FINAL REPORT

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Wayne J. Book, Principal Investigator

Final Report of Period November 15, 1985 to December 31, 1986

# Under

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### HIGH SPEED, PRECISION MOTION STRATEGIES FOR LIGHTWEIGHT STRUCTURES

Final Report, November 15, 1985 to December 31, 1986

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Technical Officer: Donald Soloway

### Summary

The work during the period covered by this report proceeded along the lines of the proposal. One additional paper was prepared and appeared in the open literature. In addition, one paper previously presented was accepted for publication. One Ph.D. student completed his degree and attendant dissertation. The other has completed his dissertation proposal. A trip was made with both of these students to NASA Langley to visit the Grant Technical Officer, and presentations of the research performed were made at that time. A proposal for continuation of the research grant was submitted and accepted.

### Fine Motion Control

The work on fine motion control undertaken by Gordon Hastings has been completed and resulted in the Ph.D. Dissertation<sup>1</sup> by Mr. Hastings and his graduation. The results of his work are summarized by the abstract of the dissertation as follows:

"Lightweight, slender manipulators offer faster response and/or greater workspace range for the same size actuators than traditional manipulators. Lightweight construction of manipulator links results in increased structural flexibility. The increased flexibility must be considered in the design of control systems to properly account for the dynamic flexible vibrations and static deflections. This thesis experimentally investigates real time control of the flexible manipulator vibrations.

"Models intended for real-time control of distributed parameter systems such as flexible manipulators rely on modal approximation schemes. A linear model based on the application of Lagrangian dynamics to a rigid body mode and a series of separable flexible modes is examined with respect to model order requirements, and modal candidate selection.

"Balanced realizations is applied to the linear flexible model to obtain an estimate of appropriate order for a selected model.

"Describing the flexible deflections as a linear combination of modes results in measurements of beam state, (position, strain, etc.), which yield information about several modes. To realize the potential of linear systems theory, in particular to implement full state feedback, knowledge of each state must be available. Reconstruction of the time varying modal amplitudes from strain measurements is examined. Reduced order observers are utilized to obtain estimates of the modal velocities from the reconstructed modal amplitudes. State estimation is also accomplished by implementation of a Kalman Filter. "State feedback control laws are implemented based upon linear quadratic regulator design. Specification of the closed loop poles in the regulator design process is obtained by inclusion of a prescribed degree of stability in the manipulator model."

The work described above has also resulted in two publications in the open literature, and will probably result in some additional publications in the near future.<sup>2,3</sup> Abstracts to these two publications appear in the appendix.

Mr. Hastings left Georgia Tech in August and joined the faculty at Clemson U. He visited NASA Langley following the completion of his degree to describe his research. Funding of two additional students was picked up. These two students, J.W. Lee and D.S. Kwon are extending the general capabilities to two link arms. This is a critical step if the work is to have direct practical application. Both of these students have completed their Ph.D. qualifying examinations successfully.

The two link case will be studied using a large arm of two 10 ft links especially constructed for this study. Initially one of the two joints will be actuated and comparisons will be made with Dr. Hastings experiments. Cost for constructing the experiment are being shared with the industrially sponsored Computer Integrated Manufacturing Systems (CIMS) program at Georgia Tech. Control will ultimately be provided through a MicroVAX II computer, which has been enhanced with some funds obtained through this grant. Structural testing and modeling are underway.

# Gross Motion Planning and Control.

Work on gross motion planning and control by Mr. Sabri Cetinkunt has resulted in a paper<sup>4</sup> describing his modeling work. The abstract of that paper appears in Appendix I. Mr. Cetinkunt has also completed his Ph.D. Dissertation proposal and it has been accepted by his thesis advisory committee. A copy of that dissertation appears as Appendix II. The work suffered a considerable set back when computer files were lost due to a system crash and a problem in the backup procedure. Work is underway to replace those files.

It is expected that Mr. Cetinkunt's research will take slightly more than one year to complete. As described in Appendix II, this research will consider adaptive control as an approach to the large motions. The design of the appropriate reference trajectory for a flexible arm is a critical part of the gross motion control. The research will also consider the transition from gross motion to a position near objects that the manipulator might come in contact with. This is viewed as a linear terminal control problem.

### APPENDIX I

### ABSTRACTS OF PUBLISHED PAPERS AND THESES

### ABSTRACT<sup>1</sup>

### Controlling Flexible Manipulators, An Experimental Investigation

Lightweight, slender manipulators offer faster response and/or greater workspace range for the same size actuators than traditional manipulators. Lightweight construction of manipulator links results in increased structural flexibility. The increased flexibility must be considered in the design of control systems to properly account for the dynamic flexible vibrations and static deflections. This thesis experimentally investigates real time control of the flexible manipulator vibrations.

Models intended for real-time control of distributed parameter systems such as flexible manipulators rely on modal approximation schemes. A linear model based on the application of Lagrangian dynamics to a rigid body mode and a series of separable flexible modes is examined with respect to model order requirements, and modal candidate selection.

Balanced realizations is applied to the linear flexible model to obtain an estimate of appropriate order for a selected model.

Describing the flexible deflections as a linear combination of modes results in measurements of beam state, (position, strain, etc.), which yield information about several modes. To realize the potential of linear systems theory, in particular to implement full state feedback, knowledge of each state must be available. Reconstruction of the time varying modal amplitudes from strain measurements is examined. Reduced order observers are utilized to obtain estimates of the modal velocities from the reconstructed modal amplitudes. State estimation is also accomplished by implementation of a Kalman Filter.

State feedback control laws are implemented based upon linear quadratic regulator design. Specification of the closed loop poles in the regulator design process is obtained by inclusion of a prescribed degree of stability in the manipulator model.

# ABSTRACT<sup>2</sup>

# Verification of a Linear Dynamic Model for Flexible Robotic Manipulators

This paper describes a linear state-space model for a flexible single link manipulator arm. The resultant model is compared to an experimental four foot long direct drive manipulator. The method employed to generate the model utilizes a separable formulation of assumed modes to represent the transverse displacement due to bending. Lagrangian dynamics are applied to determine the kinetic and potential energies for the system. The resultant dynamic equations are then organized into a state space model suitable for use in linear control system design procedures. The performance of the model is considered for different model orders and assumed modes. Several important aspects of candidate mode selection, and results for different model orders are discussed. The final section of the paper provides a brief summary and describes ongoing and future work.

# ABSTRACT<sup>3</sup> Reconstruction and Robust Reduced-Order Observation of Flexible Variables

Most models intended for real-time control of distributed parameter systems such as flexible manipulators rely on N-modal approximation schemes. Measurements made on flexible systems yield time varying quantities which are linear combinations of the system states. This paper discusses reconstruction and estimation of flexible variables from multiple strain measurements for use in state feedback control of flexible manipulators. Reconstruction is proposed for obtaining flexible mode amplitudes from the measurements, and estimation for the modal velocities. Reduced order observers are briefly reviewed, and then application to flexible manipulators is discussed. Design of the observer for estimation of the velocities is discussed with regard to robust implementation. The performance of the observer is examined experimentally for several specifications of the error dynamics.

### ABSTRACT<sup>4</sup>

### Symbolic Modeling and Dynamic Analysis of Flexible Manipulators

This paper presents a systematic method to symbolically derive the full nonlinear dynamic equations of motion of multi-link flexible manipulators. Lagrange's assumed mode method is used for the dynamic modeling and implemented via a commercially available symbolic manipulation program. Adaptation of the method suitable for symbolic manipulation and advantages are discussed. Simulation results for a two-link planar flexible arm are presented.

Since the gross motion work inherently involves the large motions of the arm, an accurate, quick way of getting complete nonlinear models of the arm are important. This is the reason Mr. Cetinkunt's work has started out with this modeling effort. The symbolic programs will allow one to examine the form of the equations and any special structure, not just look at the numerical results which would result from general modeling programs. It should also result in much more efficient simulations for these very complex and time consuming equations.

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# APPENDIX II

# ON MOTION PLANNING AND CONTROL OF MULTI-LINK LIGHT-WEIGHT ROBOTIC MANIPULATORS

A Ph.D. Thesis Proposal

ON MOTION PLANNING AND CONTROL OF MULTI-LINK LIGHT-WEIGHT ROBOTIC MANIPULATORS

A Ph. D. Thesis Proposal

Submitted to the Graduate Committee of George W. Woodruff School of Mechanical Engineering Georgia Institute of Technology

by

Sabri Cetinkunt

In Partial Fulfillment of the Requirements for Degree of Doctor of Philosophy

Fall, 1986

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- I. The Objective of the Research .
- II. Introduction
- III. The Problem Statement
- IV. Previous Work.
- V. Proposed Approach .
- VI. Discussion of the Proposed Approach .
- VII. Literature

### 0. Abstract:

A general gross and docking motion planning and control method is needed for light-weight robotic manipulator applications such as painting, welding, material handling, surface finishing, and space craft service jobs. In these applications, a multi-degree of freedom robotic manipulator moves from one position to a distant position quickly, and finally contacts an object at the end of the motion.

An adaptive model following control method is proposed for the gross motion phase. The commanded reference trajectory and reference model structure are utilized as efficient tools to plan trajectories suitable for light-weight manipulators.

The transition from the gross motion trajectory to docking motion is formulated as an LQ terminal controller. A sequence of optimum control problems is solved and among them the best one is picked.

### I. Objective of the Research :

The amount of literature in dynamics and control aspects of rigid robotic manipulators is overwhelmingly large. However, the current emphasis of research in flexible manipulators is only on the fine motion aspect. A typical robotic application involves both gross and fine motion phases. Systematic motion planning and control methods for realistic applications are yet to be developed.

The objective of this work is to develop a general motion planning and control method for light-weight robotic manipulator applications involving a gross motion and a transition to docking motion. Thus, a realistic base for the utilization of light-weight manipulators in industrial and space applications will be established.

### 2. Introduction :

Industrial robotic manipulators are mechanisms controlled by a computer (Fig. 1). The control problem of a robotic manipulator may be divided into two parts: 1. Trajectory planning, which is usually done off-line, and 2. Trajectory tracking which requires on-line computations (Fig. 2). At the trajectory planning level the manipulator task is defined and, given the environmental and system constraints, a motion is-planned off-line based on some criterion. Then, at the tracking level, the desired trajectory is commanded to the controller, and the control vector is computed based on the control law in an attempt to follow the desired trajectory planned previously.

Assuming that, at best, the controller is capable of perfectly following the desired trajectory, the best performance of the manipulator will be the planned trajectory. Thus the trajectory planning level is the one which essentially determines the upper bound of the performance. All performance requirements and system constraints must be imposed on the planned trajectory. A controller is then designed with the intent to follow that trajectory as closely as possible. Higher productivity requirements demand manipulators that move faster and more precisely. The trajectory planning methods should utilize the system capabilities as much as possible , rather than resting on very conservative, simple planning methods. The more fundamental factor which limits the manipulator productivity is the maximum velocities and accelerations affordable by the system. These are the physical constraints of the system independent of the planning and control method. These velocity and acceleration constraints are functions of the mechanical properties of the system , such as link inertial parameters, payloads, friction, and the actuator capabilities. In order to

increase the productivity of a robot, one may consider changing these parameters so that higher velocities and accelerations can be afforded. Payload and friction are the parameters determined by the nature of the task and the actuator types.

One of the options is to increase the actuator capabilities. However, in a typical industrial robot, the actuators are located at the link joints and must be carried by the previous ones. Therefore, increasing the actuator sizes in order to increase the system capabilities is not an ultimate answer, has a limit and can be self-defeating. The major factor that limits the affordable speed of operations is the inertial properties that are to be moved. Thus the fundamental question is " can the inertial parameters be reduced by the use of light weight links leading to a light-weight structure and making higher speed operation possible". Reducing the link inertias is clearly one of the most effective way of improving the manipulator speeds, which results in more productive systems.

In many cases, a reasonable light-weight robotic manipulator motion, going from one position to another, would involve a gross motion followed by a fine motion. The gross motion should be made fast to be efficient. Towards the end of the motion, a fine motion, which is slower, is performed. Many applications require the robot end effector to contact an object. The planning and execution of the docking motion, which involves getting in contact with an object, is an interesting and important problem to be solved. A simple example would be a space craft service task (Fig. 3 and 4) where the manipulator moves from its initial position to a distant object then contacts it in a controlled way, and finally works on the object.

Current motion planning and control methods of robotic manipulators cannot be directly applied to the light-weight, high performance manipulators where structural flexibilities are significant. New motion planning and control methods, which take the structural flexibilities into account, are needed for light-weight manipulators and are discussed in the rest of this paper as follows. Section III states the problem in a more concise manner. In Section IV previous work on motion planning and control of industrial manipulators is outlined and the short-comings of these methods for light-weight manipulators are discussed. Section V presents the new motion planning and control approach appropriate for the the stated problem. The expected contribution of the proposed approach and comparison with the previous work is made in Section VI. A list of references in dynamic modelling, motion planning and control aspects of robotic manipulators is given in Section VII.

III. The Problem Statement :

A general task of a multi-link flexible robotic manipulator would consist of three phases.

Phase 1: A gross motion, typically fast for productivity, from a known initial state towards a final desired state close to an object. Phase 2: A transition from gross motion to docking motion near the object. Phase 3: Finally get in contact or dock with the object . This thesis will deal with the phase 1 and 2. The phase 3 part of the problem requires the monitoring of the contact forces. Position plus force feedback control has to be employed for the remaining part of the task.

The motion required by the task can be characterized in more detail as follows. At phase one, the arm is away from the object, the motion is large and to be done fast so that task can be performed productively. The flexible deflections and vibrations at this stage are not that important, but rather one would be satisfied with following a desired trajectory in joint space, with no explicit control action for vibration stabilization. However, the desired trajectory may be designed in such a way that if there were a perfect tracking controller, resultant vibrations would be acceptable. In phase 2, the end of the arm is close to the object and should not collide in an undesirable way. Thus, the control of flexible vibrations is important as well as accurate positioning of the joint variables. The motion may be rather slow, if necessary near the desired contact point with the object.

For a task described by phase 1 and phase 2, one needs to plan trajectories for each phase in either joint or task space as a function of time, then design controllers appropriate for each phase. Notice that every phase has a planning and control level, although in some cases the planning and control problem may

be solved simultaneously. In the rigid arm case the control problem is to drive the joint variables to follow the planned trajectories, where the number of control signal is equal to the number of controlled generalized coordinates. When the structural flexibility is significant, there are two control problem exist: 1. Joint space control, and 2. Suppression of flexible vibrations. It is the phase 2 of the motion where control problem 2 is important.

### IV. Previous Work :

Dynamics of industrial robots are governed by second order, coupled, highly nonlinear differential equations (A9). When the structural flexibilities are considered, the complexity naturally increases, nonetheless after some modal truncations, the system dynamics are still governed with the same type of equations (A11). However an important difference is that the number of inputs is less than the number of generalized coordinates controlled. The motion planning and control problem is a difficult task due to: 1. Nonlinearity, 2. Strict constraints imposed on the system, i.e. actuator saturation, and collusion avoidance problems, and 3. High system order.

Because of these difficulties, earlier work took a very conservative approach to solve the problem. For example, a desired trajectory, either in joint or task space, is planned as collection of constant velocity profiles. The transition from one constant velocity segment to another is determined by the continuity requirements. Maximum allowable acceleration bounds were imposed based on the worst possible cases (B1, B2, C8, Fig.5a). The corner points of the constant velocity segments are never exactly reached unless an overshoot is allowed (Fig. 5b). Apparently such a planning scheme rarely and only instantaneously uses the full manipulator capabilities, and does not consider the manipulator dynamics, resulting in low performance and productivity. Taylor (1979) has

developed a method to execute straight line paths in task space (C10). The method determines the number of intermediate points necessary so that the deviations from the path due to linear interpolations are bounded by a preassigned value (Fig. 6). Another method developed by Lin et al (1983) to find minimum time trajectories in joint space utilized the cubic splines (C7). A desired task is defined as a sequence of N knots in the cartesian coordinates. The corresponding joint variables are found via the solution of the inverse kinematic problem. Then these N knots in joint space are connected to each other with cubic splines minimizing the total travel time with no constraint violation. These trajectory planning methods are developed for rigid robotic manipulators and do not consider structural flexibilities.

Bobrow et al, ( also Shin and McKay ) have incorporated the full nonlinear dynamics of the manipulator to the minimum time trajectory planning level, where the cartesian coordinate path and actuator constraints are given (C1, C2, C3, C4, Fig 7a and 7b). The method essentially reduces a set of n coupled second order nonlinear differential equation system to a single second order nonlinear differential equation, and uses direct numerical integration to find minimum time trajectory in the task space.

The second step in the manipulator control system design is to find an appropriate control law which will realize the planned motion. This is the lowest level in the control system hierarchy. Today the majority of industrial robots are used as positioning devises. If the robot end effector is to move from one position to another and the path followed is not important, each joint sequentially can be moved while the others are all locked. In this case each joint can be controlled by a simple position servo, since every joint control problem is a second order linear system, provided gravity is compensated. Although such a motion makes the control problem easy, it is very inefficient

and unacceptable. When all joints are allowed to move simultaneously, performance of the simple position controllers drastically deteriorates due to the inertial coupling, gravitational torque variations, friction, centrifugal and coriolis torque effects.

Conventional controllers cancel some of these coupling effects via feedforward compensation. The inertial coupling and gravitational torques are the major disturbances and can be canceled based on the dynamic model of the manipulator. The friction effect is a nondeterministic phenomenon and compensation is made based on some experimental average values. The centrifugal and coriolis effects becomes important at high speed operations and are approximately compensated at each joint based on the dynamic model of the manipulator. Notice that the whole purpose of feedforward compensation is to reduce the system back to simple second order linear form so that linear controllers can be used. However, almost all of the feedforward compensation is based on the manipulator dynamic model or its simplified forms. This so called "inverse problem" or "computed torque method" heavily relies on the accurate knowledge of the dynamic model, system parameters and their variation, and all other external disturbances.

In robotic applications parameters can be in the range of 50-200 % of average values. External disturbances and the nature of the friction are never accurately known or are even unknown in advance. The payload may drastically vary from one task to another without advance knowledge. Moreover, the dynamic characteristics of the system may change in time. Clearly computed torque methods are not so suitable for applications where external disturbances, large unknown payload variations, and uncertainty exist. It is important to note that the "resolved rate" and " resolved acceleration" methods are also computed torque for the difference is that they generate reference

trajectories in joint variables which are resolved from a desired task space trajectory.

It is very desirable to have a control method which has the following properties

- 1. -- has good tracking accuracy
- 2. -- does not require precise knowledge of the model parameters, but rather a general structural and bound information .
- 3. -- quickly adapts itself, if necessary, due to
  - a) the variations in the system parameters(insensitive to parameter variations)
  - b) disturbances ( disturbance rejection)

4.-- is stable in the large ( Global Asymptotic Stability )

These requirements call for adaptive control methods. Adaptive control methods may be divided into three major categories:

1. Gain scheduling; 2. Self tuning regulators; and 3. Model Reference Adaptive Controllers (Gradient Methods, Lyapunov and Hyperstable design). Gain scheduling and self tuning regulators are direct generalization of linear control laws. They will not be discussed here due to their serious draw-backs. For example, gain scheduling methods require storage of the control-law parameters and use the appropriate one as the operating range changes. There are two major drawbacks. First, the problem of switching from one gain to another - how does it effect the system performance and stability. Second and more importantly, if the system dimension and possible range of operating conditions are large, the storage requirements may become prohibitive. Selftuning regulators are considered to be inappropriate due to the "persistent excitation requirements", which is in robotics a severe requirement. The MRAS (Model Reference Adaptive Systems) are attractive since they do not have above draw backs and globally asymptotically stable designs are possible. The difference between the methods in this category originates in the way the adaptation mechanism is designed (Fig. 8a). An early work by Dubowsky (1979) showed the promise offered by MRAS in robotics (D8). However this work suffered from lack of global stability proof. Balastrino et al (1983) developed a globally stable adaptive model following control method based on the hyperstability approach (D3).

When a comparison is made between Lyapunov and Hyperstability based adaptation law design methods, it is seen that theoretically they offer the same solutions for systems having bounded, piecewise continuous input signals (D15). However, finding alternative Lyapunov functions is known to be very difficult and is usually done by trial and error, whereas Hyperstability and Positivity based methods offer a wider class of admissable control laws which guarantee the global asymptotic stability of the system (D1). Besides that, the reference model and the commanded reference input serve as the on-line trajectory planning method very efficiently with no complications, and result in very little computational burden for trajectory planning. Furthermore, powerful on-line control computers are not required which reduces the cost of the control system.

It is important to note that all of the previous trajectory planning and controller design methods are for rigid manipulators. An important contribution of this thesis will be to devise a methodology which allows the application of these methods to flexible robotic manipulators.

V. Proposed Approach : Gross and Docking Motion Planning and Control

The problem being investigated is as follows: Given the initial state and final desired state, where final desired state is likely to be close to an object, develop a general motion planning method and control law for the manipulator which accomplishes the following objectives:

 Move from initial state to near a final state by following a desired joint trajectory generated by a reference model. At this step the manipulator is away from the object, and exact trajectory tracking in joint variables is emphasized.
 When close to the final state, determine when to switch from the gross motion control to transition motion control. Also determine the appropriate control law for the transition motion. At this phase achieving the final desired state as closely as possible with no overshoot is important, since overshoot may result in collision.

In the proposed approach the gross motion planning and control problem is simultaneously solved in the framework of model reference control. The reference model and the commanded reference input essentially serve as a trajectory planner, then the adaptive controller attempts to asymptotically follow the reference model response. Therefore, one may ask " Can I find a control law which follows the reference model perfectly in joint space just as could be done if the manipulator were rigid. Can I predict the flexible behavior along this motion. Can I also find the dependence of the flexible modes on the joint trajectory and analyze the planning of the gross motion that would result in small flexible vibrations".

Since during the gross motion phase manipulator is far away from objects, the concern with the flexible vibrations is not as serious as it is when it is

closer to the objects. Joint trajectory tracking is of primary importance in this phase. Therefore, the gross motion reference model is chosen as a rigid manipulator with ideal dynamics. The control system adaptively matches the joint variable response with the reference model, but takes no direct control action to control flexible vibrations. This is accomplished by the use of an additional signal synthesis into the adaptive control action which cancels the coupling effects of flexible vibrations on the joint variable response (Fig. 8b). Notice that, with this approach the perfect model following condition (D1, D18) can be asymptotically satisfied in joint variables, and therefore one can use the reference model parameters and commanded input signals for path planning and modification purposes. We propose to use the reference model explicitly as a trajectory planning and analysis tool. Thus, the dynamics dependent factors affecting the flexible modes, are reduced down to the reference model parameters (which are simpler second order linear models) and the commanded input.

In the second phase of planning and control, the first problem is to determine when to start the transition ( switch ) from the gross motion control law to transition motion control law to achieve the final desired state with no residual vibrations. The second problem is what kind of control law would be suitable for this motion. More precisely, given the desired final state and a gross motion reference trajectory, does there exist a control vector time history under which the system leaves the nominal trajectory at some point and reaches the final desired state in an optimum manner (Fig. 9).

Some more insight to the physical nature of transition motion phase is as follows. It is a motion about a nominal state (final desired state), typically slowing down for docking, and can be performed slowly. Hence,

linear formulation about the final desired state would be quite accurate. Since the final desired state may vary from task to task, the linearized models may be evaluated for different final desired states off line. The docking motion control law parameters must be calculated off line. Gain scheduling for this stage may be a very simple and practical candidate control approach. The question of when to start the transition remains a motion planning problem . This problem is simultaneously solved with the control law if an optimum control approach is used. In fact a linear quadratic (LQ) optimum terminal controller with zero or penalized terminal error fits very well with the nature of the problem, as shown below.

Find the control variable(s) time history u(t) such that the following performance index is minimized,

$$J = \phi(x(t_f)) + \int_{t_0}^{t_f} (x^T A(t) x + u^T B(t) u) dt$$

subject to manipulator dynamics:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{t}) \mathbf{x} + \mathbf{G}(\mathbf{t}) \mathbf{u}$$
,  $\mathbf{x} \in \mathbf{R}^{n}$ ,  $\mathbf{u} \in \mathbf{R}^{m}$ .

and end point constraints (variable end points):

$$x_{i}(t_{0}) = \xi_{i}(t_{0}) , i = 1, \dots, q \le n$$
  
 
$$x_{i}(t_{f}) = \psi_{i}(t_{f}) , j = 1, \dots, p \le n$$

End point constraints at t<sub>0</sub> are the nominal gross motion trajectories (Fig.9). This class of optimal control problems always results in a two point boundary value problems with appropriate essential and natural boundary conditions. The above formulation with variable end points results in a more complicated boundary value problem than the fixed end point problem due to the additional transversality boundary conditions. But the differential equation does not change due to the different boundary conditions and is the same for both type of problems. One can find the solution of this difficult problem by solving a sequence of easier problems. Following this approach, the solution of the simplified problem with fixed end points can be found by assuming that:

$$\lambda(t) = S(t) x(t) + R(t) v$$
  
$$\psi(t) = U(t) x(t) + Q(t) v$$

where  $\lambda(t)$  and  $\nu$  are associated Lagrange multipliers, and S(t), R(t), Q(t) determined from the following differential equations.

From the solution of these equations (Matrix Riccati, Linear Matrix, and quadrature differential equations, respectively), one obtains the state on the gross motion trajectory at which transition should start and, the transition motion control law. In addition to that, the Q matrix, which depends on the initial state, is an indication of the controllability of the system with the given initial condition. Using that matrix as an indicator, one can trace a region on the nominal trajectory and use the state which gives the most well conditioned Q matrix.

The planning and control problem of phase one is well formulated and proposed methods are organized in such a way that, we end up with effective analysis tools. For phase two, besides the above explained Optimal LQ type controllers, Bang-Bang type open loop control laws will be considered as well. VI. Discussion of the Proposed Approach :

Previous work on motion planning and control of robotic manipulators does not consider structural flexibilities of the system or takes very conservative measures to avoid dealing with structural flexibility related problems. Although minimum time trajectories are the best trajectories from a productivity point of view, they are not as suitable for flexible manipulators as they are for rigid manipulators. Relative merits of constant velocity segmented, cubic spline, and controlled acceleration distribution type joint trajectories for light-weight manipulators are yet to be determined. Control methods of rigid manipulators cannot be directly applied since the number of inputs are not equal to the number of generalized coordinates controlled. One cannot simply ignore the flexible vibrations, since they will disturb the controlled part of the dynamics via coupling.

In the proposed work a trajectory planning method which takes the structural flexibilities into account is to be developed. Reference model parameters are used as the trajectory planning tools. By simply changing the reference model structure and parameters, different trajectories and the resulting control system performance can be compared. Using a model based signal synthesis approach, rigid manipulator control laws can be applied to light-weight manipulators that will result in the joint variable performance as if links were rigid, if there is no concern about the flexible vibrations. Finally a transition motion from gross motion to docking motion is analyzed. As a result, the proposed work establishes a complete motion planning and control method for typical light-weight robotic manipulator applications.

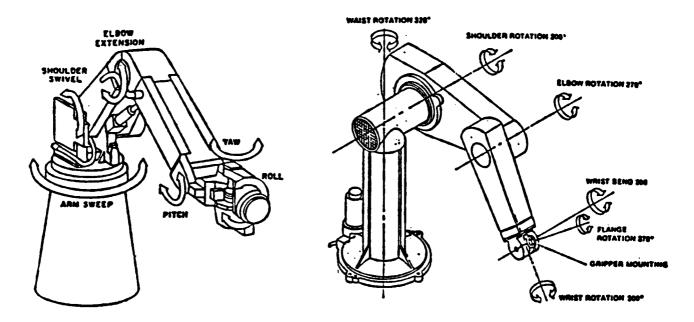
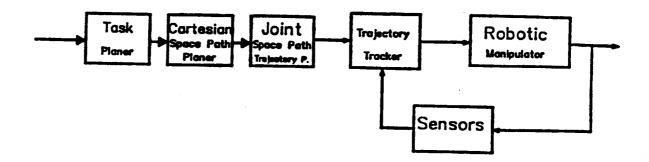
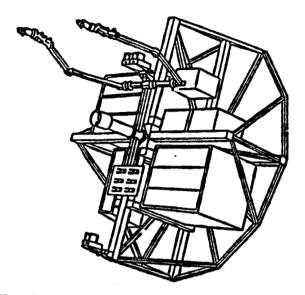


Fig.1 Examples of Industrial Robots

a) Cincinnati Milacron T3, b) Unimation PUMA 600.









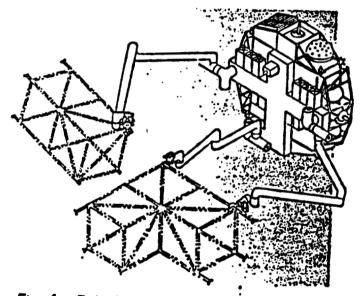
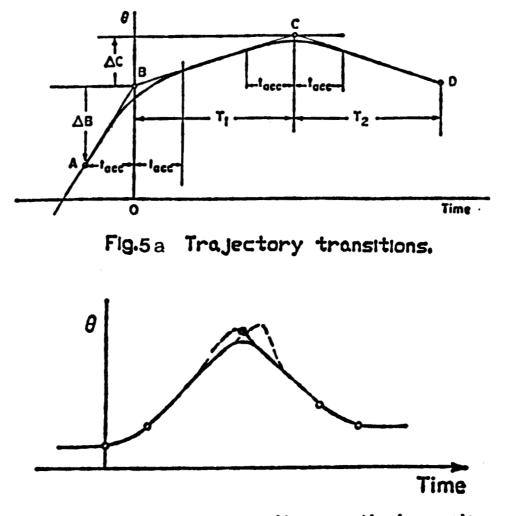
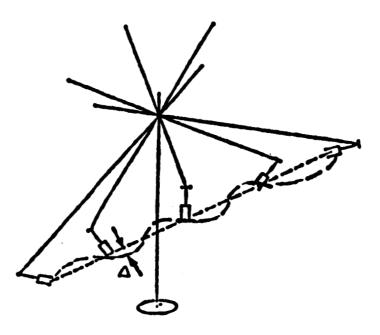


Fig. 4 Robot-aided structural assembly.



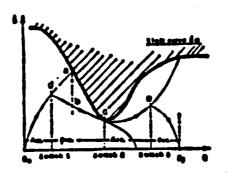






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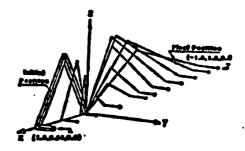
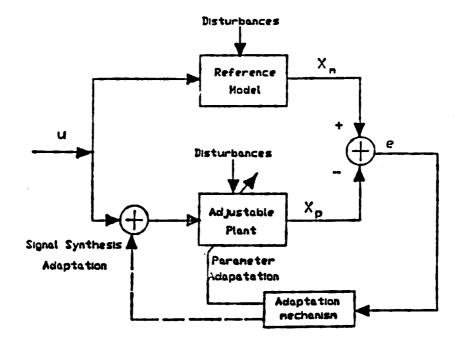
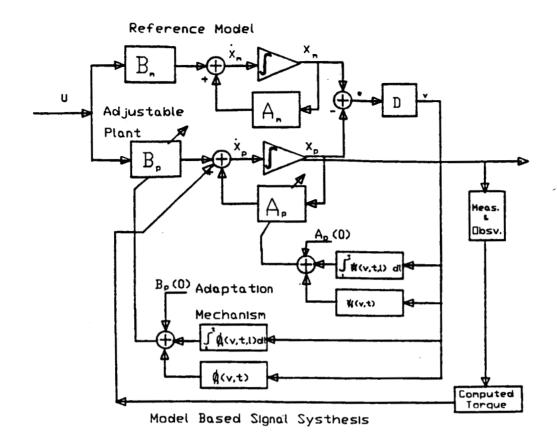


Fig.78. Finding minimum time trajectories

Fig.7b. Path in task space









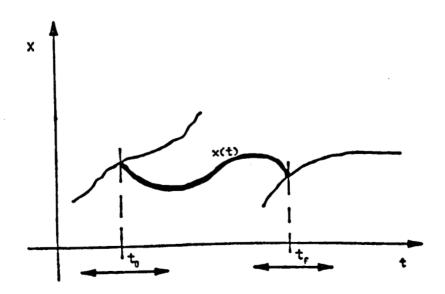


Fig.9 End points lying on curves.

19 c

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