Old Dominion University Research Foundation

DEPARTMENT OF MATHEMATICAL SCIENCES
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OLD DOMINION UNIVERSITY
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A THEORETICAL STUDY OF PHOTOVOLTAIC CONVERTERS

By

John H. Heinbockel, Principal Investigator

Final Report
For the period ending May 16, 1987

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
Research Grant NAG-1-148
Mr. Gilbert H. Walker, Technical Monitor
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SYMBOLS

A	Einstein coefficient of spontaneous emission, s^{-1}
c	velocity of light, cm/s
C_1	RI stabilized $I + I^*$ recombination rate coefficient, cm^6/s
C_2	RI stabilized $I + I$ recombination rate coefficient, cm^6/s
C_3	I_2 stabilized $I + I^*$ recombination rate coefficient, cm^6/s
C_4	I_2 stabilized $I + I$ recombination rate coefficient, cm^6/s
[I]	atomic iodine density, cm^{-3}
$[I_2]$	molecular iodine density, cm^{-3}
[I*]	electronically excited atomic iodine density, cm^{-3}
K_1	$R + I^*$ recombination rate coefficient, cm^3/s
K_2	$R + I$ recombination rate coefficient, cm^3/s
K_3	$R + R$ recombination rate coefficient, cm^3/s
L	distance between lazer end mirrors, cm
p	alkyliodide partial pressure at room temperature, torr
P	laser output power density, W/cm^2
Q_1	I^* quenching coefficient for RI, cm^3/s
Q_2	I^* quenching coefficient for I_2, cm^3/s
R_1, R_2	combined reflection coefficients of the Brewster window and end mirror at each end of the laser tube
[R]	alkyl radical density, cm^{-3}
$[R_2]$	alkyl dimer density, cm^{-3}
[RI]	alkyliodide density, cm^{-3}
S_i	maximum photodissociation rate of the ith chemical species, s^{-1}
t	time, s
t_m	output mirror transmission coefficient

z	penetration distance into the lasant gas, cm
$\Gamma(z)$	stimulated emission rate, $cm^{-3}s^{-1}$
ε_ν	radiation energy quantum from the I*, eV
ξ_i	photodissociation rate of ith chemical species at a depth z of penetration
ρ_+	photon density of photons moving in the positive direction along the z - axis, cm^{-3}
σ	stimulated emission cross section, cm^2
σ_0	photoabsorption cross section at the central frequency, cm^{-1}-$torr^{-1}$
ω	axial lasant speed, cms^{-1}
Z_{OL}	z - distance for peak illumination, cm
x_{La}	width parameter for illumination curve
ψ_1, ψ_2	source terms

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John H. Heinbockel*

INTRODUCTION

The research performed during the period March 1984 to January 1987 is summarized in the progress reports given in the Refs. [1] through [6]. The final phase of research was performed in the model simulation for the solar simulator pumped atomic iodine laser. The geometry for the steady state laser operation with axial lasant flow is illustrated in Fig. 1. The chemical kinetics for this laser are described in Refs. [7] and [8].

As a first approximation to the simulation of lasant flow and operation we have replaced the time derivatives in Ref. [7] by the material derivatives.

$$\frac{D()}{Dt} = \frac{\partial()}{\partial t} + \frac{\partial()}{\partial z}\frac{dz}{dt} \tag{1}$$

and we have assumed that the quantity $\frac{dz}{dt} = \omega$ represents the constant gas flow rate in the positive z direction. The first six of the chemical kinetic equations can then be written as

$$\frac{\partial x_i}{\partial t} + \omega \frac{\partial x_i}{\partial z} = F_i(\bar{x},t), \; i = 1, 2, \ldots, 6 \tag{2}$$

*Professor, Department of Mathematical Sciences, Old Dominion University, Norfolk, Virginia 23508.

where $\bar{x} = col(x_1, x_2, x_3, x_4, x_5, x_6)$ is a column vector and

$$x_1 = [RI],\quad x_2 = [R],\quad x_3 = [R_2],\quad x_4 = [I_2],\quad x_5 = [I^*],\quad x_6 = [I].$$

Here we are using the notation [ ] to denote the concentration (cm^{-3}) of the chemical reactant. The symbol R denotes a radical of the perflouralkyl iodides ($i\text{-}C_3F_7I$, $n - C_3F_7I$, or C_2F_5I). We use the reaction rates for $n - C_3F_7I$ and express the right hand side $F_i(\bar{x},t)$ of the system (2) as:

$$F_1 = k_1x_2x_5 + k_2x_2x_6 - \psi_1(z)x_1 - k_4x_2x_1$$

$$F_2 = \psi_1(z)x_1 - k_1x_2x_5 - k_2x_2x_6 - 2k_3x_2^2 - k_4x_2x_1$$

$$F_3 = k_3x_2^2 - k_4x_1x_2$$

$$F_4 = c_1x_1x_5x_6 + c_2x_1x_6^2 + c_3x_4x_5x_6 + c_4x_4x_6^2 - \psi_2(z)x_4$$

$$F_5 = \psi_1(z)x_1 + \psi_2(z)x_4 - k_1x_2x_5 - c_1x_1x_5x_6 - c_3x_4x_5x_6$$

$$-Q_1x_1x_5 - Q_2x_4x_5 - \Gamma_{max} - Ax_5 - A_Dx_5$$

$$F_6 = \psi_2(z)x_4 + Q_1x_1x_5 + Q_2x_4x_5 + \Gamma_{max} + Ax_5 - c_1x_1x_5x_6$$

$$-2c_2x_1x_6^2 - c_3x_4x_5x_6 - 2c_4x_4x_6^2 - k_2x_2x_6 + k_4x_1x_z\ A_Dx_6$$

where

$$\Gamma_{max} = c\sigma\rho(x_5 - \frac{1}{2}x_6)$$

$$\sigma = [2.0(10)^{17} + .443x_1]^{-1}$$

$$\rho = \rho_+ + \rho_-$$

The quantity p will be discussed in the next section. The kinetic coefficients and other constants are given by:

$$k_1 = 7.9(10)^{-13} \quad c = 3.0(10)^{11} \quad c_1 = 1.0(10)^{-33}$$

$$k_2 = 2.3(10)^{-11} \quad Q_1 = 2.0(10)^{-16} \quad c_2 = 8.5(10)^{-32}$$

$$k_3 = 2.6(10)^{-12} \quad Q_2 = 1.9(10)^{-11} \quad c_3 = 5.6(10)^{-32}$$

$$k_4 = 3.0(10)^{-16} \quad A = A_D = 0 \quad c_4 = 2.0(10)^{-30}$$

Light Flux Density. We assume a one-dimensional model for the light flux density in which monochromatic radiation propagates along the z-axis of the laser. We let $\rho_+(z,t)$ denote the photon density propagating in the positive z direction and define $\rho_-(z,t)$ as the photon density propagating in the negative z direction. The differential equation for the photon densities are given by:

$$\frac{1}{c}\frac{\partial\rho_+}{\partial t} + \frac{\partial\rho_+}{\partial z} = \sigma\rho_+(z,t)([I^*] - \frac{1}{2}[I])$$
$$\frac{1}{c}\frac{\partial\rho_-}{\partial t} - \frac{\partial\rho_-}{\partial z} = \sigma\rho_-(z,t)([I^*] - \frac{1}{2}[I]) \qquad (3)$$

where c is the speed of light in the medium. In the above equations the term $\sigma[I^*] - \frac{1}{2}[I]$ is the amplification factor for the active medium.

If R_I, R_2 denote the reflection coefficients for the faces $z = 0$ and $z = L$, respectively, the above equations are subject to the boundary conditions

$$\rho_+(0,t) = R_1\rho_-(0,t)$$
$$\rho_-(L,t) = R_2\rho_+(L,t) \qquad (4)$$

In the steady state case the solutions are denoted by $\rho_+(z)$ and $\rho_-(z)$ and must satisfy

$$\rho_+(z)\rho_-(z) = K_0 = \text{a constant} \qquad (5)$$

for all values of z between 0 and L. This condition together with the boundary conditions requires that at $z = 0$, we have

$$\rho_+(0)\rho_-(0) = R_1\rho_-^2(0) = \frac{1}{R_1}\rho_+^2(0) = K_0$$

and at $z = L$, we have

$$\rho_+(L)\rho_-(L) = \frac{1}{R_2}\rho_-^2(L) = R_2\rho_+^2(L) = K_0$$

These conditions require that

$$\rho_+(0) = \sqrt{K_0 R_1} \qquad (6)$$

and

$$\rho_+(L) = \sqrt{\frac{K_0}{R_2}} \qquad (7)$$

For fixed values R_1, R_2 we guess at an initial value for K_0 and integrate the system of steady state equations obtained from (2) and (3). This gives a calculated value of $\rho_+(L)$ which we compare with the theoretical value from (7). If these values are different we iterate on K_0 until the final value of $\rho_+(L)$ agrees with the theoretical value. For this value of K_0 and with a transmission coefficient given by $t_m = 1 - R_2$ we obtain the laser output power transmitted at the mirror where $z = L$. This power is given by

$$P = \varepsilon_\mu t_m c \rho_+(L) \quad (\mathrm{W/cm^2}) \qquad (8)$$

where ε_μ is the radiation energy quantum from I*.

The quantities $\psi_1(z)$ and $\psi_2(z)$ in the equations (2) are source terms related to the photodissociation rates of the chemical species. These functions are constructed from a function $\psi(z)$ which is assumed to be a "Gaussian type curve" with a maximum value of unity at z_{0L}. Such a curve is illustrated in Fig. 2 and can be represented in the form

$$F(z) = \exp(-\alpha(z-z_{0L})^2)$$

The spread of this probability curve is determined by the coefficient α. It is required that when $z = z_{OL} \pm \frac{x_{La}}{2}$ we have $F(z) = \frac{1}{2}$ (i.e. one half of its maximum value). This requires that

$$\frac{1}{2} = \exp\{\alpha \frac{x_{La}^2}{4}\}$$

which gives the value

$$\alpha = \frac{2.772}{x_{La}^2}$$

Hence, we can write

$$F(z) = \exp\left(-2.772 \left(\frac{z - z_{OL}}{x_{La}}\right)^2\right)$$

Further, we modify this function by subtracting a constant value. The function.

$$\psi(z) = F(z) - F(0)$$

has the property that is zero at the points $z = 0$ and $z = 2z_{OL}$. In order that the modified function have the value of unity at $z = z_{OL}$ we divide by the appropriate scale factor and obtain

$$\psi(z) = \frac{F(z) - F(0)}{1 - F(0)}$$

We use the values

$$x_{La} = 3.27 \text{ cm} \quad \text{and} \quad z_{OL} = 5.7 \text{ cm}$$

and write

$$\psi_1(z) = \xi_1 C^* \psi(z)$$

$$\psi_2(z) = \xi_2 C^* \psi(z)$$

where $C^* = 1.929(10)^4 \ W/cm^2$ denotes the lamp concentration which is adjusted for the tube geometry and ξ_1, ξ_2 are photodissociation rates of the laser gases.

A graph of maximum power vs pressure is obtained from the numerical solution of the steady-state system of equations derived from (2) and (3) and is illustrated in Fig. 3. In Fig. 3, the curve with the circles represents data from the Ref. [7]. The lower three curves represent the numerical solution for $K_0 = .5(10)^{25}$, $K_0 = .5(10)^{26}$ and $K_0 = .8(10)^{26}$.

The computer program used to calculate the data is given in Appendix A.

ACKNOWLEDGEMENT

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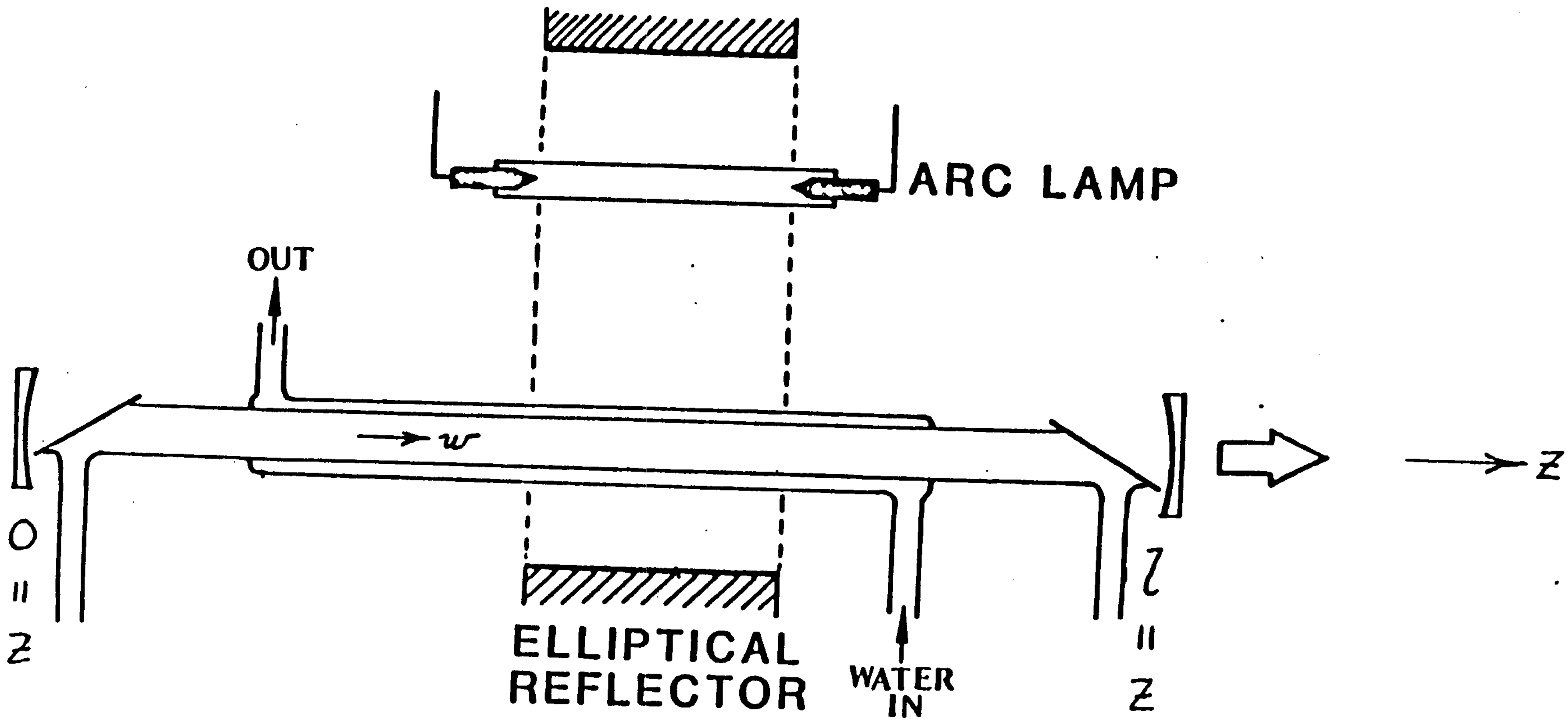

Figure 1. Steady state laser with axial lasant flow.

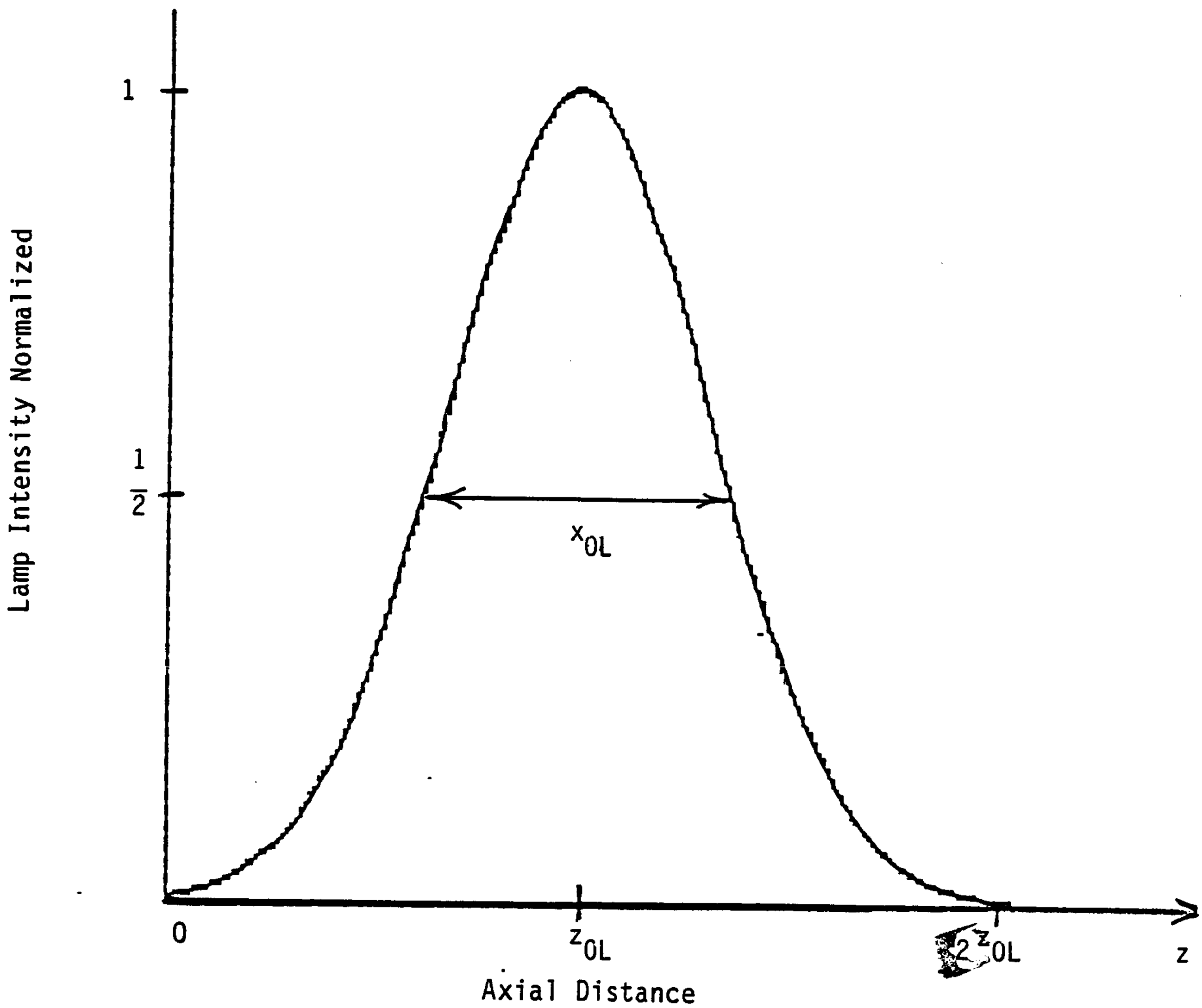

Figure 2. Lamp intensity.

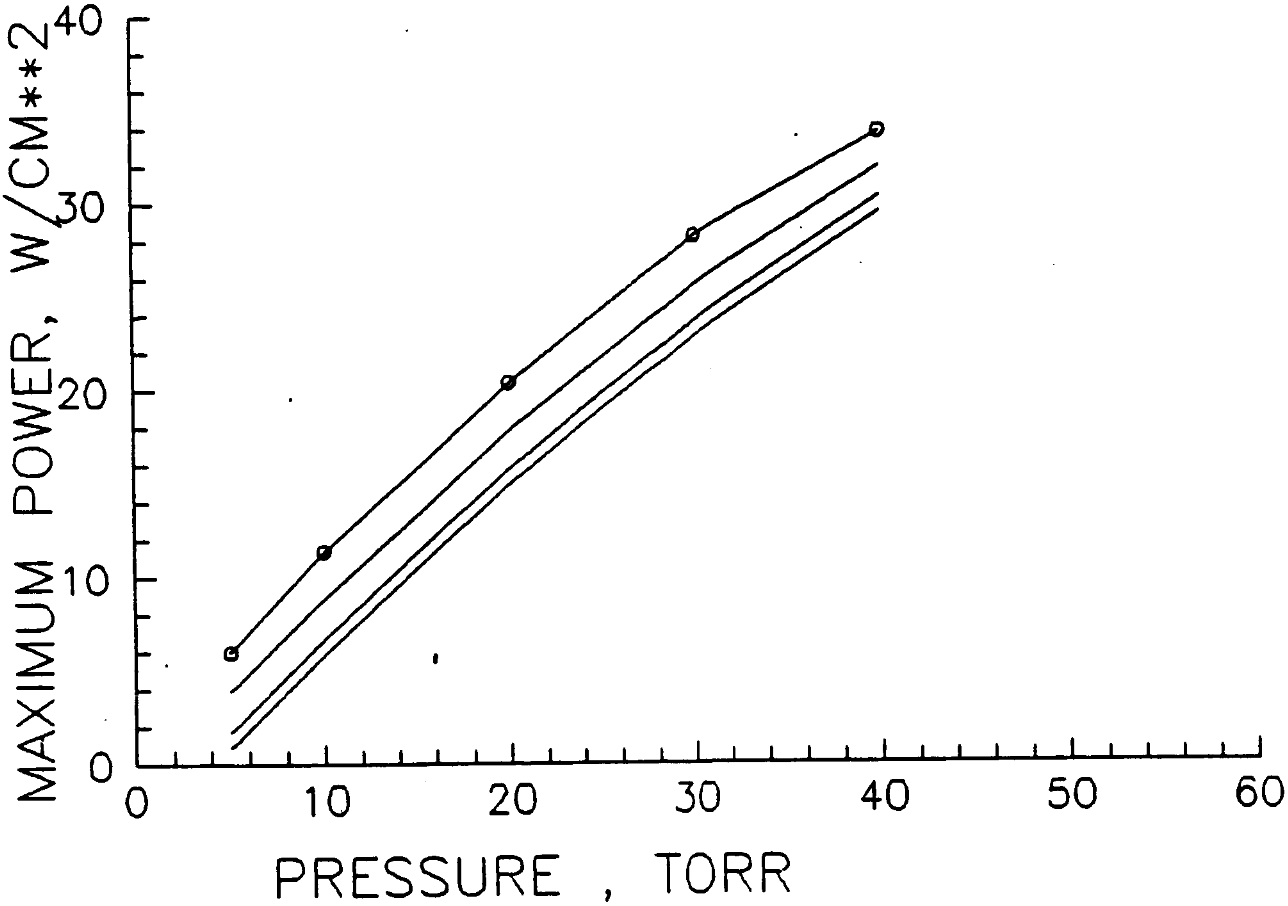

Figure 3. Maximum power versus pressure.

APPENDIX A

```
      PROGRAM LASER(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, TAPE8)
 C    MAIN PROGRAM
      COMMON/BLK3/B, B2, B3, C, AOO, BOO, EPSNU, OMEGA
      COMMON/BLK4/CHSI10, CHSI20, ABARO, Z1BAR, LC
      COMMON/BLK7/ABC, COO, CO, OMEG1, P, R1, R2, TM
      COMMON/BLK8/ZOL, XLA
      REAL LC
      WRITE(6, 123)
123   FORMAT(1X, 20H START OF PROGRAM          )
      NAMELIST/PARAM/P, OMEG1, CON, COO, R1, R2, LC
55    CONTINUE
      READ(5, PARAM)
      IF(EOF(5))600, 601
600   WRITE(6, 603)
603   FORMAT(1X, 28HEND OF FILE ENCOUNTERED-STOP)
      STOP 1313
C         P=PRESSURE,  TORR
C         OMEG1=FLOW RATE,  CM/SEC
C         CON=PEAK CONCENTRATION
C         COO=INITIAL GUESS AT VALUE OF RHO-PLUS AT ZERO --
C             WHICH IS SQUARE OF (COO*R1)
C         R1= REFLECTIVITY AT LEFT END
C         R2= REFLECTIVITY AT RIGHT END
C         TM= TRANSMISSION COEFFICIENT =1-R2
C         ZOL=POINT ALONG AXIS WHERE MAXIMUM ILLUMINATION CCCURS
C             THE POINT 2*ZOL IS WHERE ILLUMINATION CUTS OFF
C         THE ILLUMINATION IS BELL SHAPED ABOUT PT ZOL IN Z DIRECTION
C         LC=LENGTH OF CAVITY
C         XLA=WIDTH OF BELL SHAPED CURVE DEFINING LIGHT INTENSITY
C             AT THE POINT OF ONE HALF MAX CONCENTRATION.
C
601    CONTINUE
       CMIN=1.0E18
       CMAX=1.0E30
       TM=1-R2
       CO=CON
       C11=CON
       OMEGA=OMEG1
       XLA=3.27
       ZOL=5.7
       WRITE(6, 198)
198    FORMAT(///)
       WRITE(6, 199) XLA, ZOL, CON, OMEGA, COO, R1, R2, P
199    FORMAT(1X, T5, 6HXLA = , E15.7,  T30, 6HZOL = , E15.7, T55, 6HCON = ,
     1 E15.7, T80, 8HOMEGA = , E15.7, / ,
     2 1X, T5, 6HCOO = , E15.7,  T30, 6H R1 = , F10.7,  T55, 6H R2 = , F10.7,
     3 T80, 4H P = , E15.7    )
C      SET UP COEFFICIENTS IN DIFFERENTIAL EQUATIONS
       CALL COEFFS
C                    CHOOSE LC SOME MULTIPLE OF .25
C                    OUTPUT EVERY .25 STEPS 0 .LE. Z .LE. LC
       N=4*LC
C                    INTEGRATE DIFFERENTIAL EQUATIONS FROM Z=0 TO Z=LC
C                    OUTPUT RESULTS EVERY LC/N STEPS
```

```
      X1=COO
      CALL INTEG(N)
      Y1=ABC
      IF(Y1.LT.O) PER=.1
      IF(Y1.GT.O) PER=10.
702   CONTINUE
      COO=(PER)*COO
      IF(COO .LT. CMIN) STOP 5432
      IF(COO .GT. CMAX) STOP 2345
      X2=COO
      CALL INTEG(N)
      Y2=ABC
      IF((Y1*Y2).LT. O)GO  TO 701
      X1=COO
      Y1=Y2
      GO TO 702
701   CONTINUE
C      Y1,Y2 OF OPPOSITE SIGN - USE INTERVAL HALVING TO SOLVE FOR Y
      COO=(X1+X2)*.5
      CALL INTEG(N)
      X3=COO
      Y3=ABC
      IF(ABS(Y3).LT.0.001) GO TO 55
704   CONTINUE
      IF((Y1*Y3).LT. O) GO TO 705
C                Y1 & Y3 ARE OF THE SAME SIGN
      X1=X3
      Y1=Y3
      GO TO 701
705   CONTINUE
C               Y1 & Y3 ARE OF OPPOSITE SIGN
      X2=X3
      Y2=Y3
      GO TO 701
C
      END




      FUNCTION CHSI1(Z)
C     IMPLICIT REAL*8(A-H, K, L, O-Z)
      COMMON/BLK4/CHSI1O, CHSI2O, ABARO, Z1BAR, LC
      COMMON/BLK8/ZOL, XLA
      REAL K1, K2, K3, K4, LC
      IF(Z.LT.ABARO) GO TO 100
      IF(Z.LT.Z1BAR) GO TO 200
C     Z GREATER THAN Z1BAR
100   CHSI1=0.0
      RETURN
200   AA1=EXP(-2.77*(ZOL/XLA)**2)
      AA2=EXP(-2.77*((Z-ZOL)/XLA)**2)
      FUNZ=(AA2-AA1)/(1.-AA1)
      CHSI1=CHSI1O*FUNZ
      RETURN
      END
```

```
      FUNCTION CHSI2(Z)
C     IMPLICIT REAL*8(A-H, K, L, O-Z)
      COMMON/BLK4/CHSI10, CHSI20, ABARO, Z1BAR, LC
      COMMON/BLK8/ZOL, XLA
      REAL K1, K2, K3, K4, LC
      IF(Z.LT.ABARO) GO TO 100
      IF(Z.LT.ZqBAR) GO TO 200
C     Z GREATER THAN Z1BAR
100   CHSI2=0.0
      RETURN
200   AA1=EXP(-2.77*(ZOL/XLA)**2)
      AA2=EXP(-2.77*((Z-ZOL)/XLA)**2)
      FUNZ=(AA2-AA1)/(1.-AA1)
      CHSI2=CHSI20*FUNZ
      RETURN
      END


      SUBROUTINE COEFFS
C     IMPLICIT REAL*8(A-H, K, L, O-Z)
      COMMON/BLK2/K1, K2, K3, K4, C1, C2, C3, C4, Q1, Q2, A, AD, TAUC
      COMMON/BLK3/B, B2, B3, C, AOO, BOO, EPSNU, OMEGA
      COMMON/BLK4/CHSI10, CHSI20, ABARO, Z1BAR, LC
      COMMON/BLK7/ABC, COO, CO, OMEG1, P, R1, R2, TM
      COMMON/BLK8/ZOL, XLA
      REAL K1, K2, K3, K4, LC
C
C     COEFFICIENTS IN THE DIFFERENTIAL EQUATIONS
C
      OMEGA=OMEG1
      ABARO=0.0
C     ABARO= START OF ILLUMINATION
C     Z1BAR=2*ZOL = POINT ON AXIS WHERE ILLUMINATION STOPS
      CHSI10=(3.04E-3)*CO
      CHSI20=(3.38E-2)*CO
      Z1BAR=2*ZOL
      EPSNU=1.5E-19
C     WATTS/CM*CM
      AOO=2.0E17
      BOO=.443
      K1=7.9E-13
      K2=2.3E-11
      K3=2.6E-12
      K4=3.0E-16
      C=3.0E10
      Q1=2.0E-16
      Q2=1.9E-11
      B=P*(3.5E16)
      C1=1.0E-33
      C2=8.5E-32
      C3=5.6E-32
      C4=2.0E-30
      A=0
      AD=1.2E-3
      B2=B*B
      B3=B2*B
      RETURN
      END
```

```
      SUBROUTINE FUN(Z,Y,F)
C     IMPLICIT REAL*8(A-H,K,L,O-Z)
      DIMENSION Y(7),F(7)
      COMMON/BLK1/X7,POWER
      EXTERNAL CHSI1,CHSI2
      COMMON/BLK2/K1,K2,K3,K4,C1,C2,C3,C4,Q1,Q2,A,AD,TAUC
      COMMON/BLK3/B,B2,B3,C,A00,B00,EPSNU,OMEGA
      COMMON/BLK4/CHSI10,CHSI20,ABAR0,Z1BAR,LC
      COMMON/BLK7/ABC,C00,C0,OMEG1,P,R1,R2,TM
      REAL K1,K2,K3,K4,LC
C
      X8=C00/(Y(7)*B)
      SIG=1./(A00+B00*B*Y(1))
      X7STAR=Y(7)*B+X8
      DIF=Y(5)-.5*Y(6)
      F(1)=K1*B*Y(2)*Y(5)+K2*B*Y(2)*Y(6)-CHSI1(Z)*Y(1)-K4*B*Y(1)*Y(2)
      F(2)=CHSI1(Z)*Y(1)-K1*B*Y(2)*Y(5)-K2*B*Y(2)*Y(6)-2*K3*B*Y(2)*Y(2)
     1 -K4*B*Y(1)*Y(2)
      F(3)=K3*B*Y(2)*Y(2)+K4*B*Y(1)*Y(2)
      A1=C1*B2*Y(1)*Y(5)*Y(6)+C2*B2*Y(1)*Y(6)*Y(6)+C3*B2*Y(4)*Y(5)*Y(6)
      A2=C4*B2*Y(4)*Y(6)*Y(6)-CHSI2(Z)*Y(4)
      F(4)=A1+A2
      A3=CHSI1(Z)*Y(1)+CHSI2(Z)*Y(4)-K1*B*Y(2)*Y(5)
      A4=-C1*B2*Y(1)*Y(5)*Y(6)-C3*B2*Y(4)*Y(5)*Y(6)-Q1*B*Y(1)*Y(5)
      A5=-Q2*B*Y(4)*Y(5)-C*SIG*X7STAR*DIF-A*Y(5)-AD*Y(5)
      F(5)=A3+A4+A5
      A6=CHSI2(Z)*Y(4)+Q1*B*Y(1)*Y(5)+Q2*B*Y(4)*Y(5)
      A7=C*SIG*X7STAR*DIF+A*Y(5)-C1*B2*Y(1)*Y(5)*Y(6)
      A8=-2*C2*B2*Y(1)*Y(6)*Y(6)-C3*B2*Y(4)*Y(5)*Y(6)  -AD*Y(6)
      A9=-2*C4*B2*Y(4)*Y(6)*Y(6)-K2*B*Y(2)*Y(6)+K4*B*Y(1)*Y(2)
      F(6)=A6+A7+A8+A9
      DO 10 I=1,6
10    F(I)=F(I)/OMEGA
      F(7)=Y(7)*DIF*B*SIG
      RETURN
      END

      SUBROUTINE INTEG(N)
C     IMPLICIT REAL*8(A-H,K,L,O-Z)
      DIMENSION Y(7),F(7),Y0(7),X(7),AA1(7),AA2(7),AA3(7),AA4(7),U(7)
      COMMON/BLK1/X7,POWER
      COMMON/BLK3/B,B2,B3,C,A00,B00,EPSNU,OMEGA
      COMMON/BLK4/CHSI10,CHSI20,ABAR0,Z1BAR,LC
      COMMON/BLK7/ABC,C00,C0,OMEG1,P,R1,R2,TM
      REAL K1,K2,K3,K4,LC
C
C     INTEGRATE SYSTEM FROM Z=0 TO Z=LC USING RUNGE-KUTTA METHOD
C
55    CONTINUE
      STEP=LC/FLOAT(N)
      NTIME=500
      H=STEP/FLOAT(NTIME)
      NPRINT=500
C     INITIAL CONDITIONS
      Z0=0.0
      Y0(1)=1.0
      DO 9 I=2,6
9     Y0(I)=0.0
```

```
C        GUESS AT INITIAL CONDITIONS FOR X(7) AND X(8)
         X70=SQRT(R1*COO)
         YO(7)=X70/B
         WRITE(6,191)
191      FORMAT(///,T7,1HZ,T20,4HX(1),T32,4HX(2),T45,4HX(3),T57,4HX(4),
      1  T69,4HX(5),T80,4HX(6),T91,4HX(7),T103,4HX(8)      )
300      CONTINUE
         II=0
         CALL FUN(ZO,YO,F)
C        PRINT OUT
         DO 10 I=1,7
10       X(I)=B*YO(I)
         X8=COO/X(7)
         WRITE(6,199)ZO,(X(I),I=1,7),X8
199      FORMAT(1X,E12.5,8E12.5,E12.5 )
C
C        INTEGRATE USING STEP SIZE H XTIMES THEN PRINT OUT AGAIN
100      II=II+1
         CALL FUN(ZO,YO,F)
         DO 11 I=1,7
11       AA1(I)=H*F(I)
         DO 12 I=1,7
12       U(I)=YO(I)+.5*AA1(I)
         Z1=Z4+.5*H
         CALL FUN(Z1,U,F)
         DO 13 I=1,7
13       AA2(I)=H*F(I)
         DO 14 I=1,7
14       U(I)=YO(I)+.5*AA2(I)
         CALL FUN(Z1,U,F)
         DO 15 I=1,7
15       AA3(I)=H*F(I)
         Z1=ZO+H
         DO 16 I=1,7
16       U(I)=YO(I)+AA3(I)
         CALL FUN(Z1,U,F)
         DO 17 I=1,7
17       AA4(I)=H*F(I)
         DO 18 I=1,7
18       Y(I)=YO(I)+(AA1(I)+AA4(I)+2*(AA2(I)+AA3(I)))/6.
         Z=ZO+H
```

```
C       UPDATE ZO,YO VALUES
        ZO=Z
        DO 19 I=1,7
19      YO(I)=Y(I)
        IF(II.GE.NPRINT)GO TO 200
        GO TO 100
200     CONTINUE
        X(7)=B*YO(7)
        X8=COO/X(7)
        IF((ZO+.5*H).GE.LC) GO TO 500
        GO TO 300
500     CONTINUE
        CALL FUN(ZO,YO,F)
        DO 110 I=1,7
110     X(I)=B*YO(I)
        XX7L=B*Y(7)
        X8=COO/X(7)
        RHOPL=X70/SQRT(R1*R2)
C       RHOPL=RHO-PLUS AT Z=L THEORETICAL VALUE
C       XX7L= CALCULATED VALUE OF RHO-PLUS AT Z= L
C       ABC=DIFFERENCE=XX7L-RHOPL
        DIF=(((XX7L-RHOPL)/RHOPL)*100)
        ABC=DIF
        WRITE(6,202)DIF,RHOPL,XX7L,COO
202     FORMAT(1X,13HDIFFERENCE = ,E18.9,2X,12HTHEORETICAL=,E18.9,2X,
     1 10H ACTUAL = ,  E18.9,2X,6HCOO =    ,E18.8 )
        CALL OUTPUT(YO(7),ZO)
        RETURN
        END


        SUBROUTINE OUTPUT(YY,ZZ)
        COMMON/BLK3/B,B2,B3,C,AOO,BOO,EPSNU,OMEGA
        COMMON/BLK4/CHSI10,CHSI20,ABARO,Z1BAR,LC
        COMMON/BLK7/ABC,COO,CO,OMEG1,P,R1,R2,TM
        REAL LC
        XX7L=B*YY
        POWER=EPSNU*TM*C*XX7L
        WRITE(6,193)R1,R2,POWER,TM,ZZ
193     FORMAT(1X,5HR1 = ,F10.7,1X,5HR2 = ,F10.7,1X,7HPOWER = ,E18.10,
     1 1X,5HTM = ,F10.8,1X,4HL = ,F15.7  )
        RETURN
        END
```