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GRID GENERATION ON SURFACES
IN THREE DIMENSIONS

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TECHNICAL CONTENT

Under the NAVSEA project entitled "Grid Generations on Surfaces in Three-Dimensions" and administered by NASA as grant NAG-1-617, the development of a surface grid generation algorithm was initiated. In the initial research, the basic adaptive movement technique of mean-value-relaxation (1) was extended from the viewpoint of a single coordinate grid over a surface described by a simple scalar function to that of a surface more generally defined by vector functions and covered by a collection of smoothly connected grids. Within the multiconnected assemblage, the application of control was examined in several instances. In the first case, a smooth brick-like object was formed by an exponent 5 superellipsoid and was covered by 6 coordinate patches. From an algebraically generated initial grid system, a uniform and then a curvature clustered system was generated. As a second and more complex case, the same experiment was repeated for the abrupt intersection of two such brick-like objects. The abrupt intersection was successfully treated by employing a numerical filter for the weights. This was essential when strong curvature attraction was exercised. The reason is that the grid movement molecules straddling the intersection have weights that can suddenly acquire large values from small displacements of the points: this problem comes from the appearance of an infinite curvature along a curve. The weight filtering is the resonable cure and caused coordinate curves to smoothly follow the intersection rather than to smoothly weave back and forth through the intersection in an almost random fashion. The weaving effect had been witnessed prior to the filtering. In a similar manner, a stability restriction on the clustering intensity was witnessed and was then virtually eliminated by employing a new interpolation procedure (2) for the mean value relaxation process. Moreover, under

strong curvature attraction, an interplay was observed between the number of grid points in the patch grids and the pattern of high curvature regions. The result was a loose constraint on the number of points.

To examine the effect of the generatin process a little more directly, we return to the second case where two of the brick-like objects intersected. As mentioned earlier, each such object is defined by a superellipsoid. This was done for convenience so that a simple geometry description could be given for surfaces with local high curvature regions. The superellipsoid retains the essential simplicity of the usual ellipsoid by only replacing the exponents of 2 with an arbitrary number. As that number increases in size from 2, the so called superellipsoid converges towards a rigid Cartesian box. As a consequence, high curvature regions arise and increase in the limit. In our case, the exponent of 5 is already quite curved. Altogether, the specification of two such superellipsoids that also intersect is readily given and does not involve the often typical meticulous accounting that comes from the more commonly employed patch by patch approach (eg. Coons patches). As a consequence, a ready format was provided to test the basic grid generation procedure without the peripheral problems which could arise from a more elaborate geometry discription.

As a starting point for the process, an initial grid was obtained by inward projections from a Cartesian box that entirely contained the object. Accordingly, the Cartesian grids on the six faces of the box were sent onto six corresponding coordinate patches on the object surface. This grid system inherits the grid topology of the surrounding box and is displayed in Figure 1. Because of the choice of topology, the grid has larger cells at the bulge in the center. Because of the constructional rigidity, the grid contains slope discontinuities and abrupt changes in cell size across the junctures between distinct coordinate systems.

From the initial grid, the mean value relaxation algorithm (1) with local barycentric interpolation (2) was employed within the general surface context to move the grids points into new positions on the basis of unit weights. The result was a push towards an equalization of cell areas subject to the constraint imposed by our choice of grip topology and the concurrent choice of the number of points in each coordinate system. The latter choice is, of course, restricted by the continuity conditions between distinct coordinate systems. The associate grid is displayed in Figure 2. The push towards an equal distribution of cell areas is readily observed to occur within the grid. The opposing force arising from the choice of topology is also witnessed at the bulge where larger cells still occur. However, in a local sense, the area equalization is evident about each point in the entire grid. Also evident in the entire grid is a level of smoothness that was not present in the initial grid.

To come closer to an area equalization in a global sense, the constraining topological force must be relaxed. This can be done by inserting new coordinate patches along the flat portions of the bulge. Looking down upon the flat portion, we roughly put a uniform 4 x 4 Cartesian array of macro cells upon it. This gives three macro cells in each direction. The center cell is, however, consumed by the other intersecting superellipsoid. This then leaves 8 surrounding macro cells which now represent distinct coordinate systems. It is the corner coordinate patches that then provide a relaxation of the topological constraining force.

Relative to the uniform conditions provided by the topologically constrained area equalization, the next step is to provide a means to automatically obtain curvature clustering. In our development, this occurs directly from the choice of weight functions employed in the

mean-value-relaxation procedure. By adding a curvature term to the previous weight of unity, we get a new weight for curvature clustering relative to the unity term which represents the cell area equalization force. The result from an application of curvature attraction is given in Figure 3. There, the high curvature regions are seen to be resolved. Those regions correspond to the curve of intersection and the superellipsoid edges which in the limit of infinity exponent would become the edges of corresponding certain boxes.

REFERENCES

(1) Eiseman, P.R., "Adaptive Grid Generation by Mean-Value Relaxation," <u>ASME J. of Fluids Engr.</u>, Vol. 107, 1985, pp. 477-483.

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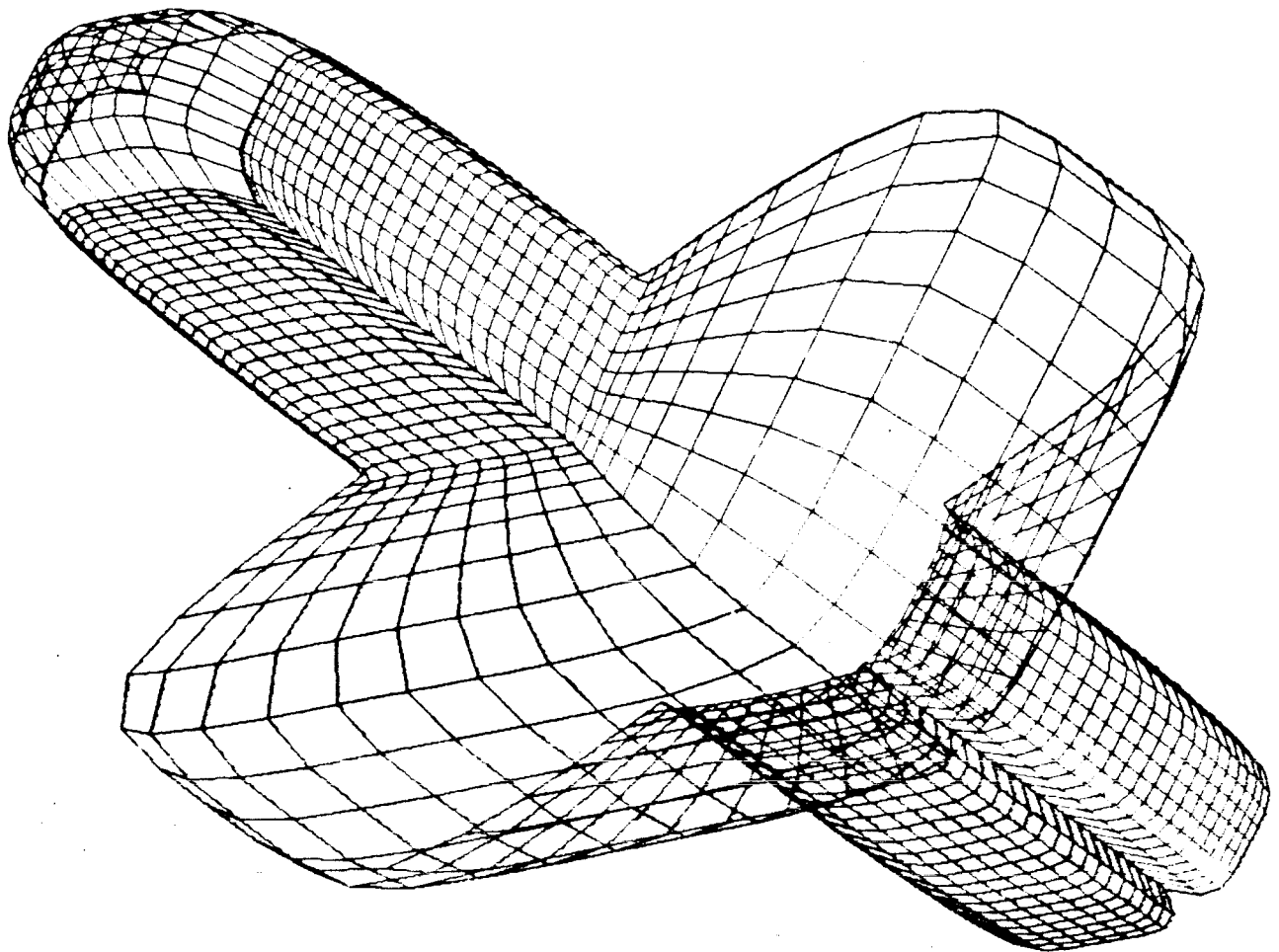
Figure 1: Initial grid constructed by projecting Cartesian grids from the faces of a surrounding Cartesian box onto the combination of intersecting superellipsoids. For simplicity, only half of the object is displayed.
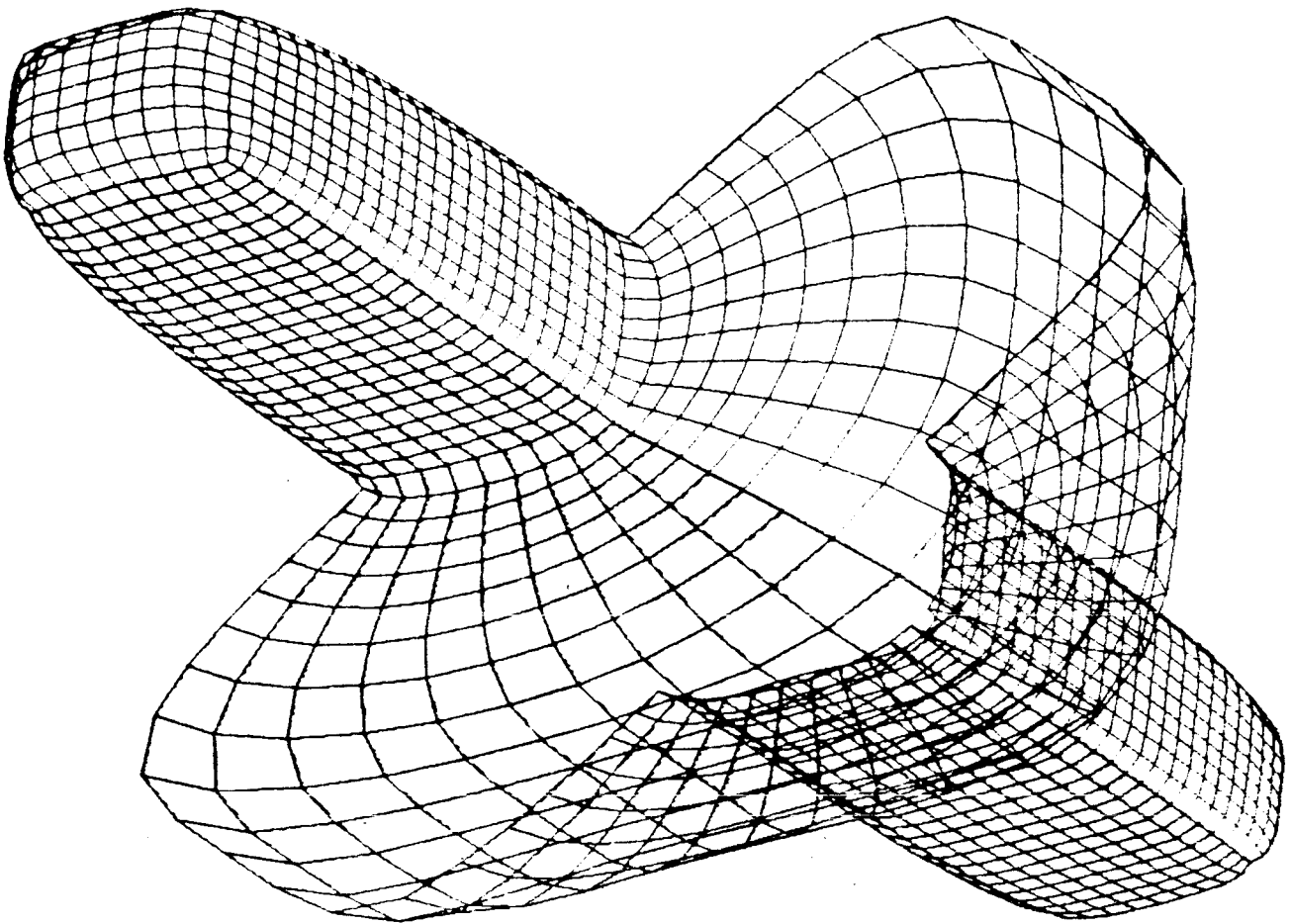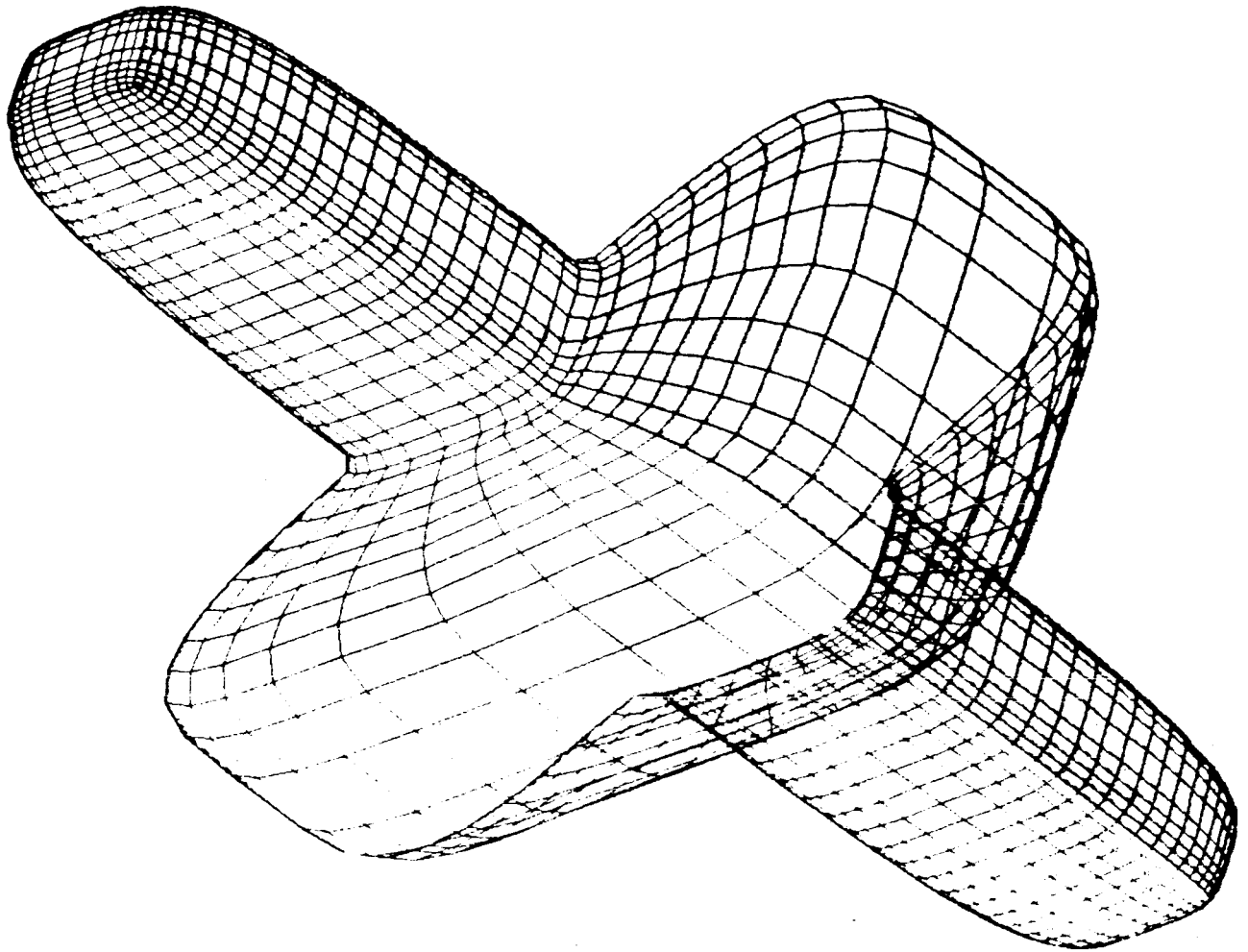
Figure 2: A uniform grid for the chosen grid topology

Figure 3:   A curvature clustered grid