DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION OF BUILT-UP STRUCTURES

Final Technical Report

by

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1. Summary of the Project

The purpose of this report is to summarize research findings from NASA Project NAG-1-215, Design Sensitivity Analysis and Optimization of Built-Up Structures, during the period October 1981 to December 1986.

Technical progress has been made in five directions, leading to publication of one book, eight papers, six technical reports, and award of one M.S. and four Ph.D. degrees. Personnel actively involved in this research project included K.K. Choi, E.J. Haug, H.G. Seong, H.G. Lee, B. Dopker, T.M. Yao, M.C. Frederick, J.H. Lee, and J.S. Park. Messrs. Seong, Lee, Dopker and Yao received Ph.D. degrees and Mrs. Frederick received an M.S. degree. The papers, reports, and book published during the course of the research are cited in the attached list of publications.

Developments during the course of the research in design sensitivity analysis and optimization of built-up structures, with both sizing and shape design variables, show clearly that a unified variational approach to design sensitivity analysis can yield derivatives of structural response with respect to design. Rigorous and practically computable results for structural components (trusses, beams, plane elastic solids, and three dimensional elastic solids) and built-up structures that are made of these components have been demonstrated and used to solve design optimization problems. A particularly important result obtained in this research is that the distributed parameter structural design sensitivity analysis approach allows one to retain the continuum elasticity formulation throughout the derivation of design sensitivity analysis results. Design sensitivity analysis expressions do not depend on the finite element formulation that is used for computation. This allows numerical implementation of design sensitivity analysis results using established finite element analysis codes, such as

ANSYS, EAL, IFAD, and NASTRAN. The method developed will allow the practicing designer to use the code with which he is already familiar and to obtain design sensitivities, along with analysis results.

Section 1.1 of this report summarizes theoretical developments of continuum design sensitivity analysis of built-up structures in this project. To obtain accurate shape design sensitivity analysis results, when numerically implemented with finite element analysis method, a domain method is developed and summarized in Section 1.2. In Section 1.3, a design component method that has been developed for sensitivity analysis of built-up structures with both sizing and shape design variables is summarized. The numerical method that has been developed to implement a unified structural design sensitivity analysis method of Sections 1.1-1.3, using established finite element codes, is summarized in Section 1.4. For shape design sensitivity analysis, a general method of defining and computing a design velocity field in the domain, in terms of perturbations of the parameters that are used for parameterization of the boundary, is summarized in Section 1.5.

1.1 Theoretical Developments in Design Sensitivity Analysis of Built-Up Structures

A substantial literature has developed in the field of design sensitivity analysis and optimization of structural components [1] over the past few years. Contributions to this field have been made using two fundamentally different approaches to structural modeling and analysis. The first approach uses a discretized structural model, based on finite element analysis, and proceeds to carry out design sensitivity analysis by differentiating the algebraic finite element equations. While this approach permits direct application of classical algebraic design sensitivity analysis methods that have been used in structural optimization, it leads to algebraic complexity and difficulties in accuracy for shape design problems. A distributed parameter design sensitivity analysis method is used in which the continuum elasticity formulation is retained throughout the derivation of design sensitivity analysis results.

For shape design sensitivity analysis, the latter approach uses an elasticity model of the structure and the material derivative method of continuum mechanics to account for changes in shape of the structure [1-4]. Using this approach, expressions for design sensitivity in terms of domain shape change are derived in the continuous setting and evaluated using any available method of structural analysis; e.g., finite element analysis, boundary element analysis, photoelasticity, etc. While the theory underlying development of design sensitivity expressions using this approach is more complex than the discretization approach, better theoretical insights and more accurate results have been obtained.

The principle objective of the project was to extend the theory of structural components, distributed parameter sizing, and shape design optimization to treat built-up structures that are made up of interconnected components. The approach originally proposed was to use recent developments in functional analysis theory of boundary-value problems to obtain a unified variational formulation for the built-up structure. As shown in Refs. 1, 4, 5, and 6, this result was achieved. In the process, it was discovered that one need not resort to abstract techniques of functional analysis, but may obtain the needed variation formulation directly from energy principles of mechanics. This finding is particularly valuable in treating built-up structures, since concepts of mechanics can guide development of the technique. Theoretical results are presented in Chapter 4 of Ref. 1, using Hamilton's principle to obtain a variational formulation of the governing equations for built-up structures that are employed for design sensitivity analysis. The variational methods presented in Chapters 2 and 3 of Ref. 1, for design sensitivity analysis with respect to conventional design variables and shape, are then combined, using the general variational formulation obtained from Hamilton's principle, to obtain expressions for design sensitivity of functionals with respect to both conventional and shape design variations. The adjoint variable method used in Chapters 2 and 3 of Ref. 1 is extended directly to built-up structures.

One of the most important developments in design sensitivity analysis with respect to shape for built-up structures is the domain method [7,8]. In Refs. 1-3 and 6, shape sensitivity information for each component is explicitly expressed as boundary integrals, using integration by parts and boundary conditions, to obtain identities for transformation of domain integrals to boundary integrals. To numerically calculate design sensitivity information using the boundary integral sensitivity formulas, one must use stresses, strains, and/or normal derivatives of state and adjoint variables on the boundary. Thus, accurate evaluation of this information on the boundary is crucial. For built-up structures, shape design sensitivity information is given as integrals on the interfaces between components. However, it is well known that results of finite element analysis at interfaces may not be accurate.

Several methods were considered to overcome this difficulty. One approach the research team used, in a related project under NSF support, is to obtain accurate finite element analysis results on the boundary [9]. In this approach a smooth boundary parameterization and isoparametric finite elements were used to avoid the "Babuska Paradox". Boundary stresses and strains were calculated by linearly extrapolating values at optimal Gaussian points to the boundary, to obtain accurate values on the boundary. This method was effective for single structural components, as shown in Ref. 9. However, the method was not effective for built-up structures.

A second method the research team employed, in the related project under NSF support, for shape design sensitivity analysis is the boundary element method [10,11]. In the finite element method, the unknown function; e.g., displacement, is approximated by trial functions that do not satisfy the governing equations, but usually satisfy kinematic boundary conditions. Nodal parameters z^i; e.g., nodal displacements, are then determined by approximate satisfaction of both differential equations and nonkinematic boundary conditions, in a domain integral mean sense. On the other hand, in the boundary element method, approximating functions satisfy the governing equations in the domain, but not the boundary conditions. Nodal parameters are determined by approximate satisfaction of boundary conditions, in a weighted boundary integral sense. An important advantage of the boundary element method in shape design sensitivity analysis is that it better represents boundary conditions and is usually more accurate in determining stress at the boundary. In Refs. 10 and 11, it was demonstrated that the boundary element method provides accurate shape design sensitivity results. However, it was found that the boundary element method is not appropriate for built-up structures.

The method developed in this project for shape design sensitivity analysis of built-up structures is a domain method [7,8], in which design sensitivity information is expressed as domain integrals, instead of boundary integrals. This formulation thus best utilizes the basic character of finite element analysis that gives accurate information, not on the boundary, but in the domain.

1.2 Domain Method for Shape Design Sensitivity Analysis

Design sensitivity analysis results obtained with the domain and boundary methods are analytically equivalent. However, when an approximate method, such as finite element analysis, is used to evaluate design sensitivity expressions, the resulting design sensitivity approximations may give quite different numerical values. The boundary method is best suited with the boundary element method and the domain method is best suited with the finite element method. The domain method for shape design sensitivity analysis has been successfully implemented in Ref. 7 for plane-stress interface and simple box problems. It is shown in Ref. 7 that when the finite element method is used for analysis, results obtained with the domain method are excellent, whereas results obtained with the boundary method are not acceptable.

Moreover, the domain method offers striking simplification in derivation of shape design sensitivity formulas for built-up structures; one simply adds contributions from individual components. That is, one need not specialize design sensitivity expressions for different adjacent components, since interface conditions are not used to transform domain integrals to boundary integrals. This gives a method for the systematic organization of shape design sensitivity analysis of built-up structures. That is, one can derive shape design sensitivity formulas for each standard component type, including truss, beam, plane elastic solid, plate, and three-dimensional elastic solid. The result will then be standard formulas that can be used for many structural types, by simply adding contributions from each component. This simple addition of contributions from each component gives a design component method with systematic organization of both sizing and shape design sensitivity analysis of built-up structures. That is, one can define a library of basic structural components that may be assembled to carry out design sensitivity analysis of built-up structures, very much like built-up structures are formed from elements in finite element analysis.

One disadvantage of the domain method is that numerical evaluation of sensitivity information is less efficient than with the boundary method, since the domain method requires integration over the entire domain, whereas the boundary method requires integration over only the variable boundary. To reduce this inefficiency, under a related NSF project, a boundary layer method [12] has been developed and successfully implemented. As shown in Ref. 12, using the boundary layer method, one can obtain direct control over the velocity field within the domain and reduce computing cost, without sacrificing accuracy of the domain method. For determination of the boundary layer, it is suggested in Ref. 12 to measure strain energy density near the varied boundary.

1.3 Design Component Method for Sensitivity Analysis of Built-Up Structures

Using results of the domain method for shape design sensitivity analysis, the design component method has been developed for design sensitivity analysis of built-up structures, with both sizing and shape design variables [4,13]. The design component concept for built-up structures is based on defining a library of basic structural components, such as truss, beam, plate, plane elastic solid, and three-dimensional elastic solid, that can be assembled to form a built-up structure. It is important to clearly distinguish between a design component and a finite element. Each design component will generally be subdivided into many finite elements for stress, displacement, vibration, and buckling analysis. The focus of the design component method is on whole components and design parameters that define their material, section, and shape properties. The continuum design sensitivity analysis formulation developed in Sections 1.1 and 1.2 is used to obtain sizing and shape design variations of energy bilinear and load linear forms of each component. The result is standard expressions that can be used to define the contribution from each component to design sensitivity analysis of the overall built-up structure. Computations are organized in a systematic way, much as computations are organized within a finite element code. Again, it is important to make the clear distinction between the micro-finite elements that are needed for analysis and the macro-design components that are employed to characterize and optimize design. As mentioned in Section 1.2, use of the domain method of shape design sensitivity analysis allows development of the design component method for built-up structures. The beauty and basis of practicality of this method rests on the ability to decompose expressions across component boundaries. Whereas complex boundary interface terms were required in the boundary method of built-up structure shape design sensitivity analysis, the domain method makes no such requirement, hence it allows for systematic assembly of total system design sensitivity expressions.

A systematic component identification scheme has been organized, allowing for definition of a variety of component design parameterizations, to allow automated assembly of design sensitivity expressions. In the actual formulation presented in Refs. 4 and 13, truss and beam components that include both bending and torsion of the beam, have been incorporated into a single component. Similarly, plate and plane elastic panel components have been combined as a single component. A modular computer program has been prepared for carrying out experimental calculations in this research. All computations required for calculation of adjoint loads and design sensitivity expressions for a given component type have been consolidated in individual modules, to allow easy modification of the characteristics of design components and their design parameterization in numerical experiments that were performed throughout the research. While no attempt has been made to prepare a commercially oriented code, care has been taken to identify trade-offs and lessons learned that may serve as a guide in future work that is directed toward large scale implementation.

Feasibility of the method is shown through a truss-beam-plate built-up structure, with excellent numerical results [4,13]. Design sensitivity results for the truss-beam-plate built-up structure have been used for optimization, using a PRIME 750 supermini computer. One quarter of the built-up structure has 400 plate elements, 80 beam elements, and 4 truss elements, with a total of 1281 degrees-of-freedom. For design purposes, it has 292 design variables and 251 stress constraints, in addition to 292 constraints on design variables.

Design optimization has been carried out using a sparse matrix symbolic factorization technique [14] for iterative structural optimization and Pshenichny's linearization method [15]. The sparse matrix symbolic factorization technique offers substantial numerical advantage for iterative structural optimization. The standard Harwell sparse matrix library is used, in conjunction with finite element structural models. The importance of this approach is accentuated when one considers built-up structures that are difficult to model with finite element grids that minimize bandwidth. With sparse matrix techniques, one need not concern himself with bandwidth minimization, since the code uses a general sparse matrix technique that is independent of node numbering.

With the PRIME 750, it took 30,000 CPU seconds per iteration. Use of a Cray supercomputer, funded by NSF, has been investigated for large scale computation, using the truss-beam-plate built-up structure [4].

With the Cray supercomputer, it takes 26 CPU seconds per iteration for the same problem.

Results obtained with the design component method indicate that the method can be implemented with established finite element codes, by assembling a modular computer program that will carry out calculations outside existing finite element codes, using postprocessing data only [16-21].

1.4 Geometric Modeling and Automatic Regridding for Shape Design Sensitivity Analysis

In structural shape design, the varying domain is treated as the design variable. Therefore it is necessary to characterize the shape of domain; i.e., parameterization of the boundary shape. The result of shape optimization is naturally limited by the design parameterization used. To reach a better optimal shape design, the design parameterization must be general and flexible enough to represent large classes of structural shapes. It is desirable that the parameterization method has the following properties: smoothness, fairness, required order of continuity, controllability in global and local senses, and a variation diminishing property. Among several parameterization methods, Bezier and B-spline surfaces were used in this research [4,22,23]. Both Bezier and B-spline surfaces use a set of blending functions and are defined in terms of characteristic polyhedra. When Bezier and B-spline surfaces are used, positions of the control points are shape design parameters.

With the parameterization of boundary carried out, a general method of defining and computing a velocity field in the domain, in terms of a perturbation of the boundary Γ has been developed. It is shown in Ref. 4 that a C^0-regular velocity field with an integrable first derivative can be used for truss, plane elastic solid, and three-dimensional solid components and a C^1-regular velocity with an integrable second derivative can be used for beam and plate components. Therefore, regularity of the velocity field must be at least at the level of regularity of the displacement field of the structural component considered. Based on this observation the displacement shape functions are used to systematically define the velocity field in the domain.

Moreover, a velocity field that obeys the governing elliptic equation of the structure has been selected. That is, the perturbation of the boundary is considered as a displacement at the boundary. With no additional external forces and a given displacement at the boundary, the finite element code has been used to find the displacement (domain velocity) field that satisfies the required regularity conditions.

An automatic regridding method has been used with the boundary layer approach very effectively [22,23]. That is, if a large portion of the structure is fixed, except for the boundary layer (or substructure), then the dimension of equation to be solved to obtain domain velocity field is reduced. In this study it was found the regridding method developed tends to maintain orthogonality of the finite element grid. Thus if the initial grid is optimized using an adaptive method, the regridding method will tend to avoid distortion of the finite elements. Also, it was demonstrated in Refs. 22 and 23 that the method developed can be used as a mesh generator. That is, starting from a regular shape with a regularly patterned mesh, the method can be used to generate a mesh directly for a given shape. The method has been tested in Refs. 22 and 23, using three-dimensional problems such as engine bearing cap, arch dam, and a 3-D interface problem, with excellent results.

1.5 Numerical Implementation of Design Sensitivity Analysis with Established Finite Element Analysis Codes

One major objective of the research was to develop and implement structural design sensitivity analysis by taking advantage of the versatility and convenience of established finite element structural analysis programs and the theoretical foundation in structural design sensitivity analysis that is reflected in Sections 1.1-1.4.

Based on the results obtained with the design component method, numerical methods have been developed to implement a unified structural design sensitivity analysis theory, using ANSYS, EAL, and IFAD [16-21]. Sizing design variables, such as thickness and cross-sectional areas, and shape design variables of components of built-up structures were considered. Structural performance functionals considered include displacement, stress, and eigenvalue.

Evaluation of design sensitivity expressions for built-up structures were implemented in a modular program, using data generated by established finite element analysis codes. Finite element grids that are generated for each component in the built-up structure have been maintained and modified in a module that contains data for that component, as sizing and shape designs are modified. Finite element mesh and load data generated within each of the modules has been passed to established finite element analysis codes for displacement and stress analysis. Numerical results obtained from the finite element analysis code are passed back to the modules for adjoint load calculation.

Adjoint loads are then formed in each module and passed back to the finite element code for adjoint displacement and strain analysis, using previously factored stiffness matrices. Numerical data are then passed back to the modules for design sensitivity computation.

Contributions to design sensitivity expressions from each component are evaluated in the appropriate module, using state and adjoint information that has been generated. Computations have been carried out at the component level, using the finite element mesh, shape functions, and component design variables that are available in each module. Upon completion of calculations at the module level, individual contributions from each module to the total design sensitivity expression are passed to a central processor and design sensitivity of the entire built-up structure is calculated.

The method allows calculations to be carried out outside established finite element codes, using postprocessing data only. Thus, design sensitivity analysis calculations do not have to be embedded in an existing finite element code. The method does not require differentiation of stiffness and mass matrices in conventional finite element models and one can obtain accurate design sensitivity information without the uncertainty of numerical accuracy associated with selection of finite difference perturbations. Under the project, numerical implementation of design sensitivity analysis has been carried out with two established finite element codes. Both conventional and shape design analyses have been implemented using EAL (Hybrid Method) and ANSYS (Displacement Method). Implementation was done for a few finite elements for each code, to demonstrate feasibility of the method.

1.5.1 Finite Element Code Using Displacement Method

The displacement method finite element code used in implementation of design sensitivity analysis is ANSYS. To implement the adjoint variable method described, one calculates an adjoint load for each constraint functional, which is written in terms of displacement, stress, compliance, and natural frequency. To calculate the adjoint load associated with a stress constraint, one must know the shape functions of the finite element analysis code [16-18]. For ANSYS, which uses a displacement method, the shape functions of the code are used.

Implementation for sizing design has been carried out and tested for finite elements such as STIF1 (2-D truss), STIF3 (2-D beam), STIF4 (3-D beam), STIF8 (3-D truss), STIF41 (3-D membrane), and STIF43 (shell). Sizing design variables treated are cross-sectional area and thickness.

For shape design sensitivity analysis of 3-dimensional solids, element STIF95 (20-noded isoparametric solid) have been used. Using element STIF95, computation of the domain velocity field and automatic regridding of Section 1.4 have been employed for shape design sensitivity analysis. This capability has been tested using 3-dimensional problems such as an engine bearing cap, an arch dam, and a 3-D interface problem [22,23].

The design sensitivity analysis method developed under this project has been extended to pointwise stress constraints. Sensitivity accuracy of pointwise stress has been tested on the three problems mentioned, with excellent results. The boundary layer method, combined with the automatic regridding method, has been successfully tested for the engine bearing cap [22,23].

1.5.2 Finite Element Code Using Hybrid Method

The hybrid method finite element code used in implementation of design sensitivity analysis was EAL [19-21]. Implementation was carried out using the database management system and runstream of EAL, without writing a separate program and a separate database. To implement the adjoint variable method of design sensitivity analysis developed, as mentioned before, displacement shape functions are necessary to compute adjoint loads and to numerically evaluate design sensitivity expressions. To calculate equivalent nodal forces for the adjoint load in a consistent way, it is desirable to use the same displacement shape functions that are used in the code. However, EAL is based on a Hybrid Method and no shape function is defined for displacement on the domain of elements. To overcome this difficulty, a library of shape functions was selected for external adjoint load calculation. Selection of the shape function is based on the finite element analysis code used. That is, once the degrees of freedom (nodal displacements) of the finite element analysis code are known, one can select a compatible shape function that is defined on the same finite element and has the same degrees of freedom. With adjoint loads calculated externally, using the selected shape function, one can proceed to use the adjoint variable method. An argument that supports this method is that, with the same degrees of freedom, different methods of approximation give comparable results, if both approximation methods are acceptable, as is the case in contemporary finite element analysis codes. These selected displacement shape functions are also used for computation of gradients of displacements that are needed for evaluation of shape design sensitivity expressions. It is shown in Refs. 19-21 that excellent design sensitivity results are obtained using these selected displacement shape functions.

Implementation for conventional design has been carried out and tested for finite elements such as E21 (general beam), E41 (membrane), E42 (plate), and E43 (membrane plus plate) [20,21]. Pointwise stress constraints were also tested using EAL and excellent results were obtained. For shape design sensitivity analysis, element E41 was used and tested for simple box and interface problems [19,21] and accurate results were obtained. Even though a limited number of elements have been tested, feasibility and accuracy are clearly demonstrated.

1.6 Results from Related Projects

The unified design sensitivity analysis method of Ref. 1 has been used and implemented in this project. Under NSF sponsorship, this method has been extended to handle geometric and material structural nonlinearities, under the kinematic assumption of infinitesimal strains. Sizing design variables, such as thicknesses and cross-sectional areas of components of individual members and built-up structures, are considered.

As in linear structural systems studied in this project, a distributed parameter structural design sensitivity analysis approach is used that retains the continuum elasticity formulation throughout derivation of design sensitivity analysis results. Using this approach and the same adjoint variable method as used in this project, explicit expressions for design sensitivity in terms of design variations are derived in the continuous setting and evaluated numerically, using established finite element analysis codes. A very interesting result is that the adjoint equation is linear. Thus, the computational effort of evaluating sensitivity expressions is the same as in linear structural systems. This means that the ratio of computational effort for sensitivity analysis and structural analysis is very low. This is very attractive, compared to the finite difference method, which will be very inefficient.

To test the new nonlinear design sensitivity analysis capability, implementation has been carried out using ANSYS finite elements STIF1 (2-D truss), STIF3 (2-D beam), STIF8 (3-D truss), and STIF41 (3-D membrane). Preliminary experiments indicate excellent results [24]. In one test problem, STIF8 and STIF41 have been used to set up a swept wing example of Ref. 25. Using the data of Ref. 25, both linear and nonlinear finite element models are made and sensitivity analyses are carried out. Both linear and nonlinear sensitivity analyses give accurate results. However, when displacements of the tip of the wing are compared, the result of linear analysis is 1.4 times the result of nonlinear analysis. Moreover, even though the sensitivity vectors are accurate for both cases, they are quite different, even opposite in direction in some cases. Hence, it is anticipated that optimum design results using linear and nonlinear models will be quite different. Future directions of research under this NSF project will include extension of design sensitivity analysis for nonlinear systems to include shape design variables.

2. References

1. Haug, E.J., Choi, K.K. and Komkov, V., Design Sensitivity Analysis of Structural Systems, Academic Press, New York, N.Y., 1986.

2. Choi, K.K. and Haug, E.J., "Shape Design Sensitivity Analysis of Elastic Structures", J. of Struct. Mechanics, Vol. 11, No. 2, 1983, pp. 231-269.

3. Choi, K.K., "Shape Design Sensitivity Analysis of Displacement and Stress Constraints", J. of Struct. Mechanics, Vol. 13, No. 1, 1985, pp. 27-41.

4. Choi, K.K., "Shape Design Sensitivity Analysis and Optimal Design of Structural Systems", Computer Aided Optimal Design, (Ed. C.A. Mota Soares), Springer-Verlag, Heidelberg, 1987.

5. Choi, K.K., "Shape Design Sensitivity Analysis of Built-Up Structures", Recent Experiences in Multidisciplinary Analysis and Optimization, NASA Conference publication 2327, 1984, pp. 423-437.

6. Lee, H.G., Choi, K.K., and Haug, E.J., Design Sensitivity Analysis and Optimization of Built-Up Structures, Technical Report 84-12, Center for Computer Aided Design, The University of Iowa, December 1984.

7. Choi, K.K. and Seong, H.G., "A Domain Method for Shape Design Sensitivity Analysis of Built-Up Structures", Computer Methods in Applied Mechanics and Engineering, Vol. 57, No. 1, 1986, pp. 1-15.

8. Seong, H.G. and Choi, K.K., Shape Design Sensitivity Analysis Using Domain Information, Technical Report 85-3, Center for Computer Aided Design, The University of Iowa, April 1985.

9. Yang, R.J. and Choi, K.K., "Accuracy of Finite Element Based Design Sensitivity Analysis", J. of Struct. Mechanics, Vol. 13, No. 2, 1985, pp. 223-239.

10. Mota Soares, C.A., Rodrigues, H.C., and Choi, K.K., "Shape Optimal Structural Design Using Boundary Element and Minimum Compliance Techniques", ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 106, No. 4, 1984, pp. 516-521.

11. Mota Soares, C.A. and Choi, K.K., "Boundary Elements in Shape Optimal Design of Structures", The Optimum Shape: Automated Structural Design, (Eds. J.A. Bennett and M.E. Botkin), Plenum Press, New York, 1986, pp. 199-231; also in Computer Aided Optimal Design, (Ed. C.A. Mota Soares), Springer-Verlag, Heidelberg, 1987.

12. Seong, H.G. and Choi, K.K., "Boundary Layer Approach To Shape Design Sensitivity Analysis", Mechanics of Structures and Machines, Vol. 15, No. 2, 1987, pp. 241-263.

13. Choi, K.K., and Seong, H.G., "Design Component Method for Sensitivity Analysis of Built-Up Structures", J. of Struct. Mechanics, Vol. 14, No. 3, 1986, pp. 379-399.

14. Lam, H.L., Choi, K.K., and Haug, E.J., "A Sparse Matrix Finite Element Technique for Iterative Structural Optimization", Computers and Structures, Vol. 16, No. 1-4, 1983, pp. 289-295.

15. Choi, K.K., Haug, E.J., Hou, J.W., and Sohoni, V.N., "Pshenichny's Linearization Method for Mechanical System Optimization", ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 105, No. 1, 1983, pp. 97-103.

16. Choi, K.K., Santos, J.L.T., and Frederick, M.C., "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes", ASME Journal of Mechanisms, Transmissions, and Automation in Design, 85-DET-77, 9p.

17. Choi, K.K., Santos, J.L.T., and Frederick, M.C., "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes", ANSYS Conference, Pittsburgh, PA, April 23-25, 1985.

18. Frederick, M. and Choi, K.K., Design Sensitivity Analysis with an Existing Code Using the Adjoint Variable Method, Technical Report 84-17, Center for Computer Aided Design, The University of Iowa, December 1984.

19. Dopker, B. and Choi, K.K., "Sizing and Shape Design Sensitivity Analysis Using a Hybrid Finite Element Code", Finite Elements in Analysis and Design, to appear, 1987.

20. Dopker, B., Choi, K.K., and Lee, J., Design Sensitivity Analysis Using EAL: Part I: Conventional Design Parameters, Technical Report 86-2, Center for Computer Aided Design, The University of Iowa, January 1986.

21. Dopker, B. and Choi, K.K., Design Sensitivity Analysis Using EAL: Part II: Shape Design Parameters, Technical Report 86-8, Center for Computer Aided Design, The University of Iowa, June 1986.

22. Choi, K.K. and Yao, T.M., "3-D Modeling and Automatic Regridding in Shape Design Sensitivity Analysis", Sensitivity Analysis in Engineering, NASA Conference publication 2457, 1987, pp. 329-345.

23. Yao, T.M. and Choi, K.K., Shape Design Sensitivity Analysis and Optimization of Three-Dimensional Elastic Solids Using Geometric Modeling and Automatic Regridding, Technical Report 87-1, Center for Computer Aided Design, The University of Iowa, January 1987.

24. Santos, J.L.T. and Choi, K.K., "Design Sensitivity Analysis of Nonlinear Structural Systems with Existing Finite Element Code", Proceedings, ANSYS Conference, pp. 1.35-1.56.

25. Camarda, C.J. and Adelman, H.M., Static and Dynamic Structural-Sensitivity Derivative Calculations in the Finite-Element-Based Engineering Analysis Language (EAL) System, NASA TM-83181, March 1984.

3. **Publications, NASA Project NAG-1-215**

Haug, E.J., Choi, K.K. and Komkov, V., Design Sensitivity Analysis of Structural Systems, Academic Press, New York, N.Y., 1986.

Choi, K.K., "Shape Design Sensitivity Analysis and Optimal Design of Structural Systems", Computer Aided Optimal Design, (Ed. C.A. Mota Soares), Springer-Verlag, Heidelberg, 1987.

Choi, K.K., "Shape Design Sensitivity Analysis of Built-Up Structures", Recent Experiences in Multidisciplinary Analysis and Optimization, NASA Conference publication 2327, 1984, pp. 423-437.

Lee, H.G., Choi, K.K., and Haug, E.J., Design Sensitivity Analysis and Optimization of Built-Up Structures, Technical Report 84-12, Center for Computer Aided Design, The University of Iowa, December 1984.

Choi, K.K. and Seong, H.G., "A Domain Method for Shape Design Sensitivity Analysis of Built-Up Structures", Computer Methods in Applied Mechanics and Engineering, Vol. 57, No. 1, 1986, pp. 1-15.

Seong, H.G. and Choi, K.K., Shape Design Sensitivity Analysis Using Domain Information, Technical Report 85-3, Center for Computer Aided Design, The University of Iowa, April 1985.

Choi, K.K. and Seong, H.G., "Design Component Method for Sensitivity Analysis of Built-Up Structures", J. of Struct. Mechanics, Vol. 14, No. 3, 1986, pp. 379-399.

Choi, K.K., Santos, J.L.T., and Frederick, M.C., "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes", ASME Journal of Mechanisms, Transmissions, and Automation in Design, 85-DET-77, 9p.

Choi, K.K., Santos, J.L.T., and Frederick, M.C., "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes", ANSYS Conference, Pittsburgh, PA, April 23-25, 1985.

Frederick, M. and Choi, K.K., Design Sensitivity Analysis with an Existing Code Using the Adjoint Variable Method, Technical Report 84-17, Center for Computer Aided Design, The University of Iowa, December 1984.

Dopker, B. and Choi, K.K., "Sizing and Shape Design Sensitivity Analysis Using a Hybrid Finite Element Code", Finite Elements in Analysis and Design, to appear, 1987.

Dopker, B., Choi, K.K., and Lee, J., Design Sensitivity Analysis Using EAL: Part I: Conventional Design Parameters, Technical Report 86-2, Center for Computer Aided Design, The University of Iowa, January 1986.

Dopker, B. and Choi, K.K., Design Sensitivity Analysis Using EAL: Part II: Shape Design Parameters, Technical Report 86-8, Center for Computer Aided Design, The University of Iowa, June 1986.

Choi, K.K. and Yao T.M., "3-D Modeling and Automatic Regridding in Shape Design Sensitivity Analysis", Sensitivity Analysis in Engineering, NASA Conference publication 2457, 1987, pp. 329-345.

Yao, T.M. and Choi, K.K., Shape Design Sensitivity Analysis and Optimization of Three-Dimensional Elastic Solids Using Geometric Modeling and Automatic Regridding, Technical Report 87-1, Center for Computer Aided Design, The University of Iowa, January 1987.