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OPTIMIZATION OF AIRCRAFT TRAJECTORIES THROUGH SEVERE MICROBURSTS

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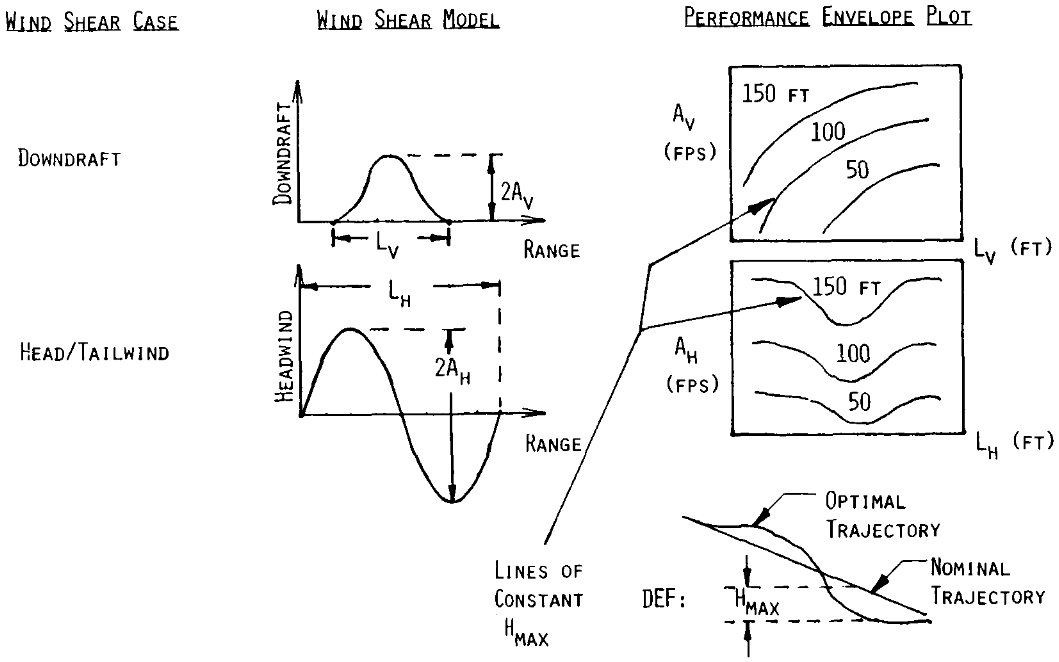
PERFORMANCE ENVELOPES, MICROBURST PENETRATION

A method of defining performance envelopes for aircraft microburst penetration is being developed. A trajectory is computed for a given aircraft/control law configuration and given microburst parameters (either a downdraft or a head/tailwind type microburst). The maximum deviation from the nominal altitude is recorded for that trajectory. Then the microburst parameters are varied, and the process is repeated. Thus a three-dimensional plot of maximum altitude deviation versus microburst range scale and intensity is generated. Finally, a certain maximum altitude deviation, say 50 feet, is defined as the safe penetration limit. Then the 50 foot level contour becomes the performance limit for safe operation as a function of microburst intensity and range.

Control inputs from deterministic trajectory optimization are used in the above described calculations to define the maximal (least conservative for a given airframe/power plant) performance limit. These limits provide targets for the designer of practical control laws. A practical control law with microburst performance limits near the maximal limits is a good control law from this standpoint.



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During the past four months work has been done in four areas. At first attempts were made to develop faster optimization procedures for use with the full-nonlinear-aerodynamics General Aviation (GA) model. Next, efforts were made towards generating the maximal performance envelopes for the Jet Transport model. Batch software for the IBM PC-XT was needed to do this efficiently. Each trajectory optimization takes 1 to 2 hours. The batch software allows one PC to do 10 or more optimizations overnight. The first of several iterations began by attempting to calculate maximal performance envelopes, having the optimization algorithm run into difficulties, and finding a fix for the difficulties. In the process two major changes to the optimization algorithm were incorporated. One was a switch from numerical to analytical evaluation of the sensitivities of the discrete-time optimization problem. The other eliminated the penalty function formulation of the control saturation and stall limit inequality constraints. It was replaced by a Lagrange multiplier formulation with a modified Newton's method search procedure.

- ATTEMPT TO SPEED UP NAVION MODEL OPTIMIZATION
- GENERATION OF MULTI-OPTIMIZATION BATCH SOFTWARE
- ATTEMPTS TO CALCULATE BOEING 727 MAXIMAL PERFORMANCE ENVELOPES
- OPTIMIZATION PROCEDURE IMPROVEMENTS

    ANALYTIC DERIVATIVES OF DISCRETE-TIME SYSTEM

    LAGRANGE MULTIPLIER FORMULATION OF INEQUALITY CONSTRAINTS.
    EXTENSION OF NEWTON'S METHOD

ANALYTIC DERIVATIVES OF DISCRETE-TIME SYSTEM

The problem was transformed into discrete-time form in order to solve the deterministic trajectory optimization on a digital computer. The discrete-time nonlinear difference equation was generated from the continuous-time differential equation by numerically solving the initial value problem (I.V.P.) shown under the definition of the difference equation's right hand side (RHS).

First and second derivatives of the RHS of the difference equation and the cost function are needed in order to do trajectory optimization using Newton's method. Previously these derivatives had been calculated numerically using a three-point scheme. When microburst intensities got large, problems cropped up in the convergence of the optimization algorithm near the solution. A simple example showed that roundoff error in numerical differentiation could be the cause. When analytic derivatives were substituted the problem went away.

The analytic derivatives of the nonlinear difference equation were generated as follows: The RHS of the difference equation is defined as the solution of an I.V.P. At each instant of time the solution to the I.V.P. depends on the arguments of the RHS of the difference equation. Differentiation of the I.V.P. -- both initial conditions and differential equation -- any number of times with respect to the arguments of the difference equation yields a new I.V.P. This new I.V.P. might be a matrix or a tensor I.V.P. depending on the number of differentiations. It, too, can be solved numerically to yield the corresponding derivative of the RHS of the difference equation.

- <u>DEFINITION:</u> $\underline{x}_{K+1} = \hat{\underline{E}}(\underline{x}_K, \underline{u}_K, K)$

  WHERE $\hat{\underline{E}}(\underline{x}_K, \underline{u}_K, K) = \underline{x}(T_{K+1})$, $\underline{x}(T)$ THE SOLUTION

  OF THE I.V.P.

$$\dot{\underline{x}}(T) = \underline{E}\,[\underline{x}(T),\ \underline{u}_K, T]$$
$$\underline{x}(T_K) = \underline{x}_K$$

- <u>EXAMPLE DISCRETE TIME SENSITIVITY (DIFFERENTIATE THE I.V.P.):</u>

$$\frac{\partial \hat{\underline{E}}}{\partial \underline{x}_K} = \frac{\partial \underline{x}(T_{K+1})}{\partial \underline{x}_K}\ ,\ \frac{\partial \underline{x}(T)}{\partial \underline{x}_K}\ \text{THE SOLUTION}$$

OF THE I.V.P.

$$\frac{D}{DT}\left(\frac{\partial \underline{x}(T)}{\partial \underline{x}_K}\right) = \frac{\partial \underline{E}}{\partial \underline{x}}\,[\underline{x}(T),\ \underline{u}_K, T]\,\frac{\partial \underline{x}(T)}{\partial \underline{x}_K}$$

$$\frac{\partial \underline{x}(T_K)}{\partial \underline{x}_K} = I$$

TRAJECTORY OPTIMIZATION WITH INEQUALITY CONSTRAINTS

The optimization problem with inequality constraints is one of picking a control time history, u(k) for k = 1...N-1 which minimizes the cost, J, subject to the discrete-time plant difference equation and the discrete-time inequality constraints. Inequality constraints can be used to deal with control saturation and to limit the angle of attack to be below stall. Proper handling of control saturation and the stall limit is essential to maximal performance envelope determination.

This optimization problem can be solved by the penalty function method. A penalty cost is added to L whenever one of the inequality constraints is violated. With this modified cost function, an unconstrained optimal trajectory is determined. As the penalty is increased, the solution approaches that of the original problem. In practice there is a limit to how fast the penalty cost can be increased. Many unconstrained optima must be computed before an answer sufficiently close to the constrained optimum is attained.

Direct necessary conditions for the inequality-constrained optimum are attributed to Kuhn and Tucker *. According to these necessary conditions, the inequality constraints are ignored when the optimal solution satisfies them but does not lie on the boundary. The constraint is treated as an equality constraint when the optimal solution does lie on the boundary. Then the Lagrange multipliers must be non-negative, so that no better solution lies in the admissible region. An extension of Newton's method is used to search for a solution to the necessary conditions. These conditions are linearized about the current guess. Then a quadratic programming problem is solved to get the next guess for the optimal time histories. This procedure yields "exact" solutions to the inequality-constrained problem in as much time as is required to solve an unconstrained problem, reducing computation by a factor of three or more from the penalty function method.

The quadratic programming algorithm so far implemented to determine the extended Newton's method increment has proven to be less than fully reliable. Therefore, a new algorithm has been identified and is under development.

* Luenberger, D.G., *Optimization by Vector Space Methods,* John Wiley and Sons, (New York, 1969), pp. 247-253.

TRAJECTORY OPTIMIZATION WITH INEQUALITY CONSTRAINTS

PROBLEM: MINIMIZE $J = \sum_{K=1}^{N-1} L(\underline{x}_K, \underline{u}_K, K) + V(\underline{x}_N)$

SUBJECT TO: $\underline{x}_{K+1} = \hat{\underline{F}}(\underline{x}_K, \underline{u}_K, K) \qquad K = 1 \ldots N-1$

$\underline{x}_1$ GIVEN

$\underline{C}(\underline{x}_K, \underline{u}_K, K) \leq 0 \qquad K = 1 \ldots N-1$

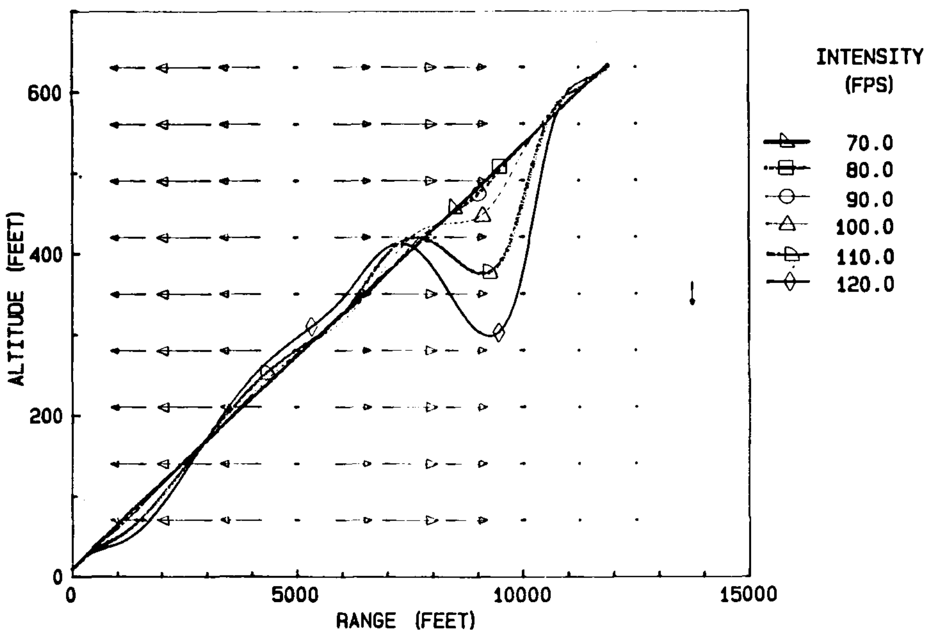
PENALTY FUNCTION METHOD: ADD COST TO $L(\underline{x}_K, \underline{u}_K, K)$ IF CONSTRAINT IS VIOLATED

LAGRANGE MULTIPLIER METHOD: TREAT INEQUALITY CONSTRAINT AS EQUALITY CONSTRAINT IF BETTER SOLUTION VIOLATES CONSTRAINT, OTHERWISE IGNORE CONSTRAINT.

OPTIMAL TRAJECTORIES THROUGH SEVERAL MICROBURSTS

Six of the many optimal trajectories so far calculated are displayed on this graph. These six are all Jet Transport take-off trajectories through 10,000 ft. microbursts. The engineering approximation microbursts have only longitudinal winds: the head/tailwind varies as one period of 10,000 ft. sine wave. The arrows represent the relative wind magnitudes and directions. The six different curves represent optimal trajectories for the same cost function (a cost function which weights altitude deviations from the nominal most highly) and six different wind intensities. At the highest intensity, 120 fps maximum head/tailwind (240 fps longitudinal wind differential in 5,000 ft)., the maximum altitude deviation reaches 200 ft. Appreciable maximum altitude deviations (greater than 40 ft.) occur at microburst intensities above 100 fps maximum head/tailwind.

SIX DIFFERENT SINUSOIDAL HEAD/TAILWIND INTENSITIES



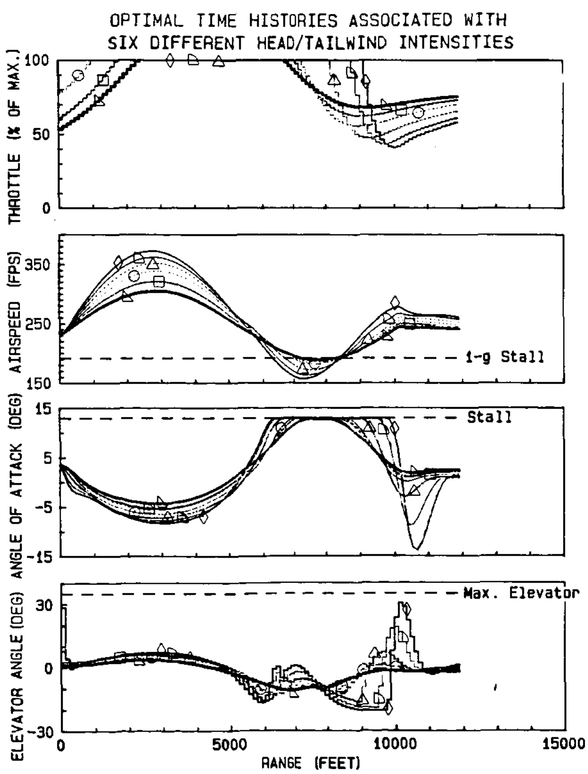
TAKE-OFF THROUGH SEVERAL MICROBURSTS

Time (range) histories of throttle setting, airspeed, angle of attack and elevator angle are shown. Several features are noteworthy:

1. Full throttle is not commanded until some time after the beginning of the optimization for the three microbursts of lowest intensity. Full throttle is commanded immediately upon initiation of the optimization for the other three cases. Maximum altitude deviation is significant in the latter three cases. The optimization would have anticipated the microburst by commanding full throttle even earlier if it could have done so. Thus, the degree to which any controller can anticipate the airspeed loss due to a microburst directly relates to the degree of flight path tracking.

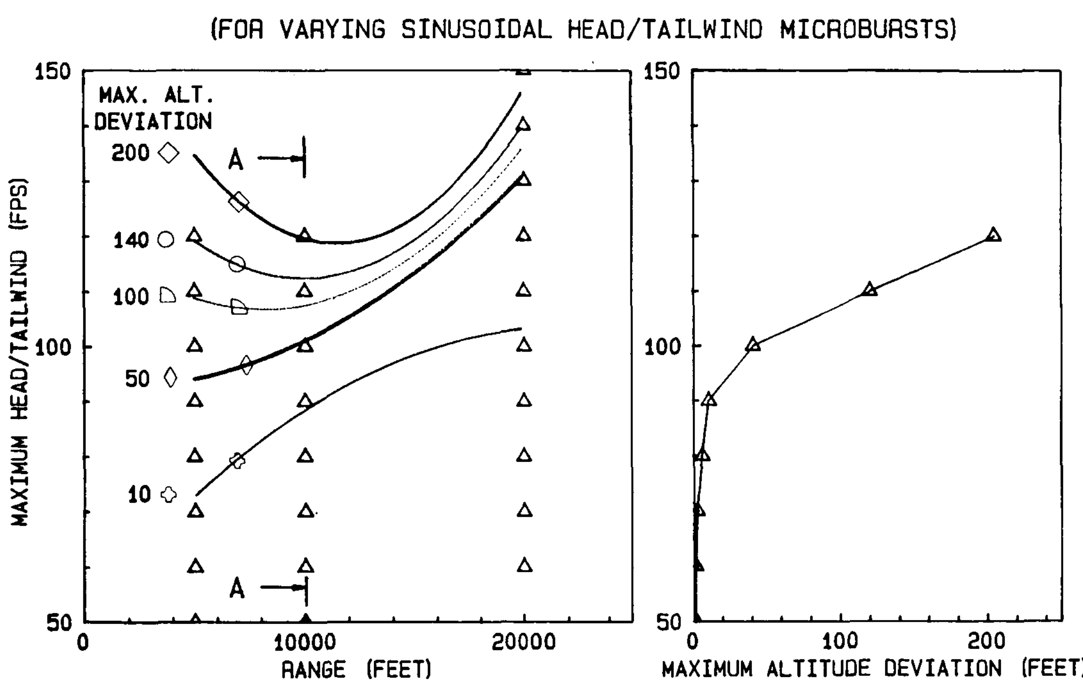
2. In those cases where airspeed fell below the 1-g stall limit, the angle of attack was raised to the stall boundary at approximately the same time. Subsequent regaining of airspeed margin above stall, however, did not dictate coincident lowering of angle of attack below the stall. Rather, the angle of attack remained at its stall limit for some time after sufficient airspeed was regained for trim at the nominal flight path angle. This was necessary to regain altitude lost during the period of airspeed deficiency. The fact that stall angle of attack did not precede stall airspeed indicates little or no microburst anticipation in the optimal pitch steering scheme. Therefore, near optimal, practical (causal) pitch steering may be possible.

3. The plots of elevator angle indicate that constant stall angle of attack can only be maintained through a varying command. The pilot has to manuever to stay at this angle of attack.



TAKE-OFF TRAJECTORY TRACKING PERFORMANCE OF A JET TRANSPORT

The plot to the left on this figure shows actual results for maximal microburst penetration performance during take-off of a Boeing-727. The microbursts under consideration were the head/tailwind sine wave type. Each triangle on the plot indicates an actual data point: for each triangle, an optimization was run through a sinusoidal head/tailwind microburst of the corresponding range and intensity, and the maximal altitude deviation from nominal was recorded. Contours of equal maximum altitude deviation were then generated via linear interpolation (extrapolation in one case) in the microburst intensity direction and parabolic curve fitting in the microburst range direction. The minima of the three upper contours occurred at microburst range scales near that of the open-loop phugoid mode. If the plot on the left were a three dimensional (3D) plot of maximal altitude deviation (positive out of the paper) versus microburst range and intensity, then the plot to the right would be the cross section of the 3D plot taken at the microburst range 10,000 ft. The "knee" in this curve corresponds to the microburst intensity where full throttle is commanded at the beginning of optimization.



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CONCLUSIONS

The improvements to the nonlinear deterministic trajectory optimization algorithm were significant. The use of analytic derivatives solved some numerical problems. The extension of Newton's method to handle inequality constraints may represent an important contribution to the general field of deterministic trajectory optimization. In any case, it contributed greatly to the feasibility of doing the performance envelope calculations at hand, reducing computation time by a factor of three and higher. What would have taken 3 hours on the IBM 3081 a year ago now takes 1.5 hours on an IBM PC-XT.

Examination of optimal trajectories through extreme microbursts and the strategies employed to achieve them shed light on the control problem. A relationship was found between anticipatory throttle activity and the ultimate maximum altitude deviation. Therefore, the ultimate performance of an optimized trajectory will depend significantly upon the point at which one chooses to begin optimization. In order to normalize for this effect, we chose to begin our optimizations at the leading edge of each microburst. The optimal angle of attack strategy showed a close correspondence to the optimal airspeed policy with little anticipation, so it seems feasible to develop a controller that would vary angle of attack to minimize the effect of airspeed variations upon lift up to the stall saturation limit, at which point it would hold the stall angle of attack until the aircraft was well on the way to recovery.

- TRAJECTORY OPTIMIZATION ALGORITHM

  UNIQUENESS AND EFFECTIVENESS OF NEWTON'S METHOD APPLIED TO INEQUALITY CONSTRAINTS

  FEASIBILITY OF MAXIMAL PERFORMANCE ENVELOPE CALCULATIONS

- OPTIMAL MICROBURST PENETRATION

  DEPENDENCE UPON INITIAL RANGE

  ANTICIPATION THROTTLE STRATEGY

  FEASIBILITY OF ANGLE OF ATTACK STRATEGY

PLANNED FUTURE WORK

The first task is to finish development of the new optimization procedure, especially the quadratic programming portion of the extended Newton's method. Hildreth's quadratic programming procedure * adapted to use the Conjugate Gradient method should yield an efficient and robust algorithm. The GA model still needs to be worked in with the new optimization procedure in order to calculate GA performance envelopes. As software is developed performance envelopes will be calculated for downdraft type and head/tailwind type microbursts, landing and take-off flight phases, and Jet Transport and General Aviation type aircraft. As time permits, practical control laws will be developed which approach the performance of deterministic optimal microburst penetration.

* Luenberger, D.G., *Optimization by Vector Space Methods*, John Wiley and Sons, (New York, 1969), pp. 299-300.

- OPTIMIZATION ALGORITHM

  PERFECT QUADRATIC PROGRAMMING PORTION OF NEWTON'S METHOD

  GET NAVION MODEL WORKING

- PERFORMANCE ENVELOPE CALCULATIONS

  BOEING 727 AND NAVION AIRCRAFT

  LANDING AND TAKE-OFF PHASES

  HEAD/TAIL WIND AND DOWNDRAFT TYPE MICROBURSTS

- PRACTICAL CONTROL LAW DESIGN