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Investigations Into the F-106 Lightning Strike Environment

As Functions of Altitude and Storm Phase

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1. INTRODUCTION

This report summarizes the research activities accomplished during the 6 month period from 15 March through 14 September 1986. It includes a summary of work performed by personnel of Electro Magnetic Applications, Inc. (EMA) under their subcontract to this grant.

2. WORK PERFORMED DURING THE REPORTING PERIOD

2.1 IAS Research

The work done during this period has centered on the completion of the first order parameterization scheme for the intracloud lightning discharge and its incorporation within the framework of the Storm Electrification Model (SEM). Previously, we established the criteria for the initiation, termination, and direction of propagation of the lightning channel path. This aspect of the parameterization scheme was incorporated into the framework of the SEM and tested in the context of the 19 July 1981 CCOPE simulation, with which we have been working. The aspect which remained to be dealt with was the redistribution of charge, created by the lightning, along the simulated channel and, finally, into the space surrounding the channel. It is this charge which, when converted into ions, interacts with the other charges in the model and allows us to investigate the resultant effects of lightning on the subsequent electrical evolution of the model cloud.

The charge redistribution problem was approached in the following manner. Using ideas from Kasemir (1964, 1980), we assume that the lightning channel is going to be electrically neutral over its entire length, indicating the deposition of equal amounts of positive and negative charge along the two portions of the simulated discharge. In the model, this charge deposition is accomplished by assuming that the charge density per unit length along the channel is proportional to the difference between the ambient potential, ϕ (in Volts), at the grid point in question and the potential, ϕ_o, at the initiation point of the discharge. Thus

$$Q_o = -k(\phi-\phi_o) \tag{1}$$

where Q_o is the linear charge density at a point along the channel, and k is a proportionality constant (farads/m). The negative sign assures that the charge deposited along that portion of the channel is basically opposite in sign to the ambient net charge in that grid volume, since the sign of the potential and the charge responsible for that potential are generally the same.

The application of Eq. (1) results in a charge distribution along the channel similar to that shown in Fig. 1 between +1.2 and -1.5 km.

The two tails extending beyond these points will be explained subsequently. Since we are working with a discrete grid and the termination criterion forces the channel path to stop at a specific grid point (not at the exact place where this criterion is just met), charge neutrality is not guaranteed by this process. In order to overcome this difficulty, we arbitrarily extend the channel path four grid points beyond the designated termination point at each end. We first integrate over the channel path to find the total charge deposited on the sections of opposite polarity. We then compare these two and, at the end of the path segment having the greatest total charge magnitude, we cause an exponentially decreasing amount of charge to be deposited over the four additional grid points using the following expression

$$Q_j = Q_n \exp(-k\{[j-n]\Delta\}^2) \quad (2)$$

where $j = n\pm1,\ldots,n\pm4$, Q_n is the linear charge density (C/m) at the termination point of that segment [as determined by Eq. (1)], Δ is the grid spacing, and k is chosen such that $Q_{n\pm4} = Q_n/1000$. This new segment of charge density is integrated to obtain the total charge deposited along the segment of that polarity. We then calculate the absolute difference between the charge residing on the completed segment and the charge residing on the segment of opposite polarity. This difference is used as a basis for calculating a charge density decrease over the four grid points at the end of the remaining segment.

As with the charge calculations on the initial channel, a trapezoidal integration is performed using Eq. (2) to represent the charge decrease over the four grid points. An iterative process (Newton-Raphson) is used on the integration scheme with the exponential factor k being the unknown quantity. By varying k, the rate of decrease of charge density away from the initial endpoint of the segment is controlled and the total additional charge distributed off the end of the segment can be set so that the total charge of that segment just balances the total charge on the other segment. By this procedure, the neutrality of the discharge channel is obtained over the discrete grid.

Figure 1 shows the linear charge density calculated along the channel using Eq. (1) and a value of $k = 2.5 \times 10^{-11}$ farads/m. The original channel is delineated by the 2 vertical bars on the horizontal axis. The effect of the charge balancing procedure is evident at the 2 tails. The total channel length is nearly 4.5 km. For this particular discharge (value of k), the total charge transferred is 12 C. This may be a bit high for an intracloud discharge, however it is not an unreasonable value.

To this point, we have the lightning discharge represented as a path on the discrete grid with a linear charge density at every grid point along the path. In order to allow this discharge to interact with the other model parameters, we need to make some further modifications. The discrete discharge path represents a line (1D) source in the 2D model. Because of the numerical methods employed in the model calculations, the sudden appearance of a line source of ions would represent a shock which the model could not accommodate. In order to make the channel ion production tractable for the model numerics, the charge must be distributed to some of the grid points adjacent to the channel path. A minimum of three grid points must be involved, the channel path itself and one grid point on each side. We have chosen to employ a total of 9 grid points (the path grid point plus four on each side) to represent the charge distribution. An expression identical to Eq. (2) is used to specify the decay of the magnitude as a function of distance away from the channel, again with $Q_{n\pm4} = Q_n/1000$. For either a vertical or diagonal path orientation, the charge spreading is done in a horizontal direction.

One additional consideration must be made. The charge density along the channel is expressed in terms of C/m. This must ultimately be converted into an ion density (ions/m^3). The linear charge density, expressed by Q_n, represents the charge density through a grid volume which surrounds the grid point and, therefore, can be converted to an equivalent volumetric charge density, assuming that the charge is distributed uniformly throughout the grid volume in question. In addition, when we undertake to distribute the line charge laterally away from the discharge path, we would be creating charge if we do not take steps to maintain charge conservation. However, charge conservation can be accomplished during the spreading by adopting the following procedure.

Figure 2 is a representation of the charge conservation concept. Q_n represents the linear charge density at a particular grid point along the discharge path. Assuming that Q_n represents the charge distribution in the grid volume $\Delta x \Delta y \Delta z$ surrounding the grid point, then the total charge in the grid volume is

$$Q_T = Q_n \Delta l \tag{3}$$

where Δl is the length of the discharge path segment for that grid volume. Because we are using a two-dimensional model, we must generate a fictitious volume with which to work. Although, in a strict sense, the adoption of slab symmetry for the 2D model implies an infinite extension to all model features in the y-direction, we choose here to adopt a concept of infinity which encompasses only the range of influence of the process being considered. Since we are employing a 9 grid point spreading procedure in the x-direction, we hypothesize a 9 grid point spreading in the y-direction also to comprise the influence volume. In Fig. 2, the rectangular volume at the center represents the total charge associated with the grid volume in question. The two Gaussian type curves represent the redistribution of that charge over the 9x9 rectangular grid array surrounding the grid point where the total charge in both distributions is the same.

Mathematically the procedure is developed as follows. The total charge under the two distributions is equated such that

$$Q_T = Q_n \Delta l = Q' \int_0^{\Delta l} \int_{-4\Delta y}^{4\Delta y} \int_{-4\Delta x}^{4\Delta x} e^{-k_1 x^2} e^{-k_2 y^2} dx dy dz \tag{4}$$

which becomes

$$Q_T = Q_n \Delta l = 4Q' \Delta l \int_0^{4\Delta y} \int_0^{4\Delta x} e^{-k_1 x^2} e^{-k_2 y^2} dx dy \tag{5}$$

which can be separated to yield

$$Q_T = Q_n \Delta l = 4Q' \Delta l \int_0^{4\Delta y} e^{-k_2 y^2} dy \int_0^{4\Delta x} e^{-k_1 x^2} dx \tag{6}$$

Here, Q' is the magnitude of the charge density (C/m^3) at the grid point under consideration which will result in the same total charge, Q_T, when distributed under the exponential distribution rule.

The two integrals are evaluated using 6 point Gauss-Legendre Quadrature with $k_1 = k_2$, yielding a fall off to 1/1000 of the central point value at the 4th grid point [see discussion following Eq. (2)]. The result of this integration yields a relationship between Q' and Q_n such that $Q' = Q_n/2.909 \times 10^5$. Therefore, the linear charge density at a grid point may easily be converted to a volumetric charge density spread out laterally away from the discharge path in a manner identical to Eq. (2), but with Q' replacing Q_n. Figure 3 shows the results of the horizontal spreading of the charge distribution depicted in Fig. 2. Because of the exponential fall off, most of the charge is concentrated nearest to the main channel. The upper portion of the channel contains negative charge and the lower portion is positive.

This process conserves the original total charge transferred by the discharge. Each calculated grid point charge density must be converted to an equivalent ion density. This is accomplished by dividing each charge magnitude by the electronic charge $e = 1.6 \times 10^{-19}$ C/electron assuming that all the ions produced are singly charged.

The ions resulting from the redistribution process are then added into the ambient ion fields as they exist at that point in the simulation. With the ion fields modified, a new total charge density is calculated, then Poisson's equation is employed to diagnose a new potential field from which the modified electric field components are calculated. This completes the discharge process and the time marching is resumed. The ions which were injected into the cloud along the discharge path interact with the charged hydrometeors in subsequent time steps and continue to modify the charge distribution and the resulting electric fields.

The first experiment performed with the parameterization scheme used the value of k referred to above and resulted in a very strong influence on the subsequent electrification. Because of this, we decided to try a series of k values to see how differing amounts of charge transfer affected the electrical behavior. Using 10 values of k ranging from 2.5×10^{-12} (weak charge transfer) to 2.5×10^{-11} (strong charge transfer) we found a vastly different character to the electrical structure of the cloud. These results ranged from a minor perturbation for the weak cases to a complete domination of the charge structure and electrical evolution for the strongest cases.

For demonstration purposes, some aspects of 4 of the cases have been chosen to illustrate these differences. In the following graphs, the numerical labels represent the following values of k [in Eq. (1)] and amounts of charge transferred:

1 -> $k = 2.5 \times 10^{-11}$, 12 Coul

2 -> $k = 2.5 \times 10^{-12}$, 1.2 Coul

5 -> $k = 1.0 \times 10^{-11}$, 4.8 Coul

7 -> $k = 1.5 \times 10^{-11}$, 7.2 Coul.

Figure 4 shows the variation of the maximum snow charge density (a), the maximum negative vertical (b) and maximum negative horizontal (c) electric field strengths for a period of 30 seconds following the time of the simulated lightning discharge for each of the four cases listed above.

The majority of the snow in the simulated cloud is charged positively as a result of riming electrification. The lightning discharge occurs in the region of maximum snow charge density with both positive and negative ions being deposited amongst the positively charged snow. The negative lightning ions are formed in the region of maximally charged snow. Figure 4a shows that, in all four cases, the negative ions attach to the snow particles to reduce the maximum charge density. For the weakest case (2) the effect is very small and the electrical processes tend to return to their previous state. For the intermediate cases (5 and 7) there are sufficient negative ions for the attachment process to continue to erode the charge on the snow. For the strongest case (1) the trend is the same initially, but then shows a large increase in the snow charge. Because the data used to make up this plot are the domain maximum values, this large increase represents the effect of the attachment of positive ions from the lower portion of the channel to snow particles lower in the cloud. While the original maximum charge region has been eroded by the negative ions, a new region of positive maximum has been created.

Figure 4b shows the effect of the discharge ions on the maximum negative vertical electric field component. The weak and intermediate discharges all exhibit a tendency to reduce the field strength, with stronger discharges exhibiting the stronger tendency. This seems intuitively satisfying because we tend to think of lightning as acting to reduce the electrical stress in a cloud. For the strong case, however, the results are somewhat unexpected. Here there is an initial increase in the magnitude of the negative field component followed by a return to a near steady state at almost the initial value. Examination of the structure of the vertical field component for cases 2, 5 and 7 shows that the lightning causes an ever increasing perturbation on that structure; however, for case 1 an entirely different structure is established by the discharge.

Figure 4c, which shows the maximum negative horizontal electric field component, further demonstrates the transition from perturbation to domination. Here, only the weak case shows a decrease in the field strength with a rapid return to the slow field buildup. The two intermediate cases both show an increase in the field strength immediately following the discharge; however, case 5 then shows a field reduction below the initial value, while case 7 continues to maintain field strengths greater than the initial value. The strong case shows a dramatic increase in this field component and maintains the increased magnitudes.

While the parameterization scheme developed thus far is admittedly crude, it does seem to produce a lightning channel with realistic character. The parameter study indicates that the effects of a lightning discharge on the electrical evolution of a small thunderstorm depends quite heavily on the amount of charge transferred by the discharge. These effects range from being a minor perturbation on the system to dominating the electrical structure. For a vertical discharge, the most sensitive parameter to the lightning effects appears to be the horizontal electric field component. Because the actual nature of intracloud lightning is not well understood, a study of this type raises more questions than it answers. It is hoped that the studies to be carried out during the remainder of this grant will lead to an improved understanding of the lightning process and thereby lead to an improved parameterization.

2.2 EMA Subcontract - The Lightning Propagation Model

Under subcontract to this grant, EMA has developed a Lightning Discharge Propagation Model (LDPM) for incorporation into the SEM.

The purpose of the LDPM is to account for the tortuous path that is usually followed by a lightning channel. One of the weaknesses of the SEM discharge parameterization scheme has been the propagation criteria which artificially constrain the channel to follow the side of a grid box or its diagonal depending on the ambient electric field direction. The LDPM scheme takes into account the electric fields created by the leader tip and the previous leader segment. Also included are the effects of random free electrons in the vicinity of the leader tip as an influence in determining the direction of propagation of the next segment.

The basic idea employed involves consideration of conditions in proximity to the leader tip. The leader tip contains a high concentration of charge. In the region around the tip, free electrons are randomly generated by the action of cosmic rays. These electrons are accelerated in the combined electric fields of the ambient atmosphere and the leader. If the acceleration is strong enough, such electrons can collide with and ionize neutral air molecules creating secondary free electrons which are subsequently influenced by the field. As the process proceeds, an avalanche of electrons will occur and the air will become highly conductive in the avalanche region. The LDPM algorithm determines which free electrons will result in the conducting path that will become the next leader step. By using random seeds, the LDPM produces lightning channels which propagate in the same general direction as the original scheme used in the SEM, but show deviations from the path more reminiscent of actual lightning. A report (EMA-86-R-55) covering the details of the LDPM is included as Appendix B.

After completion of the LDPM development, a copy of the Fortran code was forwarded to us on magnetic tape. The code must be integrated into the current structure of the SEM.

3. WORK TO BE ACCOMPLISHED

In the time remaining under the grant, several tasks remain to be accomplished. The cloud-to-ground discharge parameterization scheme must be worked out (using the intracloud scheme as a basis) and incorporated into the model. The LDPM must be worked into the model code and the termination criterion must be examined with consideration given to the electric field associated with the leader tip. A modification to the termination criterion is necessary because cloud-to-ground discharges are known to propagate through the subcloud region where the ambient field is often considerably below the threshold of 150 kV/m.

In addition, the details of the effects of the intracloud discharge on the electrification in the 19 July case will be examined because of the intriguing behavior exhibited during the variation of the charge transfer values in the parameter study. We think that this approach is necessary because of the lack of current knowledge on the details of the lightning discharge. In short, what we have done thus far looks reasonable; however, we have no way to know that what we have done is right.

Finally, the Wallops Island case and the 11 July 1978 TRIP case will be run with the lightning discharge active to see how the occurrence of lightning, interacting with the other model variables, effects the subsequent electrification and what types of discharges will occur within the model. The 11 July TRIP case was chosen because it was a well documented, isolated storm and has been the subject of investigation by several other groups.

The task remaining for EMA to accomplish under their subcontract involves the investigation of modifications to the lightning initiation criterion used in the SEM employing information they are acquiring on the effects of particle ensembles and their charges on the initiation of lightning. Because of the lack of field data on the locations and ambient conditions existent at the point of actual lightning, these studies will be speculative in nature. It is our hope that some new insights can be gained from this effort.

4. CONFERENCE PAPERS AND PUBLICATIONS

During the period covered by this report, one conference paper was submitted and subsequently accepted for oral presentation. The paper was entitled "A Lightning Parameterization Scheme in a Two-Dimensional, Time-Dependent Storm Electrification Model" authored by J. H. Helsdon, Jr., R. D. Farley, and G. Wu. It will be presented at the Conference on Cloud Physics sponsored by the American Meteorological Society to be held in Snowmass, CO, in September 1986.

Also, a paper entitled "A Numerical Modeling Study of a Montana Thunderstorm: Part II. Model Results vs. Observations Involving Electrical Aspects" was completed and submitted for publication in the Journal of Geophysical Research. The paper was reviewed and accepted for publication after modification in light of the reviewer's comments. These modifications will be accomplished during the remainder of the grant period. Abstracts of the conference and journal papers are included in Appendix A.

Finally, a Master's Thesis is being prepared by Mr. G. Wu. While Mr. Wu has not been supported directly by the NASA grant, the computations that he has used in preparing his research have been supported by the grant. The topic of the thesis involves the lightning parameterization scheme and its use in the SEM. Mr. Wu plans to finish his degree in time for December graduation.

5. REFERENCES

Kasemir, H., 1960: A contribution to the electrostatic theory of a lightning discharge. J. Geophys. Res., 65, 1873-1878.

____, 1984: Theoretical and experimental determination of field, charge and current on an aircraft hit by natural and triggered lightning. Presented at Intnl. Aerospace and Ground Conf. on Lightning and Static Electricity, Orlando, FL.

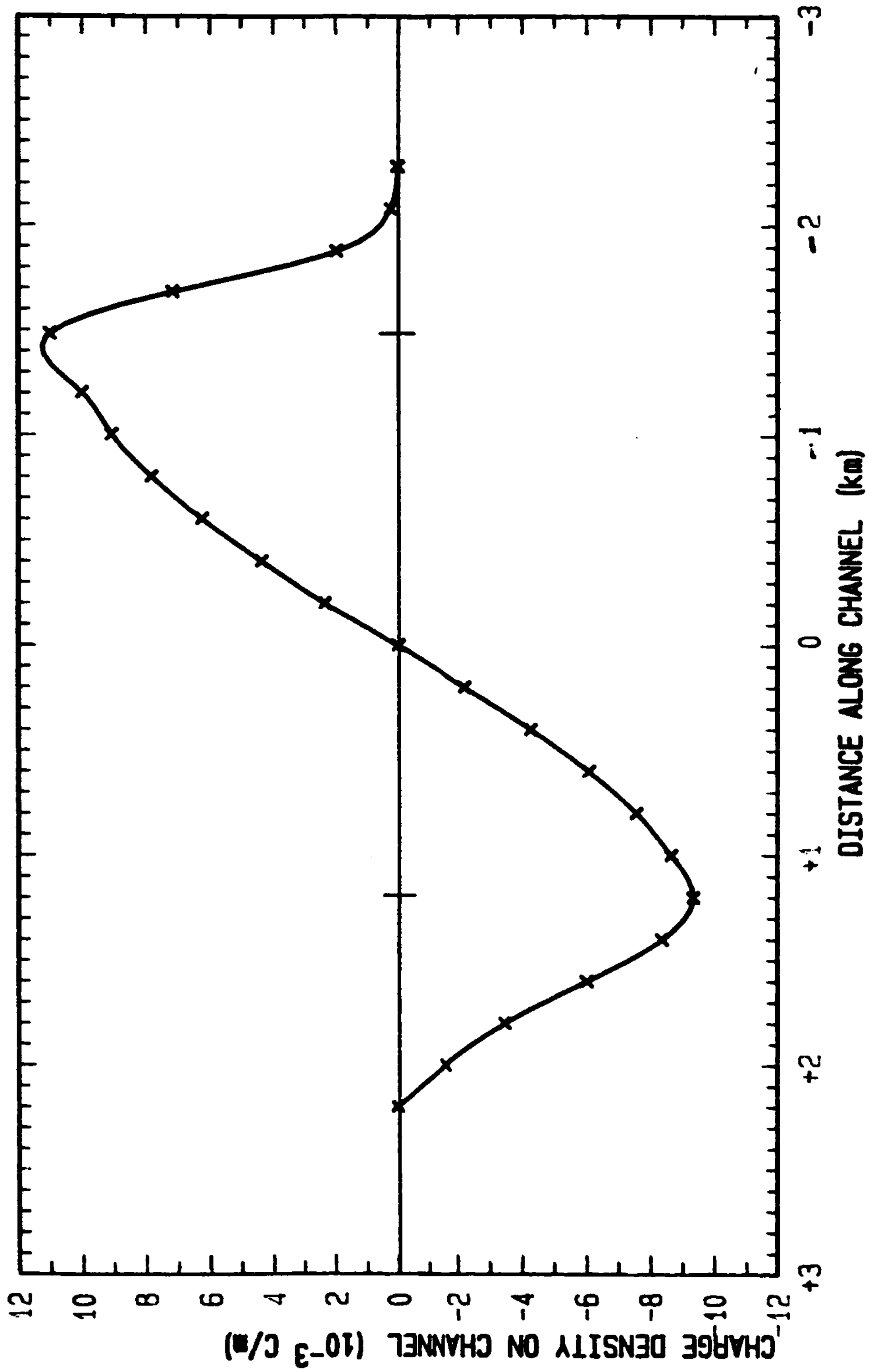

Fig. 1: Linear charge density along the channel path using Eq. (1) and a value of $k = 2.5 \times 10^{-11}$ f/m. The original channel is delineated by the vertical bars on the horizontal axis.

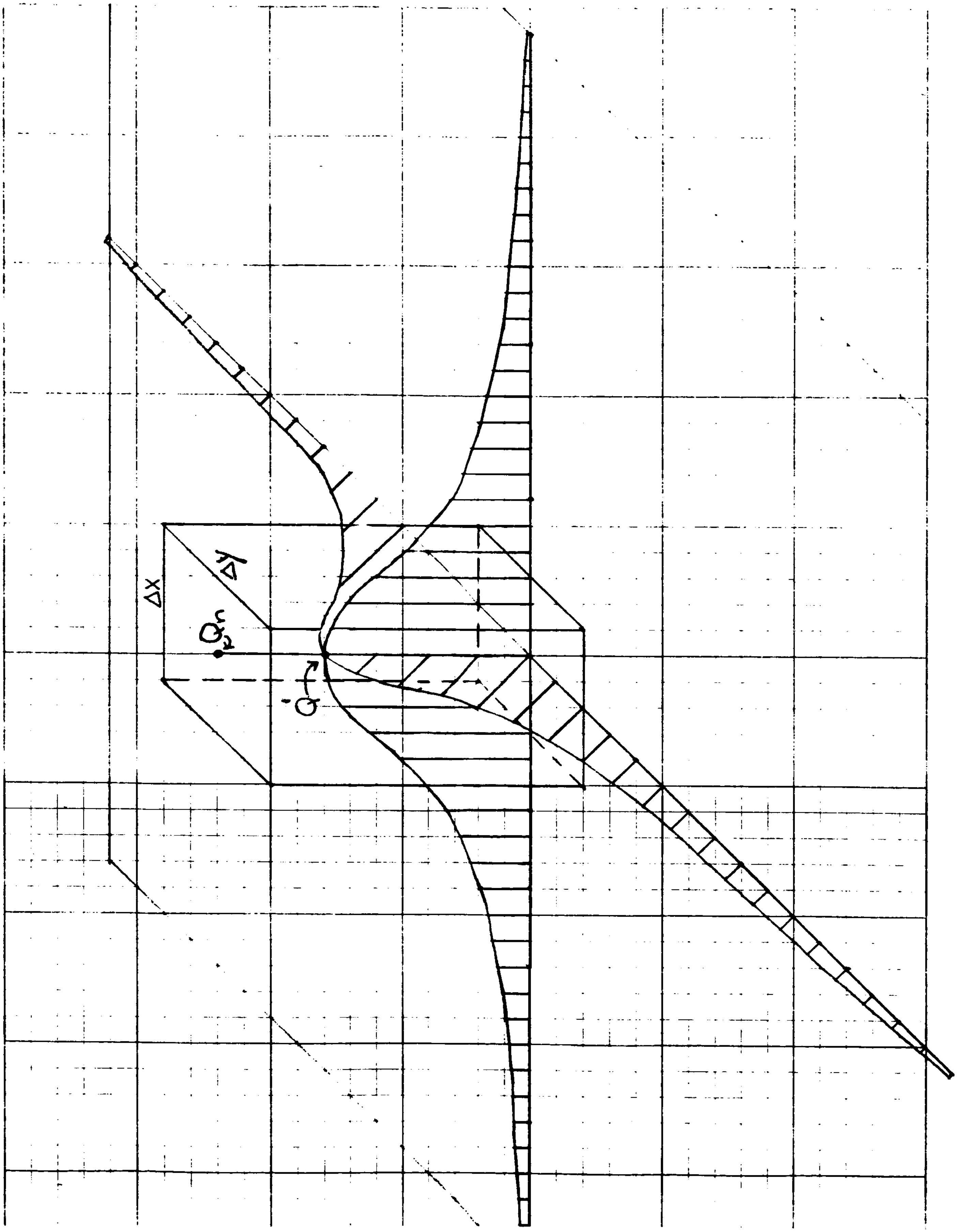

Fig. 2: Depiction of the 3 dimensional charge spreading procedure. The total charge in the rectangular volume and under the surface defined by the Gaussian curves are equal.

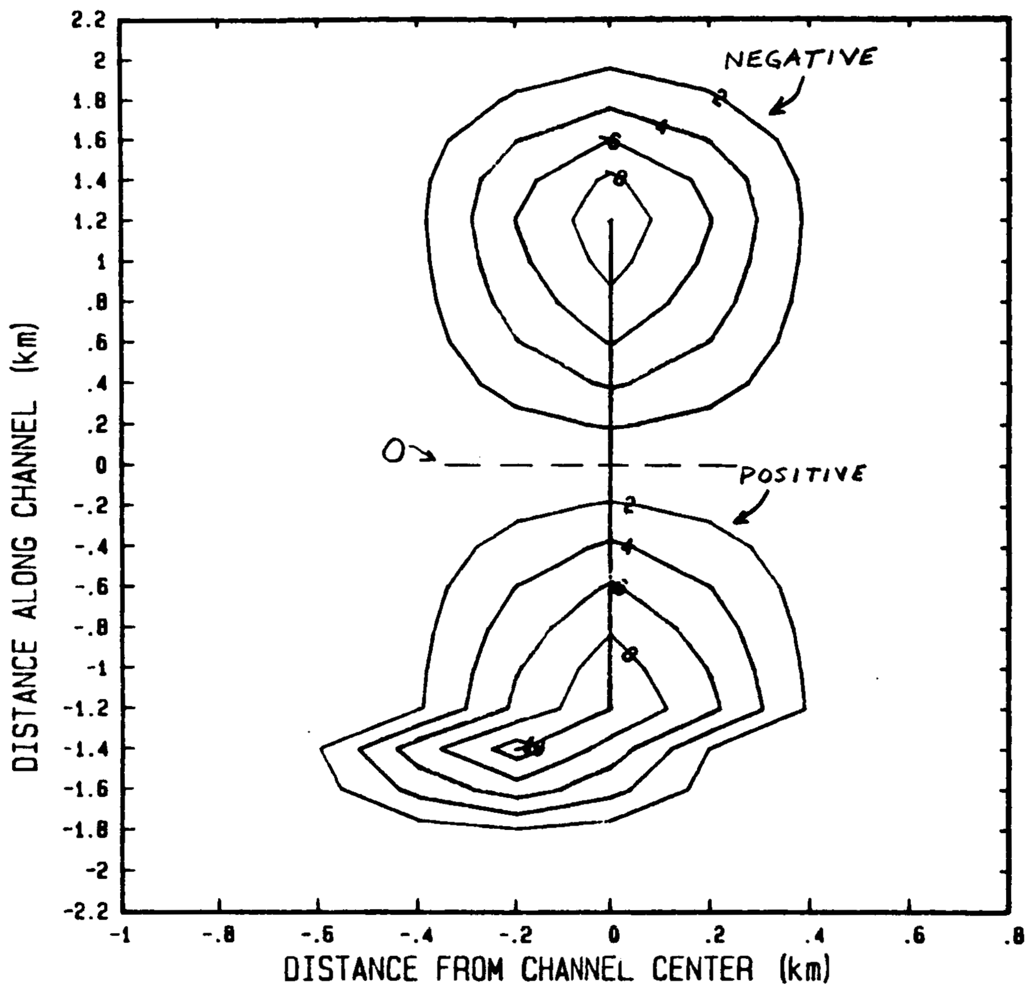

Fig. 3: Horizontal distribution of the linear charge in Fig. 1 after applying the procedure in Fig. 2.

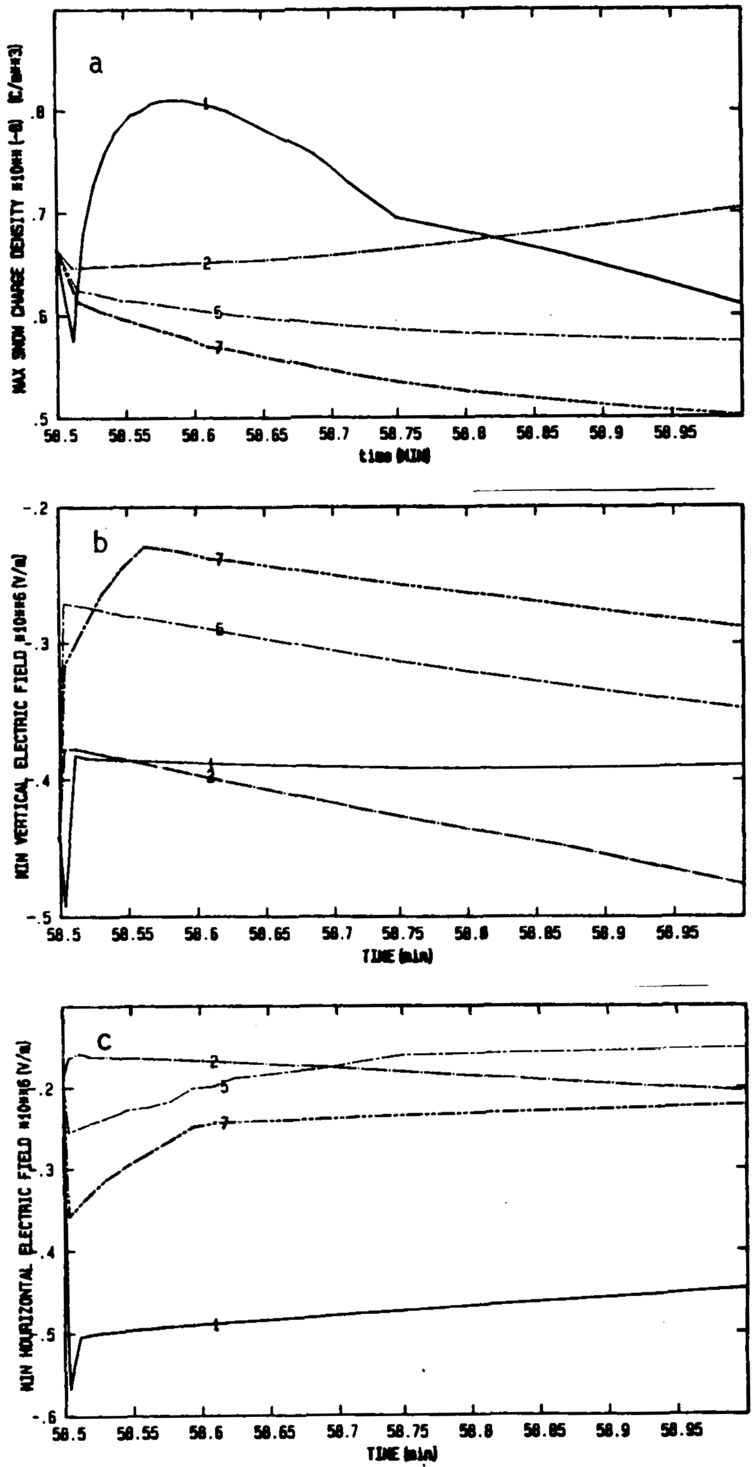

Fig. 4: Domain maximum positive show charge density (a), maximum negative vertical electric field (b), and maximum negative horizontal electric field (c) for 30 sec following the discharge for four experiments.

APPENDIX A

A LIGHTNING PARAMETERIZATION SCHEME IN A TWO-DIMENSIONAL, TIME-DEPENDENT STORM ELECTRIFICATION MODEL

John H. Helsdon, Jr., Richard D. Farley,
and Gang Wu

ABSTRACT

An important consideration in the modeling of thunderstorm electrification is the existence of lightning in nature. A model without lightning is only capable of simulating the development of a storm in its early stages (up to the time of first lightning). Since lightning accounts for significant charge rearrangement within a storm, it becomes necessary to include such processes in any modeling effort which hopes to shed light on the electrical evolution of such storms. In an attempt to make allowance for the lightning discharge in our Storm Electrification Model (SEM), we have developed a simplified parameterization scheme to simulate the lightning channel and its attendant charge redistribution.

This parameterization scheme has criteria for the initiation, propagation, termination and charge redistribution associated with lightning. In order to simplify the scheme as much as possible, the first three criteria are all dependent on the strength and direction of the electric field vector. The propagation of the channel is bidirectional and the charge distribution along the channel is calculated so as to maintain overall charge neutrality for an intracloud discharge. The charge deposited along the channel is then spread in a conservative manner laterally away from the channel over several grid points in order to maintain computational stability. This distribution of charges is then added to the small ion population and allowed to interact with the other charge carriers as the integration proceeds.

A simulation of the 19 July 1981 CCOPE case has been run with the lightning discharge included. A parametric study has been undertaken to determine the effects of different amounts of charge transfer on the subsequent electrical evolution. The results of these simulations will be discussed.

A NUMERICAL MODELING STUDY OF A MONTANA THUNDERSTORM: PART II. MODEL RESULTS VS. OBSERVATIONS INVOLVING ELECTRICAL ASPECTS

John H. Helsdon, Jr. and Richard D. Farley

ABSTRACT

In this investigation, we compare the results of the Storm Electrification Model (SEM) simulation of the 19 July CCOPE case study cloud against the actual observations with respect to the cloud's electrical characteristics, as deduced from the data of two aircraft.

It is found that the SEM reproduces the basic charge and electric field structure of the cloud on a similar time scale to that observed. The comparability of the modeled and observed values of field strength and charge within the cloud is directly related to the proximity of the aircraft to the main active core of the cloud. The character of the electric field external to the cloud appears to be well modeled, at least at the time of observation. A feature which was not observed, but appears in the model, is a charge screening layer at the tip of the anvil.

In addition, the model predicts electric field strengths sufficient to initiate a lightning discharge at approximately the same time in cloud evolution as that when an intracloud discharge was recorded.

APPENDIX B

EMA-86-R-55

A STOCHASTIC MODEL FOR THE TORTUOUS PROPAGATION OF A LIGHTNING STEPPED LEADER

by

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A STOCHASTIC MODEL FOR THE TORTUOUS PROPAGATION OF A LIGHTNING STEPPED LEADER

The results of Task 1 produced by EMA for the referenced contract "Completion of a Lightning Discharge Propagation Model (LDPM)" are discussed in this report. A FORTRAN code has been written for this model which can be integrated into the South Dakota School of Mines (SDSM) time-dependent Storm Electrification Model (SEM). The LDPM has been structured to enable its use in either a two or three-dimensional SEM with minor modifications.

1. INTRODUCTION

The LDPM is an approximation of the tortuous path of a lightning channel in a thunderstorm environment. Location and direction for the initial discharge point must be supplied as input data to the model. The discharge channel is then established as a series of connected stepped leader segments and propagates until certain termination conditions are met. All leader segments are assumed to be the same length and have equal charge densities. These parameters can be set to any values desired. While testing the LDPM, parameter values of 50 meters and 1 millicoulomb/meter were used based on the average values published by Uman[1].

The primary parameter calculated in the model is the direction for a segment of the stepped leader. This computation is based on the effects of random free electrons in the vicinity of a leader segment tip which determine the direction of the next stepped leader segment. The details of the direction parameters for a stepped leader segment will be discussed in the next section. Once this direction is established, position vectors for a segment can be calculated. The model is then able to continue the computation of the positions and directions of following stepped leader segments.

[1] Uman, M.A., "Lightning," McGraw-Hill, New York, 1969.

The remainder of this report discusses the calculations used in the LDPM, presents results obtained during the model tests, and provides a discussion on how the code is used. In addition, a listing of the code is provided.

2. PHYSICAL CONSIDERATIONS

Uman[1] discusses several theories which have been presented since the late 1930's to explain observed stepped leader characteristics. Although basic differences exist between these theories, several fundamental issues are common for the theories and are listed below.

1. A stepped leader has an inner conducting core wherein most of the current flows.

2. The core is surrounded by a corona region in which the electric field exceeds the breakdown field of air (approximately 3×10^6 volts/meter for standard temperature and pressure at sea level).

3. A mechanism exists which precedes the stepped leader to create an ionized channel in "virgin air" which provides continuation of the stepped leader propagation.

4. As current flow increases into the ionized channel, a condition occurs whereby the conductivity increases until the channel becomes a new segment of the stepped leader and the end of the channel becomes the new tip of the leader.

These theories agree that stepped leader segment directions are randomly distributed. Based on lightning channel tortuosity studies by Hill[2] , the mean absolute direction change is nearly constant from flash-to-flash and on the order of 16 degrees. For purposes of the LDPM, it is assumed that the randomness of the stepped leader direction is related to the random creation of free electrons in air due to cosmic ray bombardment.

[2] Hill, R.D., "Analysis of the Irregular Paths of Lightning Channels," J. Geophys. Res., Vol. 73, 1968 (pp. 1897 - 1906).

Issue 3 is of primary concern for this model. The stepped leader propagation theories often vary when describing this mechanism. The process occurring which results in the formation of the ionization channel ahead of the stepped leader tip has been more recently described by Rudolph[3] from studies involving the interaction of lightning with aircraft in flight.

The leader tip is a region which contains a large charge density and high electric field levels. Free electrons (generated by cosmic rays) in the air are accelerated in this high field until they collide with a neutral atom or molecule. If the electron's kinetic energy is large enough at the time of the collision, the neutral particle can have an electron separated from it, producing a second free electron and a positive ion. The free electrons are then again accelerated by the field, possibly suffering more collisions and producing more free electrons and ions. If the rate of production of free electrons is larger than the rate of loss (by recombination, and attachment to form negative ions), an electron "avalanche" occurs, in which sufficient numbers of electrons and ions are produced and substantially alter the electrical conductivity of the air.

Figures 1 and 2 illustrate how the effects of avalanching result in the generation of a new leader step from an existing leader tip. Figures 1 and 2 also illustrate the ionization path mechanisms for a negatively charged and for a positively charged leader tip, respectively.

In Figure 1, a free electron (at time t_1) is accelerated by the electric field towards the leader tip. As avalanching occurs (time t_2), a net positive charge is formed in the region of the initial free electron position. This results from positive ions which have a much lower mobility or drift velocity than electrons. The net effect is neutralization of the electric field at the leader tip (time t_3). The conductivity in the region approaches the conductivity in the inner core of the stepped leader. Thus, the region in which positive ions were first produced becomes the new leader tip.

In Figure 2, a free electron (at time t_1) is accelerated by the electric field away from the leader tip. As avalanching occurs (time t_2), a net positive charge is

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[3] Rudolph, T., and R.A. Perala, "Linear and Non-Linear Interpretation of the Direct Strike Lightning Response of the NASA F106B Thunderstorm Research Aircraft," Electro Magnetic Applications - Denver, EMA-83-R-21, March 21, 1983.

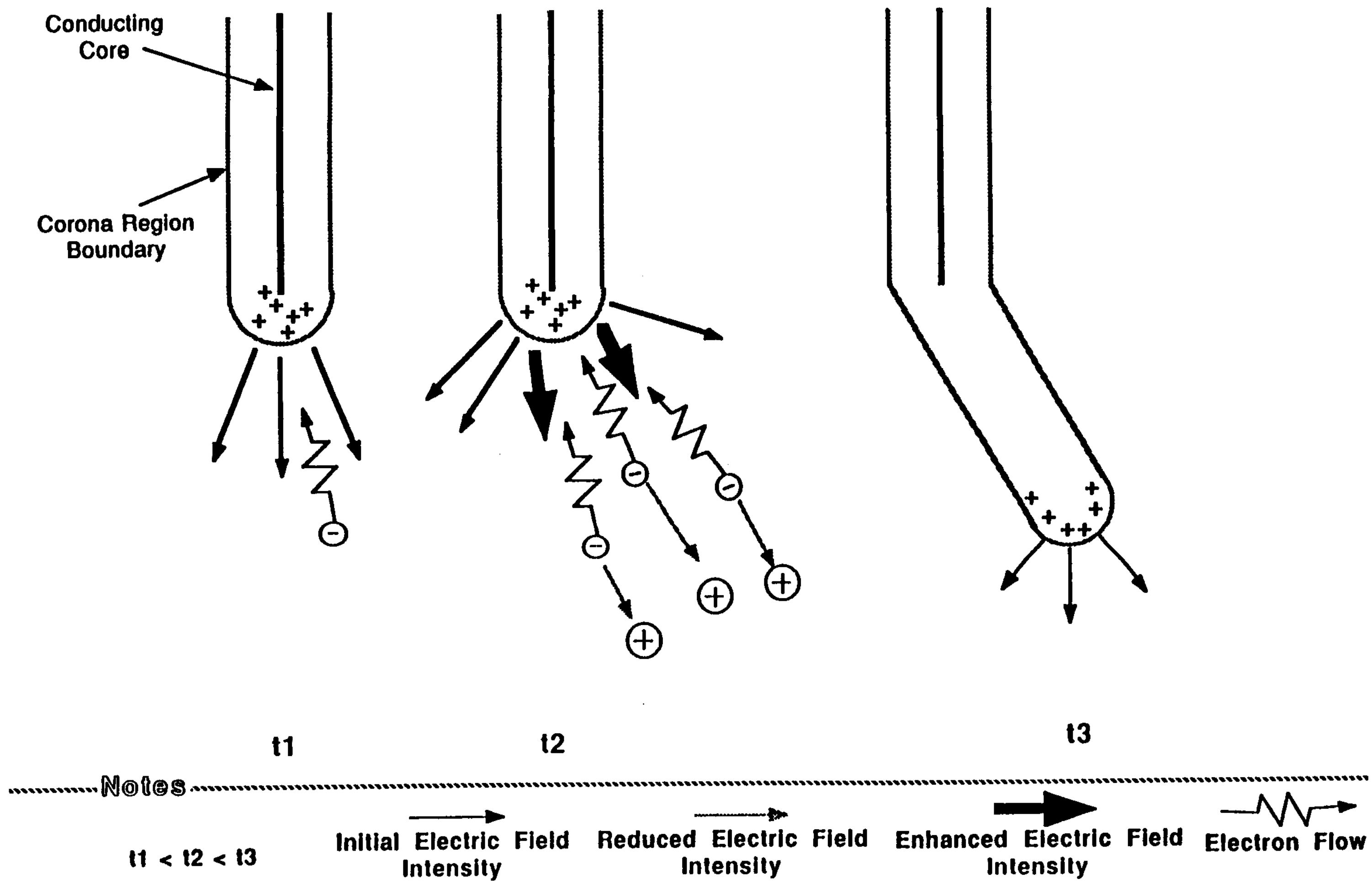


Figure 1. Time Scenario of Free Electron Avalanching for Positively Charged Leader Tip

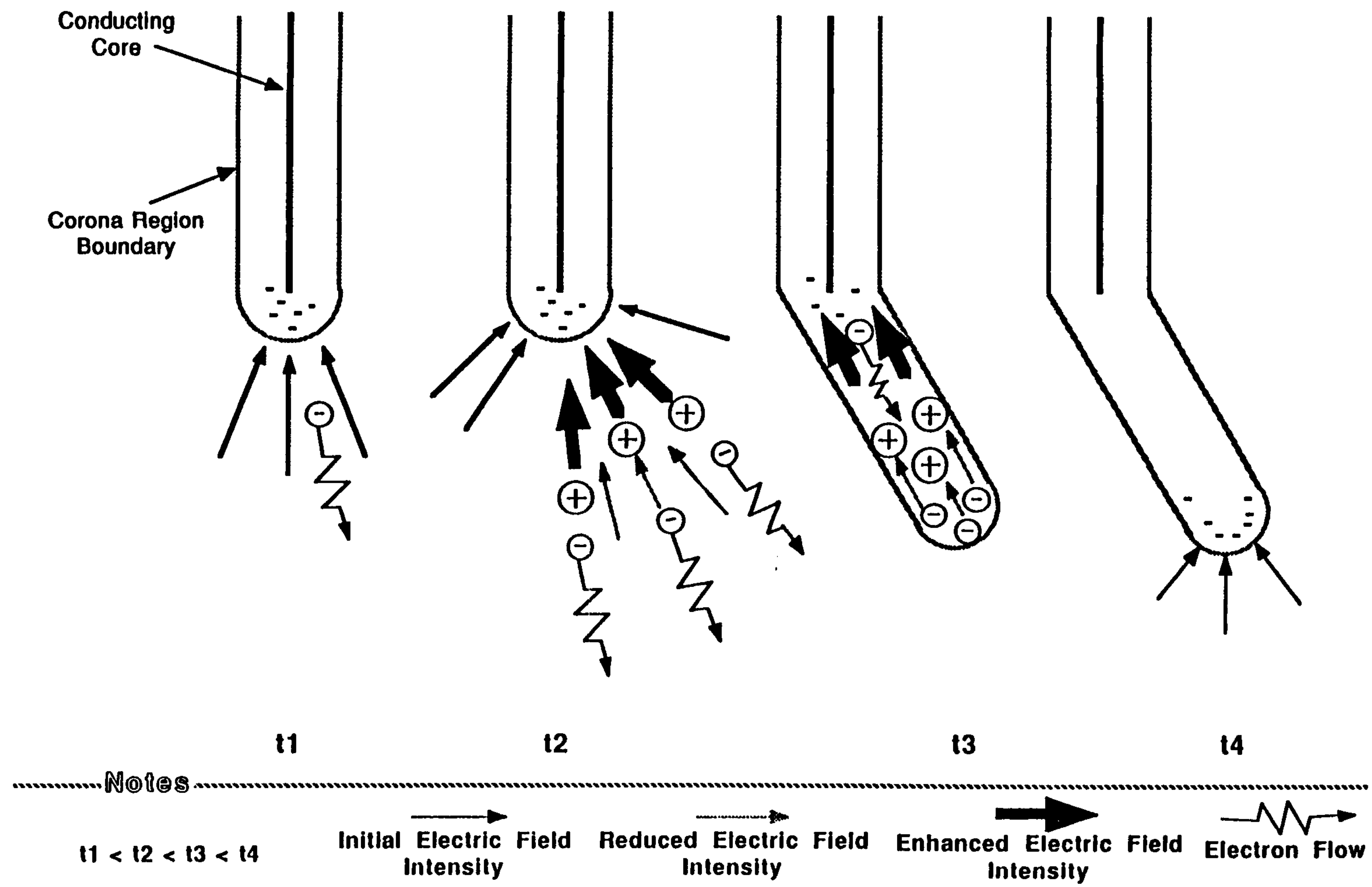


Figure 2. Time Scenario of Free Electron Avalanching for Negatively Charged Leader Tip

formed in the region of the initial position of the free electron. Moreover, as the electrons move away from this region, a conducting channel is generated between the positive ions and electrons until the electric field between the two opposite charge regions is neutralized. However, the electric field is enhanced between the leader tip and the positive charge region and induces additional forces on the negative charge region around the tip. In addition, the breakdown region is extended toward the positive charge region. This effect is illustrated in Figure 2 at time t_3 as a new conductive region extends the stepped leader. The leader tip occurs at the negative charge region created by the electrons which were first accelerated away from the original free electron position.

The mechanism for extending the stepped leader is dependent on the location of free electrons which create a conductive channel to the leader tip. Thus, it is necessary to consider all free electrons and determine which one will initiate the first complete ionized path to the leader tip. Moreover, the algorithm used must determine which electron will initiate this complete ionized path in the minimum time (compared to other electrons).

A rigorous mathematical treatment of the mechanisms just discussed would require involved calculations for the avalanche process and resultant conductivity in the stepped leader tip vicinity. These calculations would also involve significant computer time when used in the SEM. A simplified algorithm which has similar dependencies on the position of random electrons and the electric field around the tip of a stepped leader segment was developed.

Since the drift velocities of the positive ions are significantly lower than those of electrons, the first simplification does not account for the mobility of resultant positive ions. The equation of motion for electrons generated by the avalanche is then given by:

$$\frac{d\vec{V}_e}{dt} = - \frac{q_e}{m_e} (\vec{E}_T + \vec{V}_e \times \vec{B}) - \nu \vec{V}_e \qquad (1)$$

where: q_e = electron charge,

m_e = electron mass,

$\vec{E}_T$ = total electric field,
$\vec{V}_e$ = electron velocity,
$\vec{B}$ = magnetic field, and
ν = an approximate collision frequency.

To further simplify this equation, it is assumed that the creation of electrons due to avalanching has progressed to a level where a collision dominated, plasma region exists. In turn, the electron velocity is constant and equal to the drift velocity, and:

$$\frac{d\vec{V}_e}{dt} = 0 \ . \tag{2}$$

Thus,

$$\frac{q_e}{m_e} \vec{E} - \nu \vec{V}_d = 0, \quad \text{or} \quad \vec{V}_d = - \mu_e \vec{E} \tag{3}$$

where: μ_e = electron mobility, and
V_d = drift velocity.

It is assumed that the vector $\vec{r}_e$ between the leader tip and the electron position can be approximated by:

$$\vec{r}_e = \vec{V}_d t, \tag{4}$$

or the time necessary to create a channel can be approximated:

$$t = \frac{|\vec{r}_e|}{|\vec{V}_d|} \propto \frac{|\vec{r}_e|}{|\vec{E}_T|} \ . \tag{5}$$

It is assumed in the model that the ionized channel induced by the avalanching process (resulting from randomly occurring electrons) will initiate the next stepped leader segment in the same direction for either a positively or negatively charged stepped leader segment. Thus, the model is independent of the leader tip charge. Based on these assumptions, the new stepped leader segment direction is determined by identifying the free electron in the vicinity of the previous stepped leader segment tip which will create an ionization path in a minimum time (compared to other free electrons).

The model requires the following computations which will be discussed in detail in the next sections.

a. Random generation of electrons in the vicinity of each stepped leader tip.

b. Total electric field strength and direction at each electron location.

c. The direction vector of the new stepped leader.

3. LIGHTNING DISCHARGE PROPAGATION MODEL CALCULATIONS

3.1 Description

The LDPM code was written to predict the tortuous path of a lightning channel based on the physical considerations in Section 2. The routine is programmed to calculate the parameters for each stepped leader segment.

3.1.1 Random Electron Generation

A Monte Carlo technique was used to generate the random locations of free electrons within a sphere around each leader segment tip. The ambient electron density is assumed to be .2 electrons/m^3. The size of the sphere is dependent upon the number of free electrons. Each model run in this report considered 500 random electrons around each tip. Thus, the sphere centered at the leader tip was required to have a radius of 8.419 meters.

A seed integer is also required for the Monte Carlo routine. This seed controls the random placement of electrons. Different seeds will produce different sets of electron locations.

3.1.2 Total Electric Field Calculation

The total electric field strength and direction was computed for each electron location generated by the Monte Carlo technique. The electric field at any electron location should be the resultant of the ambient field and the fields due to the previous leader segments. The model code was tested to incorporate the effects of the previous leader segments and the ambient field in the electric field calculation. It was observed that this calculation only required the ambient electric field and the electric field components due to the two leader segments closest to the electron. Electric fields caused by segments further away did not affect the modeled lightning path significantly. The total resultant electric field at a location (x,y,z) in the vicinity of the ith stepped leader segment is stated below:

$$E_{total}(x,y,z) = E_{ambient}(x,y,z) + E_{i\ segment}(x,y,z) + E_{i-1\ segment}(x,y,z)\ . \qquad (6)$$

The general equation for the electric field at any point in space due to a line charge is given below and is illustrated by Figure 3.

The electric field at any point in space due to the line charge element λdl is:

$$d\vec{E} = \frac{\lambda dl\,(\vec{r} - \vec{r}')}{4\pi\varepsilon_0\,|\vec{r} - \vec{r}'|^3}\ ;\ \lambda = \text{charge/unit length}\ . \qquad (7)$$

Thus, the electric field at point p due to the total line charge is:

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{\vec{r} - (\vec{R} + l\vec{u})}{|\vec{r} - (\vec{R} + l\vec{u})|^3}\,dl\ . \qquad (8)$$

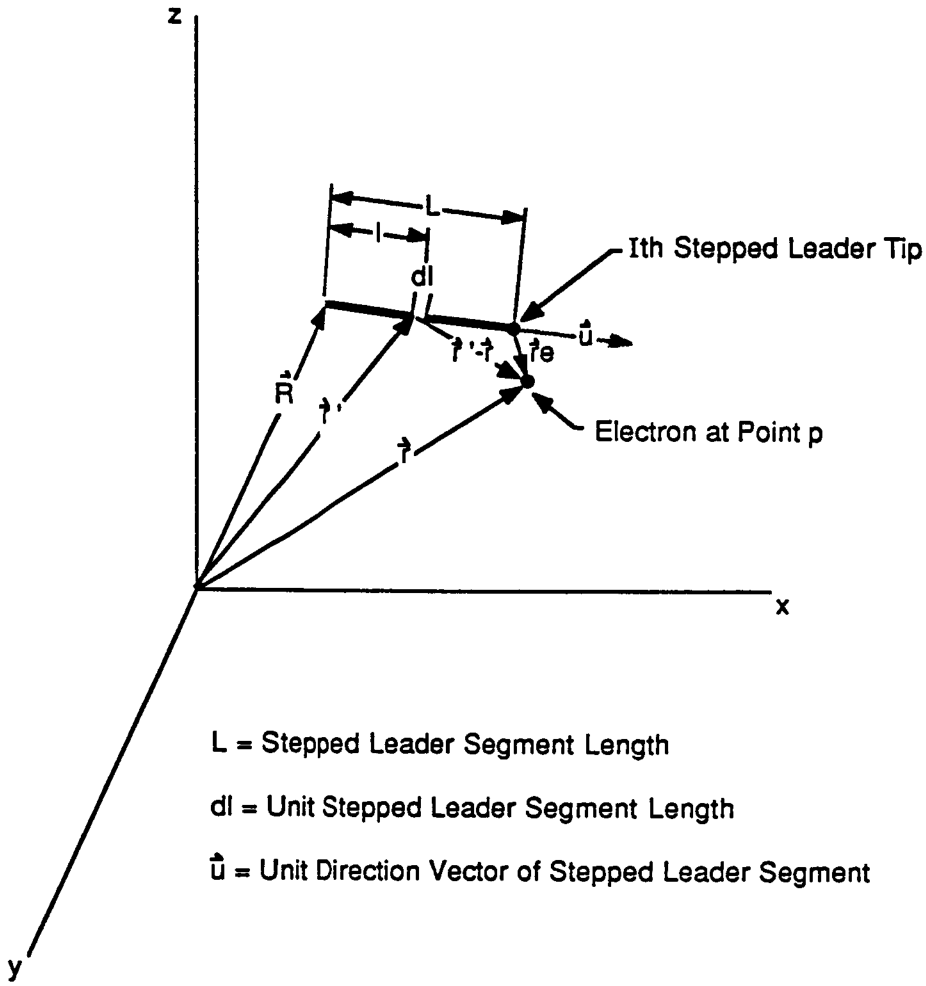


Figure 3. Stepped Leader Segment Line Charge and Its Parameters in World Coordinates

The general world coordinate solution for the electric field due to the line charge becomes:

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 (A^2 - B^2)} \left\{ \left[\frac{\vec{A}(L - B) + \vec{u}(A^2 - BL)}{(A^2 - 2BL + L^2)^{1/2}} \right] - \left[\frac{A^2\vec{u} - \vec{A}B}{|A|} \right] \right\} \quad (9)$$

where: $\vec{A} = \vec{r} - \vec{R}$, and
$B = \vec{A} \bullet \vec{u}$.

Figure 4 depicts the constant electric field contours due to a line charge. The arrows show the direction of the electric field.

3.1.3 Direction Vector of the New Stepped Leader Segment

Only one of the electrons generated in the vicinity of the leader tip will properly satisfy the model requirements and determine the direction of the new stepped leader segment. For simplification, each stepped leader segment is assumed to be positively charged (as discussed in Section 2) for the LDPM. The new stepped leader segment is assumed to be initiated by the ionized path formed in the least time. The time to generate the ionized path is proportional to the ratio formed from the distance of the leader tip to the electron (r_e) and the resultant electric field strength (E_T) at the electron location. This relationship was determined in Section 2 and is stated in Equation 5. Thus, the new stepped leader segment direction is chosen to be the electric field direction at the initial location of the free electron which initiates an ionized path in the minimum time (compared to other electrons).

If the angle θ_e, formed by the E_T and r_e vectors at the initial electron location is within a certain angle range, that electron will create an ionized channel to the leader tip along a "relatively" straight path. The time it takes to establish the ionization channel can be approximated using Equation (5). As θ_e increases beyond this certain angle range, the actual path of the ionization channel along electric field lines will deviate from a straight line. Therefore, the actual time it takes to establish the ionized channel to the leader tip may be significantly greater than the time approximated by Equation (5). In fact, if θ_e is large enough, the ionized path may not attach to the leader segment tip, but to some point behind the tip instead. The cosine of θ_e formed by E_T and r_e at any electron location is computed as follows:

$$\cos \theta_e = \frac{\vec{r}_e \bullet \vec{E}_T}{|\vec{r}_e| \; |\vec{E}_T|} \quad . \tag{10}$$

A stepped leader segment is defined by the computer code as a finite length with its tip being a single point. In nature, the leader tip is not a single point but is actually defined as a volume. In terms of the LDPM, this volume exists around the

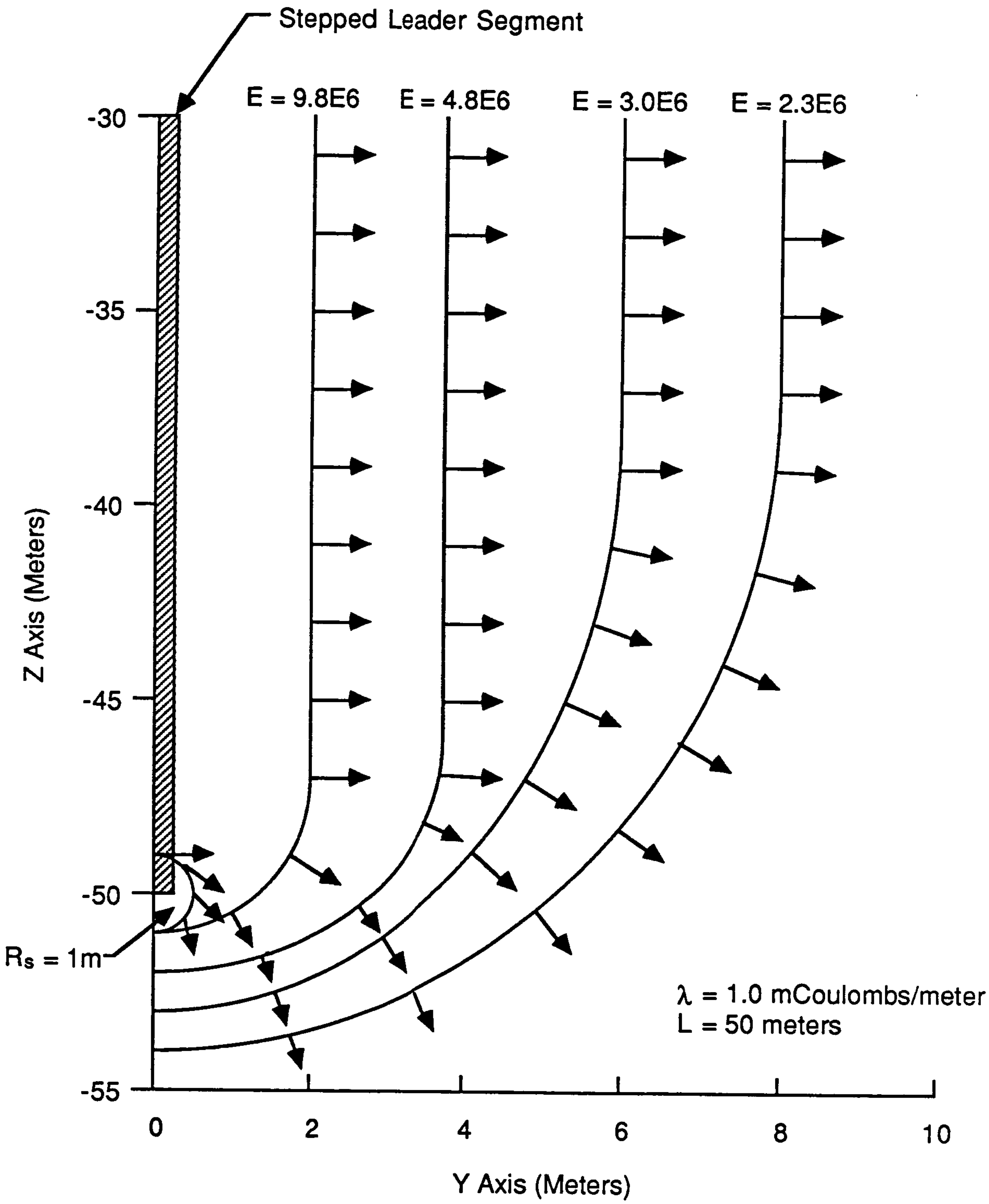


Figure 4. Constant Electric Field Amplitude Contours for Line Charge

leader tip point and has an electric field strength greater than 3×10^6 V/m everywhere inside.

Figure 5 depicts the above concepts. The angle range parameter is determined by the radius of the corona region. This radius is on the order of 2 meters and is assumed to be the same magnitude for all leader segments in the LDPM. The angle range parameter was adjusted to an absolute value of 12.5 degrees. This value appeared to allow the lightning channel path to be reasonably tortuous. The angle range parameter can be changed in the LDPM as desired. However, 12.5 degrees is recommended, since smaller angle parameter values limit the tortuousity, while larger angle parameter values result in erratic tortuousity which exceeds the findings of Hill[2].

Each of the 500 electrons generated by the Monte Carlo routine is tested in the code to determine which one is used to establish the direction of the new stepped leader segment. The one electron chosen must meet the following conditions:

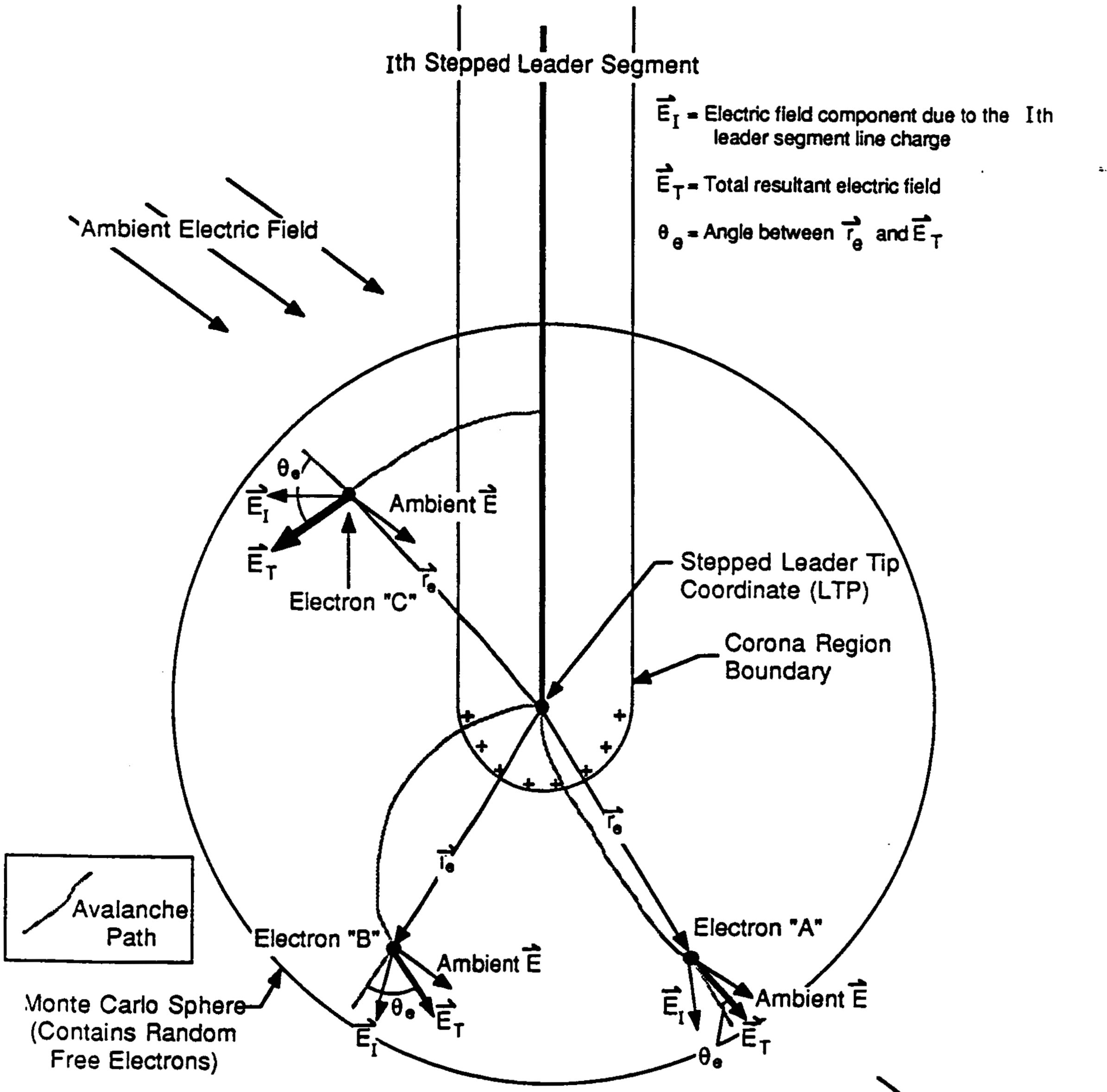
1. The initial free electron which initiates an avalanche must originate in a location where the electric field strength is less than 3×10^6 V/m. (If the field was larger, the electron was assumed to be inside the corona region of the leader segment and was not considered).

2. The angle θ_e formed by E_T and r_e for an electron location must be less than the angle range parameter (absolute value of 12.5 degrees) (If θ_e was greater, the electron was not considered).

3. Of the remaining electrons, the electron which creates an ionization channel to the leader tip in the least time, t (as computed in Equation (5)) was identified and used to calculate the new direction.

The new stepped leader segment direction is then taken to be the direction of the total electric field at this "identified" electron location.

It is important to note that, if the Monte Carlo sphere is too small, most or all of the random electrons will lie in a region where the electric field strength is above 3×10^6 V/m. In turn, the model may not function properly. It was observed that a sphere containing 500 electrons and having a radius of 8.419 meters worked best



Electron "A" will be drawn to the leader tip in time, t.
t can be adequately approximated in equation [5] since θ_e <12.5°.

Electron "B" will be drawn to the leader tip along the path illustrated.
The time will be significantly greater than t computed by equation [5] since θ_e > 12.5°.

Electron "C" will not be drawn to the leader tip at all and equation [5] does not apply.
However, θ_e >12.5° and the electron will not be considered.

Figure 5. Paths of Free Electrons in Vicinity of Stepped Leader Tip

during the LDPM development. This sphere size contained enough electrons in a region where the electric field was below breakdown and allowed the LDPM to function properly and efficiently.

3.2 Model Termination Conditions

The LDPM will terminate if any of the following conditions are met:

1. The stepped leader enters a defined region (grid cell) with an ambient electric field less than 1.5×10^5 V/m. This termination criterion was based on work done by Griffiths[4] involving the propagation of positive streamers in the laboratory,

2. The stepped leader propagates outside the designated problem space,

3. (Optional) The stepped leader enters a defined charged region (Specified grid cell).

3.3 MODEL INPUT AND OUTPUT REQUIREMENTS

3.3.1 Input

The problem space is depicted as a rectangular grid either in two or three dimensions. The code is written to accommodate two or three-dimensional data. This version of the LDPM is adjusted for a two-dimensional SEM. Each grid cell has side lengths equal to 200 meters. This routine requires the number of grid cells along the x-axis (horizontal), along the z-axis (vertical), and the initial discharge coordinate (x,z). Ambient electric field components are required for each grid cell. The routine also requires a seed integer for the Monte Carlo scheme. These variables are:

1.	NCELX	Number of cells along the x-axis
	NCELZ	Number of cells along the z-axis,

[4] Griffiths, R.F. and C.T. Phelps, "The Effects of Air Pressure and Water Vapor Content on the Propagation of Positive Corona Streamers and Their Implications to Lightning Initiation," Quart. J. Roy. Meteor. Soc., Vol 102, 1976 (p. 419).

2. ICX, ICZ — Initial discharge point (x,z),

3. FEX(K), FEZ(K) — One-dimensional array of x,z ambient field components, and

4. ISEED — Monte Carlo seed integer; must be less than 32,000.

For the NCAR Electricity Model:

1.	NCELX = 97;	NCELZ = 97
2.	ICX = 86;	ICZ = 31
3.	FEX(K), K = 1, 9409;	FEZ(K), K = 1, 9409

3.3.2 Output

The routine generates the (x,z) position in world coordinates for each stepped leader segment tip in the propagation path. The x and z arguments are written in the one-dimensional RX and RZ arrays. Coordinate values x and z are given in meters from the origin (x = 0, z = 0). These arguments can be passed back into the main program and are as follows:

1. RX(NL), RZ(NL) — One-dimensional arrays of the x and z values in meters

   NL corresponds to the leader number (not the grid cell number), and

2. NFL — Integer; total number of NL leader segments.

3.4 MODEL EXAMPLES

Several lightning discharge propagation paths were generated and are shown in Figures 6 through 8. Figure 6 shows the lightning paths modeled for the NCAR electricity data at 58.5 minutes. The solid and dashed lines are the models

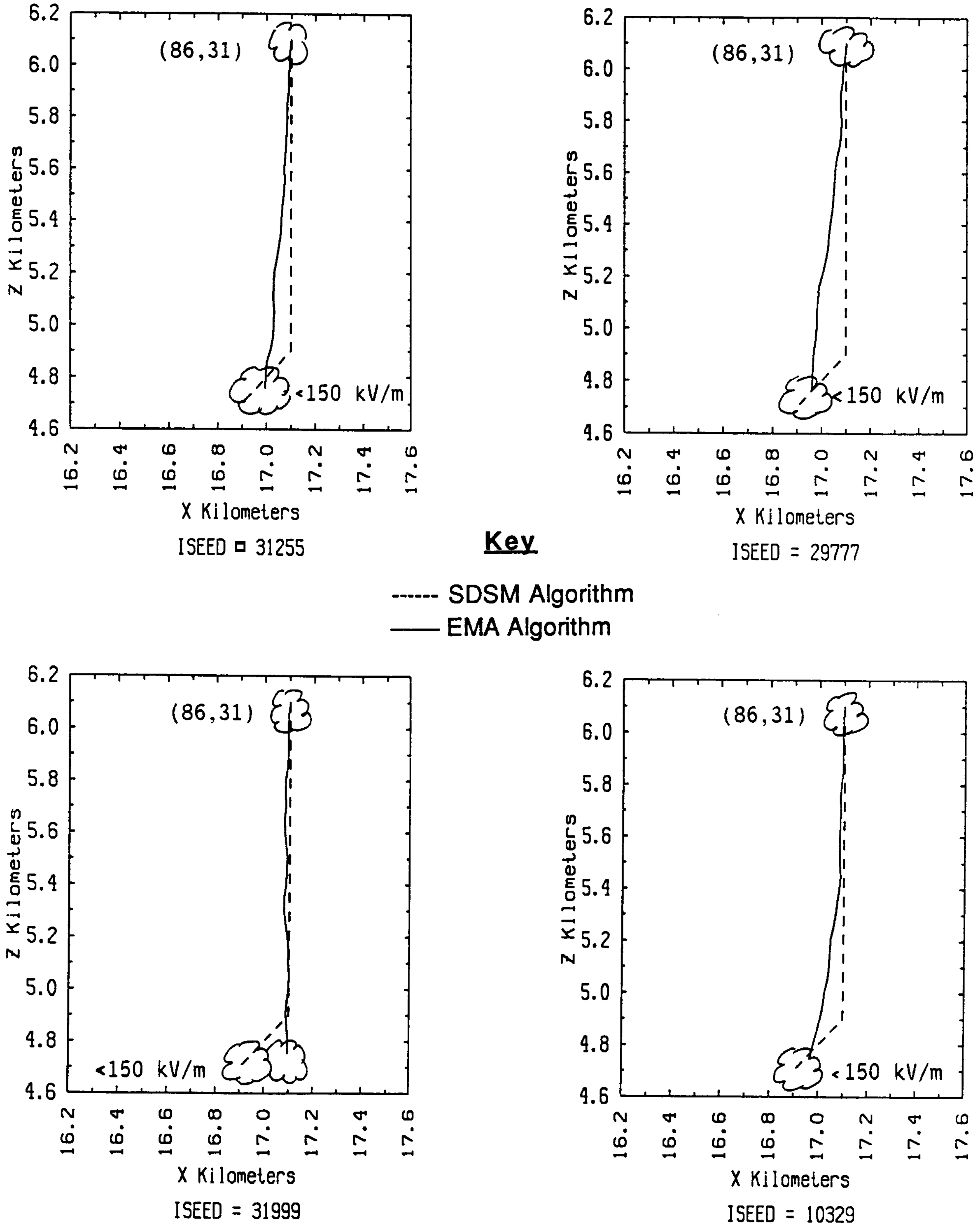


Figure 6. NCAR Electricity Model Data (58.5 Minutes) (Comparison of Monte Carlo Seeds)

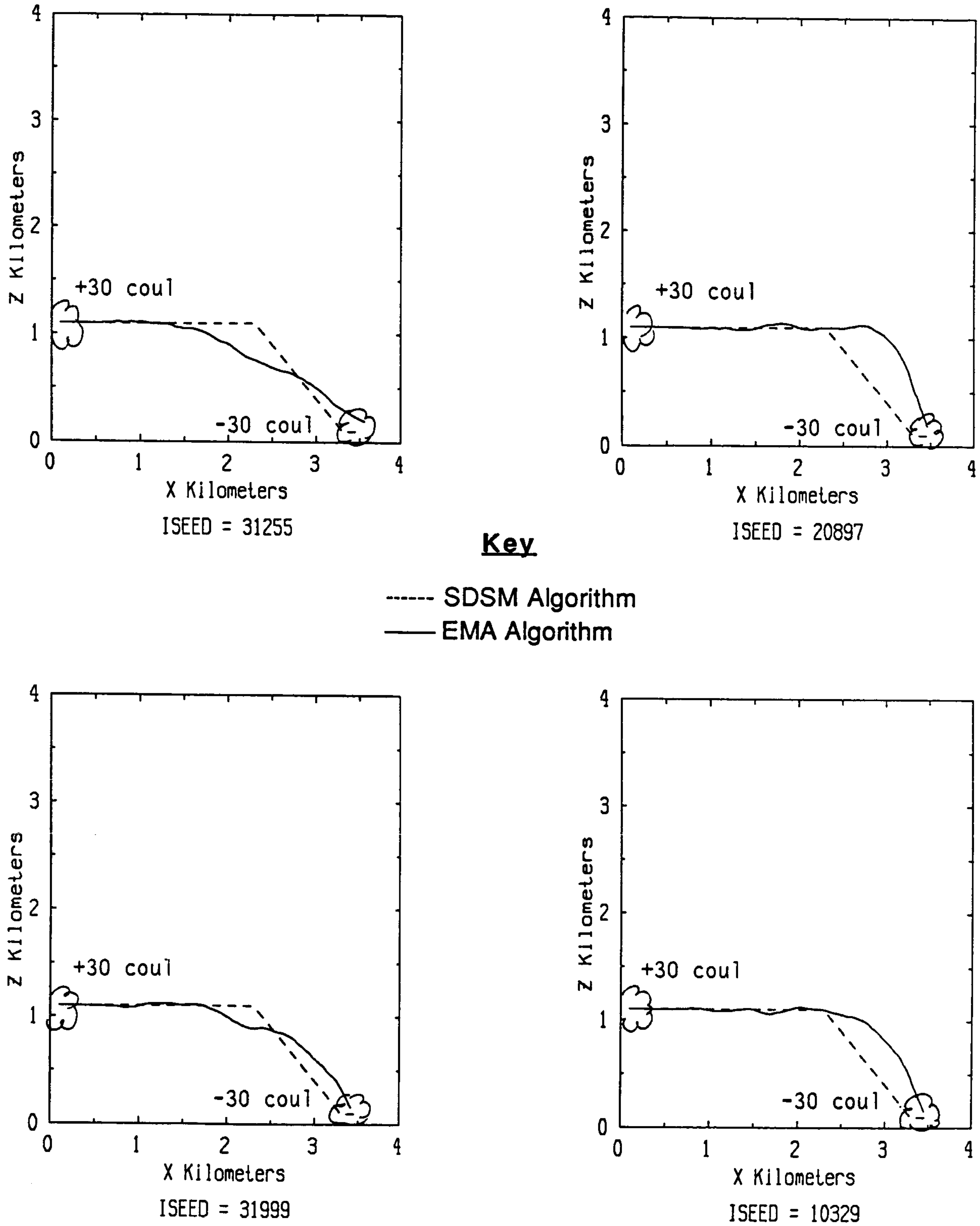


Figure 7. EMA Electricity Model Data - Initial Discharge in the Positively Charged Grid Cell (Comparison of Monte Carlo Seeds)

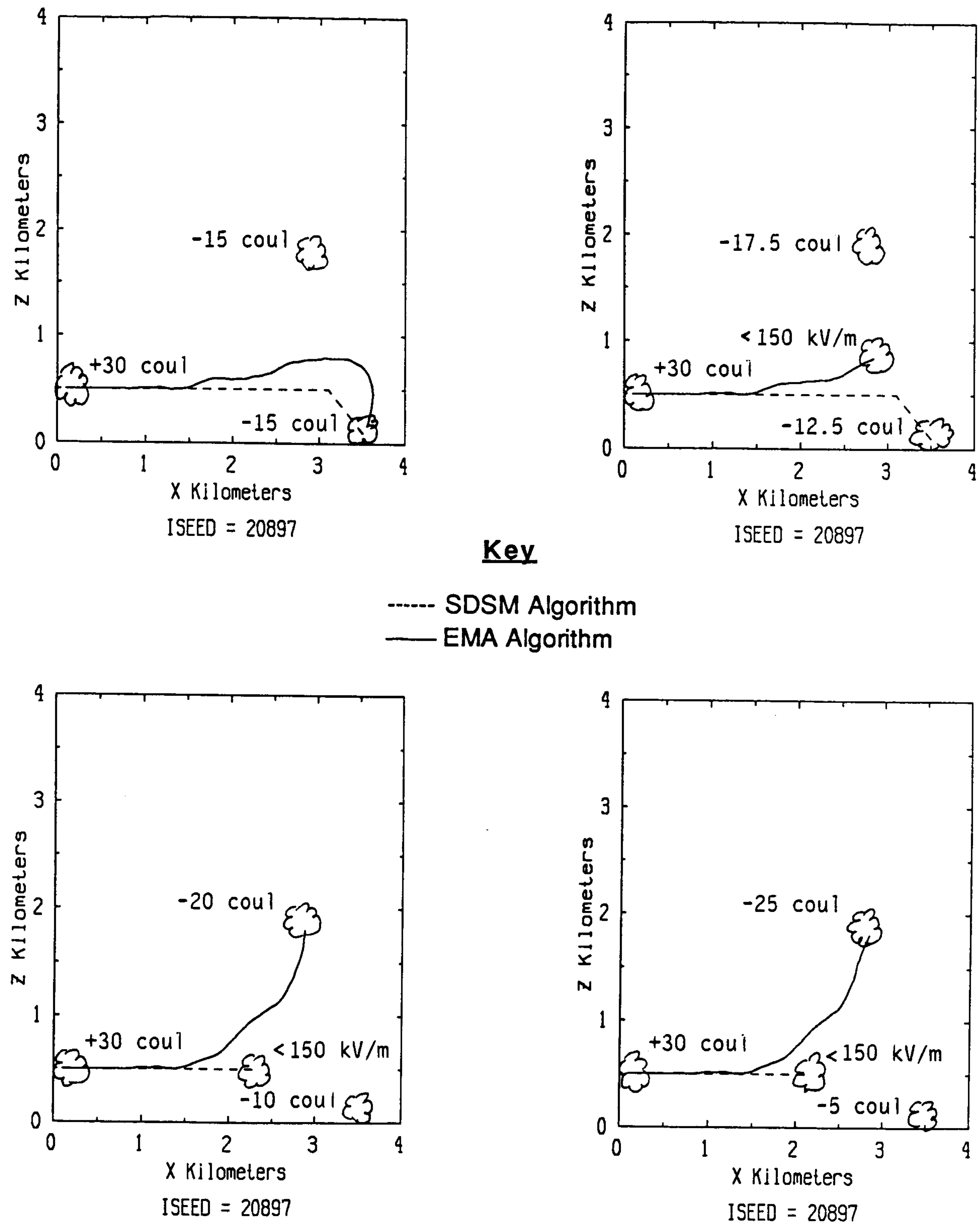


Figure 8. EMA Electricity Model Data - Initial Discharge in the Positively Charged Grid Cell (Comparison of Different Ambient Electric Fields)

produced from the LDPM and the algorithm described in Helsdon[5], respectively. The four plots in Figure 6 demonstrate the effects of varying ISEED (Monte Carlo Seed for Generating Random Electron Locations).

Figure 7 shows the effects of ISEED for ambient electric fields generated from an in-house routine. This routine creates electric fields from a set of discrete charge centers. Figure 8 shows the effects of varying the charge magnitudes at specified cells. These ambient fields were computed from the in-house routine.

4. MODEL VERSIONS AND DATA

The listings for three versions of LDPM are included in Appendix A of this report. In addition, the code used to generate ambient electric fields is included. The listings are as follows:

1. LDPM - Subroutine
   Subroutine with common statements; requires the number of x cells (NCELX), z cells (NCELZ), the initial discharge coordinate, ambient electric field components at each cell, and ISEED. Passes leader tip locations RX(I), RZ(I) and NFL, the number of total leaders back into main program.

2. LDPM.FOR
   Routine reads in ambient electric fields; requires NCELX, NCELZ, initial discharge coordinate, and ISEED. Calculates leader tip locations.

3. LDPM - Interactive (not required by contract; used as test program)
   Interactive routine; generates ambient electric field components over grid from specified cells and their charge magnitudes. Requires NCELX, NCELZ, the initial discharge coordinate, and ISEED. Routine also requires coordinates of specified charge cells. Calculates leader tip locations.

5 Helsdon, J.H., R.D. Farley and G. Wu, "A Lightning Parameterization Scheme in a Two-Dimensional, Time-Dependent Storm Electrification Model," presented at the 1986 Conference on Cloud Physics, held Sept. 22-26, 1986 in Snowmass, Colorado.

4. EFS.DT

NCAR ambient electric field data (58.5 minutes, unformatted)

FEX(K), K = 1, 9409;   FEZ(K), K = 1, 9409.

ADDITIONAL REFERENCES

1. Krider, E.P., C.D. Weidman, and R.C. Noggle, "The Electric Fields Produced by Lightning Stepped Leaders", J. Geophy. Res., 82, 1981 (pp.951-960).

2. Fleagle, R.G., J.A. Busingeil, " An Introduction to Atmospheric Physics", Academic Press, New York, 1963.

3. Golde, R.H., "Physics of Lightning - Volume 1," Academic Press, New York, 1972.

4. Hill, R.D., "Analysis of Irregular Paths of Lightning Channels," J. Geophys. Res., Vol. 73, 1968 (pp. 1897 - 1906).

APPENDIX A

LDPM CODE VERSIONS

LDPM -SUBROUTINE

```
      SUBROUTINE LDPM
C   DEVELOPED BY E.M.A. - AUGUST 1986
C   SUB-PROGRAM DETERMINES DIRECTION OF NEXT LEADER SEGMENT DUE TO
C   ELECTRON WHICH WILL FORM AN IONIZED PATH TO
C   THE STEPPED LEADER SEGMENT TIP IN THE LEAST TIME
C   E FIELD TOTAL = E FIELD LEADER + E FIELD PREVIOUS LEADER
C            + AMBIENT E FIELD
C
C   NCEL= # CELLS IN X AND IN Z
C   ICX,ICZ= RX,RZ COORD OF INITIAL CHARGE REGION
C   NFL= TOTAL NUMBER OF STEPPED LEADER SEGMENTS
      COMMON/MAIN/RX(0:500),RZ(0:500),FEX(9409),FEZ(9409)
      COMMON/VAR/NCELX,NCELZ,ICX,ICZ,NFL
      CHARACTER*40 OFILE1,OFILE2
C
C   ISEED=RANDOM FUNCTION INTEGER
      ISEED=30987
C
      OFILE1='LDPM.DAT'     ! BINARY PLOTFILE OF LIGHTNING PATH
      OFILE2='LDPM.PRINT'   ! PRINT VALUES OF LIGHTNING MODEL
C
C     INPUT VARIABLES
C   CL=SEGMENT LENGTH      ELAMBDA=LEADER CHARGE DENSITY
C   BDF=BREAK DOWN FIELD   EK=1/4PIEpso        NL=LEADER #
C   DZ=Z CELL HEIGHT       DX=X CELL WIDTH
C   NE=# FREE ELECTRONS    ED=ELECTRON DENSITY (.2/m**3)
C   Q=MAX ALLOWABLE ANGLE BTW EFIELD & DIRECTION VECTORS
C
      DATA CL,ELAMBDA,BDF/50.,.001,1.5E5/
      DATA PI,EK,NL/3.1415927,9.E9,0/
      DATA DZ,DX,NE,ED,Q/200.,200.,500,.2,12.5/
      DATA EXM1,EYM1,EZM1/3*0./
      TWOPI=2.*PI
      QP=Q*PI/180.
      CONST=EK*ELAMBDA
      VOL=FLOAT(NE)/ED
      RADIUS=(.75*VOL/PI)**(1./3.)
      KTOT=NCELX*NCELZ
      XTOT=NCELX*DX
C
C   INITIALIZE STEPPED LEADER PROPAGATION
C   INPUT DATA: R1, AMBIENT E FIELD, U1
C   COORDINATES OF INITIAL POINT ON GRID PROBLEM SPACE
      RX(0)=DX*(FLOAT(ICX)-.5)
      RY=0.  ! FOR 2D CASE;  RY(0)=DY*FLOAT(ICY)-.5), FOR 3D CASE
      RZ(0)=DZ*(FLOAT(ICZ)-.5)
C
      IC=ICX+NCELX*(ICZ-1)
C   FOR 2D CASE
      ETOT=SQRT(FEX(IC)*FEX(IC)+FEZ(IC)*FEZ(IC))
C   FOR 3D CASE
C     ETOT=SQRT(FEX(IC)*FEX(IC)+FEY(IC)*FEY(IC)+FEZ(IC)*FEZ(IC))
C   DEFINE UNIT VECTOR
      UX=FEX(IC)/ETOT
      UY=0.  ! FOR 2D CASE; UY=FEY(IC)/ETOT, FOR 3D CASE
      UZ=FEZ(IC)/ETOT
C
      OPEN(1,FILE=OFILE1,FORM='UNFORMATTED',STATUS='NEW')
      OPEN(2,FILE=OFILE2,STATUS='NEW')
C
```

```
C   START LOOP
 2000   NL=NL+1
C
C   DETERMINE CELL POSITION OF NEW LEADER SEGMENT TIP ON OVERLAY
C   USE APPROPRIATE AMBIENT E FIELD MAGNITUDE AND DIRECTION
C   K IS THE CELL NUMBER OF THE LEADER SEGMENT TIP LOCATION
        KX=ANINT((RX(NL-1)/DX)+.5)
        KZ=ANINT((RZ(NL-1)/DZ)+.5)
        K=KX+NCELX*(KZ-1)  ! NEED KY FOR 3D CASE
C   FOR 2D CASE
        EFAMB=SQRT(FEX(K)*FEX(K)+FEZ(K)*FEZ(K))
C   FOR 3D CASE
C       EFAMB=SQRT(FEX(K)*FEX(K)+FEY(K)*FEY(K)+FEZ(K)*FEZ(K))
C
C   TERMINATE IF
C   LEADER SEGMENT HAS PROPAGATED OUTSIDE THE GRID
C   LEADER SEGMENT ENTERING CELL WITH AN AMBIENT FIELD LT 1.5M V/M
        IF(K.LT.1.OR.K.GT.KTOT.OR.
     +     RX(NL-1).LT.0.OR.RX(NL-1).GT.XTOT)GOTO 3000
C
        WRITE(2,*)'LEADER SEGMENT NUMBER',NL
        WRITE(2,*)'CELL NUMBER  ',K,'     (X,Z) COORD',KX,KZ
        WRITE(2,*)'TOT E FIELD  ',ETOT,' AMB E FIELD ',EFAMB
        WRITE(2,*)'AMB EX       ',FEX(K),' AMB EZ      ',FEZ(K)
        WRITE(2,*)'X COORD      ',RX(NL-1),' Z COORD     ',RZ(NL-1)
        WRITE(2,*)'                                          '
        WRITE(1)RX(NL-1),RZ(NL-1)
C
        IF(EFAMB.LT.BDF)GOTO 3000
C
C   SHORTEST TIME OF ELECTRON TO FORM IONIZED CHANNEL
C   THIS TIME DETERMINES DIRECTION OF NEW LEADER SEGMENT
        TMIN=1.E20
C
C   USE MONTE CARLO METHOD TO GENERATE RANDOM FREE ELECTRONS
C   TO CALCULATE ELECTRON DISTRIBUTION NEAR LEADER SEGMENT TIP
        RXP=RX(NL-1)+UX*CL
        RYP=RY+UY*CL  ! USE RY(NL-1) FOR 3D CASE
        RZP=RZ(NL-1)+UZ*CL
C
        DO 100 I=1,NE
C
C   IN WORLD COORDINATES
        CALL RANDOM(ISEED,RAND)
        RE=RAND*RADIUS
C       RE=SQRT(RAND*RADIUS*RADIUS) ! FOR ACTUAL RANDOM DISTRIBUTION
        CALL RANDOM(ISEED,RAND)
        TH=RAND*PI
C       TH=ACOS((RAND*2)-1)         ! FOR ACTUAL RANDOM DISTRIBUTION
        CALL RANDOM(ISEED,RAND)
        PH=RAND*TWOPI
        RMX=RE*SIN(TH)*COS(PH)
        RMY=RE*SIN(TH)*SIN(PH)
        RMZ=RE*COS(TH)
        RM=SQRT(RMX*RMX+RMY*RMY+RMZ*RMZ)
        X=RMX+RXP
        Y=RMY+RYP
        Z=RMZ+RZP
C
C   COMPUTE E FIELD COMPONENTS FOR Ith ELECTRON
```

```
      AX=X-RX(NL-1)
      AY=Y-RY  ! USE RY(NL-1) FOR 3D CASE
      AZ=Z-RZ(NL-1)
      A=SQRT(AX*AX+AY*AY+AZ*AZ)
      IF(A.LT.1.E-6)GOTO 100
      B=AX*UX+AY*UY+AZ*UZ
      AS=A*A
      BS=B*B
      IF(AS-BS.LT.1.E-6)GOTO 100
      P=CONST/(AS-BS)
      D=CL*CL-2.*B*CL+AS
      IF(D.LE.0.)GOTO 100
      D=SQRT(D)
C
C   E FIELD COMPONENTS DUE TO LEADER SEGMENT AND AMBIENT FIELD
      EX=P*(((AX*(CL-B)+UX*(AS-B*CL))/D)-
     +    ((UX*AS-AX*B)/A))+FEX(K)
      EY=P*(((AY*(CL-B)+UY*(AS-B*CL))/D)-
     +    ((UY*AS-AY*B)/A)) !+FEY(K) FOR 3D CASE
      EZ=P*(((AZ*(CL-B)+UZ*(AS-B*CL))/D)-
     +    ((UZ*AS-AZ*B)/A))+FEZ(K)
C
      IF(NL.LE.1)GOTO 1000
      AXM1=X-RX(NL-2)
      AYM1=Y-RYM1  ! FOR 2D CASE;  RYM1=RY(NL-2), FOR 3D CASE
      AZM1=Z-RZ(NL-2)
      AM1=SQRT(AXM1*AXM1+AYM1*AYM1+AZM1*AZM1)
      IF(AM1.LT.1.E-6)GOTO 100
      BM1=AXM1*UXM1+AYM1*UYM1+AZM1*UZM1
      ASM1=AM1*AM1
      BSM1=BM1*BM1
      IF(ASM1-BSM1.LT.1.E-6)GOTO 100
      PM1=CONST/(ASM1-BSM1)
      DM1=CL*CL-2.*BM1*CL+ASM1
      IF(DM1.LE.0.)GOTO 100
      DM1=SQRT(DM1)
C
C   E FIELD COMPONENTS DUE TO PREVIOUS LEADER SEGMENT
      EXM1=PM1*(((AXM1*(CL-BM1)+UXM1*(ASM1-BM1*CL))/DM1)-
     +    ((UXM1*ASM1-AXM1*BM1)/AM1))
      EYM1=PM1*(((AYM1*(CL-BM1)+UYM1*(ASM1-BM1*CL))/DM1)-
     +    ((UYM1*ASM1-AYM1*BM1)/AM1))
      EZM1=PM1*(((AZM1*(CL-BM1)+UZM1*(ASM1-BM1*CL))/DM1)-
     +    ((UZM1*ASM1-AZM1*BM1)/AM1))
C
C   TOTAL E FIELD COMPONENTS FOR Ith ELECTRON
 1000 EX=EX+EXM1
      EY=EY+EYM1
      EZ=EZ+EZM1
      ET=SQRT(EX*EX+EY*EY+EZ*EZ)+.01
      DL=(RMX*EX+RMY*EY+RMZ*EZ)/(RM*ET)
C
C   CHOOSE APPROPRIATE ELECTRON TO FORM IONIZED CHANNEL
C   TO THE LEADER SEGMENT TIP IN SHORTEST TIME
      IF(ET.LT.(3.0E6).AND.(RM/ET).LT.TMIN.
     + AND.ACOS(ABS(DL)).LE.QP)THEN
      TMIN=RM/ET
      ETOT=ET
      DIRX=EX
      DIRY=EY
```

```
      DIRZ=EZ
      END IF
 100  CONTINUE
C
C   CALCULATE DIRECTION FOR NEXT LEADER SEGMENT
      DRX=CL*UX
      DRY=CL*UY
      DRZ=CL*UZ
      RYM1=RY
      RY=RY+DRY   ! FOR 2D CASE, RY(NL)=RY(NL-1)+DRY, FOR 3D CASE
      RX(NL)=RX(NL-1)+DRX
      RZ(NL)=RZ(NL-1)+DRZ
C
C   NEW UNIT VECTOR
C   UNIT VECTOR IS PROJECTED ONTO X,Z PLANE FOR 2D SPACE
C   FOR 3D CASE
C     DIR=SQRT(DIRX*DIRX+DIRY*DIRY+DIRZ*DIRZ)
C   FOR 2D CASE
      DIR=SQRT(DIRX*DIRX+DIRZ*DIRZ)
      IF(DIR.GE.1.E-6)THEN
      UXM1=UX
      UYM1=UY
      UZM1=UZ
      UX=DIRX/DIR
      UY=0.   ! FOR 2D CASE;  UY=DIRY/DIR, FOR 3D CASE
      UZ=DIRZ/DIR
      END IF
C
      GO TO 2000
C
 3000 NFL=NL
      IF(K.LT.1.OR.K.GT.KTOT.OR.
     +    RX(NL-1).LT.0.OR.RX(NL-1).GT.XTOT)THEN
      WRITE(2,*)'LEADER OUT OF BOUNDS'
      END IF
      IF(EFAMB.LT.BDF)THEN
      WRITE(2,*)'LEADER SEGMENT ENTERED REGION LT BREAK DOWN'
      END IF
C
      CLOSE(1)
      CLOSE(2)
      RETURN
      END
C
C
      SUBROUTINE RANDOM(ISEED,RAND)
C   RANDOM NUMBER GENERATOR
C
      DOUBLE PRECISION E31,E32,SI,SJ
      E31=2.**31
      E32=2.**32
      SI=DFLOAT(ISEED)
      SJ=MOD(69069.*SI+1.,E32)
      IF(ABS(SJ).LT.E31)THEN
      SI=SJ
      ELSEIF(SJ.GE.E31)THEN
      SI=SJ-E32
      ELSE
      SI=SJ+E32
      END IF
```

```
IF(SI.GE.0.)THEN
RAND=SI/E32
ELSE
RAND=1.+SI/E32
END IF
ISEED=INT(SI)
RETURN
END
```

LDPM.FOR

```
C   DEVELOPED BY E.M.A. -  AUGUST 1986
C   PROGRAM DETERMINES DIRECTION OF NEXT LEADER SEGMENT DUE TO
C   ELECTRON WHICH WILL FORM AN IONIZED PATH TO THE SEGMENT TIP IN
C   THE LEAST TIME
C   E FIELD TOTAL = E FIELD SEGMENT + E FIELD PREVIOUS SEGMENT
C            + AMBIENT E FIELD
C
C   NCEL= # CELLS IN X AND IN Z
C   ICX,ICZ= X,Y COORD OF INITIAL CHARGE REGION
      PARAMETER(NCELX=97,NCELZ=97,ICX=86,ICZ=31)
      DIMENSION FEX(9409),FEZ(9409)
      CHARACTER*40 INFILE,OFILE1,OFILE3
C
C   ISEED=RANDOM FUNCTION INTEGER
      ISEED=20897
C   SPECIFIED CHARGED REGION CELLS (OTHER THAN INITIAL CHARGED CELL)
      DATA NC1,NC2/195,18/
C
      INFILE='EFS.DT'        ! INPUT AMBIENT E FIELDS
      OFILE1='LDPM3.DAT'     ! BINARY PLOTFILE OF LIGHTNING PATH
      OFILE3='LDPM.PRINT'    ! PRINT VALUES OF LIGHTNING MODEL
C
C     INPUT VARIABLES
C   CL=SEGMENT LENGTH      ELAMBDA= SEGMENT CHARGE DENSITY
C   BDF=BREAK DOWN FIELD  EK=1/4PIEpso         NL=LEADER SEGMENT #
C   DZ=Z CELL HEIGHT       DX=X CELL WIDTH
C   NE=# FREE ELECTRONS    ED=ELECTRON DENSITY (.2/m**3)
C   Q=MAX ALLOWABLE ANGLE BTW EFIELD & DIRECTION VECTORS
C
      DATA CL,ELAMBDA,BDF/50.,.001,1.5E5/
      DATA PI,EK,NL/3.1415927,9.E9,0/
      DATA DZ,DX,NE,ED,Q/200.,200.,500,.2,12.5/
      DATA EXM1,EYM1,EZM1/3*0./
      TWOPI=2.*PI
      QP=Q*PI/180.
      CONST=EK*ELAMBDA
      VOL=FLOAT(NE)/ED
      RADIUS=(.75*VOL/PI)**(1./3.)
      KTOT=NCELX*NCELZ
      XTOT=NCELX*DX
C
C   READ IN AMBIENT E FIELDS
      OPEN(2,FILE=INFILE,FORM='UNFORMATTED',STATUS='OLD')
      READ(2)(FEX(K),K=1,KTOT),(FEZ(K),K=1,KTOT)
      CLOSE(2)
C
C   INITIALIZE LEADER SEGMENT PROPAGATION
C   INPUT DATA: R1, AMBIENT E FIELD, U1
C   COORDINATES OF INITIAL POINT ON GRID PROBLEM SPACE
      RX=DX*(FLOAT(ICX)-.5)
      RY=0.
      RZ=DZ*(FLOAT(ICZ)-.5)
C
      IC=ICX+NCELX*(ICZ-1)
      ETOT=SQRT(FEX(IC)**2+FEZ(IC)**2)
C   DEFINE UNIT VECTOR
      UX=FEX(IC)/ETOT
      UY=0.
      UZ=FEZ(IC)/ETOT
C
```

```
      OPEN(1,FILE=OFILE1,FORM='UNFORMATTED',STATUS='NEW')
      OPEN(3,FILE=OFILE3,STATUS='NEW')
C
C   START LOOP
 2000   NL=NL+1
C
C   DETERMINE CELL POSITION OF NEW LEADER SEGMENT TIP ON OVERLAY
C   USE APPROPRIATE AMBIENT E FIELD MAGNITUDE AND DIRECTION
C   K IS THE CELL NUMBER OF THE LEADER SEGMENT TIP LOCATION
      KX=ANINT((RX/DX)+.5)
      KZ=ANINT((RZ/DZ)+.5)
      K=KX+NCELX*(KZ-1)
      EFAMB=SQRT(FEX(K)**2+FEZ(K)**2)
C
C   TERMINATE IF
C   LEADER SEGMENT HAS PROPAGATED OUTSIDE THE GRID
C   LEADER SEGMENT ENTERING CELL WITH AN AMBIENT FIELD LT 1.5M V/M
C   LEADER SEGMENT ENTERS INTO CHARGED REGION
      IF(K.LT.1.OR.K.GT.KTOT.OR.
     +   RX.LT.0.OR.RX.GT.XTOT)GOTO 3000
C
      WRITE(3,*)NL,K,KX,KZ,RX,RZ,ETOT,EFAMB,FEX(K),FEZ(K)
      PRINT*, NL,K,KX,KZ,RX,RZ,ETOT,EFAMB,FEX(K),FEZ(K)
      WRITE(1)RX,RZ
C

      IF(EFAMB.LT.BDF)GOTO 3000
C
C   SHORTEST TIME PATH OF ELECTRON TO LEADER SEGMENT TIP
C   THIS TIME PATH DETERMINES DIRECTION OF NEW LEADER SEGMENT
      TMIN=1.E20
C
C   USE MONTE CARLO METHOD FOR FREE ELECTRONS
C   TO CALCULATE ELECTRON BISTRIBUTION NEAR LEADER SEGMENT TIP
      RXP=RX+UX*CL
      RYP=RY+UY*CL
      RZP=RZ+UZ*CL
C
      DO 100 I=1,NE
C
C   IN WORLD COORDINATES
      CALL RANDOM(ISEED,RAND)
      RE=RAND*RADIUS
C     RE=SQRT(RAND*(RADIUS**2))
      CALL RANDOM(ISEED,RAND)
      TH=RAND*PI
C     TH=ACOS(RAND*2-1)
      CALL RANDOM(ISEED,RAND)
      PH=RAND*TWOPI
C     RE=RAN(ISEED)*RADIUS
C     TH=RAN(ISEED)*PI
C     PH=RAN(ISEED)*2.*PI
      RMX=RE*SIN(TH)*COS(PH)
      RMY=RE*SIN(TH)*SIN(PH)
      RMZ=RE*COS(TH)
      X=RMX+RXP
      Y=RMY+RYP
      Z=RMZ+RZP
      RM=SQRT(RMX**2+RMY**2+RMZ**2)
C
```

```
C     COMPUTE E FIELD COMPONENTS FOR Ith ELECTRON
          AX=X-RX
          AY=Y-RY
          AZ=Z-RZ
          A=SQRT(AX**2+AY**2+AZ**2)
          IF(A.LT.1.E-6)GOTO 100
          B=AX*UX+AY*UY+AZ*UZ
          AS=A**2
          BS=B**2
          IF(AS-BS.LT.1.E-6)GOTO 100
          P=CONST/(AS-BS)
          D=CL**2-2.*B*CL+AS
          IF(D.LE.0.)GOTO 100
          D=SQRT(D)
C
C     E FIELD COMPONENTS DUE TO LEADER SEGMENT AND AMBIENT FIELD
          EX=P*(((AX*(CL-B)+UX*(AS-B*CL))/D)-
       +     ((UX*AS-AX*B)/A))+FEX(K)
          EY=P*(((AY*(CL-B)+UY*(AS-B*CL))/D)-
       +     ((UY*AS-AY*B)/A)) !+FEY(K)
          EZ=P*(((AZ*(CL-B)+UZ*(AS-B*CL))/D)-
       +     ((UZ*AS-AZ*B)/A))+FEZ(K)
C
          IF(NL.LE.1)GOTO 1000
          AXM1=X-RXM1
          AYM1=Y-RYM1
          AZM1=Z-RZM1
          AM1=SQRT(AXM1**2+AYM1**2+AZM1**2)
          IF(AM1.LT.1.E-6)GOTO 100
          BM1=AXM1*UXM1+AYM1*UYM1+AZM1*UZM1
          ASM1=AM1**2
          BSM1=BM1**2
          IF(ASM1-BSM1.LT.1.E-6)GOTO 100
          PM1=CONST/(ASM1-BSM1)
          DM1=CL**2-2.*BM1*CL+ASM1
          IF(DM1.LE.0.)GOTO 100
          DM1=SQRT(DM1)
C
C     E FIELD COMPONENTS DUE TO PREVIOUS LEADER SEGMENT
          EXM1=PM1*(((AXM1*(CL-BM1)+UXM1*(ASM1-BM1*CL))/DM1)-
       +     ((UXM1*ASM1-AXM1*BM1)/AM1))
          EYM1=PM1*(((AYM1*(CL-BM1)+UYM1*(ASM1-BM1*CL))/DM1)-
       +     ((UYM1*ASM1-AYM1*BM1)/AM1))
          EZM1=PM1*(((AZM1*(CL-BM1)+UZM1*(ASM1-BM1*CL))/DM1)-
       +     ((UZM1*ASM1-AZM1*BM1)/AM1))
C
C     TOTAL E FIELD COMPONENTS FOR Ith ELECTRON
 1000     EX=EX+EXM1
          EY=EY+EYM1
          EZ=EZ+EZM1
          ET=SQRT(EX**2+EY**2+EZ**2)+.01
          DL=(RMX*EX+RMY*EY+RMZ*EZ)/(RM*ET)
C
C     CHOOSE APPROPRIATE ELECTRON TO BE DRAWN TOWARD
C     LEADER SEGMENT TIP IN SHORTEST TIME
          IF(ET.LT.(3.0E6).AND.(RM/ET).LT.TMIN.
       + AND.ACOS(ABS(DL)).LE.QP)THEN
          TMIN=RM/ET
          ETOT=ET
          DIRX=EX
```

```
      DIRY=EY
      DIRZ=EZ
      END IF
 100  CONTINUE
C
C   CALCULATE DIRECTION FOR NEXT LEADER SEGMENT
      DRX=CL*UX
      DRY=CL*UY
      DRZ=CL*UZ
      RXM1=RX
      RYM1=RY
      RZMZ=RZ
      RX=RX+DRX
      RY=RY+DRY
      RZ=RZ+DRZ
C
C   NEW UNIT VECTOR
C     DIR=SQRT(DIRX**2+DIRY**2+DIRZ**2)
      DIR=SQRT(DIRX**2+DIRZ**2)   ! PROJ'N ONTO 2D GRID
      IF(DIR.LT.1.E-6)GOTO 2000
      UXM1=UX
      UYM1=UY
      UZM1=UZ
      UX=DIRX/DIR
C     UY=DIRY/DIR
      UY=0.
      UZ=DIRZ/DIR
C
      GO TO 2000
C
 3000 IF(K.LT.1.OR.K.GT.KTOT.OR.
     +   RX.LT.0.OR.RX.GT.XTOT)THEN
      PRINT*, 'LEADER SEGMENT OUT OF BOUNDS'
      WRITE(3,*)'LEADER SEGMENT OUT OF BOUNDS'
      END IF
      IF(EFAMB.LT.BDF)THEN
      PRINT*, 'LEADER SEGMENT ENTERED REGION LT BREAK DOWN'
      WRITE(3,*)'LEADER SEGMENT ENTERED REGION LT BREAK DOWN'
      END IF
C
      CLOSE(1)
      CLOSE(3)
      STOP
      END
C
C
      SUBROUTINE RANDOM(ISEED,RAND)
C   RANDOM NUMBER GENERATOR
C
      DOUBLE PRECISION E31,E32,SI,SJ
      E31=2.**31
      E32=2.**32
      SI=DFLOAT(ISEED)
      SJ=MOD(69069.*SI+1.,E32)
      IF(ABS(SJ).LT.E31)THEN
      SI=SJ
      ELSEIF(SJ.GE.E31)THEN
      SI=SJ-E32
      ELSE
      SI=SJ+E32
```

```
END IF
IF(SI.GE.0.)THEN
RAND=SI/E32
ELSE
RAND=1.+SI/E32
END IF
ISEED=INT(SI)
RETURN
END
```

LDPM-INTERACTIVE

```
C   INTERACTIVE VERSION OF LDPM
C
C   PROGRAM DETERMINES DIRECTION OF NEXT LEADER SEGMENT DUE TO
C   ELECTRON WHICH WILL FORM AN IONIZED PATH TO
C   THE STEPPED LEADER SEGMENT TIP IN THE LEAST TIME
C   E FIELD TOTAL = E FIELD LEADER + E FIELD PREVIOUS LEADER
C           + AMBIENT E FIELD
C
      COMMON/MAIN/FEX(9409),FEZ(9409)
      COMMON/VAR/NCX(10),NCZ(10),NCK(10),QN(10)
      COMMON/DAT/NCELX,NCELZ,ICX,ICZ,NC,QP
      CHARACTER*40 INFILE,OFILE1,OFILE3
C
      OFILE1='LDPM.DAT'     ! BINARY PLOTFILE OF LIGHTNING PATH
      OFILE3='LDPM.PRINT'   ! PRINT VALUES OF LIGHTNING MODEL
C
C
      PRINT*, 'LIGHTNING PROPAGATION - TORTUOSITY MODEL'
      PRINT*, 'DEVELOPED BY E.M.A.  -   AUGUST 1986'
      PRINT*, '      '
      PRINT*, 'SPECIFY GRID INPUT DATA'
      PRINT*, '      '
      PRINT*, 'GRID SIZE - ENTER THE TOTAL NUMBER OF CELLS'
      PRINT*, 'EACH CELL IS 200m x 200m'
      PRINT*, 'ALONG THE X (HORZ AXIS), ALONG THE Z (VERT AXIS)'
      PRINT*, 'ENTER  X,Z   (MAX IS 97,97)'
      READ*, NCELX,NCELZ
C
      PRINT*, '      '
      PRINT*, 'ENTER THE INITIAL CELL COORD (X,Z) OF DISCHARGE'
      PRINT*, 'ENTER  X,Z'
      READ*, ICX,ICZ
      PRINT*, 'ENTER THE CHARGE(+,-COUL)'
      READ*, QP
C
      PRINT*, '      '
      PRINT*, 'YOU MAY SPECIFY ANY CHARGE REGION COORDINATE '
      PRINT*, 'OTHER THAN THE INITIAL DISCHARGE COORDINATE. '
      PRINT*, 'IF YOU DONT WANT THESE REGIONS ENTER  0 '
      PRINT*, 'IT IS SUGGESTED THAT OVER THE ENTIRE GRID '
      PRINT*, 'THE SUM OF THE TOTAL CHARGE = 0  '
      PRINT*, 'ENTER THE # OF CHARGE REGIONS  (MAX IS 10)'
      PRINT*, '(NOT INCLUDING THE INITIAL DISCHARGE COORDINATE)'
      READ*, NC
      IF(NC.EQ.0)GOTO 101
      PRINT*, 'ENTER THE X,Z PAIRS AND THEIR CHARGE(-,+COUL)'
      PRINT*, 'ie:  X1,Z1,CHARGE1'
      PRINT*, '     X2,Z2,CHARGE2'
      DO 10 I=1,NC
 10   READ*, NCX(I),NCZ(I),QN(I)
      DO 11 I=1,NC
 11   NCK(I)=NCX(I)+NCELX*(NCZ(I)-1)
C
 101  PRINT*, '      '
      PRINT*, 'THE PROGRAM USES A MONTE CARLO METHOD TO'
      PRINT*, 'GENERATE FREE ELECTRONS AROUND EACH STEPPED'
      PRINT*, 'LEADER TIP. AN ASSUMED DENSITY IS .2elec/m**3'
      PRINT*, '500 ELECTRONS IS RECOMMENDED'
      PRINT*, 'LESS THAN 100 WILL LIMIT TORTUOSITY'
      PRINT*, 'ENTER THE NUMBER OF ELECTRONS DESIRED'
```

```
      READ*, NE
C
      PRINT*, '      '
      PRINT*, 'THE MONTE CARLO ROUTINE REQUIRES A SEED INTEGER'
      PRINT*, 'ENTER ANY INTEGER UP TO 5 DIGITS & LT 32000.'
      READ*, ISEED
C
      PRINT*, '      '
      PRINT*, 'PROGRAM GENERATES A PRINT FILE OF THE MODEL'
      PRINT*, 'VALUES & A BINARY PLOTFILE IN AN X,Z OUTPUT'
      PRINT*, 'AMBIENT E FIELDS ARE PRINTED OUT'
      PRINT*, 'THESE FILES ARE  LDPM.PRINT, LDPM.DAT, AMB.DAT'
      PRINT*, '      '
      PRINT*, '****************************************'
      PRINT*, '      '
C
C
C   CALCULATE AMBIENT E FIELDS
      CALL EGRID
C     INPUT VARIABLES
C   CL=LEADER LENGTH      ELAMBDA=LEADER CHARGE DENSITY
C   BDF=BREAK DOWN FIELD  EK=1/4PIEpso         NL=LEADER #
C   DZ=Z CELL HEIGHT      DX=X CELL WIDTH
C   NE=# FREE ELECTRONS   ED=ELECTRON DENSITY (.2/m**3)
C   Q=MAX ALLOWABLE ANGLE BTW EFIELD & DIRECTION VECTORS
C
      DATA CL,ELAMBDA,BDF/50.,.001,1.5E5/
      DATA PI,EK,NL/3.1415927,9.E9,0/
      DATA DZ,DX,NE,ED,Q/200.,200.,500,.2,12.5/
      DATA EXM1,EYM1,EZM1/3*0./
      CONST=EK*ELAMBDA
      VOL=FLOAT(NE)/ED
      RADIUS=(.75*VOL/PI)**(1./3.)
      KTOT=NCELX*NCELZ
      XTOT=NCELX*DX
C
C   INITIALIZE LEADER PROPAGATION
C   INPUT DATA: R1, AMBIENT E FIELD, U1
C   COORDINATES OF INITIAL POINT ON GRID PROBLEM SPACE
      RX=DX*(FLOAT(ICX)-.5)
      RY=0.
      RZ=DZ*(FLOAT(ICZ)-.5)
C
      IC=ICX+NCELX*(ICZ-1)
      ETOT=SQRT(FEX(IC)**2+FEZ(IC)**2)
C   DEFINE UNIT VECTOR
      UX=FEX(IC)/ETOT
      UY=0.
      UZ=FEZ(IC)/ETOT
C
      OPEN(1,FILE=OFILE1,FORM='UNFORMATTED',STATUS='NEW')
      OPEN(3,FILE=OFILE3,STATUS='NEW')
C
C   START LOOP
 2000   NL=NL+1
C
C   DETERMINE CELL POSITION OF NEW LEADER SEGMENT TIP ON OVERLAY
C   USE APPROPRIATE AMBIENT E FIELD MAGNITUDE AND DIRECTION
C   K IS THE CELL NUMBER OF THE LEADER SEGMENT TIP LOCATION
      KX=ANINT((RX/DX)+.5)
```

```
      KZ=ANINT((RZ/DZ)+.5)
      K=KX+NCELX*(KZ-1)
      EFAMB=SQRT(FEX(K)**2+FEZ(K)**2)
C
C   TERMINATE IF
C   LEADER HAS PROPAGATED OUTSIDE THE GRID
C   LEADER ENTERING CELL WITH AN AMBIENT FIELD LT 1.5M V/M
C   LEADER ENTERS INTO CHARGED REGION
      IF(K.LT.1.OR.K.GT.KTOT.OR.
     +   RX.LT.0.OR.RX.GT.XTOT)GOTO 3000
C
      WRITE(3,*)'LEADER SEGMENT NUMBER ',NL
      WRITE(3,*)'CELL NUMBER   ',K,'     (X,Z) COORD',KX,KZ
      WRITE(3,*)'TOT E FIELD V/m',ETOT,' AMB E FIELD ',EFAMB
      WRITE(3,*)'AMB EX          ',FEX(K),' AMB EZ      ',FEZ(K)
      WRITE(3,*)'X met. from 0,0',RX,' Z met. from 0,0   ',RZ
      WRITE(3,*)'                                     '
      PRINT*, 'LEADER SEGMENT NUMBER ',NL
      PRINT*, 'CELL NUMBER   ',K,'     (X,Z) COORD',KX,KZ
      PRINT*, 'TOT E FIELD V/m',ETOT,' AMB E FIELD ',EFAMB
      PRINT*, 'AMB EX          ',FEX(K),' AMB EZ      ',FEZ(K)
      PRINT*, 'X met. from 0,0',RX,' Z met. from 0,0   ',RZ
      PRINT*, '                                     '
      WRITE(1)RX,RZ
C
      IF(EFAMB.LT.BDF)GOTO 3000
      IF(NC.EQ.0.)GOTO 55
      DO 111 II=1,NC
 111  IF(K.EQ.NCK(II))GOTO 3000
C
C   SHORTEST TIME OF ELECTRON TO FORM IONIZATION CHANNEL
C   THIS TIME DETERMINES DIRECTION OF NEW LEADER SEGMENT
 55   TMIN=1.E20
C
C   USE MONTE CARLO METHOD TO GENERATE RANDOM FREE ELECTRONS
C   CALCULATE ELECTRON POSITIONS NEAR LEADER TIP
      DO 100 I=1,NE
C
C   IN WORLD COORDINATES
      CALL RANDOM(ISEED,RAND)
      RE=RAND*RADIUS
C     RE=SQRT(RAND*(RADIUS**2))
      CALL RANDOM(ISEED,RAND)
      TH=(RAND)*PI
C     TH=ACOS((RAND*2)-1)
      CALL RANDOM(ISEED,RAND)
      PH=(RAND)*2.*PI
      X=RE*SIN(TH)*COS(PH)+RX+UX*CL
      Y=RE*SIN(TH)*SIN(PH)+RY+UY*CL
      Z=RE*COS(TH)+RZ+UZ*CL
      RMX=X-RX-UX*CL
      RMY=Y-RY-UY*CL
      RMZ=Z-RZ-UZ*CL
      RM=SQRT(RMX**2+RMY**2+RMZ**2)
C
C   COMPUTE E FIELD COMPONENTS FOR Ith ELECTRON
      AX=X-RX
      AY=Y-RY
      AZ=Z-RZ
      A=SQRT(AX**2+AY**2+AZ**2)
```

```
      IF(A.LT.1.E-6)GOTO 100
      B=AX*UX+AY*UY+AZ*UZ
      AS=A**2
      BS=B**2
      IF(AS-BS.LT.1.E-6)GOTO 100
      P=CONST/(AS-BS)
      D=CL**2-2.*B*CL+AS
      IF(D.LE.0.)GOTO 100
      D=SQRT(D)
C
C   E FIELD COMPONENTS DUE TO LEADER SEGMENT AND AMBIENT FIELD
      EX=P*(((AX*(CL-B)+UX*(AS-B*CL))/D)-
     +   ((UX*AS-AX*B)/A))+FEX(K)
      EY=P*(((AY*(CL-B)+UY*(AS-B*CL))/D)-
     +   ((UY*AS-AY*B)/A)) !+FEY(K)
      EZ=P*(((AZ*(CL-B)+UZ*(AS-B*CL))/D)-
     +   ((UZ*AS-AZ*B)/A))+FEZ(K)
C
      IF(NL.LE.1)GOTO 1000
      AXM1=X-RXM1
      AYM1=Y-RYM1
      AZM1=Z-RZM1
      AM1=SQRT(AXM1**2+AYM1**2+AZM1**2)
      IF(AM1.LT.1.E-6)GOTO 100
      BM1=AXM1*UXM1+AYM1*UYM1+AZM1*UZM1
      ASM1=AM1**2
      BSM1=BM1**2
      IF(ASM1-BSM1.LT.1.E-6)GOTO 100
      PM1=CONST/(ASM1-BSM1)
      DM1=CL**2-2.*BM1*CL+ASM1
      IF(DM1.LE.0.)GOTO 100
      DM1=SQRT(DM1)
C
C   E FIELD COMPONENTS DUE TO PREVIOUS LEADER SEGMENT
      EXM1=PM1*(((AXM1*(CL-BM1)+UXM1*(ASM1-BM1*CL))/DM1)-
     +   ((UXM1*ASM1-AXM1*BM1)/AM1))
      EYM1=PM1*(((AYM1*(CL-BM1)+UYM1*(ASM1-BM1*CL))/DM1)-
     +   ((UYM1*ASM1-AYM1*BM1)/AM1))
      EZM1=PM1*(((AZM1*(CL-BM1)+UZM1*(ASM1-BM1*CL))/DM1)-
     +   ((UZM1*ASM1-AZM1*BM1)/AM1))
C
C   TOTAL E FIELD COMPONENTS FOR Ith ELECTRON
 1000 EX=EX+EXM1
      EY=EY+EYM1
      EZ=EZ+EZM1
      ET=SQRT(EX**2+EY**2+EZ**2)+.01
      DL=(RMX*EX+RMY*EY+RMZ*EZ)/(RM*ET)
C
C   CHOOSE APPROPRIATE ELECTRON TO FORM IONIZED CHANNEL
C   TO THE LEADER TIP IN THE SHORTEST TIME
      IF(ET.LT.(3.0E6).AND.(RM/ET).LT.TMIN.
     + AND.ACOS(ABS(DL)).LE.Q*PI/180.)THEN
      TMIN=RM/ET
      ETOT=ET
      DIRX=EX
      DIRY=EY
      DIRZ=EZ
      END IF
 100  CONTINUE
C
```

```
C   CALCULATE DIRECTION FOR NEXT LEADER SEGMENT
      DRX=CL*UX
      DRY=CL*UY
      DRZ=CL*UZ
      RXM1=RX
      RYM1=RY
      RZMZ=RZ
      RX=RX+DRX
      RY=RY+DRY
      RZ=RZ+DRZ
C
C   NEW UNIT VECTOR
C     DIR=SQRT(DIRX**2+DIRY**2+DIRZ**2)
      DIR=SQRT(DIRX**2+DIRZ**2)   ! PROJ'N ONTO 2D GRID
      IF(DIR.LT.1.E-6)GOTO 2000
      UXM1=UX
      UYM1=UY
      UZM1=UZ
      UX=DIRX/DIR
C     UY=DIRY/DIR
      UY=0.
      UZ=DIRZ/DIR
C
      GO TO 2000
C
 3000 IF(K.LT.1.OR.K.GT.KTOT.OR.
     +   RX.LT.0.OR.RX.GT.XTOT)THEN
      PRINT*, 'LEADER OUT OF BOUNDS'
      WRITE(3,*)'LEADER OUT OF BOUNDS'
      END IF
      IF(EFAMB.LT.BDF)THEN
      PRINT*, 'LEADER ENTERED REGION LT BREAK DOWN'
      WRITE(3,*)'LEADER ENTERED REGION LT BREAK DOWN'
      END IF
      DO 112 II=1,NC
      IF(K.EQ.NCK(II))THEN
      PRINT*, 'LEADER ENTERED CHARGED REGION'
      WRITE(3,*)'LEADER ENTERED CHARGED REGION'
      END IF
 112  CONTINUE
C
      CLOSE(1)
      CLOSE(3)
      STOP
      END
C
C
      SUBROUTINE RANDOM(ISEED,RAND)
C   RANDOM NUMBER GENERATOR
C
      DOUBLE PRECISION E31,E32,SI,SJ
      E31=2.**31
      E32=2.**32
      SI=DFLOAT(ISEED)
      SJ=MOD(69069.*SI+1.,E32)
      IF(ABS(SJ).LT.E31)THEN
      SI=SJ
      ELSEIF(SJ.GE.E31)THEN
      SI=SJ-E32
      ELSE
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      SI=SJ+E32
      END IF
      IF(SI.GE.0.)THEN
      RAND=SI/E32
      ELSE
      RAND=1.+SI/E32
      END IF
      ISEED=INT(SI)
      RETURN
      END
C
C
      SUBROUTINE EGRID
C   CALCULATION OF AMBIENT ELECTRIC FIELDS
C
      DIMENSION X(9409),Z(9409)
      DIMENSION XMRN(5),B(5)
      COMMON/MAIN/FEX(9409),FEZ(9409)
      COMMON/VAR/NCX(10),NCZ(10),NCK(10),QN(10)
      COMMON/DAT/NCELX,NCELZ,ICX,ICZ,NC,QP
      DATA DX,DZ,DP,PI/200.,200.,9.E9,3.1415927/
C
      OPEN(4,FILE='AMB.DAT',STATUS='NEW')
C
      DO 10 J=1,NCELZ
      DO 10 I=1,NCELX
      K=I+NCELX*(J-1)
      X(K)=DX*(FLOAT(I)-.5)
      Z(K)=DZ*(FLOAT(J)-.5)
      K1=K
 10   CONTINUE
C
      LP=ICX+NCELX*(ICZ-1)
C   CALCULATE RESULTANT E FIELDS FROM CHARGE CENTERS
      DO 20 K=1,K1
      IF(NC.EQ.0)THEN
      IF(X(K).EQ.X(LP).AND.Z(K).EQ.Z(LP))GOTO 20
      END IF
      IF(NC.EQ.0)GOTO 23
      DO 333 I=1,NC
      IF(X(K).EQ.X(LP).AND.Z(K).EQ.Z(LP).OR.
     +   X(K).EQ.X(NCK(I)).AND.Z(K).EQ.Z(NCK(I)))GOTO 20
 333  CONTINUE
C
 23   XMRP=SQRT((X(K)-X(LP))**2+(Z(K)-Z(LP))**2)
      A=DP*QP/XMRP**3
      FEX(K)=0.
      FEZ(K)=0.
      IF(NC.EQ.0)GOTO 24
      DO 444 I=1,NC
      XMRN(I)=SQRT((X(K)-X(NCK(I)))**2+(Z(K)-Z(NCK(I)))**2)
      B(I)=DP*QN(I)/XMRN(I)**3
      FEX(K)=FEX(K)+B(I)*(X(K)-X(NCK(I)))
      FEZ(K)=FEZ(K)+B(I)*(Z(K)-Z(NCK(I)))
 444  CONTINUE
C
 24   FEX(K)=FEX(K)+A*(X(K)-X(LP))
      FEZ(K)=FEZ(K)+A*(Z(K)-Z(LP))
 20   CONTINUE
      FEX(LP)=FEX(LP+1)
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      FEZ(LP)=FEZ(LP+1)
      IF(NC.EQ.0)GOTO 25
      DO 555 I=1,NC
      FEX(NCK(I))=FEX(NCK(I)-1)
      FEZ(NCK(I))=FEZ(NCK(I)-1)
555   CONTINUE
25    DO 26 K=1,K1
      WRITE(4,*)K,FEX(K),FEZ(K),X(K),Z(K)
26    CONTINUE
      CLOSE(4)
      RETURN
      END
```