1):15 BBB E41 "ICOU" "CLC"
TRANSFERR(SOURCE=A2, ILIM=5, SBASE="B1")
!B1=FREE()
$
DEFINE BB1=14 PMAT E41 "ICOU" 0
DEFINE BB2=15 BBB E41 "ICOU" "CLC"
!SAGR=DS 8,"ICOU", 1 (14 CONN E41 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
15 GPST E41 "ICOU" "CLC" = RPROD ("OOTH"  BB1,BB2)
!OOTH=FREE()
!SAGR=FREE()
$
DEFINE B11=14 SIEP E41 "ICOU" 0
DEFINE B12=15 GPST E41 "ICOU" "CLC"
14 GPEP E41 "ICOU" "CLC"= RPROD (B11,B12)
$
!SI1 =DS 1,1,1 (14 GPST E41 "ICOU" 0)
!SI2 =DS 2,1,1 (14 GPST E41 "ICOU" 0)
!SI3 =DS 3,1,1 (14 GPST E41 "ICOU" 0)
!SI4 =DS 4,1,1 (14 GPST E41 "ICOU" 0)
!SI5 =DS 5,1,1 (14 GPST E41 "ICOU" 0)
!SI6 =DS 6,1,1 (14 GPST E41 "ICOU" 0)
!SI7 =DS 7,1,1 (14 GPST E41 "ICOU" 0)
!SI8 =DS 8,1,1 (14 GPST E41 "ICOU" 0)
!SI9 =DS 9,1,1 (14 GPST E41 "ICOU" 0)
!SI10=DS 10,1,1 (14 GPST E41 "ICOU" 0)
!SI11=DS 11,1,1 (14 GPST E41 "ICOU" 0)
!SI12=DS 12,1,1 (14 GPST E41 "ICOU" 0)
!EP1 =DS 1,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP2 =DS 2,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP3 =DS 3,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP4 =DS 4,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP5 =DS 5,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP6 =DS 6,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP7 =DS 7,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP8 =DS 8,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP9 =DS 9,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP10=DS 10,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP11=DS 11,1,1 (14 GPEP E41 "ICOU" "CLC" )
!EP12=DS 12,1,1 (14 GPEP E41 "ICOU" "CLC" )
$
!EP1=SI1*EP1
!EP2=SI2*EP2
```

```
!EP3=2.0*SI3*EP3
!DETE=DS 1,1,1 (14 DETE E41 "ICOU" 0)
!EP3=EP1+EP2+EP3*DETE
!EP4=SI4*EP4
!EP5=SI5*EP5
!EP6=2.0*SI6*EP6
!DETE=DS 2,1,1 (14 DETE E41 "ICOU" 0)
!EP6=EP4+EP5+EP6*DETE
!EP7=SI7*EP7
!EP8=SI8*EP8
!EP9=2.0*SI9*EP9
!DETE=DS 3,1,1 (14 DETE E41 "ICOU" 0)
!EP9=EP7+EP8+EP9*DETE
!EP10=SI10*EP10
!EP11=SI11*EP11
!EP12=2.0*SI12*EP12
!DETE=DS 4,1,1 (14 DETE E41 "ICOU" 0)
!EP12=EP10+EP11+EP12*DETE
!PSIS=EP3+EP6+EP9+EP12
!NU=DS 1,"ICOU",1  (14 MATP E41  0 0)
!SAGR=DS 8,"ICOU", 1 (14 CONN E41 0 0)
!OUTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
!PSIS=-PSIS/NU
!PSI=PSI+PSIS
!ICOU
!PSIS
!PSIS=FREE()
!SAGR=FREE()
!OUTH=FREE()
!NU=FREE()
$
!SI21=SI2*SI1
!SI54=SI5*SI4
!SI87=SI8*SI7
!SI01=SI10*SI11
$
!SI1=SI1*SI1
!SI2=SI2*SI2
!SI3=SI3*SI3
!SI4=SI4*SI4
!SI5=SI5*SI5
!SI6=SI6*SI6
!SI7=SI7*SI7
!SI8=SI8*SI8
!SI9=SI9*SI9
!SI10=SI10*SI10
!SI11=SI11*SI11
!SI12=SI12*SI12
$
!SI3=SI3*3.0+SI1+SI2-SI21
!DETE=DS 1,1,1 (14 DETE E41 "ICOU" 0)
!SI3=SI3**0.5*DETE
!SI6=SI6*3.0+SI4+SI5-SI54
!DETE=DS 2,1,1 (14 DETE E41 "ICOU" 0)
!SI6=SI6**0.5*DETE
!SI9=SI9*3.0+SI7+SI8-SI87
!DETE=DS 3,1,1 (14 DETE E41 "ICOU" 0)
!SI9=SI9**0.5*DETE
!SI12=SI12*3.0+SI10+SI11-SI01
!DETE=DS 4,1,1 (14 DETE E41 "ICOU" 0)
```

```
!SI12=SI12**0.5*DETE
!VMST=SI3+SI6+SI9+SI12
!AREA=DS 1,"ICOU",1 (14 AREA E41 0 0)
!VMST=VMST/AREA
!AREA=FREE()
!VMST
$
!SI21=FREE()
!SI54=FREE()
!SI87=FREE()
!SI01=FREE()
!SI1 =FREE()
!SI2 =FREE()
!SI3 =FREE()
!SI4 =FREE()
!SI5 =FREE()
!SI6 =FREE()
!SI7 =FREE()
!SI8 =FREE()
!SI9 =FREE()
!SI10=FREE()
!SI11=FREE()
!SI12=FREE()
!EP1 =FREE()
!EP2 =FREE()
!EP3 =FREE()
!EP4 =FREE()
!EP5 =FREE()
!EP6 =FREE()
!EP7 =FREE()
!EP8 =FREE()
!EP9 =FREE()
!EP10=FREE()
!EP11=FREE()
!EP12=FREE()
!DETE=FREE()
$
*LABEL 195
!ICOU=ICOU+1
!TEMP=DE41-ICOU+1
*JGZ(TEMP,200)
!B1=FREE()
!ICOU=FREE()
!TEMP=FREE()
$
*XQT U1
*SHOW
*RETURN
*                                                    ENDSC41
$
$-------------------------------------------------------------
$
*(29 ADJO E41 0 0)                                   ENDAD41
$
*XQT AUS
!B1=JENT-1*25
DEFINE A1=14 HIT E41 0 0
TABLE(NI=5, NJ=5):15 HITE ELEM "ADLC" 1
TRANSFERR(SOURCE=A1, ILIM=25, SBASE="B1")
!B1=FREE()
```

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```
$  DEFINE MATRIX A
!X21=DS 3,"JENT",1 (14 ELRF E41 0 0)
!X31=DS 5,"JENT",1 (14 ELRF E41 0 0)
!X32=DS 6,"JENT",1 (14 ELRF E41 0 0)
!X41=DS 7,"JENT",1 (14 ELRF E41 0 0)
!X42=DS 8,"JENT",1 (14 ELRF E41 0 0)
!EN22=-1.0*X32/X21
!EN24=X32/X21
!EN32=X31/X21-1.0
!EN34=-X31/X21
!EN42=-1.0*X42/X21
!EN44=X42/X21
!EN52=X41/X21-1.0
!EN54=-X41/X21
*XQT U1
*TI (15 AMAT E41 "ADLC" 0)
-1.0    -1.0    0.0     -1.0    0.0
0.0     "EN22"  "EN32"  "EN42"  "EN52"
1.0     0.0     0.0     0.0     0.0
0.0     "EN24"  "EN34"  "EN44"  "EN54"
0.0     1.0     0.0     0.0     0.0
0.0     0.0     1.0     0.0     0.0
0.0     0.0     0.0     1.0     0.0
0 0     0.0     0.0     0.0     1.0
!X21=FREE()
!X31=FREE()
!X32=FREE()
!X41=FREE()
!X42=FREE()
!EN22=FREE()
!EN24=FREE()
!EN32=FREE()
!EN34=FREE()
!EN42=FREE()
!EN44=FREE()
!EN52=FREE()
!EN54=FREE()
$
*XQT AUS
DEFINE BAQ=15 HITE ELEM "ADLC" 1
DEFINE BAW=15 AMAT E41 "ADLC" 0
15 HITF E41 "ADLC" 0=RPROD(BAQ ,BAW)
DEFINE BAQ=14 PMAT E41 "JENT" 0)
DEFINE BAW=15 HITF E41 "ADLC" 0)
!SAGR=DS 8,"JENT", 1 (14 CONN E41 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
15 PHIT E41 "ADLC" 0=RPROD ("OOTH" BAQ,BAW)
!OOTH=FREE()
!SAGR=FREE()
$
!IDOF=1
*LABEL 400
!IDOF
!SI1 =DS 1,1,1 (14 GPST E41 "JENT" 0)
!SI2 =DS 2,1,1 (14 GPST E41 "JENT" 0)
!SI3 =DS 3,1,1 (14 GPST E41 "JENT" 0)
!SI4 =DS 4,1,1 (14 GPST E41 "JENT" 0)
!SI5 =DS 5,1,1 (14 GPST E41 "JENT" 0)
!SI6 =DS 6,1,1 (14 GPST E41 "JENT" 0)
!SI7 =DS 7,1,1 (14 GPST E41 "JENT" 0)
```

```
!SI8 =DS 8,1,1 (14 GPST E41 "JENT" 0)
!SI9 =DS 9,1,1 (14 GPST E41 "JENT" 0)
!SI10=DS 10,1,1 (14 GPST E41 "JENT" 0)
!SI11=DS 11,1,1 (14 GPST E41 "JENT" 0)
!SI12=DS 12,1,1 (14 GPST E41 "JENT" 0)
!EP1 =DS 1,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP2 =DS 2,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP3 =DS 3,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP4 =DS 4,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP5 =DS 5,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP6 =DS 6,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP7 =DS 7,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP8 =DS 8,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP9 =DS 9,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP10=DS 10,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP11=DS 11,"IDOF",1 (15 PHIT E41 "ADLC" 0)
!EP12=DS 12,"IDOF",1 (15 PHIT E41 "ADLC" 0)
$
!VM1=2.0*SI1-SI2
!VM1=0.5*VM1
!VM2=2.0*SI2-SI1
!VM2=0.5*VM2
!VM3=3.0*SI3
!VM4=2.0*SI4-SI5
!VM4=0.5*VM4
!VM5=2.0*SI5-SI4
!VM5=0.5*VM5
!VM6=3.0*SI6
!VM7=2.0*SI7-SI8
!VM7=0.5*VM7
!VM8=2.0*SI8-SI7
!VM8=0.5*VM8
!VM9=3.0*SI9
!VM10=2.0*SI10-SI11.
!VM10=0.5*VM10
!VM11=2.0*SI11-SI10
!VM11=0.5*VM11
!VM12=3.0*SI12
$
!SI21=SI2*SI1
!SI54=SI5*SI4
!SI87=SI8*SI7
!SI01=SI10*SI11
$
!SI1=SI1*SI1
!SI2=SI2*SI2
!SI3=SI3*SI3
!SI4=SI4*SI4
!SI5=SI5*SI5
!SI6=SI6*SI6
!SI7=SI7*SI7
!SI8=SI8*SI8
!SI9=SI9*SI9
!SI10=SI10*SI10
!SI11=SI11*SI11
!SI12=SI12*SI12
$
!SI3=SI3*3.0+SI1+SI2-SI21
!SI3=SI3**0.5
!SI6=SI6*3.0+SI4+SI5-SI54
```

```
!SI6=SI6**0.5
!SI9=SI9*3.0+SI7+SI8-SI87
!SI9=SI9**0.5
!SI12=SI12*3.0+SI10+SI11-SI01
!SI12=SI12**0.5
$
!VM1=VM1/SI3*EP1
!VM2=VM2/SI3*EP2
!VM3=VM3/SI3*EP3
!VM4=VM4/SI6*EP4
!VM5=VM5/SI6*EP5
!VM6=VM6/SI6*EP6
!VM7=VM7/SI9*EP7
!VM8=VM8/SI9*EP8
!VM9=VM9/SI9*EP9
!VM10=VM10/SI12*EP10
!VM11=VM11/SI12*EP11
!VM12=VM12/SI12*EP12
$
!DETE=DS 1,1,1 (14 DETE E41 "JENT" 0)
!VM3=VM3+VM2+VM1*DETE
!DETE=DS 2,1,1 (14 DETE E41 "JENT" 0)
!VM6=VM6+VM5+VM4*DETE
!DETE=DS 3,1,1 (14 DETE E41 "JENT" 0)
!VM9=VM9+VM8+VM7*DETE
!DETE=DS 4,1,1 (14 DETE E41 "JENT" 0)
!VM12=VM12+VM11+VM10*DETE
!AREA=DS 1,"JENT",1 (14 AREA E41 0 0)
!VM12=VM12+VM9+VM6+VM3
!VM12=VM12/AREA
!AREA=FREE()
$
!TEM2=IDOF-1
*JNZ(TEM2,100)
!AD1=1.0*VM12
*JUMP 300
*LABEL 100
!TEM2=IDOF-2
*JNZ(TEM2,110)
!AD2=1.0*VM12
*JUMP 300
*LABEL 110
!TEM2=IDOF-3
*JNZ(TEM2,120)
!AD3=1.0*VM12
*JUMP 300
*LABEL 120
!TEM2=IDOF-4
*JNZ(TEM2,130)
!AD4=1.0*VM12
*JUMP 300
*LABEL 130
!TEM2=IDOF-5
*JNZ(TEM2,140)
!AD5=1.0*VM12
*JUMP 300
*LABEL 140
!TEM2=IDOF-6
*JNZ(TEM2,150)
!AD6=1.0*VM12
```

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```
*JUMP 300
*LABEL 150
!TEM2=IDOF-7
*JNZ(TEM2,160)
!AD7=1.0*VM12
*JUMP 300
*LABEL 160
!TEM2=IDOF-8
*JNZ(TEM2,170)
!AD8=1.0*VM12
*LABEL 170
$
*LABEL 300
!IDOF=IDOF+1
!TEM2=IDOF-9
*JNZ (TEM2, 400)
!TEM2=FREE()
!IDOF=FREE()
!SI21=FREE()
!S154=FREE()
!SI87=FREE()
!SI01=FREE()
!SI1 =FREE()
!SI2 =FREE()
!SI3 =FREE()
!SI4 =FREE()
!SI5 =FREE()
!SI6 =FREE()
!SI7 =FREE()
!SI8 =FREE()
!SI9 =FREE()
!SI10=FREE()
!SI11=FREE()
!SI12=FREE()
!EP1 =FREE()
!EP2 =FREE()
!EP3 =FREE()
!EP4 =FREE()
!EP5 =FREE()
!EP6 =FREE()
!EP7 =FREE()
!EP8 =FREE()
!EP9 =FREE()
!EP10=FREE()
!EP11=FREE()
!EP12=FREE()
!VM1=FREE()
!VM2=FREE()
!VM3=FREE()
!VM4=FREE()
!VM5=FREE()
!VM6=FREE()
!VM7=FREE()
!VM8=FREE()
!VM9=FREE()
!VM10=FREE()
!VM11=FREE()
!VM12=FREE()
!DETE=FREE()
$
```

```
$  NOW ASSIGN LOAD TO PROPER DEGREES OF FREEDOM
!VMST
DEFINE BB2=14 ELNO E41 0 0
TABLE(NI=12, NJ=12):15 ROTA ELEM "ADLC" 99
!A1=JENT-1*36
!B1=0
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+3
!B1=12
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+6
!B1=24
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+9
!B1=36+3
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+12
!B1=48+3
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+15
!B1=60+3
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+18
!B1=72+6
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+21
!B1=84+6
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+24
!B1=96+6
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+27
!B1=108+9
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+30
!B1=120+9
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+33
!B1=132+9
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=FREE()
!B1=FREE()
TABLE(NI=12, NJ=1):15 ELLO E41 "ADLC" 1
J=1: "AD1" "AD2" 0.0 "AD3" "AD4" 0.0 "AD5" "AD6" 0.0 "AD7" "AD8" 0.0
DEFINE A1 = 15 ROTA ELEM "ADLC" 99
15 ROTA ELEM "ADLC" 1 =RTRAN( A1 )
DEFINE BB2= 15 ELLO E41 "ADLC" 1
DEFINE A1 = 15 ROTA ELEM "ADLC" 1
14 ADLO VE41 "ADLC" 0 = RPROD ( A1, BB2)
!AD1=DS 1,1,1  (14 ADLO VE41 "ADLC" 0)
!AD2=DS 2,1,1  (14 ADLO VE41 "ADLC" 0)
!AD3=DS 3,1,1  (14 ADLO VE41 "ADLC" 0)
!AD4=DS 4,1,1  (14 ADLO VE41 "ADLC" 0)
!AD5=DS 5,1,1  (14 ADLO VE41 "ADLC" 0)
!AD6=DS 6,1,1  (14 ADLO VE41 "ADLC" 0)
!AD7=DS 7,1,1  (14 ADLO VE41 "ADLC" 0)
!AD8=DS 8,1,1  (14 ADLO VE41 "ADLC" 0)
!AD9=DS 9,1,1  (14 ADLO VE41 "ADLC" 0)
!AD10=DS 10,1,1  (14 ADLO VE41 "ADLC" 0)
!AD11=DS 11,1,1  (14 ADLO VE41 "ADLC" 0)
```

```
!AD12=DS 12,1,1   (14 ADLO VE41 "ADLC" 0)
$
!J1=DS 13,"JENT",1   (14 CONN E41 0 0)
!J2=DS 14,"JENT",1   (14 CONN E41 0 0)
!J3=DS 15,"JENT",1   (14 CONN E41 0 0)
!J4=DS 16,"JENT",1   (14 CONN E41 0 0)
SYSVEC:  APPL FORC "ADLC" 1
I=1
J="J1"  :  "AD1"
J="J2"  :  "AD4"
J="J3"  :  "AD7"
J="J4"  :  "AD10"
I=2
J="J1"  :  "AD2"
J="J2"  :  "AD5"
J="J3"  :  "AD8"
J="J4"  :  "AD11"
I=3
J="J1"  :  "AD3"
J="J2"  :  "AD6"
J="J3"  :  "AD9"
J="J4"  :  "AD12"
!AD1=FREE()
!AD2=FREE()
!AD3=FREE()
!AD4=FREE()
!AD5=FREE()
!AD6=FREE()
!AD7=FREE()
!AD8=FREE()
!AD9=FREE()
!AD10=FREE()
!AD11=FREE()
!AD12=FREE()
!J1=FREE()
!J2=FREE()
!J3=FREE()
!J4=FREE()
$
*XQT DCU
ERASE 15
*RETURN
*
$------------------------------------ENDAD41
$-------------------------------------------------------------------------
*(29 PREP E42 0 0)
$                                        ENDPE42
$   THIS SUBROUTINE IS FOR ELEMENT TYP E42
$
$    THE PRINCIPAL LIBRARY IS 16
$    TEMPORY LIBRARY IS 17     CAN BE DELETED AT THE END OF THIS ROUTINE
$
*XQT EI1
$
EXTRACT:   SOURCE=E42:    CONTENT SPEC:   GEOM   1 $   AREA OF ELEMENTS
CREATE 16 AREA E42 0 0
$
EXTRACT:   SOURCE=E42:    CONTENT SPEC:   GEOM   5,12 $ LOCAL ELEM. REF. FRAME
CREATE 16 ELRF E42 0 0
$
```

```
EXTRACT:  SOURCE=E42:   CONTENT SPEC:  GEOM  22,57 $ ELEMENT-NODAL RELATION
CREATE 16 ELNO E42 0 0
$
EXTRACT:  SOURCE=E42:   CONTENT SPEC:  MATE 1,2 $ MATERIAL PROPERTIES
CREATE 16 MATP E42 0 0
$
EXTRACT:  SOURCE=E42:   CONTENT SPEC:  INTE 1,16  $ CONNECTIVITY
CREATE 16 CONN E42 0 0
$
*XQT DCU
TOCC 16 CONN E42 0 0 :  TYPE=0
*XQT U1
!ENT1=0.6220084
!ENT2=1.0/6.0
!ENT3=0.0446582
*TI(16 SHAP FUNC 1 1)
"ENT1"  "ENT2"  "ENT3"  "ENT2"
"ENT2"  "ENT1"  "ENT2"  "ENT3"
"ENT3"  "ENT2"  "ENT1"  "ENT2"
"ENT2"  "ENT3"  "ENT2"  "ENT1"
!ENT1=FREE()
!ENT2=FREE()
!ENT3=FREE()
$
*XQT AUS
!ICOU=1
!TEMP=0
*LABEL 200
!TEMP=FREE()
!B1=ICOU-1*8
DEFINE A1=16 ELRF E42 0 0
TABLE(NI=2,NJ=4):17 ELEM REFT "ICOU" 99
TRANSFER (SOURCE=A1,ILIM=8,SBASE="B1")
!B1=FREE()
DEFINE BQ1=17 ELEM REFT "ICOU" 99
16 ELEM REFE "ICOU" O=RTRAN(BQ1)
$
$    COMPUTE DETERMINATES FOR INTEGRATION
!IL=1
!GP1=0.57735
*LABEL 55
!PXI=-1.0*GP1
!PET=-1.0*GP1
!TEMP=IL-2
*JNZ(TEMP,35)
!PXI=+1.0*GP1
!PET=-1.0*GP1
*LABEL 35
!TEMP=IL-3
*JNZ(TEMP,37)
!PXI=+1.0*GP1
!PET=+1.0*GP1
*LABEL 37
!TEMP=IL-4
*JNZ(TEMP,39)
!PXI=-1.0*GP1
!PET=+1.0*GP1
*LABEL 39
!DUM1=-1.0+PET*0.25
!DUM2=-1.0+PXI*0.25
```

```
!DUM3=+1.0-PET*0.25
!DUM4=-1.0-PXI*0.25
!DUM5=+1.0+PET*0.25
!DUM6=+1.0+PXI*0.25
!DUM7=-1.0-PET*0.25
!DUM8=+1.0-PXI*0.25
TABLE(NI=2,NJ=4): 17 HELP E42 "IL" 99
J=1 :   "DUM1"   "DUM2"
J=2 :   "DUM3"   "DUM4"
J=3     "DUM5"   "DUM6"
J=4 :   "DUM7"   "DUM8"
DEFINE E1=17 HELP E42 "IL" 99
DEFINE BB2=16 ELEM REFE "ICOU" 0
17 JAC E42 "ICOU" "IL"=RPROD (E1,BB2)
!DUM1=DS 1,1,1 (17 JAC E42 "ICOU" "IL")
!DUM2=DS 1,2,1 (17 JAC E42 "ICOU" "IL")
!DUM3=DS 2,1,1 (17 JAC E42 "ICOU" "IL")
!DUM4=DS 2,2,1 (17 JAC E42 "ICOU" "IL")
!DUM5=DUM1*DUM4
!DUM6=DUM2*DUM3
!DUM8=DUM5-DUM6
!DUM8
!TEMP=IL-1
*JNZ(TEMP,45)
!DET1=DUM8
*JUMP 53
*LABEL 45
!TEMP=IL-2
*JNZ(TEMP,47)
!DET2=DUM8
*JUMP 53
*LABEL 47
!TEMP=IL-3
*JNZ(TEMP,49)
!DET3=DUM8
*JUMP 53
*LABEL 49
!TEMP=IL-4
*JNZ(TEMP,53)
!DET4=DUM8
*LABEL 53
!IL=IL+1
!TEMP=IL-5
*JNZ(TEMP,55)
TABLE(NI=4,NJ=1):16 DETE E42 "ICOU" 0
J=1:  "DET1"   "DET2"   "DET3"   "DET4"
!DET1=FREE()
!DET2=FREE()
!DET3=FREE()
!DET4=FREE()
!DUM1=FREE()
!DUM2=FREE()
!DUM3=FREE()
!DUM4=FREE()
!DUM5=FREE()
!DUM6=FREE()
!DUM7=FREE()
!DUM8=FREE()
!IL=FREE()
!PXI=FREE()
```

```
!PET=FREE()
!GP1=FREE()
$
DEFINE BB1=16 SHAP FUNC 1 1
DEFINE BB2=16 ELEM REFE "ICOU" 0
17 GAUS POIN "ICOU" 0= RPROD (BB1,BB2)
$
!XGP1=DS 1,1,1 (17 GAUS POIN "ICOU" 0)
!YGP1=DS 1,2,1 (17 GAUS POIN "ICOU" 0)
!XGP2=DS 2,1,1 (17 GAUS POIN "ICOU" 0)
!YGP2=DS 2,2,1 (17 GAUS POIN "ICOU" 0)
!XGP3=DS 3,1,1 (17 GAUS POIN "ICOU" 0)
!YGP3=DS 3,2,1 (17 GAUS POIN "ICOU" 0)
!XGP4=DS 4,1,1 (17 GAUS POIN "ICOU" 0)
!YGP4=DS 4,2,1 (17 GAUS POIN "ICOU" 0)
!XYG1=XGP1*YGP1
!XYG2=XGP2*YGP2
!XYG3=XGP3*YGP3
!XYG4=XGP4*YGP4
$
TABLE(NI=12,NJ=11): 16 PMAT E42 "ICOU" 0
J=1:  1.0    0.0    0.0  1.0    0.0    0.0    1.0    0.0  0.0    1.0  0.0  0.0
J=2:  0.0    1.0    0.0  0.0    1.0    0.0    0.0    1.0  0.0    0.0  1.0  0.0
J=3:  0.0    0.0    1.0  0.0    0.0    1.0    0.0    0.0  1.0    0.0  0.0  1.0
J=4:  "YGP1" 0.0  0.0 "YGP2" 0.0  0.0   "YGP3" 0.0  0.0 "YGP4" 0.0  0.0
J=5:  0.0 "XGP1" 0.0  0.0  "XGP2" 0.0   0.0  "XGP3" 0.0  0.0 "XGP4" 0.0
J=6.  "XGP1" 0.0  0.0 "XGP2" 0.0  0.0   "XGP3" 0.0  0.0 "XGP4" 0.0  0.0
J=7:  0.0 "YGP1" 0.0  0.0  "YGP2" 0.0   0.0  "YGP3" 0.0  0.0 "YGP4" 0.0
J=8:  0.0 0.0  "YGP1" 0.0  0.0  "YGP2"  0.0  0.0  "YGP3" 0.0 0.0  "YGP4"
J=9:  0.0 0.0  "XGP1" 0.0  0.0  "XGP2"  0.0  0.0  "XGP3" 0.0 0.0  "XGP4"
J=10:"XYG1" 0.0  0.0 "XYG2" 0.0   0.0   "XYG3" 0.0  0.0 "XYG4" 0.0  0.0
J=11:  0.0 "XYG1" 0.0  0.0  "XYG2" 0.0   0.0  "XYG3" 0.0  0.0 "XYG4" 0.0
!XGP1=FREE()
!YGP1=FREE()
!XGP2=FREE()
!YGP2=FREE()
!XGP3=FREE()
!YGP3=FREE()
!XGP4=FREE()
!YGP4=FREE()
!XYG1=FREE()
!XYG2=FREE()
!XYG3=FREE()
!XYG4=FREE()
$
!NU=DS 2,"ICOU",1  (16 MATP E42  0 0)
!NUP1=NU+1.0
!NU=-1.0*NU
$
TABLE(NI=12,NJ=12): 16 SIEP E42 "ICOU" 0
J=1:  1.0  "NU"  0.0    0.0 0.0 0.0   0.0 0.0 0.0   0.0 0.0 0.0
J=2:  "NU" 1.0   0.0    0.0 0.0 0.0   0.0 0.0 0.0   0.0 0.0 0.0
J=3:  0.0  0.0 "NUP1"   0.0 0.0 0.0   0.0 0.0 0.0   0.0 0.0 0.0
J=4:  0.0 0.0 0.0   1.0 "NU"   0.0    0.0 0.0 0.0   0.0 0.0 0.0
J=5:  0.0 0.0 0.0  "NU" 1.0    0.0    0.0 0.0 0.0   0.0 0.0 0.0
J=6:  0.0 0.0 0.0   0.0 0.0 "NUP1"    0.0 0.0 0.0   0.0 0.0 0.0
J=7:  0.0 0.0 0.0   0.0 0.0 0.0   1.0 "NU"    0.0   0.0 0.0 0.0
J=8:  0.0 0.0 0.0   0.0 0.0 0.0   "NU" 1.0    0.0   0.0 0.0 0.0
J=9:  0.0 0.0 0.0   0.0 0.0 0.0   0.0 0.0 "NUP1"    0.0 0.0 0.0
J=10:  0.0 0.0 0.0   0.0 0.0 0.0   0.0 0.0 0.0   1.0 "NU" 0.0
```

```
J=11:  0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0   "NU" 1.0 0.0
J=12:  0.0 0.0 0.0   0.0 0.0 0.0  0.0 0.0 0.0   0.0 0.0 "NUP1"
!NU=FREE()
!NUP1=FREE()
$
!TEMP=FREE()
!B1=ICOU-1*11
DEFINE A2=1 ESB E42 1 1
TABLE(NI=11, NJ=1):17 BBB E42 "ICOU" 0
TRANSFERR(SOURCE=A2, ILIM=11, SBASE="B1")
!B1=FREE()
$
DEFINE BB1=16 PMAT E42 "ICOU" 0
DEFINE BB2=17 BBB E42 "ICOU" 0
!SAGR=DS 8,"ICOU", 1 (16 CONN E42 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
!OOTH=OOTH*OOTH*6.0
16 GPST E42 "ICOU" O= RPROD ("OOTH" BB1,BB2)
!OOTH=FREE()
!SAGR=FREE()
$
!ICOU=ICOU+1
!TEMP=DE42-ICOU+1
*JGZ(TEMP,200)
!B1=FREE()
!ICOU=FREE()
!TEMP=FREE()
$
*XQT DCU
ERASE 17
*RETURN
*                         ENDPE42
$
$-----------------------------------------------------------------------
*(29 SECA E42 0 0)                  ENDSC42
$
*XQT AUS
$
!ICOU=1
!TEMP=0
*LABEL 200
$
!ELGR=DS 1,"ICOU",1   (1 ED42 REL 0 0 )
!TEMP=ELGR-ICDV
!ELGR
!ICDV
!ELGR=FREE()
*JNZ(TEMP,195)
$
!B1=ICOU-1*11
DEFINE A2=1 ESB E42 "ADLC"  1
TABLE(NI=11, NJ=1):17 BBB E42 "ICOU" "CLC"
TRANSFER(SOURCE=A2, ILIM=11, SBASE="B1")
!B1=FREE()
$
DEFINE BB1=16 PMAT E42 "ICOU" 0
DEFINE BB2=17 BBB E42 "ICOU" "CLC"
!SAGR=DS 8,"ICOU", 1 (16 CONN E42 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
!OOTH=OOTH*OOTH*6.0
```

```
17 GPST E42 "ICOU" "CLC" = RPROD ("OOTH"  BB1,BB2)
!OOTH=FREE()
!SAGR=FREE()
$
DEFINE B11=16 SIEP E42 "ICOU" O
DEFINE B12=17 GPST E42 "ICOU" "CLC"
16 GPEP E42 "ICOU" "CLC"= RPROD (B11,B12)
$
!SI1 =DS 1,1,1 (16 GPST E42 "ICOU" O)
!SI2 =DS 2,1,1 (16 GPST E42 "ICOU" O)
!SI3 =DS 3,1,1 (16 GPST E42 "ICOU" O)
!SI4 =DS 4,1,1 (16 GPST E42 "ICOU" O)
!SI5 =DS 5,1,1 (16 GPST E42 "ICOU" O)
!SI6 =DS 6,1,1 (16 GPST E42 "ICOU" O)
!SI7 =DS 7,1,1 (16 GPST E42 "ICOU" O)
!SI8 =DS 8,1,1 (16 GPST E42 "ICOU" O)
!SI9 =DS 9,1,1 (16 GPST E42 "ICOU" O)
!SI10=DS 10,1,1 (16 GPST E42 "ICOU" O)
!SI11=DS 11,1,1 (16 GPST E42 "ICOU" O)
!SI12=DS 12,1,1 (16 GPST E42 "ICOU" O)
!EP1 =DS 1,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP2 =DS 2,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP3 =DS 3,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP4 =DS 4,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP5 =DS 5,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP6 =DS 6,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP7 =DS 7,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP8 =DS 8,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP9 =DS 9,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP10=DS 10,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP11=DS 11,1,1 (16 GPEP E42 "ICOU" "CLC" )
!EP12=DS 12,1,1 (16 GPEP E42 "ICOU" "CLC" )
!SI3=-1.0*SI3
!EP3=-1.0*EP3
!SI6=-1.0*SI6
!EP6=-1.0*EP6
!SI9=-1.0*SI9
!EP9=-1.0*EP9
!SI12=-1.0*SI12
!EP12=-1.0*EP12
$
!EP1=SI1*EP1
!EP2=SI2*EP2
!EP3=2.0*SI3*EP3
!DETE=DS 1,1,1 (16 DETE E42 "ICOU" O)
!EP3=EP1+EP2+EP3*DETE
!EP4=SI4*EP4
!EP5=SI5*EP5
!EP6=2.0*SI6*EP6
!DETE=DS 2,1,1 (16 DETE E42 "ICOU" O)
!EP6=EP4+EP5+EP6*DETE
!EP7=SI7*EP7
!EP8=SI8*EP8
!EP9=2.0*SI9*EP9
!DETE=DS 3,1,1 (16 DETE E42 "ICOU" O)
!EP9=EP7+EP8+EP9*DETE
!EP10=SI10*EP10
!EP11=SI11*EP11
!EP12=2.0*SI12*EP12
!DETE=DS 4,1,1 (16 DETE E42 "ICOU" O)
```

```
!EP12=EP10+EP11+EP12*DETE
!PSIS=1.0*0.0
!PSIS
!PSIS=EP3+EP6+EP9+EP12
!NU=DS 1,"ICOU",1  (16 MATP E42  0 0)
!PSIS=-PSIS/NU
!PSI=PSI+PSIS
!ICOU
!NU=FREE()
!SAGR=FREE()
!OOTH=FREE()
$
!SI21=SI2*SI1
!SI54=SI5*SI4
!SI87=SI8*SI7
!SI01=SI10*SI11
$
!SI1=SI1*SI1
!SI2=SI2*SI2
!SI3=SI3*SI3
!SI4=SI4*SI4
!SI5=SI5*SI5
!SI6=SI6*SI6
!SI7=SI7*SI7
!SI8=SI8*SI8
!SI9=SI9*SI9
!SI10=SI10*SI10
!SI11=SI11*SI11
!SI12=SI12*SI12
$
!SI3=SI3*3.0+SI1+SI2-SI21
!DETE=DS 1,1,1 (16 DETE E42 "ICOU" 0)
!SI3=SI3**0.5*DETE
!SI6=SI6*3.0+SI4+SI5-SI54
!DETE=DS 2,1,1 (16 DETE E42 "ICOU" 0)
!SI6=SI6**0.5*DETE
!SI9=SI9*3.0+SI7+SI8-SI87
!DETE=DS 3,1,1 (16 DETE E42 "ICOU" 0)
!SI9=SI9**0.5*DETE
!SI12=SI12*3.0+SI10+SI11-SI01
!DETE=DS 4,1,1 (16 DETE E42 "ICOU" 0)
!SI12=SI12**0.5*DETE
!VMST=SI3+SI6+SI9+SI12
!AREA=DS 1,"ICOU",1 (16 AREA E42 0 0)
!VMST=VMST/AREA
!VMST
!TEMP=CLC-CCOM-CDIS-CS21-CS41-1
*JLZ(TEMP,155)
!TEMP=TEMP+1-CS42
*JOZ(TEMP,155)
!TEMP=CLC-CCOM-CDIS-CS21-CS41
!JENT=DS "TEMP",1 ,1  (1  ST42 LIST 1 1)
!JENT
!TEMP=ICOU-JENT
!JENT=FREE()
*JNZ(TEMP,155)
$
!SAGR=DS 8,"ICOU", 1 (16 CONN E42 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
!PSI
```

ORIGINAL PAGE IS
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```
!PSIS
!PSIS=VMST*OOTH+PSIS
!PSI=VMST*OOTH+PSI
!PSI
!PSIS
!SAGR=FREE()
!OOTH=FREE()
!DELT=FREE()
$
*LABEL 155
!TEMP=FREE()
$
!PSIS
!PSIS=FREE()
!SI21=FREE()
!SI54=FREE()
!SI87=FREE()
!SIO1=FREE()
!TEM1=FREE()
!SI1 =FREE()
!SI2 =FREE()
!SI3 =FREE()
!SI4 =FREE()
!SI5 =FREE()
!SI6 =FREE()
!SI7 =FREE()
!SI8 =FREE()
!SI9 =FREE()
!SI10=FREE()
!SI11=FREE()
!SI12=FREE()
!EP1 =FREE()
!EP2 =FREE()
!EP3 =FREE()
!EP4 =FREE()
!EP5 =FREE()
!EP6 =FREE()
!EP7 =FREE()
!EP8 =FREE()
!EP9 =FREE()
!EP10=FREE()
!EP11=FREE()
!EP12=FREE()
!DETE=FREE()
!AREA=FREE()
$
*LABEL 195
!ICOU=ICOU+1
!TEMP=DE42-ICOU+1
*JGZ(TEMP,200)
!B1=FREE()
!ICOU=FREE()
!TEMP=FREE()
$
*XQT U1
*SHOW
*RETURN
*                                              ENDSC42
$
$----------------------------------------------------------------------
```

```
$
*(29 ADJO E42 0 0)                           ENDAD42
$
*XQT U1
!GP1=0.57735*0.5
!JENT
!A0=DS 2,1,1 (16 ELEM REFE "JENT" 0)
!B0=DS 4,2,1 (16 ELEM REFE "JENT" 0)
!A00=A0*A0
!B00=B0*B0
!A01=2.0/A0
!B01=2.0/B0
!AB1=A0*B0
!AB0=2.0/AB1
!AB1=FREE()
$
!ROW=0
!PXI=0.5-GP1
!PET=0.5-GP1
*LABEL 55
!A123=12.0*PXI-6.0
!A2=1.0-PET
!B11=A123*A2/A00
!B12=0.0
!B13=-6.0*PXI+4.0*A2/A0
!B14=-B11
!B15=0.0
!B16=-6.0*PXI+2.0*A2/A0
!B17=-2.0*PXI+1.0*PET*6.0/A00
!B18=0.0
!B19=-3.0*PXI+1.0*PET*2.0/A0
!B110=12.0*PXI-6.0*PET/A00
!B111=0.0
!B112=-3.0*PXI+2.0*PET*2.0/A0
!ROW=ROW+1
*II(17 AROW E42 "ROW" 0)
"B11"
"B12"
"B13"
"B14"
"B15"
"B16"
"B17"
"B18"
"B19"
"B110"
"B111"
"B112"
!A123=12.0*PET-6.0
!A2=1.0-PXI
!B11=A123*A2/B00
!B12=6.0*PET-4.0*A2/B0
!B13=0.0
!B14=2.0*PET-1.0*PXI*6.0/B00
!B15=3.0*PET-2.0*PXI*2.0/B0
!B16=0.0
!B17=-B14
!B18=3.0*PET-1.0*PXI*2.0/B0
!B19=0.0
!B110=-B11
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
!B111=6.0*PET-2.0*A2/BO
!B112=0.0
!ROW=ROW+1
*TI(17 AROW E42 "ROW" O)
"B11"
"B12"
"B13"
"B14"
"B15"
"B16"
"B17"
"B18"
"B19"
"B110"
"B111"
"B112"
!A123=4.0*PET
!A2=4.0*PXI
!A3=PET-1.0*PET*6.0
!A4=PXI-1.0*PXI*6.0
!B11=-1.0-A3-A4*AB0
!B12=-PET*PET*3.0+A123-1.0*A01
!B13-PXI*PXI*3.0-A2+1.0*B01
!B14=-B11
!B15=-B12
!B16=3.0*PXI-2.0*PXI*B01
!B17=-B14
!B18=3.0*PET-2.0*PET*A01
!B19=-B16
!B110=B14
!B111=-B18
!B112=-B13
!ROW=ROW+1
*TI(17 AROW E42 "ROW" O)
"B11"
"B12"
"B13"
"B14"
"B15"
"B16"
"B17"
"B18"
"B19"
"B110"
"B111"
"B112"
!TEMP=ROW-3
*JNZ(TEMP,67)
!PXI=0.5+CP1
!PET=0.5-CP1
*JUMP 77
*LABEL 67
!TEMP=ROW-6
*JNZ(TEMP,69)
!PXI-0.5+CP1
!PET=0.5+CP1
*JUMP 77
*LABEL 69
!TEMP=ROW-9
*JNZ(TEMP,77)
```

ORIGINAL PAGE IS
OF POOR QUALITY

ORIGINAL PAGE IS
OF POOR QUALITY

```
!PXI=0.5-GP1
!PET=0.5+GP1
*LABEL 77
!TEMP=ROW-12
*JNZ(TEMP,55)
$
!ROW=FREE()
!A123=FREE()
!A2=FREE()
!A3=FREE()
!A4=FREE()
!A0=FREE()
!B0=FREE()
!A00=FREE()
!B00=FREE()
!A01=FREE()
!B01=FREE()
!AB0=FREE()
!GP1=FREE()
!PXI=FREE()
!PET=FREE()
!B11=FREE()
!B12=FREE()
!B13=FREE()
!B14=FREE()
!B15=FREE()
!B16=FREE()
!B17=FREE()
!B18=FREE()
!B19=FREE()
!B110=FREE()
!B111=FREE()
!B112=FREE()
$
!EMOD=DS 1,"JENT",1   (16 MATP E42  0 0)
!NUE=DS 2,"JENT",1   (16 MATP E42  0 0)
!NUE2=NUE*NUE
!NUE2=1.0-NUE2
!E11=EMOD/NUE2
!E12=E11*NUE
!NUE2=1.0+NUE
!E33=0.5*EMOD/NUE2
!EMOD=FREE()
!NUE=FREE()
!NUE2=FREE()
*TI(17 DMAT E42 "ADLC" 0)
"E11" "E12" 0.0   0.0  0.0  0.0   0.0  0.0  0.0   0.0  0.0  0.0
"E12" "E11" 0.0   0.0  0.0  0.0   0.0  0.0  0.0   0.0  0.0  0.0
 0.0   0.0 "E33"  0.0  0.0  0.0   0.0  0.0  0.0   0.0  0.0  0.0
 0.0   0.0  0.0  "E11" "E12" 0.0  0.0  0.0  0.0   0.0  0.0  0.0
 0.0   0.0  0.0  "E12" "E11" 0.0  0.0  0.0  0.0   0.0  0.0  0.0
 0.0   0.0  0.0   0.0  0.0 "E33"  0.0  0.0  0.0   0.0  0.0  0.0
 0.0   0.0  0.0   0.0  0.0  0.0  "E11" "E12" 0.0  0.0  0.0  0.0
 0.0   0.0  0.0   0.0  0.0  0.0  "E12" "E11" 0.0  0.0  0.0  0.0
 0.0   0.0  0.0   0.0  0.0  0.0   0.0  0.0 "E33"  0.0  0.0  0.0
 0.0   0.0  0.0   0.0  0.0  0.0   0.0  0.0  0.0  "E11" "E12" 0.0
 0.0   0.0  0.0   0.0  0.0  0.0   0.0  0.0  0.0  "E12" "E11" 0.0
 0.0   0.0  0.0   0.0  0.0  0.0   0.0  0.0  0.0   0.0  0.0 "E33"
!E11=FREE()
!E12=FREE()
```

```
!E33=FREE()
$
*XQT AUS
DEFINE A1=17 AROW E42 1 0
DEFINE A2=17 AROW E42 2 0
DEFINE A3=17 AROW E42 3 0
DEFINE A4=17 AROW E42 4 0
DEFINE A5=17 AROW E42 5 0
DEFINE A6=17 AROW E42 6 0
DEFINE A7=17 AROW E42 7 0
DEFINE A8=17 AROW E42 8 0
DEFINE A9=17 AROW E42 9 0
DEFINE A10=17 AROW E42 10 0
DEFINE A11=17 AROW E42 11 0
DEFINE A12=17 AROW E42 12 0
TABLE(NI=12, NJ=12):16 BB E42 "ADLC" 99
TRANSFER(SOURCE=A1, ILIM=12, DBASE=0)
TRANSFER(SOURCE=A2, ILIM=12, DBASE=12)
TRANSFER(SOURCE=A3, ILIM=12, DBASE=24)
TRANSFER(SOURCE=A4, ILIM=12, DBASE=36)
TRANSFER(SOURCE=A5, ILIM=12, DBASE=48)
TRANSFER(SOURCE=A6, ILIM=12, DBASE=60)
TRANSFER(SOURCE=A7, ILIM=12, DBASE=72)
TRANSFER(SOURCE=A8, ILIM=12, DBASE=84)
TRANSFER(SOURCE=A9, ILIM=12, DBASE=96)
TRANSFER(SOURCE=A10, ILIM=12, DBASE=108)
TRANSFER(SOURCE=A11, ILIM=12, DBASE=120)
TRANSFER(SOURCE=A12, ILIM=12, DBASE=132)
DEFINE A2=16 BB E42 "ADLC" 99
16 BB E42 "ADLC" 0=RTRAN(A2)
$
DEFINE A1=17 DMAT E42 "ADLC" 0
DEFINE A2=16 BB E42 "ADLC" 0
17 DB E42 "ADLC" 0=RPROD(A1,A2)
!IDOF=1
*LABEL 400
!SI1 =DS 1,1,1 (16 GPST E42 "JENT" 0)
!SI2 =DS 2,1,1 (16 GPST E42 "JENT" 0)
!SI3 =DS 3,1,1 (16 GPST E42 "JENT" 0)
!SI4 =DS 4,1,1 (16 GPST E42 "JENT" 0)
!SI5 =DS 5,1,1 (16 GPST E42 "JENT" 0)
!SI6 =DS 6,1,1 (16 GPST E42 "JENT" 0)
!SI7 =DS 7,1,1 (16 GPST E42 "JENT" 0)
!SI8 =DS 8,1,1 (16 GPST E42 "JENT" 0)
!SI9 =DS 9,1,1 (16 GPST E42 "JENT" 0)
!SI10=DS 10,1,1 (16 GPST E42 "JENT" 0)
!SI11=DS 11,1,1 (16 GPST E42 "JENT" 0)
!SI12=DS 12,1,1 (16 GPST E42 "JENT" 0)
!EP1 =DS 1,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP2 =DS 2,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP3 =DS 3,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP4 =DS 4,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP5 =DS 5,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP6 =DS 6,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP7 =DS 7,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP8 =DS 8,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP9 =DS 9,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP10=DS 10,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP11=DS 11,"IDOF",1 (17 DB E42 "ADLC" 0)
!EP12=DS 12, "IDOF",1 (17 DB E42 "ADLC" 0)
```

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```
$
!SI3=-1.0*SI3
!SI6=-1.0*SI6
!SI9=-1.0*SI9
!SI12=-1.0*SI12
!OOS2=1.0/2.0
!VM1=2.0*SI1-SI2
!VM1=OOS2*VM1
!VM2=2.0*SI2-SI1
!VM2=OOS2*VM2
!VM3=6.0*SI3*OOS2
!VM4=2.0*SI4-SI5
!VM4=OOS2*VM4
!VM5=2.0*SI5-SI4
!VM5=OOS2*VM5
!VM6=6.0*SI6*OOS2
!VM7=2.0*SI7-SI8
!VM7=OOS2*VM7
!VM8=2.0*SI8-SI7
!VM8=OOS2*VM8
!VM9=6.0*SI9*OOS2
!VM10=2.0*SI10-SI11
!VM10=OOS2*VM10
!VM11=2.0*SI11-SI10
!VM11=OOS2*VM11
!VM12=6.0*SI12*OOS2
!OOS2=FREE()
$
!SI21=SI2*SI1
!SI54=SI5*SI4
!SI87=SI8*SI7
!SI01=SI10*SI11
$
!SI1=SI1*SI1
!SI2=SI2*SI2
!SI3=SI3*SI3
!SI4=SI4*SI4
!SI5=SI5*SI5
!SI6=SI6*SI6
!SI7=SI7*SI7
!SI8=SI8*SI8
!SI9=SI9*SI9
!SI10=SI10*SI10
!SI11=SI11*SI11
!SI12=SI12*SI12
$
!SI3=SI3*3.0+SI1+SI2-SI21
!SI3=SI3**0.5
!SI6=SI6*3.0+SI4+SI5-SI54
!SI6=SI6**0.5
!SI9=SI9*3.0+SI7+SI8-SI87
!SI9=SI9**0.5
!SI12=SI12*3.0+SI10+SI11-SI01
!SI12=SI12**0.5
$
!VM1=VM1/SI3
!VM2=VM2/SI3
!VM3=VM3/SI3
!VM4=VM4/SI6
!VM5=VM5/SI6
```

```
!VM6=VM6/SI6
!VM7=VM7/SI9
!VM8=VM8/SI9
!VM9=VM9/SI9
!VM10=VM10/SI12
!VM11=VM11/SI12
!VM12=VM12/SI12
!VM1=VM1*EP1
!VM2=VM2*EP2
!VM3=VM3*EP3
!VM4=VM4*EP4
!VM5=VM5*EP5
!VM6=VM6*EP6
!VM7=VM7*EP7
!VM8=VM8*EP8
!VM9=VM9*EP9
!VM10=VM10*EP10
!VM11=VM11*EP11
!VM12=VM12*EP12
!DETE=DS 1,1,1 (16 DETE E42 "JENT" 0)
!VM3=VM3+VM2+VM1*DETE
!DETE=DS 2,1,1 (16 DETE E42 "JENT" 0)
!VM6=VM6+VM5+VM4*DETE
!DETE=DS 3,1,1 (16 DETE E42 "JENT" 0)
!VM9=VM9+VM8+VM7*DETE
!DETE=DS 4,1,1 (16 DETE E42 "JENT" 0)
!VM12=VM12+VM11+VM10*DETE
!AREA=DS 1,"JENT",1 (16 AREA E42 0 0)
!SAGR=DS 8,"JENT",1 (16 CONN E42 0 0)
!OOTH=DS 26,"SAGR",1 (1 SA BTAB 2 13)
!VM12=VM12+VM9+VM6+VM3
!VM12=VM12/AREA
!VM12=-0.5*VM12
!VM12=VM12/OOTH
!AREA=FREE()
!SAGR=FREE()
!OOTH=FREE()
$
!TEM2=IDOF-1
*JNZ(TEM2,100)
!AD1=1.0*VM12
*JUMP 300
*LABEL 100
!TEM2=IDOF-2
*JNZ(TEM2,110)
!AD2=1.0*VM12
*JUMP 300
*LABEL 110
!TEM2=IDOF-3
*JNZ(TEM2,120)
!AD3=1.0*VM12
*JUMP 300
*LABEL 120
!TEM2=IDOF-4
*JNZ(TEM2,130)
!AD4=1.0*VM12
*JUMP 300
*LABEL 130
!TEM2=IDOF-5
*JNZ(TEM2,140)
```

```
!AD5=1.0*VM12
*JUMP 300
*LABEL 140
!TEM2=IDOF-6
*JNZ(TEM2,150)
!AD6=1.0*VM12
*JUMP 300
*LABEL 150
!TEM2=IDOF-7
*JNZ(TEM2,160)
!AD7=1.0*VM12
*JUMP 300
*LABEL 160
!TEM2=IDOF-8
*JNZ(TEM2,170)
!AD8=1.0*VM12
*JUMP 300
*LABEL 170
!TEM2=IDOF-9
*JNZ(TEM2,180)
!AD9=1.0*VM12
*JUMP 300
*LABEL 180
!TEM2=IDOF-10
*JNZ(TEM2,190)
!AD10=1.0*VM12
*JUMP 300
*LABEL 190
!TEM2=IDOF-11
*JNZ(TEM2,200)
!AD11=1.0*VM12
*JUMP 300
*LABEL 200
!TEM2=IDOF-12
*JNZ(TEM2,210)
!AD12=1.0*VM12
*LABEL 210
$
*LABEL 300
!IDOF=IDOF+1
!TEM2=IDOF-13
*JNZ (TEM2, 400)
!TEM2=FREE()
!IDOF=FREE()
!SI21=FREE()
!SI54=FREE()
!SI87=FREE()
!S101=FREE()
!SI1 =FREE()
!SI2 =FREE()
!SI3 =FREE()
!SI4 =FREE()
!SI5 =FREE()
!SI6 =FREE()
!SI7 =FREE()
!SI8 =FREE()
!SI9 =FREE()
!SI10=FREE()
!SI11=FREE()
!SI12=FREE()
```

```
!EP1 =FREE()
!EP2 =FREE()
!EP3 =FREE()
!EP4 =FREE()
!EP5 =FREE()
!EP6 =FREE()
!EP7 =FREE()
!EP8 =FREE()
!EP9 =FREE()
!EP10=FREE()
!EP11=FREE()
!EP12=FREE()
!VM1=FREE()
!VM2=FREE()
!VM3=FREE()
!VM4=FREE()
!VM5=FREE()
!VM6=FREE()
!VM7=FREE()
!VM8=FREE()
!VM9=FREE()
!VM10=FREE()
!VM11=FREE()
!VM12=FREE()
!DETE=FREE()
$
$  NOW ASSIGN LOAD TO PROPER DEGREES OF FREEDOM
!VMST
DEFINE BB2=16 ELNO E42 0 0
TABLE(NI=24, NJ=24): 17 ROTA ELEM "ADLC" 99
!A1=JENT-1*36
!B1=0
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+3
!B1=24
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+6
!B1=48
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+9
!B1=72+3
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+12
!B1=96+3
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+15
!B1=120+3
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+18
!B1=144+6
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+21
!B1=168+6
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+24
!B1=192+6
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
!A1=JENT-1*36+27
!B1=216+9
TRANSFERR(SOURCE=BB2, ILIM=3, SBASE="A1",DBASE="B1")
```

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```
!A1=JENT-1*36+30
!B1=240+9
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+33
!B1=264+9
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36
!B1=288+12
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+3
!B1=312+12
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+6
!B1=336+12
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+9
!B1=360+15
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+12
!B1=384+15
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+15
!B1=408+15
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+18
!B1=432+18
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+21
!B1=456+18
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+24
!B1=480+18
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+27
!B1=504+21
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+30
!B1=528+21
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=JENT-1*36+33
!B1=552+21
TRANSFERR(SOURCE=BB2,  ILIM=3,  SBASE="A1",DBASE="B1")
!A1=FREE()
!B1=FREE()
TABLE(NI=1,  NJ=24):17 ELLO E42 "ADLC" 99
J=1   :    0.0
J=2   :    0.0
J=3   :    "AD1"
J=4   :    "AD2"
J=5   :    "AD3"
J=6   :    0.0
J=7   :    0.0
J=8   :    0.0
J=9   :    "AD4"
J=10  :    "AD5"
J=11  :    "AD6"
J=12  :    0.0
J=13  :    0.0
J=14  :    0.0
J=15  :    "AD7"
```

```
J=16 :   "AD8"
J=17 :   "AD9"
J=18 :   0.0
J=19 :   0.0
J=20 :   0.0
J=21 :   "AD10"
J=22 :   "AD11"
J=23 :   "AD12"
J=24 :   0.0
DEFINE A1 = 17 ELLO E42 "ADLC" 99
17 ELLO E42 "ADLC" 1 =RTRAN ( A1 )
DEFINE A1 = 17 ROTA ELEM "ADLC" 99
17 ROTA ELEM "ADLC" 1 =RTRAN( A1 )
DEFINE BB2= 17 ELLO E42 "ADLC" 1
DEFINE A1 = 17 ROTA ELEM "ADLC" 1
16 ADLO VE42 "ADLC" 0 = RPROD ( A1, BB2)
!AD1=DS 1,1,1  (16 ADLO VE42 "ADLC" 0)
!AD2=DS 2,1,1  (16 ADLO VE42 "ADLC" 0)
!AD3=DS 3,1,1  (16 ADLO VE42 "ADLC" 0)
!AD4=DS 7,1,1  (16 ADLO VE42 "ADLC" 0)
!AD5=DS 8,1,1  (16 ADLO VE42 "ADLC" 0)
!AD6=DS 9,1,1  (16 ADLO VE42 "ADLC" 0)
!AD7=DS 13,1,1  (16 ADLO VE42 "ADLC" 0)
!AD8=DS 14,1,1  (16 ADLO VE42 "ADLC" 0)
!AD9=DS 15,1,1  (16 ADLO VE42 "ADLC" 0)
!AD10=DS 19,1,1  (16 ADLO VE42 "ADLC" 0)
!AD11=DS 20,1,1  (16 ADLO VE42 "ADLC" 0)
!AD12=DS 21,1,1  (16 ADLO VE42 "ADLC" 0)
$
!J1=DS 13,"JENT",1  (16 CONN E42 0 0)
!J2=DS 14,"JENT",1  (16 CONN E42 0 0)
!J3=DS 15,"JENT",1  (16 CONN E42 0 0)
!J4=DS 16,"JENT",1  (16 CONN E42 0 0)
SYSVEC: APPL FORC "ADLC" 1
I=1
J="J1" :  "AD1"
J="J2" :  "AD4"
J="J3" :  "AD7"
J="J4" :  "AD10"
I=2
J="J1" :  "AD2"
J="J2" :  "AD5"
J="J3" :  "AD8"
J="J4" :  "AD11"
I=3
J="J1" :  "AD3"
J="J2" :  "AD6"
J="J3" :  "AD9"
J="J4" :  "AD12"
!AD1=DS 4,1,1  (16 ADLO VE42 "ADLC" 0)
!AD2=DS 5,1,1  (16 ADLO VE42 "ADLC" 0)
!AD3=DS 6,1,1  (16 ADLO VE42 "ADLC" 0)
!AD4=DS 10,1,1  (16 ADLO VE42 "ADLC" 0)
!AD5=DS 11,1,1  (16 ADLO VE42 "ADLC" 0)
!AD6=DS 12,1,1  (16 ADLO VE42 "ADLC" 0)
!AD7=DS 16,1,1  (16 ADLO VE42 "ADLC" 0)
!AD8=DS 17,1,1  (16 ADLO VE42 "ADLC" 0)
!AD9=DS 18,1,1  (16 ADLO VE42 "ADLC" 0)
!AD10=DS 22,1,1  (16 ADLO VE42 "ADLC" 0)
!AD11=DS 23,1,1  (16 ADLO VE42 "ADLC" 0)
```

```
!AD12=DS 24,1,1   (16 ADLO VE42 "ADLC" 0)
I=4
J="J1"  :  "AD1"
J="J2"  :  "AD4"
J="J3"  :  "AD7"
J="J4"  :  "AD10"
I=5
J="J1"  :  "AD2"
J="J2"  :  "AD5"
J="J3"  :  "AD8"
J="J4"  :  "AD11"
I=6
J="J1"  :  "AD3"
J="J2"  :  "AD6"
J="J3"  :  "AD9"
J="J4"  :  "AD12"
!AD1=FREE()
!AD2=FREE()
!AD3=FREE()
!AD4=FREE()
!AD5=FREE()
!AD6=FREE()
!AD7=FREE()
!AD8=FREE()
!AD9=FREE()
!AD10=FREE()
!AD11=FREE()
!AD12=FREE()
!J1=FREE()
!J2=FREE()
!J3=FREE()
!J4=FREE()
$
*XQT DCU
ERASE 17
*RETURN
*                                    ENDAD42
$-----------------------------------------------------------------------
$
$  This call of a subroutine in  EAL final starts running the
$  program. Everything before is stored as subroutines in the
$  EAL database.
$
$
*DCALL,(29 DRIV GLOB 0 0)
$
*XQT EXIT
TIME
COMO -E

COMO -E
```

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Addendum to Technical Report No. 86-2

RESULTS OF THE DESIGN SENSITIVITY ANALYSIS

The results of the design sensitivity analysis program in EAL are all stored in EAL-library file L012.

If a stress constraint design sensitivity of the appropriate element type is specified in the input control parameter, the following data sets are created to store the appropriate stress constraint values:

DVAL	E21	1	0	The values of the stress constraint functionals are given for constraints listed in the input data set ST 21 LIST
DVPO	E21	1	0	A set of pointers that indicate the location of points where maximum stress occurs in elements are given for constraints listed in input data set ST21 LIST.
DVAL	E41	1	0	The values of the stress constraint functionals are given for constraint listed in input data set ST41 LIST
DVAL	E42	1	0	The values of the stress constraint functionals are given for constraints listed in input data set ST42 LIST

The names of the data sets for design sensitivity vectors are given in the EAL-library file L012 that have the following basic form:

DSVE "ETYP" "CONT" "CIND"

where

DSVE	Design sensitivity vector
"ETYP" - E21, E41, or E42:	"ETYP" indicates the element type for which the design sensitivity vector is stored. For element types E41 and E42, the data set contains one sensitivity vector, indicating design sensitivity with respect to thickness design parameter, t. For element type E21, the data set contains two sensitivity vectors, indicating design sensitivity with respect to width, b, and height, h, of the beam.
"CONT":	In the input file for each element, a control parameter has to be specified as the following arrays: ED21 REL 0 0, ED41 REL 0 0 and/or ED42 REL 0 0. The parameter "CONT" indicates the control parameter for which a design sensitivity vector is computed.

"CIND": "CIND" is the constraint indicator

10,000	Compliance Constraint
between 21,000 and 22,000	Displacement constraint in the x-direction
between 22,000 and 23,000	Displacement constraint in the y-direction
between 23,000 and 24,000	Displacement constraint in the z-direction
betweem 30,000 and 40,000	Stress constraint in beam elements
between 40,000 and 50,000	Stress constraint in membrane elements
between 50,000 and 60,000	Stress constraint in plate elements

For displacement and stress constaints, the last three digits of the constraint indicator "CIND" give the node number of the displacement constraint and the element number of the stress constraint, respectively.