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PROJECTION FILTERS FOR MODAL PARAMETER ESTIMATE FOR FLEXIBLE STRUCTURES

By

Jen-Kuang Huang, Principal Investigator

and

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Progress Report
For the period ended December 31, 1986

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under
Research Grant NAG-1-655
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By

Jen-Kuang Huang[1] and Chung-Wen Chen[2]

ABSTRACT

Single-mode projection filters are developed for eigensystem parameter estimates from both analytical results and test data. Explicit formulations of these projection filters are derived using the pseudoinverse matrices of the controllability and observability matrices in the general sense. A global minimum optimization algorithm is developed to update the filter parameters by using interval analysis method. Modal parameters can be attracted and updated in the global sense within a specific region by passing the experimental data through the projection filters. For illustration of this new approach, a numerical example is shown by using a one-dimensional global optimization algorithm to estimate modal frequencies and dampings.

[1]Assistant Professor, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508.

[2]Research Assistant, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508.

Introduction

The problems of deriving control algorithms and state estimators for maneuvering flexible structures have been investigated by many researchers in recent years. The control design demands an accurate model of the system dynamics which will adequately describe the system's behavior. System identification methods use experimental measurements to estimate dynamic properties such as natural frequencies, damping factors, mode shapes and modal masses which are referred to as modal parameters. Several different time-domain and frequency-domain methods are possible for the identification of structures. Various techniques may share the same mathematical foundation via system realization theory[1]. However, most techniques do not account explicitly for the factors which will affect the actual performance in practice significantly. These factors include nonlinearities, local modes, and system and measurement noises. In order to achieve the final purpose of identification, i.e., control of flexible structures, an on-line estimation technique needs to be developed. This technique may provide updated modal parameter estimates only for specific regions needed to be controlled. On the other hand, modal parameters can also be identified by using the analytical finite element method. The result is usually used only for the comparison with the experimental one. However, the analytical result may provide valuable initial estimate for modal parameters for an on-line estimator.

For linear time-invariant systems, optimal model-reduction and state estimation has been developed via optimal projection equations based on modified Riccati and Lyapunov equations[2]. Other filtering approaches in both time and frequency domains are easy to implement and effective in rejecting uncorrelated measurement noise from simulated data[3]. Although filtering approaches are not restricted to linear systems, time-domain filters usually involve unacceptable computational burden for multi-mode system like large flexible structure.

The objective of this paper is to introduce simple projection filters for modal state estimate. These filters are formulated with a single mode only and their explicit expressions can be derived using the pseudoinverse of the controllability and observability matrices in the general sense. Filter parameters are initially implemented from the analytical model and updated by real data using a global minimum optimization algorithm. The global minimum optimization algorithm is developed by using interval analysis method. Since the filters are developed in modal space, system modal parameters within a specific region are, as a by-product, identified.

Finally, a numerical example for a ten-mode structure is given to illustrate this new method. A one-dimensional global optimization algoritm is also developed and guarantees to find the smallest value of a cost function throughout a specific closed region of modal parameters.

Projection Filters Formulations

The projection filters are developed based on system realization theory. A finite-dimensional, linear, time-invariant dynamic system can be represented by the state-variable equations in discrete-time form:

$$x(k+1) = A\,x(k) + B\,u(k) \tag{1}$$

$$y(k) = C\,x(k) \tag{2}$$

where x is an n-dimensional state vector, u is an m-dimensional control or input vector, and y is a p-dimensional measurement or output vector. The integer k is the sample indicator. For flexible structures, the state transition matrix A is a representation of mass, stiffness, and damping properties. The control influence matrix B charaterizes the locations and type of input control vector u. The measurement influence matrix C describes the relationship between the state vector x and the output measurement vector y, and characterizes the mode shapes of the system.

For the state-variable Eqs. (1) and (2) with free pulse response, the time domain description is given by the function known as the Markov parameter

$$Y(k) = CA^{k-1} B \tag{3}$$

or in the case of initial state response

$$Y(k) = CA^{k} x(0)$$

where x(0) represents the initial conditions of state vector and k is an integer. The functions Y(k) can be obtained from the experimental data and used to form the (r+1) by (s+1) block data matrix (generalized Hankel matrix)

$$H(k-1) = \begin{bmatrix} Y(k) & Y(k+t_1) \; \ldots\ldots & Y(k+t_s) \\ Y(j_1+k) & Y(j_1+k+t_1) \; \ldots\ldots & Y(j_1+k+t_s) \\ \vdots & \vdots & \vdots \\ Y(j_r+k) & Y(j_r+k+t_1) \; \ldots\ldots & Y(j_r+k+t_s) \end{bmatrix} \tag{4}$$

where $j_i(i=1,\ldots,r)$ and $t_i(i=1,\ldots,s)$ are arbitrary integers. For the system with initial state response measurements, simply replace H(k-1) by H(k).

From Eqs. (3) and (4), it can be shown that

$$H(k) = V_r A^k W_s; \quad V_r = \begin{bmatrix} C \\ CA^{j_1} \\ \vdots \\ CA^{j_r} \end{bmatrix}$$

and

$$W_s = [B, A^{t_1} B, \ldots, A^{t_s} B] \tag{5}$$

where V_r and W_s are generalized observability and controllability matrices. The dimensions of V_r and W_s are $(r+1)p \times n$ and $n \times m(s+1)$ respectively. Now observe that

$$H(0) = V_r W_s \tag{6}$$

we can derive

$$V_r^{\#} H(0) W_s^{\#} = I_n \tag{7}$$

where $V_r^{\#}$ and $W_s^{\#}$ are the pseudoinverse matrices of V_r and W_s respectively in a general sense. I_n is an identity matrix of order n. Now, instead of having the matrix $V_r^{\#}$ and $W_s^{\#}$ for n-dimensional multi-mode system, we develop simpler forms of $V^{\#}$ and $W^{\#}$ which represent the pseudoinverse matrices of respective generalized observability and controllability matrices derived from a single-mode model only. Note that $V^{\#}$ and $W^{\#}$ are rectangular matrices with dimensions $(r+1)x2$ and $2x(s+1)$, respectively. The general explicit expressions of $V^{\#}$ and $W^{\#}$ will be derived later. The matrices $V^{\#}$ and $W^{\#}$, which are formulated only for specific modes of interest from the analytical results, will be used as the left and right projection filter respectively. The Hankel matrix H(0), which is formed by experimental data, will then run through the projection filters to attract the system modal parameters. If the projection filters have the same modal characteristics as the actual system does, then from Eq. (7) we have

$$V^{\#} H(0) W^{\#} = I_2 \tag{8}$$

where I_2 is a 2x2 identity matrix. Otherwise, we have

$$V^{\#} H(0) W^{\#} = 0 \tag{9}$$

This indicates that the modal parameters of the projection filters are different from those of the actual system. The projection filters should be tuned in order to match the actual modes. The algorithm for filter update is developed in the next section.

Now, the explicit expressions of the projection filters $V^{\#}$ and $W^{\#}$ can be derived as follows. A single mode, continuous-time, linear, time-invariant dynamical system has the state-variable equations in modal space

$$\dot{x} = \bar{A}\, x + \bar{B}\, u \tag{10}$$

$$y = \bar{C}\, x \tag{11}$$

with

$$\bar{A} = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix} \tag{12}$$

where ω is the modal frequency and σ is the damping. The corresponding discrete-time system can be represented by Eqs. (1) and (2) with

$$A = \begin{bmatrix} e^{-\sigma T} \cos\omega T & e^{-\sigma T} \sin\omega T \\ -e^{-\sigma T} \sin\omega T & e^{-\sigma T} \cos\omega T \end{bmatrix} \tag{13}$$

and

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad C = [c_1 \quad c_2] \tag{14}$$

where T is the sampling time and b_1 , b_2, c_1, c_2 are scalars. From Eq. (5) with $j_o = t_o = 0$

$$V = \begin{bmatrix} CA^{j_o} \\ CA^{j_1} \\ \vdots \\ CA^{j_r} \end{bmatrix} = [c_2 \, V_1(r) + c_1 \, V_2\,(r), \;\; - c_1 \, V_1(r) + c_2 \, V_2(r)] \qquad (15)$$

$$W = [A^{t_o} B, A^{t_1} B, \ldots, A^{t_s} B] = \begin{bmatrix} -b_2 \, V_3^T(s) + b_1 \, V_4^T(s) \\ b_1 \, V_3^T(s) + b_2 \, V_4^T(s) \end{bmatrix} \qquad (16)$$

where

$$V_1^T(r) = [\ldots, -e^{-j_i \sigma T} \sin(j_i \omega T), \ldots] \qquad (17)$$

$$V_2^T(r) = [\ldots, \; e^{-j_i \sigma T} \cos(j_i \omega T), \ldots] \qquad (18)$$

with $i = 0, 1, 2, \ldots, r$

$$V_3^T(s) = [\ldots, -e^{-t_k \sigma T} \sin(t_k \omega T), \ldots] \qquad (19)$$

$$V_4^T(s) = [\ldots, e^{-t_k \sigma T} \cos(t_k \omega T), \ldots] \qquad (20)$$

with $k = 0, 1, 2, \ldots, s$

Assume we choose j_i, t_k as follows

$$j_r - j_{r-i} = j_i \qquad i = 0, 1, 2, \ldots, \text{integer } [\tfrac{r}{2}] \qquad (21)$$

$$t_s - t_{s-k} = t_k \qquad k = 0, 1, 2, \ldots, \text{integer } [\tfrac{s}{2}] \qquad (22)$$

Then, the projection filters, $V^{\#}$ and $W^{\#}$, may have the following explicit expressions (see Appendix for proof):

$$V^{\#} = \begin{bmatrix} \frac{1}{c_1^2 + c_2^2} (c_2 V_a^{\#}(r) + c_1 V_b^{\#}(r)) \\ \frac{1}{c_1^2 + c_2^2} (-c_1 V_a^{\#}(r) + c_2 V_b^{\#}(r)) \end{bmatrix} \tag{23}$$

$$W^{\#} = [\frac{1}{b_1^2+b_2^2} (-b_2 V_c^{\#}(s) + b_1 V_d^{\#}(s))^T, \frac{1}{b_1^2+b_2^2} (b_1 V_c^{\#}(s) + b_2 V_d^{\#}(s))^T] \tag{24}$$

where

$$V_a^{\#}(r) = [\ldots, e^{j_i \sigma T} (- \frac{\sin(j_i \omega T)}{2} (\frac{1}{\lambda_1(r)} + \frac{1}{\lambda_2(r)}) - \frac{\sin((j_r - j_i)\omega T)}{2} (\frac{1}{\lambda_1(r)} - \frac{1}{\lambda_2(r)})), \ldots] \tag{25}$$

$$V_b^{\#}(r) = [\ldots, e^{j_i \sigma T} (\frac{\cos(j_i \omega T)}{2} (\frac{1}{\lambda_1(r)} + \frac{1}{\lambda_2(r)}) + \frac{\cos((j_r - j_i)\omega T)}{2} (\frac{1}{\lambda_1(r)} - \frac{1}{\lambda_2(r)})), \ldots] \tag{26}$$

with $i = 0,1,2,\ldots,r$

and

$$V_c^{\#}(s) = [\ldots, e^{t_k \sigma T} (- \frac{\sin(t_k \omega T)}{2} (\frac{1}{\lambda_3(s)} + \frac{1}{\lambda_4(s)}) - \frac{\sin((t_s - t_k)\omega T)}{2} (\frac{1}{\lambda_3(s)} - \frac{1}{\lambda_4(s)})), \ldots] \tag{27}$$

$$V_d^{\#}(s) = [\ldots, e^{t_k \sigma T} (\frac{\cos(t_k \omega T)}{2} (\frac{1}{\lambda_3(s)} + \frac{1}{\lambda_4(s)}) + \frac{\cos((t_s - t_k)\omega T)}{2} (\frac{1}{\lambda_3(s)} - \frac{1}{\lambda_4(s)})), \ldots] \tag{28}$$

with $k = 0, 1, 2, \ldots, s$

$\lambda_1(r)$ and $\lambda_2(r)$ are the eigenvalues of $V^T V$, and $\lambda_3(s)$ and $\lambda_4(s)$ are the eigenvalues of WW^T. $\lambda_1(r)$, $\lambda_2(r)$, $\lambda_3(s)$ and $\lambda_4(s)$ can be derived as follows

$$\lambda_1(r) = \begin{cases} 1+m+Y(m) & \text{if } r \text{ is even} \\ m+Y(m) & \text{if } r \text{ is odd} \end{cases} \tag{29}$$

$$\lambda_2(r) = m-Y(m) \tag{30}$$

with

$$m = \begin{cases} \frac{r}{2} & \text{if } r \text{ is even} \\ \frac{r+1}{2} & \text{if } r \text{ is odd} \end{cases}, \qquad Y(m) = \sum_{i=0}^{m-1} \cos\left((j_r - 2j_i)\omega T\right) \tag{31}$$

$$\lambda_3(s) = \begin{cases} 1+n+Z(n) & \text{if } s \text{ is even} \\ n+Z(n) & \text{if } s \text{ is odd} \end{cases} \tag{32}$$

$$\lambda_4(s) = n-Z(n) \tag{33}$$

with

$$n = \begin{cases} \frac{s}{2} & \text{if } s \text{ is even} \\ \frac{s+1}{2} & \text{if } s \text{ is odd} \end{cases}, \qquad Z(n) = \sum_{k=0}^{n-1} \cos\left((t_s - 2t_k)\omega T\right) \tag{34}$$

Note that $V^\#$ and $W^\#$ are rectangular matrices with dimensions $2x(r+1)$ and $(s+1)x2$ respectively.

Filter Update

In order to update the projection filters to attract the actual modes from experimental data within a specific range of accuracy, a cost function is formed as follows:

$$J = \frac{1}{2} U^T U \tag{35}$$

where

$$U^T = [E_{11}, E_{12}, E_{21}, E_{22}]$$

and

$$\begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix} = V^{\#} H(0) W^{\#} - I_2$$

From Eq. (8), the cost function J would go to a minimum value (ideally, zero) when the projection filters have the same modal characteristics as the actual system does. On the other hand, for a specific region of system parameters of interest, we may update those system parameters of the projection filters within the specific region so that the cost function is globally minimized. This global minimum of the cost function in the specific region will provide the best estimates for the system parameters of the actual system. Although the cost function may be corrupted by the system or the measurement noise, the system parameters corresponding to the global minimum are expected to be quite insensitive to the noise if the noise is not particularly correlated to those parameters we estimate.

Interval Analysis: Global Minimum Optimization

The method for computing a global minimum within a specific region of system parameters is based on the algorithm developed by Hansen[5-8]. Although this algorithm can deal with problems in the multi-variable case with inequality constraints[6], only single variable[5] (either modal frequency or modal damping) is considered here. The global optimization algorithm basically uses a Newton method[8] in conjunction with the interval analysis to solve a system of nonlinear equations. The term "global minimum" used herein refers to the smallest value of the cost function J throughout a closed interval of a system parameter. Because of the interval analysis, the computatinal procedure of this algorithm requires explicit expressions of the first derivative (J') and the second derivative (J'') of the cost function J shown in Eq. (35). This can be easily derived by using the explicit expressions for the modal filters shown in Eqs. (23) and (24). The algorithm developed by Hansen[5] has been slightly modified and summarized as follows:

Initial Step: The algorithm starts with an initial interval X_0. This interval is equally subdivided into subintervals which are stored in a list L_0. A list L_1 (initially empty) consists of intervals for which the width is smaller than a specified value w_1 and the corresponding width of J is smaller than a specified value w_2. Let $\bar{x}$ denote a feasible approximation to the global minimum. If the feasible point is not given, the upper limit of the cost function is set to $\bar{J} = \infty$ with $\bar{x}$ indefinite. Let $[j_L, j_R]$, $[j_L', j_R']$ and $[j_L'', j_R'']$ denote the interval resulting from evaluating J, J' and J'' in interval arithmatic using the argument X, respectively; that is

$$J(X) = [j_L\ j_R],\ J'(X) = [j_L', j_R'],\ \ J''(X) = [j_L'', j_R''] \qquad (36)$$

and

$$X = [x_L, x_R]$$

Then, use the interval analysis to find the corresponding J, J' and J'' for all the subintervals in L_0.

Main Steps:

1. If the list L_0 is empty, go to step 11. Otherwise, find the subinterval X in L_0 for which the left endpoint of J(X), i.e. j_L, is smallest.

2. If $\bar{x} \in X$, set $x = \bar{x}$. Otherwise, set x=m(X) = midpoint of X. If $j_L > \bar{J}$, the cost of any point inside the intervals in L_0 exceeds the upper limit $\bar{J}$. Then L_0 is set empty and go to step 11.

3. Concavity check.

   If $j_R'' < 0$, J is concave in X and cannot have a minimum in the interior of X. Then, X is deleted and go to step 1.

4. Monotonicity Check

   If $j_R' < 0$ or $j_L' > 0$, the gradient of J is strictly positive or strictly negative over X. Then, X is deleted and go to step 1.

5. Gaussian elimination

   Denote $E = \bar{J} - J(x)$, $\Delta = [J'(x)]^2 + 2Ej_L''$

   If $j_L'' > 0$ and $\Delta>0$, it implies that $J(y)>\bar{J}$ for any $y \in X$. Then, X is deleted and go to step 1. Note that this is true only for $j_L'' > 0$, which is not indicated in Ref. 5.

6. Interval Newton Method[5]

   If $j_L'' > 0$, denote $S' = x - J'(x)/J''(X)$ and S = intersection of S' and X. Otherwise, denote $S = S_1 \cup S_2$

   Here S_1 and S_2 are defined as follows:

   Denote $c = x - J'(x)/j_L''$ for $j_L'' \neq 0$.

   and $d = x - J'(x)/j_R''$ for $j_R'' \neq 0$.

   If $J'(x)>0$

$$S_1 = \begin{cases} [x_L,\ d] & \text{when } j_R''>0 \text{ and } d>x_L \\ \text{empty} & \text{when } j_R''=0 \text{ or } d<x_L \end{cases}$$

$$S_2 = \begin{cases} [c,\ x_R] & \text{when } j_L''<0 \text{ and } c<x_R \\ \text{empty} & \text{when } j_L''=0 \text{ or } c>x_R \end{cases}$$

   If $J'(x)<0$

$$S_1 = \begin{cases} [x_L,\ c] & \text{when } j_L''<0 \text{ and } c>x_L \\ \text{empty} & \text{when } j_L''=0 \text{ or } c<x_L \end{cases}$$

$$S_2 = \begin{cases} [d,\ x_R] & \text{when } j_R''>0 \text{ and } d<x_R \\ \text{empty} & \text{when } j_R''=0 \text{ or } d>x_R \end{cases}$$

7. If S is empty, go to step 1. If S=X, then split X in half.

8. For each new generated subinterval $\hat{X} = S_1$ or S_2, repeat steps 3 and 4.

9. Update $\bar{J}$

For each new subinterval $\hat{X}$, denote

$w[\hat{X}]$ = width of $\hat{X}$, $\hat{x}$ = mid $[\hat{X}]$ = midpoint of $\hat{X}$

$\hat{X} = [\hat{x}_L, \hat{x}_R]$ and $J(\hat{X}) = [\hat{j}_L, \hat{j}_R]$

If $J(\hat{x})<\bar{J}$, simply replace $\bar{J}$ by $J(\hat{x})$ or conduct a line search to reduce $\bar{J}$ as follows:

a. If $J'(\hat{x})>0$, denote $\hat{x}_1 = \hat{x}_L$. Otherwise, denote $\hat{x}_1 = \hat{x}_R$. Set $\hat{x}_0 = \hat{x}$.

b. Denote $\hat{x}_2 = (\hat{x}_0 + \hat{x}_1)/2$. If $J(\hat{x}_2) > \max [J(\hat{x}_0), J(\hat{x}_1)]$, go to step e.

c. If $J(\hat{x}_0) < J(\hat{x}_1)$, replace $\hat{x}_1$ by $\hat{x}_2$. Otherwise replace $\hat{x}_0$ by $\hat{x}_2$.

d. If $|\hat{x}_1-\hat{x}_0| > \frac{1}{16} w[X]$, go to step b.

e. Set $\bar{J} = \min[J(\hat{x}), J(\hat{x}_0), J(\hat{x}_1)]$ and set $\bar{x}$ to the corresponding arugument of $\bar{J}$.

10. Store new intervals

For each new interval $\hat{X}$, if $\hat{x}_L > \bar{J}$, delete $\hat{X}$.

If $w[\hat{X}] < w_1$ and $j_R - j_L < w_2$, store $\hat{X}$ in L_1. Otherwise, store $\hat{X}$ in L_0. Go to step 1.

11. If the list L_1 is empty, go to step 13. Otherwise, delete subintervals X for which $j_L > \bar{J}$ where $J(X) = [j_L, j_R]$.

12. If the list L_1 is empty, go to step 13. Otherwise, the midpoints of each interval remaining in L_1 are used as the global minimums. Note that there may exist multiple global minimums.

13. Because L_1 is empty, the global minimum is located on one of the two boundaries of the initial interval X_0, which corresponds to a smaller J.

Numerical Simulation

From Eqs. (1), (2), (13) and (14), a linear dynamic system with additive measurement noise can be represented by:

$$x(k+1) = A\,x(k) + B\,u(k)$$

$$y(k) = C\,x(k) + v(k) \tag{37}$$

with

$$A = \begin{bmatrix} A_1 & & & 0 \\ & A_2 & & \\ & & \ddots & \\ 0 & & & A_m \end{bmatrix} \qquad B^T = [B_1^T, B_2^T, \ldots, B_m^T], \; C = [C_1, C_2, \ldots, C_m]$$

$$A_j = \begin{bmatrix} e^{-\sigma_j T} \cos \omega_j T & e^{-\sigma_j T} \sin \omega_j T \\ -e^{-\sigma_j T} \sin \omega_j T & e^{-\sigma_j T} \cos \omega_j T \end{bmatrix} \qquad j=1, 2, \ldots, m \tag{38}$$

where ω_j, σ_j are the modal frequency and damping for j^{th} mode, m is the number of modes, and v(k) is a white noise. To illustrate applications of the projection filters in a single input and single output case, the actuator and sensor are chosen and located to give

$$B_j = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_j = [0 \;\; 1] \qquad j = 1, 2, \ldots, m \tag{39}$$

From Eqs. (3), (37) and (39) with free pulse response, we have

$$Y(k) = CA^{k-1}\,B + v(k)$$

$$= - \sum_{j=1}^{m} e^{-\sigma_j (k-1)T} \sin[\omega_j (k-1)T] + v(k) \tag{40}$$

This allows one to form a symmetric Hankel matrix H(0) from Eq. (4) by using:

$$r = s, \quad j_i = t_i = i, \quad i = 1,2,\ldots,r \tag{41}$$

From Eqs. (14) and (39),

$$b_1 = c_2 = 1, \quad b_2 = c_1 = 0 \tag{42}$$

Then, the projection filters, Eqs. (23)-(28), become

$$V^{\#} = \begin{bmatrix} V_a^{\#}(r) \\ V_b^{\#}(r) \end{bmatrix}, \qquad W^{\#} = [V_b^{\#}(r)^T, \; V_a^{\#}(r)^T], \tag{43}$$

$$\lambda_1(r) = \lambda_3(s), \; \lambda_2(r) = \lambda_4(s), \; V_c^{\#}(s) = V_a^{\#}(r),$$

and

$$V_d^{\#}(s) = V_b^{\#}(r)$$

Because the Hankel matrix is symmetric, the cost function shown in Eq. (35) can be simplified:

$$J = E_{11}^2 + \frac{1}{2}(E_{12}^2 + E_{21}^2) \tag{44}$$

with

$$E_{11} = V_a^{\#} \, H(0) \, V_b^{\#T} - 1 = E_{22}$$

$$E_{12} = V_a^{\#} \, H(0) \, V_a^{\#T}$$

$$E_{21} = V_b^{\#} \, H(0) \, V_b^{\#T}$$

The first and second derivative of J can be derived as follows:

$$J' = 2\, E'_{11}\, E_{11} + E'_{12}\, E_{12} + E'_{21}\, E_{21} \tag{45}$$

$$J'' = 2\, E''_{11}\, E_{11} + 2(E'_{11})^2 + E''_{12}\, E_{12} + (E'_{12})^2 + E''_{21}\, E_{21} + (E'_{21})^2 \tag{46}$$

with

$$E'_{11} = V_a^{\#}\, H(0)\, (V_b^{\#T})' + V_b^{\#}\, H(0)\, (V_a^{\#T})'$$

$$E'_{12} = 2\, V_a^{\#}\, H(0)\, (V_a^{\#T})'$$

$$E'_{21} = 2\, V_b^{\#}\, H(0)\, (V_b^{\#T})'$$

$$E''_{11} = V_b^{\#}\, H(0)\, (V_a^{\#T})'' + V_a^{\#}\, H(0)\, (V_b^{\#T})'' + 2\, (V_a^{\#})'\, H(0)\, (V_b^{\#T})'$$

$$E''_{12} = 2(V_a^{\#})'\, H(0)\, (V_a^{\#T})' + 2\, V_a^{\#}\, H(0)\, (V_a^{\#T})''$$

$$E''_{21} = 2\, (V_b^{\#})'\, H(0)\, (V_b^{\#T})' + 2\, V_b^{\#}\, H(0)\, (V_b^{\#T})''$$

One numerical example is illustrated by using the following conditions (See Eqs. (29)-(34)):

$$r = s = 49, \quad m = 10, \quad T = 0.04 \text{ sec} \tag{47}$$

$$w_1 = 0.001, \quad w_2 = 0.01$$

where w_1 and w_2 are the accuracy of system parameters (either freqency or damping) and the accuracy of cost respectively, used in the global minimum optimization. Specific modal frequencies with different damping factors and noise levels are shown in Table 1. The noise level is the ratio of the noise standard deviation with respect to the maximum value of $Y(k)$, i.e. the peak of free impulse response. For each case, the simulation starts by forming a Hankel matrix for this ten modes structure with a damping factor for all modes and a specific noise level. Corresponding to each modal frequency, ten frequency intervals are given for the projection filters:

[.1, 5], [5, 10], [10, 15], [15, 20], [20, 25], [25, 35]

[35, 40], [40, 50], [50, 60], [60, 70] in rad/sec.

For each frequency interval, with a fixed zero damping factor, the projection filters (Eq. (43)) first update their frequencies by using the interval analysis method to find the global minimum of the cost function (shown in Eq. (44)) within this frequency interval. The midpoint of the final frequency interval (width is smaller than w_1) is used for the first estimate of the acutal modal frequency. With this estimated frequency, the projection filters then update their damping factors with an initial interval from 0 to 12% by using the same interval analysis method to attract the damping factor from the Hankel matrix. The midpoint of the final damping factor interval (width is also smaller than w_1) is used for the first estimate of the actual damping factor. With this new damping factor, the whole procedures are repeated again. Since the second estimates of the modal frequency and damping factor are quite similar to the first estimates, further estimates are prohibited. The percentage errors for the second estimates of the modal frequency and damping factor are then calculated for each mode and listed in Table 1 and 2. The cost function J is plotted in Fig. 1 as the projection filters update their frequencies with a fixed zero damping factor for both cases: no noise and 30% noise level with zero damping factor in the data. The result shows that the cost function is distorted by the noise, but the minimums are not effected too much. From Table 1, as the damping factor varies from 0.3% to

10%, the errors of estimated modal frequencies fall within 2% for the noise free case. As the noise level increases, the errors increase proportionally and stay within 10% for 30% noise. For a fixed noise level, the errors increase for most of the modes as the damping factor increases. This may be caused by the fact that the signal will damp out faster for higher damping factor, especially for high frequency modes. From Table 2, similar results are found for the modal damping errors except that the percentage errors are higher. For 30% noise, the modal damping errors generally fall within 100%. For low frequency and low damping modes, the errors are higher because the contribution of the damping is comparably smaller. As a result, this numerical simulation shows that the projection filters are a promising way for the esimates of the modal frequencies and dampings.

Concluding Remarks

Two developments are presented in this paper. First, projection filters are developed for modal estimation for dynamical systems. Explicit expressions of these single-mode filters are derived using the pseudoinverse of the controllability and observability matrices in the general sense. Filter parameters are initially implemented from the analytical model and then updated to attract the actual system modal parameters within a specific region by passing the experimental data through filters. Second, a global minimum optimization algorithm is developed by using interval analysis method. Giving the first and second derivatives of the cost function, this algorithm guarantees to find the smallest value of the cost function throughout a specified closed region of system parameters. Numerical simulation with a one-dimensional global optimization algorithm shows that the errors of the estimates for modal frequencies are less than 2% for low noise case and fall within 10% for 30% measurement noise. Several sets of single-mode projection filters may be used at the same time for modal parameters estimates in different ranges for the control requirements. Multi-variable global optimization algorithm needs to be developed to imrpove the estimates for the modal parameters.

Appendix

For the controllability matrix W and the observability matrix V shown in Eqs. (15) and (16), the corresponding pseudoinverse matrices, $W^{\#}$ and $V^{\#}$, can be derived as follows. First, observe that

$$V^{\#}\ V = W\ W^{\#} = I_2. \tag{A1}$$

From Eqs. (17) and (25), it is shown that

$$V_a^{\#}(r)\ V_1(r) = \sum_{i=0}^{r} \left[\frac{\sin^2(j_i\omega T)}{2}\left(\frac{1}{\lambda_1(r)} + \frac{1}{\lambda_2(r)}\right) + \frac{\sin(j_i\omega T)\sin((j_r-j_i)\omega T)}{2}\left(\frac{1}{\lambda_1(r)} - \frac{1}{\lambda_2(r)}\right)\right] = \frac{1}{2\lambda_1(r)}\sum_{i=0}^{r}\left[\sin^2(j_i\omega T) + \sin(j_i\omega T)\sin((j_r-j_i)\omega T)\right]$$

$$+ \frac{1}{2\lambda_2(r)}\sum_{i=0}^{r}\left[\sin^2(j_i\omega T) - \sin(j_i\omega T)\sin((j_r-j_i)\omega T)\right] \tag{A2}$$

If r is even, with the aid of Eqs. (21) and (29)-(31), one obtains that

$$\sum_{i=0}^{r}\left[\sin^2(j_i\omega T) + \sin(j_i\omega T)\sin((j_r-j_i)\omega T)\right]$$

$$= \sum_{i=0}^{r/2-1}\left[\sin(j_i\omega T) + \sin((j_r - j_i)\omega T)\right]^2 + 2\sin^2(j_{r/2}\,\omega T)$$

$$= \sum_{i=0}^{r/2-1}\left[4\cos^2\left(\left(j_i-\frac{j_r}{2}\right)\omega T\right)\sin^2\frac{j_r\omega T}{2}\right] + 2\sin^2(j_r\omega T/2)$$

$$= 2\sin^2(j_r\omega T/2)\left[1 + \sum_{i=0}^{r/2-1} 2\cos^2((j_i - j_r/2)\omega T)\right]$$

$$= 2\sin^2(j_r\omega T/2)\left[1 + \sum_{i=0}^{r/2-1} (1 + \cos((j_r - 2j_i)\omega T))\right]$$

$$= 2\sin^2(j_r\omega T/2)\,\lambda_1(r) \qquad \text{(A3)}$$

$$\sum_{i=0}^{r}\left[\sin^2(j_i\omega T) - \sin(j_i\omega T)\sin((j_r - j_i)\omega T)\right]$$

$$= \sum_{i=0}^{r/2-1}\left[\sin(j_i\omega T) - \sin((j_r - j_i)\omega T)\right]^2$$

$$= \sum_{i=0}^{r/2-1}\left[4\cos^2(j_r\omega T/2)\sin^2((j_i - j_r/2)\omega T)\right]$$

$$= 2\cos^2(j_r\omega T/2)\sum_{i=0}^{r/2-1}\left[1 - \cos((j_r - 2j_i)\omega T)\right]$$

$$= 2\cos^2(j_r\omega T/2)\,\lambda_2(r) \qquad \text{(A4)}$$

If r is odd, with the aid of Eqs. (21) and (29)-(31), one arrives that

$$\sum_{i=0}^{r}\left[\sin^2(j_i\omega T) + \sin(j_i\omega T)\sin((j_r - j_i)\omega T)\right]$$

$$= \sum_{i=0}^{(r-1)/2}\left[\sin(j_i\omega T) + \sin((j_r - j_i)\omega T)\right]^2$$

$$= 2\sin^2(j_r\omega T/2)\sum_{i=0}^{(r-1)/2}\left[1 + \cos((j_r - 2j_i)\omega T)\right]$$

$$= 2\sin^2(j_r\omega T/2)\,\lambda_1(r) \qquad \text{(A5)}$$

$$\sum_{i=0}^{r} \left[\sin^2 (j_i \omega T) - \sin(j_i \omega T) \sin ((j_r - j_i) \omega T)\right]$$

$$= \sum_{i=0}^{(r-1)/2} \left[\sin(j_i \omega T) - \sin ((j_r - j_i)\omega T)\right]^2$$

$$= 2 \cos^2(j_r \omega T/2) \sum_{i=0}^{(r-1)/2} \left[1-\cos((j_r - 2j_i)\omega T)\right]$$

$$= 2\cos^2 (j_r \omega T/2)\, \lambda_2(r) \tag{A6}$$

Substitution of Eqs. (A3)-(A6) into (A2) yields

$$V_a^{\#}(r)\ V_1(r) = \frac{1}{2\lambda_1(r)} \left[2 \sin^2(j_r \omega T/2)\ \lambda_1(r)\right] + \frac{1}{2\lambda_2(r)} \left[2 \cos^2(j_r \omega T/2)\lambda_2(r)\right]$$

$$= 1 \tag{A7}$$

Similary

$$V_b^{\#}(r)\ V_2(r) = 1 \tag{A8}$$

Next, from Eqs. (18) and (25), it is shown that

$$V_a^{\#}(r)\ V_2(r) = - \frac{1}{2\lambda_1(r)} \sum_{i=0}^{r} \left[\sin(j_i \omega T) \cos(j_i \omega T) + \sin((j_r - j_i)\omega T)\cos(j_i \omega T)\right]$$

$$+ \frac{1}{2\lambda_2(r)} \sum_{i=0}^{r} \left[- \sin(j_i \omega T) \cos(j_i \omega T) + \sin((j_r - j_i)\omega T) \cos(j_i \omega T)\right] \tag{A9}$$

If r is even, with the aid of Eqs. (21) and (29)-(31), one obtains

$$\sum_{i=0}^{r} \left[\sin(j_i \omega T) \cos(j_i \omega T) + \sin((j_r - j_i)\omega T) \cos(j_i \omega T)\right]$$

$$= \sum_{i=0}^{r/2-1} [\sin(j_i\omega T) + \sin((j_r - j_i)\omega T)]\ [\cos(j_i\omega T) + \cos((j_r - j_i)\omega T)]$$

$$+ 2 \sin (j_r\omega T/2) \cos(j_r\omega T/2)$$

$$= \sum_{i=0}^{r/2-1} 4 \sin(j_r\omega T/2) \cos((j_i-j_r/2)\omega T) \cos(j_r\omega T/2) \cos((j_i-j_r/2)\omega T)$$

$$+ \sin(j_r\omega T)$$

$$= \sin(j_r\omega T)\ [1 + \sum_{i=0}^{r/2-1} 2 \cos^2((j_i - j_r/2)\omega T)]$$

$$= \sin(j_r\omega T)\ [1 + \sum_{i=0}^{r/2-1} (1 + \cos((j_r - 2\ j_i)\omega T)]$$

$$= \sin(j_r\omega T)\ \lambda_1(r) \qquad \text{(A10)}$$

$$\sum_{i=0}^{r} [- \sin(j_i\omega T) \cos(j_i\omega T) + \sin ((j_r - j_i)\omega T)\cos(j_i\omega T)]$$

$$= \sum_{i=0}^{r/2-1} [\sin((j_r - j_i)\omega T) - \sin(j_i\omega T)]\ [\cos(j_i\omega T) - \cos((j_r - j_i)\omega T)]$$

$$= \sum_{i=0}^{r/2-1} 4\ [\sin((j_r/2 - j_i)\omega T)\cos(j_r\omega T/2)]\ [- \sin(j_r\omega T/2)\sin((j_i - j_r/2)\omega T)]$$

$$= \sin(j_r\omega T) \sum_{i=0}^{r/2-1} 2 \sin^2((j_r/2 - j_i)\omega T)$$

$$= \sin(j_r\omega T) \sum_{i=0}^{r/2-1} [1 - \cos((j_r - 2\ j_i)\omega T)]$$

$$= \sin(j_r \omega T)\ \lambda_2(r) \tag{A11}$$

Substitution of Eqs. (A10) and (A11) into (A13) yields

$$V_a^{\#}(r)\ V_2(r) = -\frac{1}{2\lambda_1(r)}\left[\sin(j_r\omega T)\lambda_1(r)\right] + \frac{1}{2\lambda_2(r)}\left[\sin(j_r\omega T)\lambda_2(r)\right] = 0 \tag{A12}$$

This is also true if r is odd.

Similary, it can be proven that

$$V_b^{\#}(r)V_1(r) = 0 \tag{A13}$$

observation of Eqs. (15), (23), (A7), (A8), (A12) and (A13) leads to

$$V^{\#}\ V = I_2 \tag{A14}$$

Similar procedures can be used to verify

$$WW^{\#} = I_2 \tag{A15}$$

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Table 1. Percentage Error of Estimated Modal Frequency

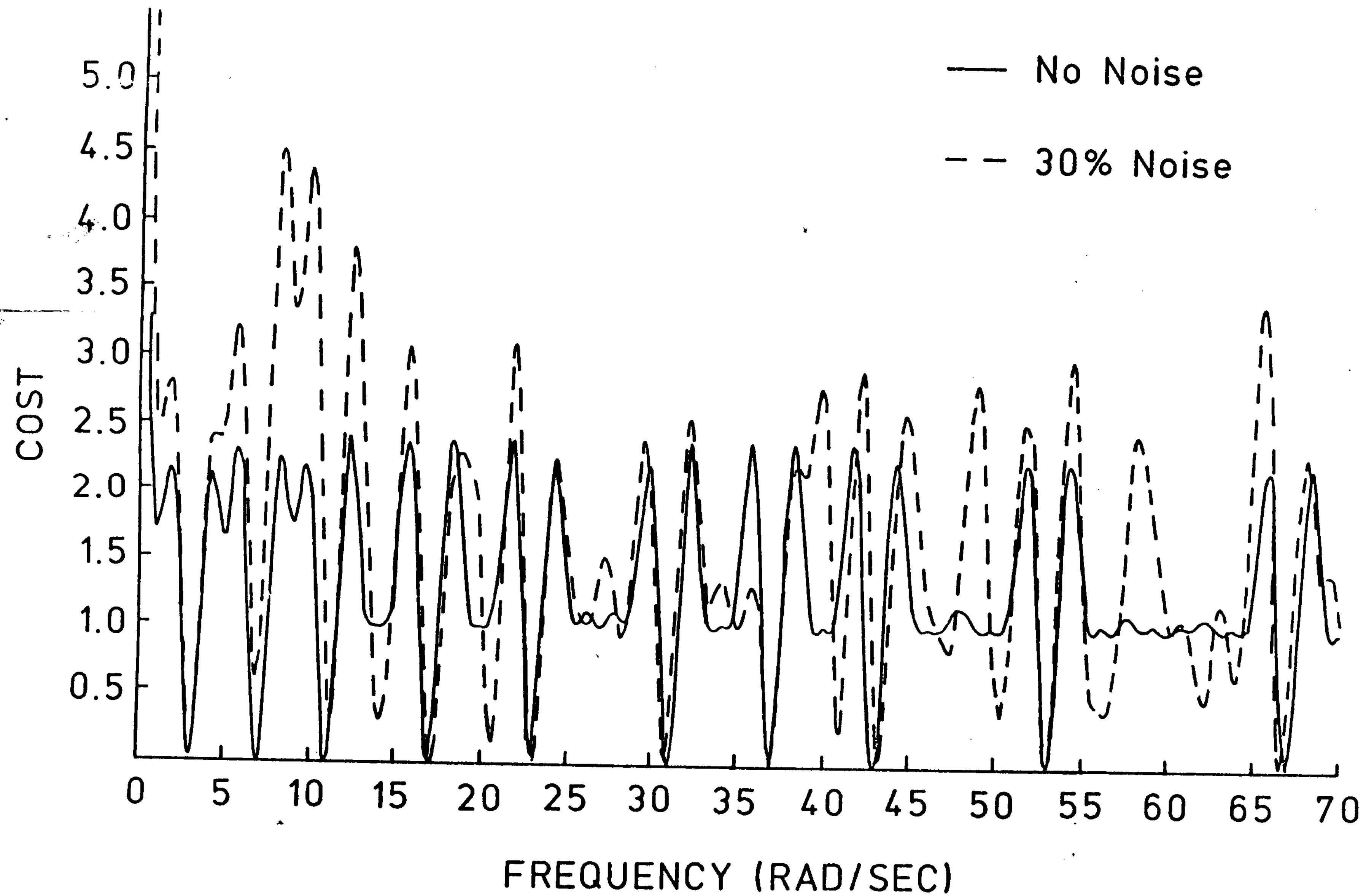
Damping / Noise / Frequency (rad/s)	0.3%				2%				5%				10%			
	0%	5%	10%	30%	0%	5%	10%	30%	0%	5%	10%	30%	0%	5%	10%	30%
3	0.85	1.09	1.34	2.28	0.75	1.00	1.26	2.23	0.62	0.89	1.16	2.22	0.61	0.80	1.09	2.22
7	0.03	0.32	0.66	1.95	0.03	0.42	0.77	2.03	0.16	0.60	0.98	2.19	0.44	0.93	1.33	2.43
11	0.16	0.24	0.59	1.63	0.17	0.38	0.81	1.93	0.18	0.74	1.34	2.42	0.15	1.79	2.26	2.93
17	0.01	0.17	0.33	0.99	0.00	0.25	0.53	1.72	0.04	0.67	1.76	3.75	0.83	4.53	4.77	4.79
23	0.02	0.08	0.13	0.31	0.01	0.12	0.21	9.78	0.05	0.11	0.24	9.62	0.97	9.45	9.53	9.60
31	0.02	0.17	0.31	0.84	0.02	0.53	0.96	1.59	0.20	1.87	1.98	2.08	1.23	2.21	2.21	2.19
37	0.01	0.02	0.07	0.35	0.02	0.43	0.98	3.72	0.12	1.79	3.67	3.85	0.15	3.69	3.76	3.90
43	0.00	0.18	0.36	0.92	0.00	0.79	1.17	1.54	0.05	1.60	1.68	1.75	0.15	1.71	1.74	1.79
53	0.00	0.03	0.06	0.14	0.02	0.13	0.21	0.36	0.02	0.45	0.33	5.10	0.07	5.43	5.33	5.13
67	0.00	0.14	0.27	0.63	0.00	0.90	0.96	1.01	0.21	1.07	1.06	1.06	0.56	1.08	1.07	1.07

Table 2. Percentage Error of Estimated Modal Damping

Damping / Noise / Frequency (rad/s)	0.3%				2%				5%				10%			
	0%	5%	10%	30%	0%	5%	10%	30%	0%	5%	10%	30%	0%	5%	10%	30%
3	127.31	100	100	100	16.97	28.29	73.20	100	5.82	14.20	34.10	100	100	9.50	21.19	65.56
7	3.44	100	100	100	0.42	53.05	100	100	0.61	28.93	54.33	100	1.91	24.88	43.66	96.38
11	13.56	100	100	100	2.93	25.28	51.83	100	1.88	100	40.18	91.47	100	31.46	49.56	84.72
17	0.27	4.81	11.34	61.36	0.02	0.36	0.09	15.83	0.43	5.79	5.75	43.41	100	26.42	42.61	70.37
23	11.55	1.73	14.95	64.84	0.76	7.37	14.38	40.29	0.65	20.67	30.86	74.64	16.18	50.09	63.62	86.74
31	11.71	1.05	19.03	100	2.72	0.26	13.08	68.64	1.66	34.63	51.46	81.49	30.92	64.84	73.61	88.89
37	0.75	23.28	46.41	127.01	0.09	13.06	9.69	27.04	5.14	36.89	51.33	75.65	41.45	68.53	75.39	87.22
43	1.16	1.00	7.07	100	0.79	12.84	34.60	85.94	9.74	55.05	69.03	91.88	49.75	76.19	83.34	94.92
53	0.56	15.90	31.94	92.01	0.20	14.57	25.74	55.21	21.76	44.34	54.87	82.94	59.46	77.30	82.58	91.08
67	1.64	9.15	5.04	76.78	0.99	40.21	58.43	88.27	33.42	74.34	81.87	94.09	69.29	86.68	90.45	96.58

Figure Legend

Fig. 1 -Numerical example for the cost function J as a function of modal frequency with a zero damping and two different measurement noises (0% and 30%) for a ten-mode structure.


No Noise
30% Noise
COST
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
FREQUENCY (RAD/SEC)