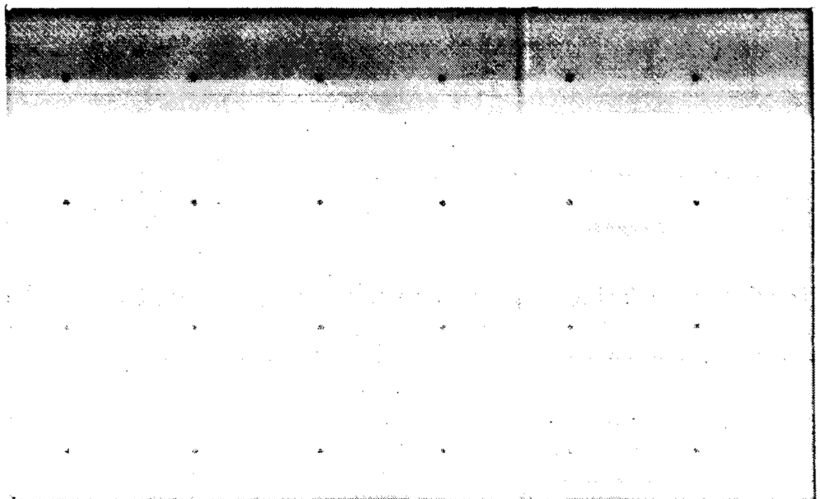
NAG1-215
IN-39 P-FILE
82564
CR
p. 37

CENTER FOR COMPUTER AIDED DESIGN



College of Engineering
The University of Iowa
Iowa City, Iowa 52242

(NASA-CR-181098) DESIGN SENSITIVITY ANALYSIS USING EAL. PART 2: SHAPE DESIGN PARAMETERS (Iowa Univ.) 37 p Avail: NTIS HC A03/MF A01 CSCL 20K

N87-27222

Unclas
G3/39 0082564

Technical Report 86-8

DESIGN SENSITIVITY ANALYSIS USING EAL:
PART II: SHAPE DESIGN PARAMETERS

by

B. Dopker and Kyung K. Choi

Center for Computer Aided Design

College of Engineering
The University of Iowa
Iowa City, Iowa 52242

Research supported by NASA Grant NAG-1-215

June 1986

ABSTRACT

A numerical implementation of shape design sensitivity analysis of built-up structures is presented, using the versatility and convenience of an existing finite element structural analysis code and its data base management system. This report is a continuation of the Technical Report 86-2, Part I: Conventional Design Parameters. The finite element code used in the implementation presented is the Engineering Analysis Language (EAL), which is based on a hybrid method of analysis. It has been shown that shape design sensitivity computations can be carried out using the database management system of EAL, without writing a separate program and a separate database.

The material derivative concept of continuum mechanics and an adjoint variable method of design sensitivity analysis are used to derive shape design sensitivity informations of structural performances. A domain method of shape design sensitivity analysis and a design component method are used. Displacement and stress functionals are considered as performance criteria.

CHAPTER I

INTRODUCTION

1.1 Purpose

To date there exist a wide variety of finite element structural analysis programs that are used as reliable tools for structural analysis. They give the designer pertinent information such as stresses, strains, and displacements of the structure being modeled. However, if this information reveals that the structure does not meet specified constraint requirements, the designer must make intuitive estimates on how to improve the design. If the structure is complex, it becomes very difficult to decide what step must be taken to improve the design. There is however, substantial literature [1] on the theory of shape design sensitivity analysis, which predicts the effect that structural shape design changes have on the performance of a structure.

The purpose of this work is to develop and implement structural shape design sensitivity analysis, using the material derivative concept of continuum mechanics and the adjoint variable method [1] that takes advantage of the versatility and convenience of an existing finite element structural analysis code and its database management system. A domain method of shape design sensitivity analysis [2] is used in which design sensitivity information is expressed as domain integrals, instead of boundary integrals, to best utilize the basic character of the finite element method that gives accurate information not on the boundary but in the domain. The finite element code that will be used is the Engineering Analysis Language EAL [3].

Using the full capabilities of the EAL system, design sensitivities can be calculated within the program, without knowing the source code of the program. This has the advantage that the user deals with only one program, with only one data base and no interfaces between different programs [4,5,6].

1.2 Continuum Approach of Design Sensitivity Analysis

A number of methods could be used to implement structural shape design sensitivity analysis, but the most powerful is the continuum approach [1]. This method can be implemented outside an existing finite element code [4,5,6], using only postprocessing data. This approach is convenient, because shape design sensitivity analysis software does not have to be embedded in an existing finite element code. The continuum approach to shape design sensitivity analysis calculation can also be implemented using a powerful database management system such as the Engineering Analysis Language (EAL). Using the database management system of EAL, only one database is necessary for computation of shape design sensitivity information. That is, it is not necessary to crease interfaces and other datafiles to compute sensitivity information. Information on element shape functions used in the finite element model is, however, necessary for design sensitivity calculation.

The continuum approach to shape design sensitivity analysis can easily be extended to complex structural systems that have more than one structural component using a design component method [7]. The shape design sensitivity vector is the derivative of a constraint functional with respect to shape design parameters. The magnitude of each component reflects how sensitive the constraint functional is to a change in the associated shape design parameter. If a component of the vector is negative, the corresponding shape design parameter should be decreased to increase the value of the constraint functional. If a component of the vector is positive, the corresponding shape design parameter should be increased to increase the value of the constraint functional. In addition, if the magnitude of a component of the vector is large, then the corresponding shape design parameter will have a more substantial effect on design improvement.

When a designer uses a finite element structural analysis code in design of a structure, it is most likely that a number of program runs are necessary before a substantially improved design is obtained. With the aide of a shape design sensitivity vector, the designer will know what direction to take to improve the design most efficiently.

CHAPTER II

DESIGN SENSITIVITY ANALYSIS METHOD

A detailed treatment of methods and procedures for calculating shape design sensitivities is given in Ref. 1, for constraint functionals such as compliance, displacement, stress and eigenvalues. For compliance and eigenvalue functionals, the adjoint equation is the same as the state equation, thus no adjoint equation needs to be solved. Each displacement and stress functional requires an adjoint load computation and an adjoint equation must be solved. Due to the symmetry of the energy bilinear forms, the state equation and the adjoint equation differ only in their load terms [1]. Using the reload or multi-load option of an existing finite element code, the adjoint equation can be solved efficiently [4]. For the displacement functional, the adjoint load is a unit load acting at the point where the displacement constraint is imposed. For the stress functional, the equivalent nodal force of the adjoint load has to be computed [1].

The flow chart of Fig. 1 shows the overall procedure. First, the model is defined by identifying the shape design parameters, constraint functionals, finite element model, and loadings. In the next step, EAL is used to obtain structural response. With the structural response obtained, an adjoint load is calculated for each constraint functional, external to EAL, using assumed displacement shape functions. The adjoint load is input to EAL, to obtain an adjoint response for each

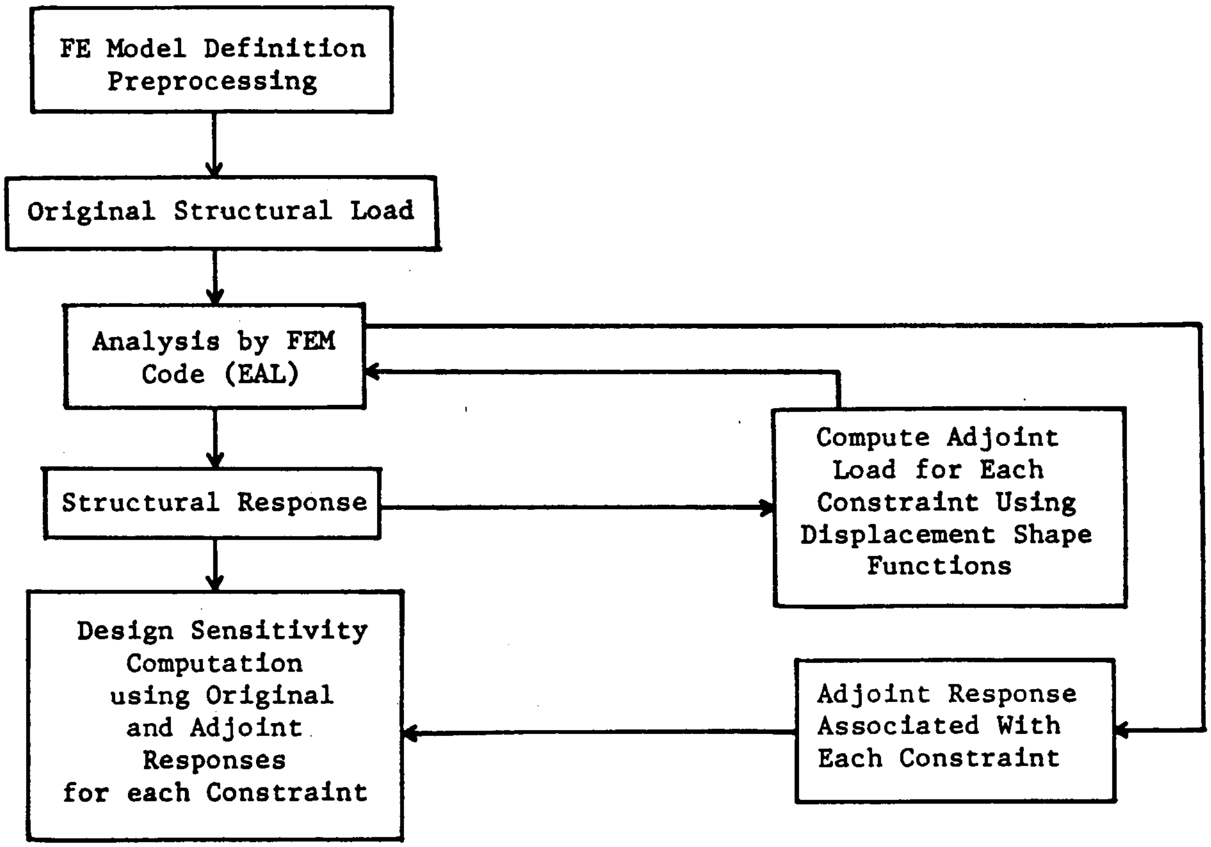


Figure 1. Flow Chart of Design Sensitivity Computation

constraint functional using reload option. Using the original structural response and the adjoint response, shape design sensitivity information is computed for each constraint, by numerically integrating the design sensitivity expressions. The process is convenient, since it uses data that are available in the database or easily computable outside EAL.

The design sensitivity expressions contain derivatives of the displacement field. Because EAL uses a hybrid formulation for membrane and plate elements, a stress field is assumed and the displacement field is unknown. So derivatives of the displacement field cannot be computed directly. To overcome this difficulty, an acceptable displacement shape function is selected for the displacement field. Selection of the shape function is based only on the degree of freedom (nodal displacement) of the EAL finite element code. With nodal displacements calculated from EAL and using the selected shape function, derivatives of the displacement field at the Gauss points are calculated. An argument that supports this method is that, with the same degrees of freedom, different methods of finite element approximation give accurate results, if both approximation methods are acceptable, as in the case in contemporary FEM codes.

To give the basic idea of implementation of shape design sensitivity analysis and computation procedures, a simple prototype structural component is investigated. Once shape design sensitivity analysis of a structural component is completed, the design component method of Ref. 6 can be used for design sensitivity analysis of built-up structures.

Consider a thin elastic solid shown in Fig. 2, with thickness h(x) of the membrane, where $x = [x_1, x_2]^T$. The design is the shape of the domain of the membrane. The energy bilinear and load linear forms are [1],

$$a_u(z,\bar{z}) = \iint_\Omega h(x) \sum_{i,j=1}^{2} \sigma^{ij}(z)\ \varepsilon^{ij}(\bar{z})d\Omega \tag{2.1}$$

and

$$\ell_u(\bar{z}) = \int_\Gamma \sum_{i=1}^{2} T^i\bar{z}^i\ d\Gamma \tag{2.2}$$

where $z = [z^1, z^2]^T$ is the displacement field, $T = [T^1, T^2]^T$ is the boundary traction, and $\sigma^{ij}(z)$ and $\varepsilon^{ij}(\bar{z})$ are the stress and strain fields associated with the displacement z and the virtual displacement $\bar{z}$, respectively. The variational state equation is [1]

$$a_u(z,\bar{z}) = \ell_u(\bar{z}) \tag{2.3}$$

for all kinematically admissible virtual displacements $\bar{z}$.

First consider the functional that represents the displacement z at a discrete point $\hat{x}$,

$$\psi_1 \equiv z(\hat{x}) = \iint_\Omega \hat{\delta}(x - \hat{x})z(x)\ d\Omega \tag{2.4}$$

where $\hat{\delta}(x)$ is the Dirac delta. The first variation of Eq. 2.4 is [1]

$$\psi_1' = \iint_\Omega \hat{\delta}(x - \hat{x})\dot{z}(x)d\Omega \tag{2.5}$$

The adjoint equation in this case is [1,3]

$$a_u(\lambda,\bar{\lambda}) = \iint_\Omega \hat{\delta}(x - \hat{x})\bar{\lambda}(x)d\Omega \tag{2.6}$$

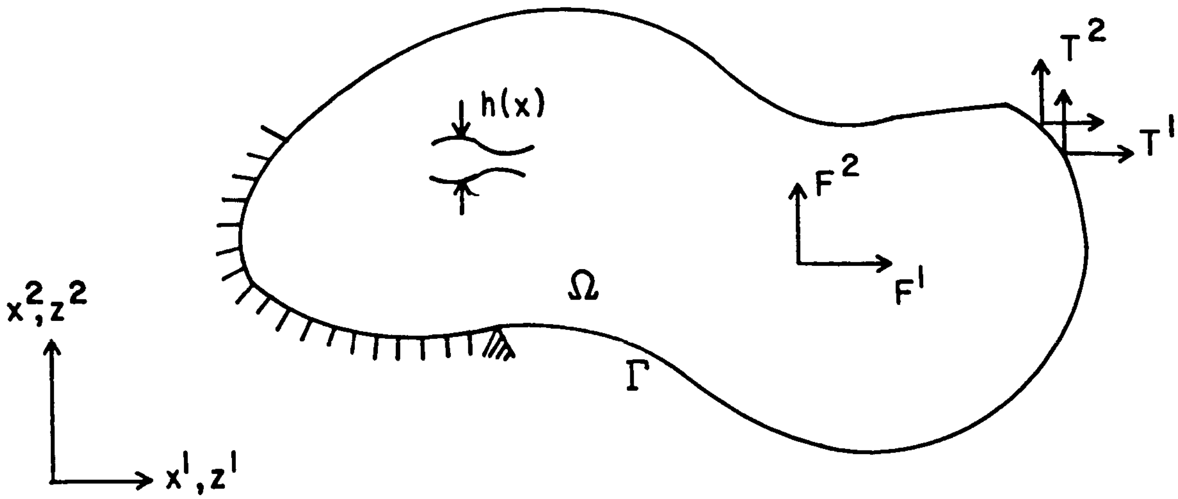


Figure 2. Clamped Plane Elastic Solid of Variable Thickness

for all kinematically admissible displacements $\bar{\lambda}$. This equation has a unique solution λ, which is the displacement field due to a unit point load acting at a point $\hat{x}$. Using the adjoint variable method, design sensitivity of the displacement functional is [1]

$$\psi_1' = \iint_\Omega \sum_{i,j=1}^{2} [\sigma^{ij}(z)(\nabla\lambda^{i^T} V_j) + \sigma^{ij}(\lambda)(\nabla z^{i^T} V_j)]d\Omega$$

$$- \iint_\Omega [\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)]\text{div } Vd\Omega \qquad (2.7)$$

where λ is the solution of Eq. 2.6, ∇ is the gradient operator defined as $\nabla = [\frac{\partial}{\partial x_1} \frac{\partial}{\partial x_2}]^T$, V is the design velocity field and div V is the divergence of the design velocity field. To numerically integrate Eq. 2.7, a two-by-two Gauss-point integration procedure is used. Using stress computation of membrane element E41 in EAL, stresses and strains can be expressed in matrix form as [3, 8]

$$\{\sigma^{11}(z),\ \sigma^{22}(z),\ \sigma^{12}(z)\}^T = [P]\{\beta\} \tag{2.8}$$

$$\{\varepsilon^{11}(z),\ \varepsilon^{22}(z),\ \varepsilon^{12}(z)\}^T = [E]^{-1}\{\sigma\} \tag{2.9}$$

where

$$[P] = \begin{bmatrix} 0 & 0 & 1 & x_2 & 0 \\ 0 & 1 & 0 & 0 & x_1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{\beta\} = \{b_1,\ b_2,\ b_3,\ b_4,\ b_5\}^T \tag{2.10}$$

$$[E] = \frac{E}{\left(1 - \nu^2\right)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \tag{2.11}$$

and [P] is the position coordinate matrix, which determines points where the stresses are obtained, $\{\beta\}$ is the stress coefficient vector, and [E] is the elasticity matrix for a plane stress problem [9, 10]. To compute the derivatives of the displacement field z and the adjoint displacement field λ which are required to evaluate Eq. 2.7, a bilinear displacement shape function is used. Because the membrane element E41 in EAL is a four noded element, a linear isoparametric element is assumed.

$$\begin{Bmatrix} z^1 \\ z^2 \end{Bmatrix} = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \\ N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} u^1 & v^1 \\ u^2 & v^2 \\ u^3 & v^3 \\ u^4 & v^4 \end{bmatrix}$$

where u^i and v^i, $i = 1,2,3,4$ are the nodal displacements of z^1 and z^2, respectively. The individual shape functions are

$$\begin{aligned} N_1 &= \frac{1}{4}(1-\xi)(1-\eta) \\ N_2 &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 &= \frac{1}{4}(1+\xi)(1+\eta) \\ N_4 &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned} \tag{2.13}$$

The mapping of the finite element from the x_1-x_2 coordinate to the ξ-η coordinate is given in Fig. 3. Using the chain rule of differentiation, the derivatives of the displacement field are

$$\begin{bmatrix} z^1_1 & z^2_1 \\ z^1_1 & z^2_2 \end{bmatrix} = [J^{-1}] \begin{bmatrix} N_{1,\xi} & N_{2,\xi} & N_{3,\xi} & N_{4,\xi} \\ N_{1,\eta} & N_{2,\eta} & N_{3,\eta} & N_{4,\eta} \end{bmatrix} \begin{bmatrix} u^1 & v^1 \\ u^2 & v^2 \\ u^3 & v^3 \\ u^4 & v^4 \end{bmatrix} \tag{2.14}$$

where [J] is the Jacobian of the mapping function from the x_1-x_2 coordinate to the ξ-η coordinate.

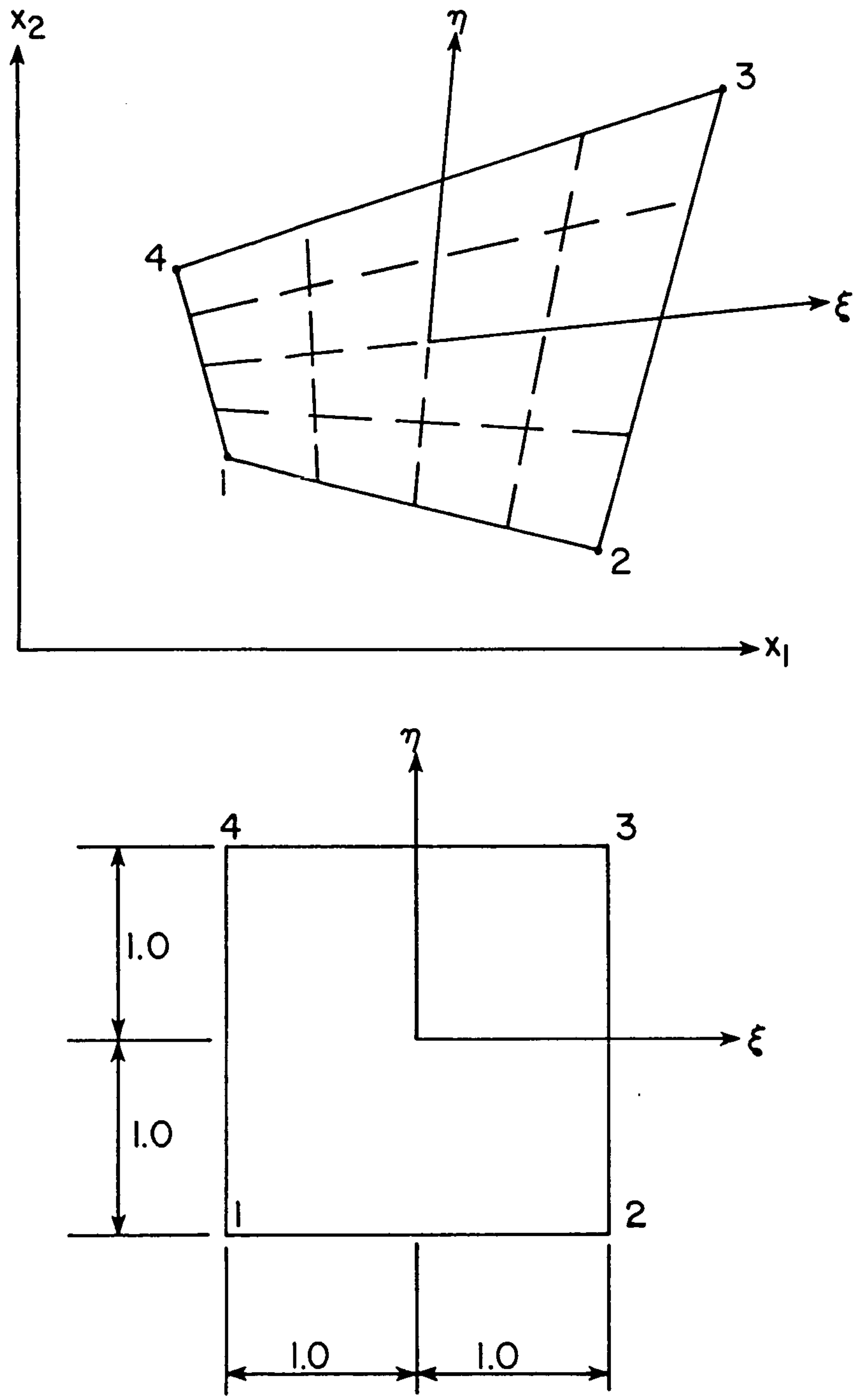


Figure 3. Mapping of the Element from the ξ-η Coordinate to the x_1-x_2 Coordinate

Consider the general functional that represents a locally averaged stress on an element as

$$\psi_2 = \iint_{\Omega} g\big(\sigma(z)\big) m_p \, d\Omega \tag{2.15}$$

where m_p is a characteristic function, defined on a finite element Ω_p as

$$m_p = \begin{cases} \dfrac{1}{\iint_{\Omega_p} d\Omega} & x \in \Omega_p \\ 0 & x \notin \Omega_p \end{cases} \tag{2.16}$$

and g is the stress function. The first variation of Eq. 2.15 is [1]

$$\begin{aligned} \psi_2' = & \iint_{\Omega} \Big[\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z) \, \sigma^{ij}(\dot{z}) \Big] m_p \, d\Omega \\ & - \iint_{\Omega} \sum_{i,j=1}^{2} \Big[\sum_{k,\ell=1}^{2} g_{\sigma^{ij}}(z) \, C^{ijk\ell} \, \big(\nabla z^{k^T} V_\ell\big) \Big] m_p \, d\Omega \\ & + \iint_{\Omega} g(z) \operatorname{div} V \, m_p \, d\Omega - \iint_{\Omega} g(z) m_p \, d\Omega \iint_{\Omega} m_p \operatorname{div} V \, d\Omega \end{aligned} \tag{2.17}$$

For the adjoint variable method, the material derivative $\dot{z}$ is replaced by a virtual displacement $\bar{\lambda}$ in the first term on the right side of Eq. 2.17, to define a load functional, and equate to the energy bilinear form to obtain the adjoint equation

$$a_u(\lambda, \bar{\lambda}) = \iint_{\Omega} \Big[\sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma^{ij}}(z) \, \sigma^{ij}(\bar{\lambda}) \Big] m_p \, d\Omega \tag{2.18}$$

for all kinematically admissible displacements $\bar{\lambda}$. Equation 2.18 has a unique solution for a displacement field λ [1]. Using the adjoint variable method, design sensitivity of the stress functional is

$$\psi_2' = \iint_\Omega \sum_{i,j=1}^{2} [\sigma^{ij}(z)(\nabla\lambda^{i^T} V_j) + \sigma^{ij}(\lambda)(\nabla z^{i^T} V_j)]d\Omega$$

$$- \iint_\Omega [\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)]\text{div } V\, d\Omega$$

$$- \iint_\Omega \sum_{i,j=1}^{2} [\sum_{k,\ell=1}^{2} g_{\sigma^{ij}}(z)C^{ijk\ell}(\nabla z^{k^T} V_\ell)]m_p d\Omega \tag{2.19}$$

$$+ \iint_\Omega g(z)\text{div } Vm_p\, d\Omega - \iint_\Omega g(z)m_p d\Omega \iint_\Omega m_p \text{ div } Vd\Omega$$

here $C^{ijk\ell}$ is the stress-strain relation defined as

$$\sigma^{ij} = C^{ijk\ell} \varepsilon^{k\ell} \quad , \qquad i,j,k,\ell = 1,2. \tag{2.20}$$

To numerically integrate Eqs. 2.18 and 2.19, a two-by-two Gauss point integration procedure is used.

If von Mises' stress criteria is selected in the constraint functional, then

$$g = [(\sigma^{11})^2 + (\sigma^{22})^2 + 3(\sigma^{12})^2 - \sigma^{11}\sigma^{22}]^{1/2} \tag{2.21}$$

and

$$\frac{\partial g}{\partial \sigma^{11}} = (2\sigma^{11} - \sigma^{22})/2g$$

$$\frac{\partial g}{\partial \sigma^{22}} = (2\sigma^{22} - \sigma^{11})/2g \tag{2.22}$$

$$\frac{\partial g}{\partial \sigma^{12}} = 3\sigma^{12}/g$$

which can be written in vector form as

$$\{\frac{\partial g}{\partial \sigma^{ij}}\} = \{\frac{\partial g}{\partial \sigma^{11}}, \frac{\partial g}{\partial \sigma^{22}}, \frac{\partial g}{\partial \sigma^{12}}\}^T \tag{2.23}$$

The equivalent nodal force is computed, based on the modified Hellinger-Reissner principle. The stress can be written as

$$\{\sigma\} = [P]\{\beta\} \tag{2.24}$$

(see also Eq. 2.8). The stress coefficients $\{\beta\}$ can be expressed in terms of the nodal displacement coefficients $\{q\}$ as [8]

$$\{\beta\} = [H^{-1}][T]\{q\} \tag{2.25}$$

where H is the flexibility matrix. Using Eqs. 2.24 and 2.25, stress can be expressed in terms of the nodal displacements as

$$\{\sigma\} = [P][H^{-1}][T]\{q\} \tag{2.26}$$

and thus the nodal equivalent forces for the adjoint load are

$$\iint_{\Omega} \{ \frac{\partial g}{\partial \delta^{ij}} \}^{T}[P][H^{-1}][T]m_{P} \, d\Omega \tag{2.27}$$

A general structure is a collection of structural components that are interconnected by kinematic constraints at their boundaries. Results stated are from Refs. 1 and 7. The energy bilinear and load linear forms of a general system, consisting of membranes can be written as

$$a_u(z,\bar{z}) = \sum_{\ell=1}^{n} [a_u(z,\bar{z})]^{\ell} \tag{2.28}$$

$$\ell_u(z) = \sum_{\ell=1}^{n} [\ell_u(\bar{z})]^{\ell} \tag{2.29}$$

where $[a_u(z,\bar{z})]$ and $[\ell_u(\bar{z})]$ are given in Eqs. 2.1 and 2.2. The state equation is [1]

$$a_u(z,\bar{z}) = \ell_u(\bar{z})$$

for all kinematically admissible virtual displacement $\bar{z}$. Since the energy bilinear and load linear forms of the state equation are just the sum of energy bilinear and load linear forms of each structural component, the design sensitivity equation of the system is a simple additive process [1, 7]. The generalized shape design sensitivity of a built-up structure for a displacement functional is

$$\begin{aligned}\psi_5' = &\sum_{\ell=1}^{n} \iint_{\Omega^\ell} \sum_{i,j=1}^{2} [\sigma^{ij}(z)(\nabla\lambda^{i^T} V_j) + \sigma^{ij}(\nabla z^{i^T} V_j)]d\Omega \\ &- \sum_{\ell=1}^{n} \iint_{\Omega^\ell} [\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)]\text{div } V \, d\Omega\end{aligned} \tag{2.30}$$

where n is the number of components in the built-up structure. The shape design sensitivity of a built-up structure for a stress functional is

$$\begin{aligned}\psi_6' = &\sum_{\ell=1}^{n} \iint_{\Omega^\ell} \sum_{i,j=1}^{2} [\sigma^{ij}(z)(\nabla\lambda^{i^T} V_j) + \sigma^{ij}(\lambda)(\nabla z^{i^T} V_j)]d\Omega \\ &- \sum_{\ell=1}^{n} \iint_{\Omega^\ell}[\sum_{i,j=1}^{2} \sigma^{ij}(z)\varepsilon^{ij}(\lambda)]\text{div } Vd\Omega \\ &- \iint_{\Omega^q} \sum_{i,j=1}^{2} [\sum_{k,\ell=1}^{2} g_{\sigma^{ij}}(z)C^{ijk\ell}(\nabla z^{k^T} V_\ell)]m_p d\Omega \\ &+ \iint_{\Omega^q} g \text{ div } V \, m_p d\Omega - \iint_{\Omega^q} g \, m_p d\Omega \iint_{\Omega^q} m_p \text{ div } Vd\Omega\end{aligned} \tag{2.31}$$

where n is the number of components in the built-up structure and the stress is averaged over a subdomain of component q.

CHAPTER III

PROGRAMMING ASPECTS

So far analytical results and numerical algorithms for shape design sensitivity analysis have been stated. An outline of the basic organization of the EAL database management system is given in Part I of this report [6].

The program can handle two types of constraint functionals; displacement and stress. So far only membrane elements can be used to model a structure and evaluate shape design sensitivities. However, the method presented can be and will be extended to include truss, beam, plate, and three-dimensional elastic solid. A flow chart of the program is given in Figure 4. To use the program, the user sets the system control parameter, gives information about the design variables, specifies the constraints, sets up the finite element model and describes the velocity field. The finite element model is described in the runstream dataset INIT MODL 0 0, the velocity field is described in the runstream dataset VELO FILD 0 0, all other information is given in the runstream dataset PARA SET 0 0. After that, the program automatically computes sensitivities for the given constraints and design variables. System control parameters are

LCAS - Actual load case number

DE41 - Number of membrane elements

NMDV - Number of design variables

CDIS - Number of displacement constraint functionals

CS41 - Number of stress constraint functionals for element type E41

CTOT - Total number of constraints

For every constraint group, a table is required to describe the location of the constraint functional. For displacement constraints, two entries are needed for each constraint. The first entry is the node number on which the displacement sensitivity is evaluated and the second entry is the direction of the displacement constraint. For the stress constraints, a table is needed that gives the element numbers on which the stress constraint functionals are evaluated. The tables are

CDIS List 0 0 - Displacement constraints

ST41 List 0 0 - Stress constraints

The velocity field is given in the runstream dataset VELO FILD 0 0. For each design parameter, one velocity field has to be given. The velocity for each nodal point has to be specified explicitly.

Note that EAL stores all element information in an intrinsic coordinate system that depends on the displacement. Because derivatives of the displacement depend on the coordinate system used, the program recomputes element displacement in the element coordinates.

The results of the shape design sensitivity analysis program in EAL are all stored in EAL-library file L12. If a stress constraint design sensitivity is specified in the input control parameter, the data set DVAL E41 1 0 is created to store the stress constraint functional

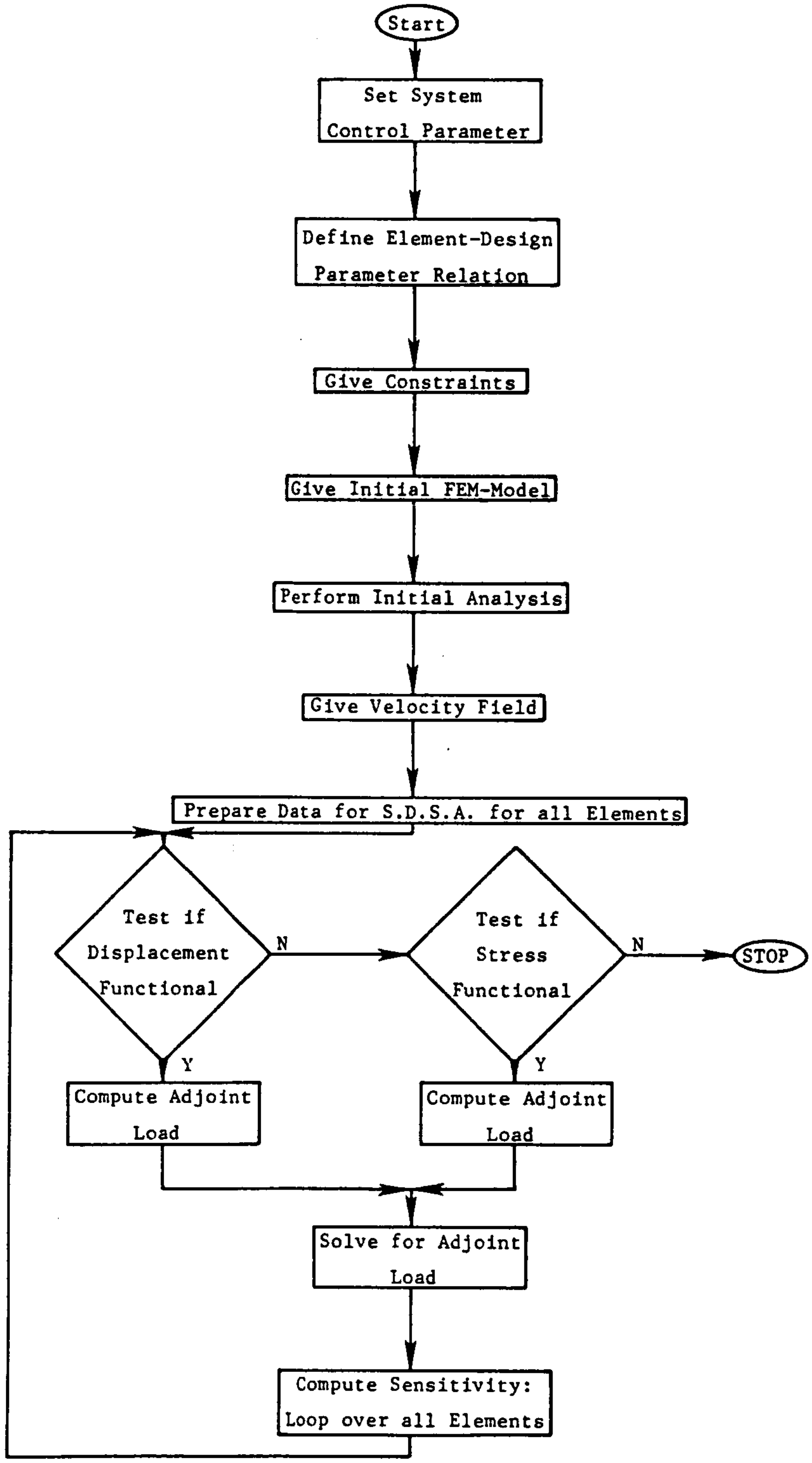


Figure 4. Program Organization

values for constraints that are listed in the input data set ST41 LIST. The shape design sensitivity parameter given in EAL-library file L12 have the following basic form:

DSVE E41 "DV" "CIND"

where

DSVE E41 Design sensitivity parameter for membranes

DV Design variable number

CIND Constraint indicator

between 21,000 and 22,000 Displacement constraint in global x_1-direction

between 22,000 and 23,000 Displacement constraint in global x_2-direction

between 23,000 and 24,000 Displacement constraint in global x_3-direction

between 40,000 and 50,000 Stress constraints in membrane elements

For displacement and stress constraints, the last three digits of the constraint indicator "CIND" give the node number of the displacement constraint and the element number of the stress constraint, respectively.

Chapter IV

Numerical Examples

In order to check whether the shape design sensitivity information obtained is accurate, a comparison is made with the finite difference $\Delta\psi$. An appropriate design perturbation Δu must be selected, in order to obtain a meaningful finite difference of the performance functional. That is, if Δu is too small, $\Delta\psi = \psi(u + \Delta u) - \psi(u)$ may be inaccurate, due to loss of significant digits in the difference. On the other hand, if Δu is too large, $\Delta\psi$ will contain nonlinear terms and the comparison with ψ' will be meaningless.

To demonstrate the capability of the program, two examples are given in this chapter. First, a cubic box is presented in Section 4.1. The cubic box is an extremely simplified model of a wing-box structure. A study of this square box can, however, provide a basis for study of the wing-box. The second example treated in Section 4.2, is a simple interface problem, composed of two plane elastic membranes with different material properties.

4.1 Cubic Box Problem

Consider a cubic box shown in Fig. 5. The box consists of five plane elastic components; top, bottom, two sides and end. Subdomains (or components) are numbered in Fig. 6 for convenience. The shape design variable of the system is height h of the box. A C°-velocity field is required on plane elastic component [11]. It suffices to use

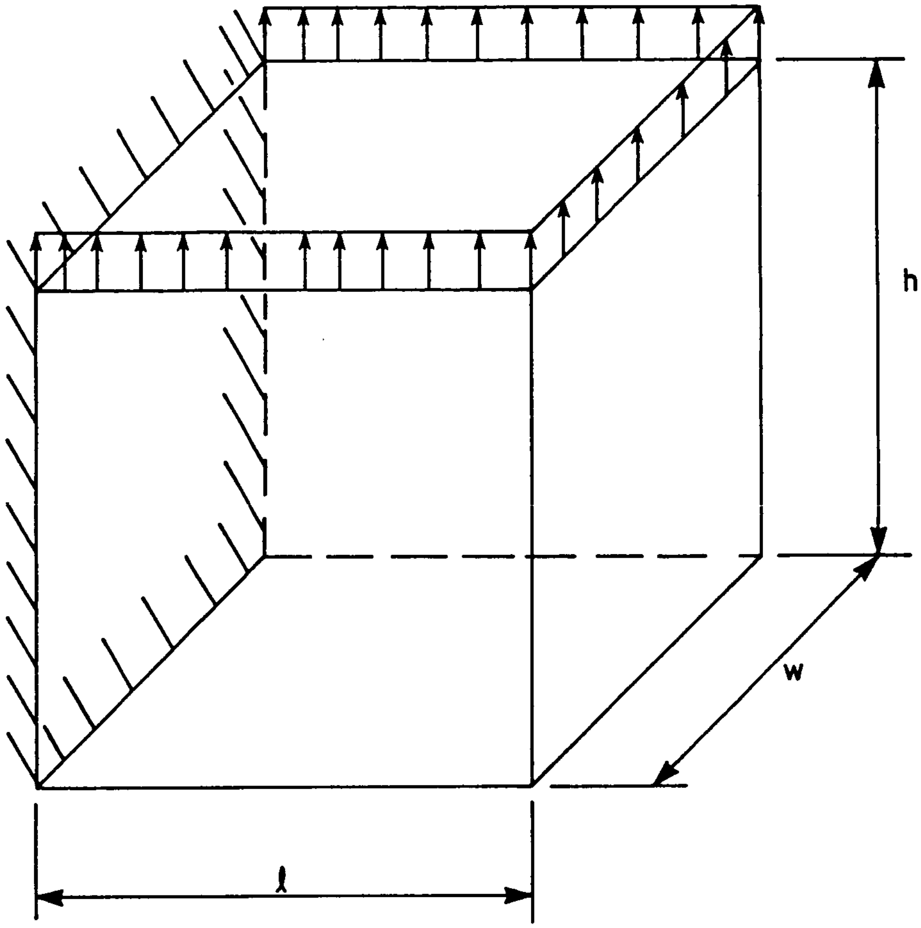


Figure 5. Cubic Box

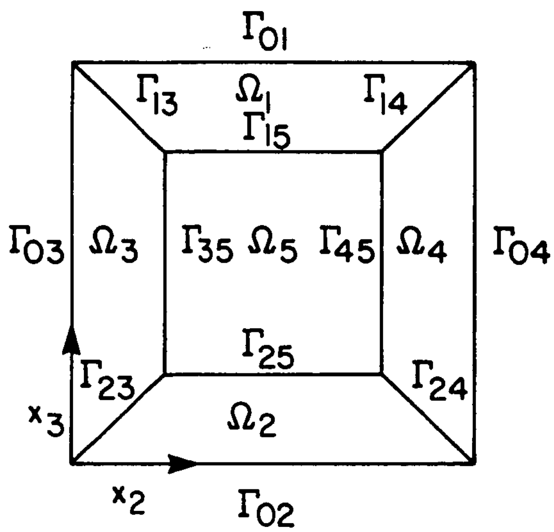


Figure 6. Numbering of Subdomains and Interfaces
(view in (-) x-direction)

piecewise linear velocity fields on each component, which are given in Table 1. Note that δh is design change.

Table 1. Velocity Fields on Each Component

Patch #	Velocity Field Definition
Ω_1	$V^1 = 0$
Ω_2	$V^2 = 0$
Ω_3	$V^3 = \frac{x_3 - h}{h} \delta h$
Ω_4	$V^4 = \frac{x_3 - h}{h} \delta h$
Ω_5	$V^5 = \frac{x_3 - h}{h} \delta h$

External loads are applied along the edges of the top surface Γ_{13}, Γ_{14}, Γ_{15} with constant magnitude of 4.77 lb/in in the positive x_1-direction. The cubic box is discretized by 320, 4-noded membrane elements (E41) having 377 nodes. Young's modulus and Poisson's ratio are 1.0×10^7 lb/in^2 and 0.316, respectively. The thickness of each component is 0.1 in.

Several nodal points are selected to check accuracy of shape design sensitivity of the displacement functional of Eq. 2.7. Design sensitivity predictions ψ_1' and finite differences $\Delta\psi_1 = \psi_1(u + \Delta u) - \psi_1(u)$ with $\delta h = 0.01h$ are given in Table 2.

Table 2. Box Problem - Displacement Sensitivity

Node	Dir.	$\psi_1(u)$ * 10^{-4}	$\psi_1(u + \Delta u)$ * 10^{-4}	$\Delta\psi_1$ * 10^{-5}	ψ_1' * 10^{-5}	Ratio %
129	x_3	0.7615	0.7725	0.1102	0.1070	97.1
153	x_3	0.9449	0.9544	0.0951	0.0917	96.5
157	x_3	0.7793	0.7899	0.1063	0.1034	97.3
257	x_1	0.3295	0.3342	0.0467	0.0448	96.0
257	x_3	1.4425	1.4649	0.2236	0.2189	97.9
261	x_1	0.1448	0.1467	0.0195	0.0190	97.1
277	x_1	-0.1694	-0.1713	-0.0186	-0.0179	96.1
281	x_1	-0.3701	-0.3745	-0.0441	-0.0435	98.6
281	x_3	1.6099	1.6302	0.2024	0.1995	98.6
285	x_3	1.4964	1.5184	0.2201	0.2148	97.6

To check accuracy of shape design sensitivity of stress functional of Eq. 2.19, 17 finite elements are selected. Design sensitivity predictions ψ_2' and finite differences $\Delta\psi_2$ with $\delta h = 0.01$ h are given in Table 3. Both performance functionals yield excellent sensitivity results.

For the computation of the shape design sensitivity analysis, the EAL runstream idea is used which is convenient, but the cost of direct disk read/write access and database overhead disk read/write access can be very high. The CPU-time and the overhead cost for the sensitivity analysis of the box problem are given in Table 4.

Table 3. Box Problem - Stress Sensitivity

El. No.	$\psi_2(u)$	$\psi_2(u + \Delta u)$	$\Delta\psi_2$	ψ_2'	Ratio %
1	85.97	87.14	1.1725	1.1598	98.9
2	47.39	48.01	0.6157	0.6288	102.1
9	129.77	131.48	1.7100	1.6932	99.0
17	100.57	101.64	1.0660	1.0544	98.9
18	53.15	53.74	0.5911	0.5842	98.8
25	175.71	177.16	1.4540	1.4418	99.2
65	64.88	65.76	0.8741	0.8644	98.9
81	70.59	71.41	0.8143	0.8046	98.8
129	53.36	54.08	0.7224	0.7142	98.9
130	39.59	40.13	0.5377	0.5316	98.9
145	57.92	58.62	0.7064	0.6980	98.8
146	44.21	44.72	0.5143	0.5081	98.8
225	20.12	20.38	0.2648	0.2621	99.0
248	28.60	28.85	0.2503	0.2474	98.8
249	55.09	55.38	0.2960	0.2919	98.6
256	32.82	33.20	0.3796	0.3756	98.9
284	35.00	34.94	-0.0532	-0.0533	100.2

Table 4. Overhead Cost

	FEM ANALYSIS	Preprocessing	Evaluation of each Performance Functional
CPU-time	474.8	3791.6	2375.4
Total number of disk write accesses	468	50817	44187
Total number of disk read accesses	1131	240575	166455
Number of database overhead disk write access	19	9842	2889
Number of database overhead disk read access	13	145445	67072

The overhead cost is given for three computational steps

- the finite element analysis
- the preprocessing step that computes the stresses and displacement derivatives at the Gauss points for the actual load and prepares all necessary data for the sensitivity analysis.
- the evaluation of each performance functional that evaluates the adjoint load, computes strain and displacement derivatives at the Gauss points for the particular adjoint load, and evaluates the sensitivity expression.

4.2 Interface Problem

Consider an interface problem, shown in Fig. 7. The structure consists of two plane elastic components, modeled with 45 nodes and 32 membrane elements (E41) (see Fig. 8). The shape variation is parameterized by two design parameter b_1 and b_2 that determine the position of the interface boundary and the height of the structure (see Table 5).

Table 5. Velocity Fields

Design Parameter	Component	Velocity Field	
1	1	$V^1 = 0.1\, x_1 \delta b_1$	$V^2 = 0.0$
	2	$V^1 = -0.1(x_1 - 20)\delta b_1$	$V^2 = 0.0$
2	1	$V^1 = 0.0$	$V^2 = 0.1(x_2 - 10)\delta b_2$
	2	$V^1 = 0.0$	$V^2 = 0.1(x_2 - 10)\delta b_2$

External load P = 100 N is applied to the structure at the upper left corner (x_1 = 20.0, x_2 = 10.0). Young's modulus and Poisson's ratio of the

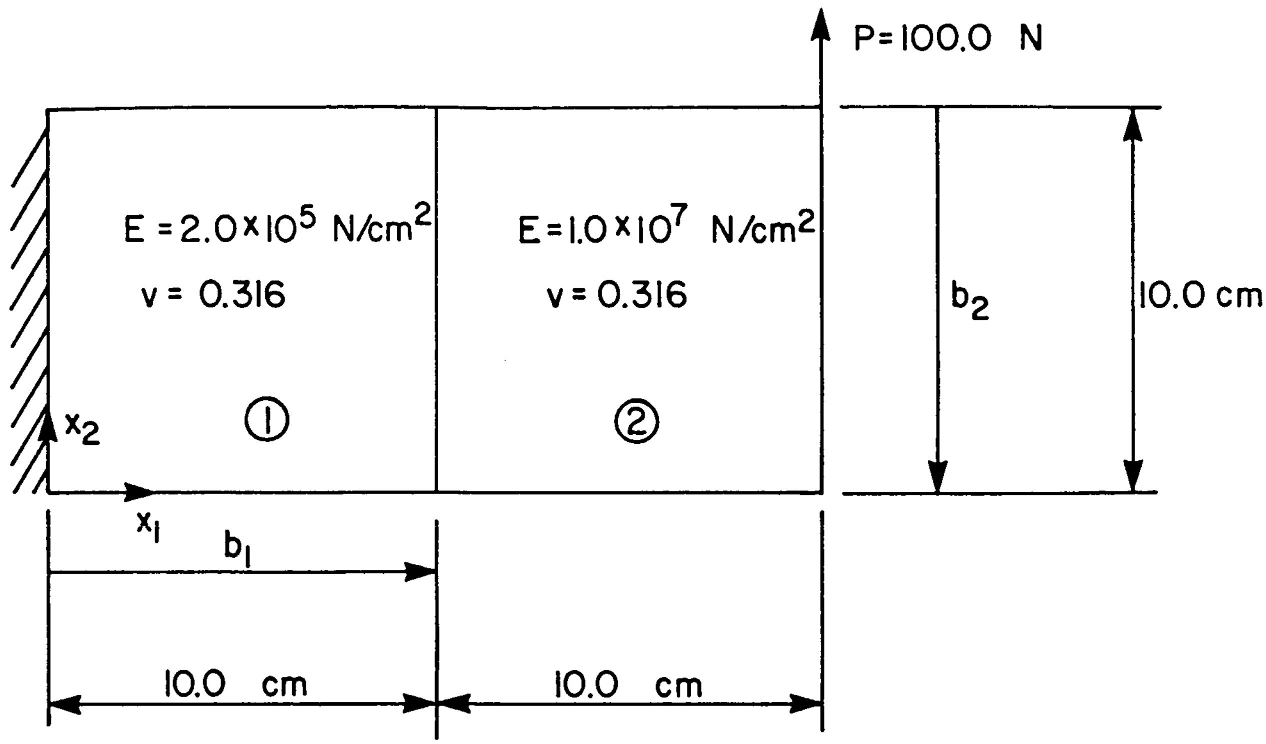


Figure 7. Interface Model

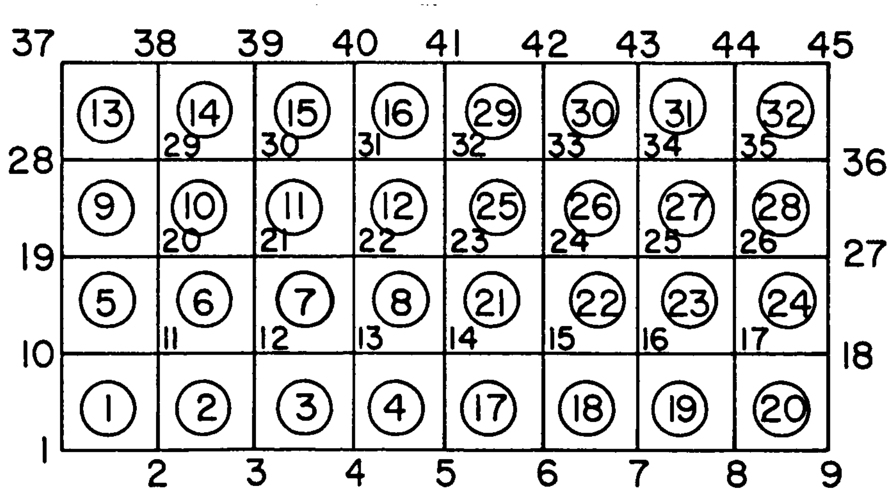


Figure 8. Element and Node Numbering

structure are $E = 2.0 \times 10^5$ N/cm^2 and $\nu = 0.316$, respectively, for the first component and $E = 1.0 \times 10^7$ N/cm^2 and $r = 0.316$, respectively, for the second component of the structure. The thickness for each member is 0.1 cm.

Several nodal points are selected to check accuracy of shape design sensitivity of displacement functional of Eq. 2.7. Design sensitivity predictions ψ_i' and finite differences $\Delta\psi_i = \psi_i(u + \Delta u) - \psi_i(u)$ with 1% design perturbation are given in Table 6.

Table 6. Interface Problem - Displacement Sensitivity

Node No.	Direction	Design Var.	ψ_i^1	ψ_i^2	$\Delta\psi_i$ * 10^{-3}	ψ_i' * 10^{-3}	Ratio %
9	1	1	0.0489747	0.0480451	-0.9296	-0.9438	101.5
18	1	1	0.0232287	0.0227568	-0.4719	-0.4791	101.5
27	1	1	-0.000187702	-0.0001882	-0.0005	-0.000535	107.0
36	1	1	-0.023455	-0.0229872	0.4678	0.4748	101.5
45	1	1	-0.0469181	-0.045982	0.9361	0.9502	101.5
9	2	1	0.162013	0.157690	-4.323	-4.4083	102.0
18	2	1	0.159341	0.155000	-4.341	-4.4269	102.0
27	2	1	0.158151	0.153809	-4.342	-4.4275	102.0
36	2	1	0.157767	0.153427	-4.340	-4.4266	102.0
45	2	1	0.157713	0.153372	-4.341	-4.4264	102.0
9	1	2	0.0489747	0.0492151	0.2404	0.2437	101.4
18	1	2	0.0232287	0.0233417	0.1130	0.1147	101.5
27	1	2	-0.000187702	-0.000188328	-0.0006	-0.000533	88.8
36	1	2	-0.023455	-0.0235724	-0.1174	-0.1189	101.3
45	1	2	-0.0469181	-0.0471523	-0.2342	-0.2373	101.3
9	2	2	0.162013	0.162603	0.590	0.6025	102.1
18	2	2	0.159341	0.159913	0.572	0.5839	102.1
27	2	2	0.158151	0.158722	0.571	0.5834	102.2
36	2	2	0.157767	0.158339	0.572	0.5843	102.2
45	2	2	0.157713	0.158285	0.572	0.5845	102.2

To check accuracy of shape design sensitivity of the stress functional of Eq. 2.19, 32 finite elements are selected. Design sensitivity predictions ψ_2' and finite differences $\Delta\psi_2$ with 1% design perturbation are given in Table 7. Both performance functionals yield excellent results, with the exception of few stress sensitivity predictions, where the sensitivity is relatively small to other sensitivities and therefore small numerical errors in the overall calculation lead to large differences in the finite difference and the shape design sensitivity prediction.

For the computation of the shape design sensitivity analysis, the EAL runstream idea is used which is convenient, but the cost of direct disk read/write accesses and database overhead disk read/write access can be very high. The CPU-time and the overhead cost are given in Table 8 for the following computational steps;

- Reading the runstream into the database.
- The finite element analysis.
- The preprocessing step that computes the stress and displacement derivatives at the Gauss points for the actual load and prepares all necessary data for the sensitivity analysis.
- The evaluation of each performance functional for two design variables that evaluates the adjoint load, computes strain and displacement derivatives at the Gauss points for the particular adjoint load, and evaluates the sensitivity expression.
- As an additional information, the overhead cost for the evaluation of each performance functional for one design variable is also given.

Table 7. Interface Problem - Stress Sensitivity

Element No.	Design variable	ψ_i^1	ψ_i^2	$\Delta\psi_i$	ψ_i'	Ratio %
1	1	824.747	808.933	-15.754	-16.071	98.0
1	2	824.747	824.075	- 0.672	- 0.570	117.9
2	1	742.343	727.787	-14.556	-14.778	98.5
2	2	742.343	740.690	- 1.653	- 1.620	102.0
3	1	632.872	620.581	-12.291	-12.479	98.5
3	2	632.872	630.073	- 2.799	- 2.770	101.0
4	1	500.639	490.968	- 9.670	- 9.817	98.5
4	2	500.639	497.113	- 3.526	- 3.514	100.3
5	1	288.532	283.359	- 5.213	- 5.257	99.1
5	2	288.532	288.948	0.416	0.505	82.3
6	1	370.417	364.384	- 6.033	- 6.120	98.5
6	2	370.417	370.315	- 0.102	- 0.037	27.3
7	1	330.002	325.300	- 4.702	- 4.759	98.5
7	2	330.002	329.159	- 0.843	- 0.816	103.4
8	1	304.815	300.636	- 4.179	- 4.228	98.5
8	2	304.815	303.576	- 1.248	- 1.218	102.5
9	1	288.385	283.201	- 5.175	- 5.267	98.2
9	2	288.385	288.798	0.413	0.501	82.5
10	1	370.176	364.152	- 6.024	- 6.112	98.6
10	2	370.176	370.075	- 0.101	- 0.039	26.2
11	1	330.275	325.668	- 4.607	- 4.665	98.7
11	2	330.275	329.495	- 0.780	- 0.754	103.4
12	1	307.525	303.585	- 3.940	- 3.990	98.7
12	2	307.525	306.530	- 0.995	- 0.980	101.1
13	1	824.798	808.992	-15.806	-16.061	98.4
13	2	824.798	824.131	- 0.667	- 0.571	116.8
14	1	742.558	728.017	-14.541	-14.762	98.5
14	2	742.558	740.917	- 1.641	- 1.611	101.9
15	1	633.305	621.021	-12.284	-12.470	98.5
15	2	633.305	630.518	- 2.787	- 2.762	100.9
16	1	500.888	491.189	- 9.699	- 9.845	98.5
16	2	500.888	497.341	- 3.547	- 3.538	100.3
17	1	419.434	411.272	- 8.162	- 8.284	98.5
17	2	419.434	415.596	- 3.838	- 3.815	100.6
18	1	314.409	308.864	- 5.545	- 5.628	98.5
18	2	314.409	311.911	- 2.498	- 2.494	100.2
19	1	221.980	218.903	- 3.077	- 3.105	99.1
19	2	221.980	221.086	- 0.894	- 0.963	92.8
20	1	607.664	605.998	- 1.666	- 1.860	89.5
20	2	607.664	612.102	4.438	4.255	104.3
21	1	286.907	282.791	- 4.116	- 4.166	98.8
21	2	286.907	285.815	- 1.092	- 1.081	101.6
22	1	220.797	218.224	- 2.573	- 2.618	98.3
22	2	220.797	220.302	- 0.495	- 0.506	97.8
23	1	283.586	282.316	- 1.270	- 1.278	99.4
23	2	283.586	285.151	1.565	1.551	100.9
24	1	373.198	370.467	- 2.731	- 3.148	86.7
24	2	373.198	374.175	0.977	0.573	170.4
25	1	317.198	314.061	- 3.137	- 3.179	98.6
25	2	317.198	317.426	0.228	0.226	101.1

Table 7--Continued

26	1	301.809	299.335	- 2.474	- 2.500	98.9
26	2	301.809	302.232	0.423	0.424	99.8
27	1	295.251	291.885	- 3.366	- 3.388	99.4
27	2	295.251	294.793	- 0.458	- 0.445	103.0
28	1	184.710	182.358	- 2.352	- 2.391	98.4
28	2	184.710	184.169	- 0.541	- 0.543	99.6
29	1	421.246	413.049	- 8.197	- 8.322	98.5
29	2	421.246	417.392	- 3.854	- 3.838	100.4
30	1	294.280	288.590	- 5.690	- 5.775	98.5
30	2	294.280	291.453	- 2.827	- 2.822	100.2
31	1	169.748	166.300	- 3.448	- 3.499	98.5
31	2	169.748	167.956	- 1.792	- 1.793	100.0
32	1	51.074	50.048	- 1.026	- 1.043	98.3
32	2	51.074	50.541	- 0.533	- 0.534	99.7

Table 8. Overhead Cost

	Reading in Program	FEM Analysis	Preprocessing	Evaluation of Each Performance Functional: Two Design Parameter	Evaluation of Each Performance Functional: One Design Parameter
CPU-time	24.4	11.0	252.2	317.0	198.8
Total number of disk write accesses	158	78	6133	7689	4817
Total number of disk read accesses	2	152	16,216	22,994	13,895
Number of database overhead disk write access	4	14	966	469	382
Number of database overhead disk read access	0	14	4391	4591	2949

Chapter V

Conclusions

Results of this study show that it is possible to combine the shape design sensitivity algorithms of Ref. 1 with a database management system of EAL. For the sensitivity computation, it is necessary to compute the derivatives of the displacement field and the adjoint displacement field. In the EAL finite element analysis, the stress and strain calculations are based on a hybrid formulation. However, assumed displacement shape functions are used for evaluating the displacement derivatives. Nevertheless, results of the shape design sensitivity analysis are very accurate, which indicates that it is not necessary that the sensitivity analysis and the finite element analysis use the same shape function.

A database management system with a finite element capability and the adjoint variable method of design sensitivity analysis, permit implementation of a shape design sensitivity analysis method that does not require differentiation of element stiffness and mass matrices. It is shown that a database management system can be used to implement shape design sensitivity analysis, so only one program with one database is necessary.

References

1. Haug, E.J., Choi, K.K., and Komkov, V., Structural Design Sensitivity Analysis, Academic Press, New York, N.Y., 1986.

2. Choi, K.K. and Seong, H.G., "A Domain Method For Shape Design Sensitivity Analysis of Built-up Structures", Computer Methods in Applied Mechanics and Engineering, to appear, 1986.

3. Whetstone, W.D., EISI-EAL Engineering Analysis Language, Reference Manual, EIS Inc., July 1983.

4. Choi, K.K., Santos, L.T., Frederick, M.C., "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes", Journal of Mechanics, Transmissions, and Automation in Design, 85-DET-77.

5. Frederick, M.C. and Choi, K.K., "Design Sensitivity Analysis with APPLICON IFAD Using the Adjoint Variable Method", Technical Report 84-17, Center for Computer Aided Design, University of Iowa, 1984.

6. Dopker, B., Choi, K.K., and Lee, J., "Design Sensitivity Analysis Using EAL: Part I: Conventional Design Parameters", Technical Report 86-2, Center for Computer Aided Design, University of Iowa, 1986.

7. Choi, K.K. and Seong, H.G., "Design Component Method for Sensitivity Analysis of Built-up Structures", Journal of Structural Mechanics, to appear, 1986.

8. Pian, T.H.H., "Derivation of Element Stiffness Matrices by Assumed Stress Distribution", AIAA, July 1964.

9. Cook, R.D., Concept and Application of the Finite Element Analysis, John Wiley and Sons Inc., New York, 1981.

10. Zienswicz, O.O., The Finite Element Method, McGraw-Hill, London, 1977.

11. Choi, K.K. and Seong, H.G., "A Numerical Method For Shape Design Sensitivity Analysis and Optimization of Built-up Structures", The Optimum Shape: Automated Structural Design, (Eds. J.A. Bennett and M.E. Botkin), Plenum Press, New York, 1986.