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EFFECTS OF JOINT RATE AND DISPLACEMENT CONSTRAINTS ON STABILITY REGIONS

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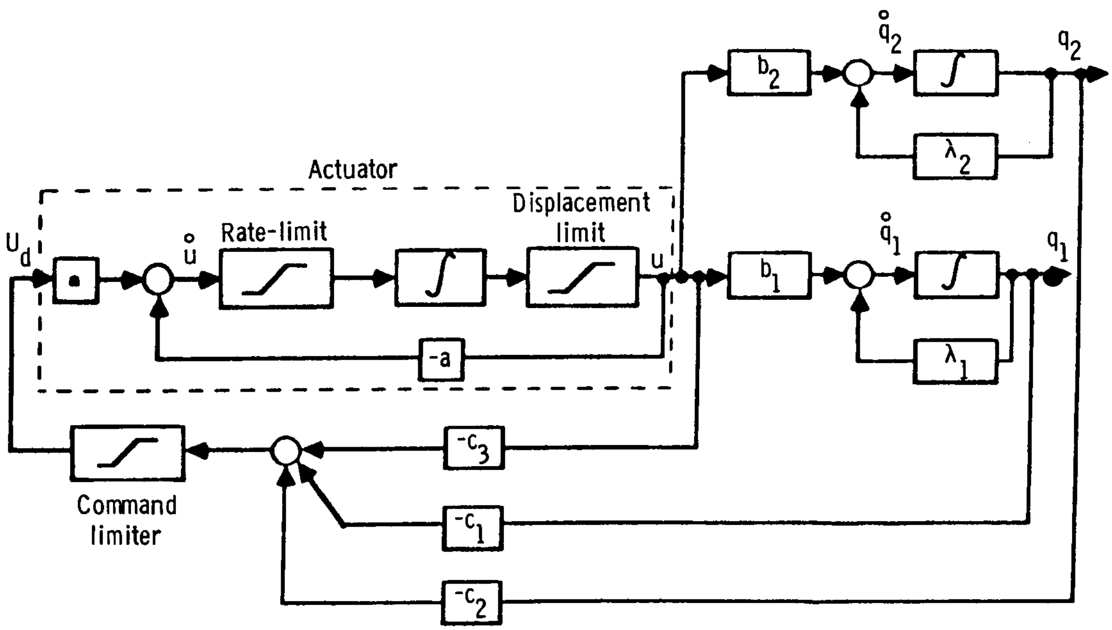
This block diagram shows the plant dynamics in normal mode coordinates, a linear feedback controller with command limits, and the actuator dynamics with rate and displacement limits. The objective is to examine the effects of joint rate and displacement saturation limits on the stability regions.

SCHEMATIC BLOCK-DIAGRAM



The unstable short-period dynamic model is transformed into normal mode coordinates, and it is augmented with actuator dynamics. The $\underline{q}$ mode is considered as the unstable mode. A linear feedback controller is used to provide closed-loop stability. We examine the stability boundaries with constrained actuator rate limits under varying bandwidth, displacement and command limits. The stability boundaries are unstable limit cycles in a phase-plane plot whose axes are the unstable mode and control deflection.

PROBLEM FORMULATION

- SHORT-PERIOD DYNAMICS

$$\dot{\alpha} = -L_\alpha \alpha + (1 - L_q/V_o)q$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta e}\delta e$$

- ACTUATOR DYNAMICS

$$\dot{u} = -au + au_d$$

- AUGMENTED SYSTEM DYNAMICS (NORMAL MODE)

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{u} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & b_1 \\ 0 & \lambda_2 & b_2 \\ 0 & 0 & -a \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix} u_d$$

- LINEAR CONTROLLER

$$u_d = -c_1 q_1 - c_2 q_2 - c_3 u$$

The closed-loop stability by feedback of only the unstable mode requires the root magnitude to be greater than that of the unstable eigenvalues. In other words, the control bandwidth must be high enough to stabilize the system. This requirement is relaxed when the unstable mode and the control deflection are fed back because the effective actuator bandwidth is increased. Furthermore, feedback of control deflection also permits reduction in the feedback gain of the unstable mode.

CLOSED-LOOP DYNAMICS

- FEEDBACK OF UNSTABLE MODE (q_1 - MODE) [CONTROL LAW 1]
  - CHARACTERISTIC EQUATION:

$$(S - \lambda_2)\left[S^2 + (a - \lambda_1)\,S + (ab_1C_1 - a\lambda_1)\right] = 0$$

  - STABILITY CONDITIONS:

$$C_1 > \lambda_1/b_1$$

$$a > \lambda_1$$

- FEEDBACK OF UNSTABLE MODE AND CONTROL DEFLECTION (q_1 AND u) [CONTROL LAW 2]
  - CHARACTERISTIC EQUATION:

$$(S - \lambda_2)\left[S^2 + (a + aC_3 - \lambda_1)\,S + a(b_1C_1 - C_3\lambda_1 - \lambda_1)\right] = 0$$

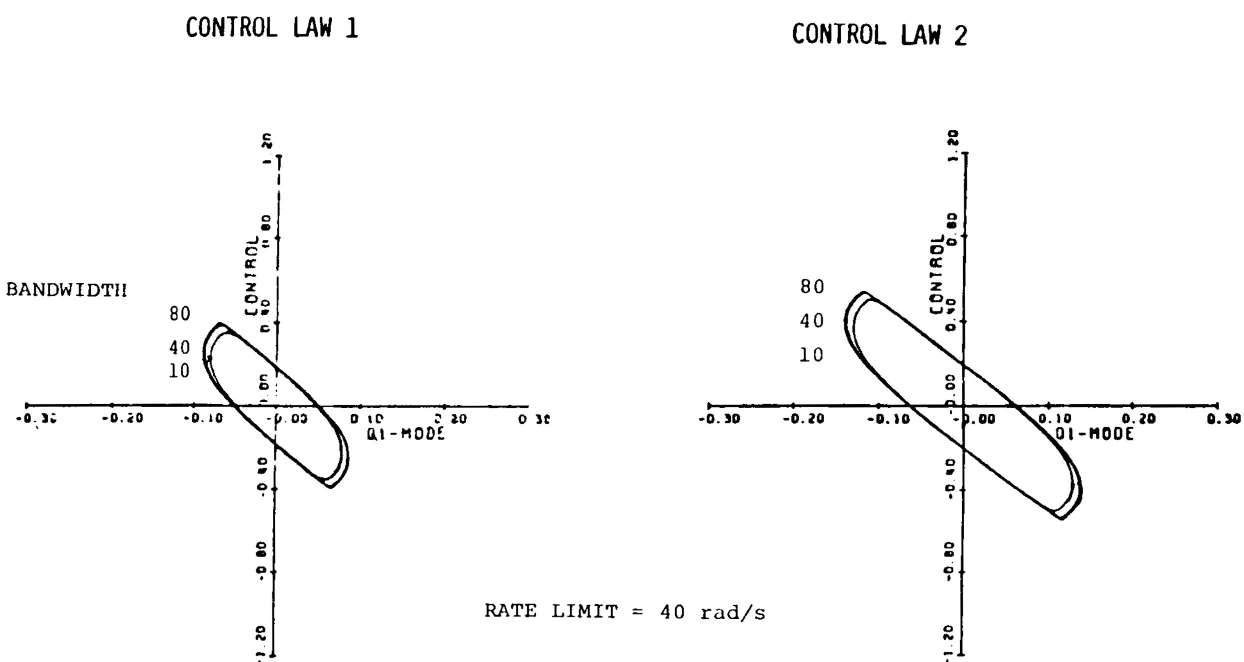
  - STABILITY CONDITIONS

$$a(1 + C_3) > \lambda_1 \Rightarrow a > \frac{\lambda_1}{(1+C_3)}$$

$$b_1C_1 > \lambda_1(1 + C_3) \Rightarrow b_1C_1 > \lambda_1^2/a \qquad \text{SIGN OF } C_3?$$

The increase in the sizes of stability regions is not proportional to the bandwidth. At high bandwidths, the sizes of stability regions become almost independent of the bandwidth because the actuator is almost always rate-saturated. Feedback of control deflection permits increased region of rate-unsaturated operation due to reduced feedback gain requirement on the unstable mode. This helps increase the size of stability region size.

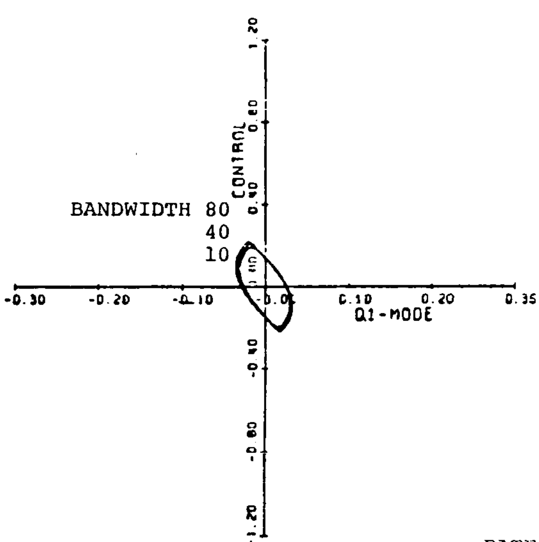
EFFECTS OF BANDWIDTH VARIATIONS (MCE GAIN)



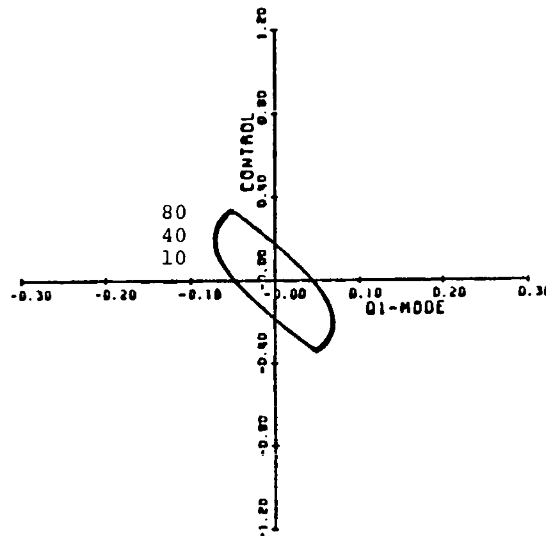
While increase in the system feedback increases the closed-loop stability, it causes early rate saturation thereby reducing the sizes of stability regions.

EFFECTS OF BANDWIDTH VARIATIONS (TWICE MCE GAIN)

CONTROL LAW 1



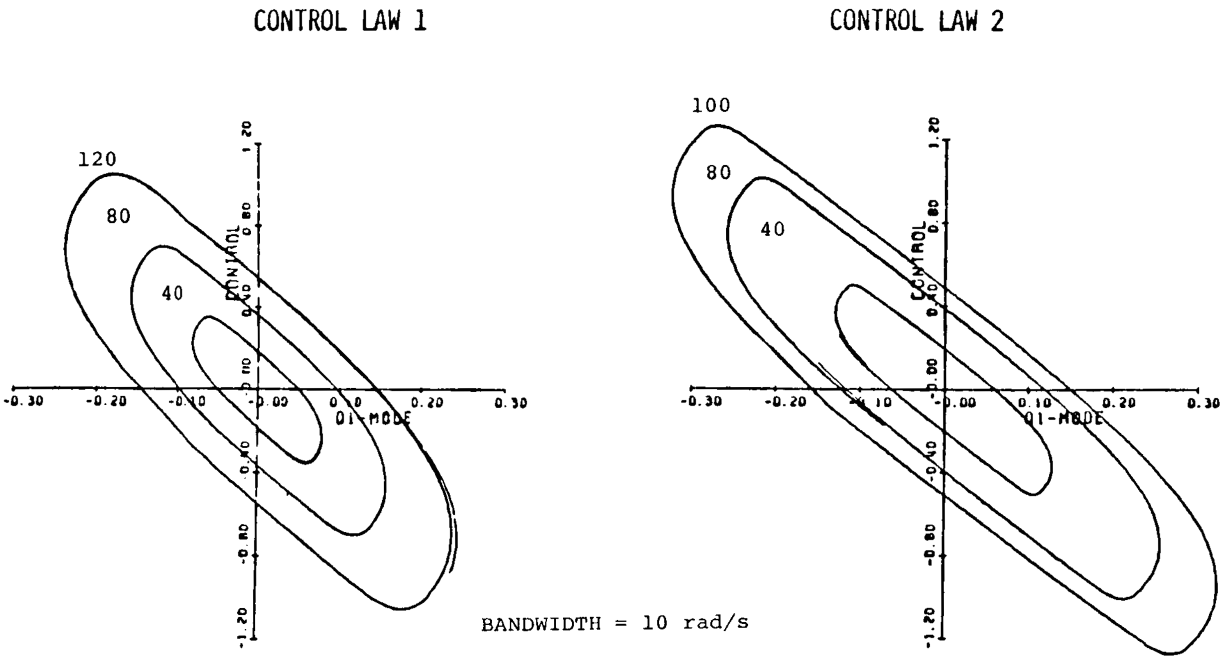
CONTROL LAW 2



RATE LIMIT = 40 rad/s

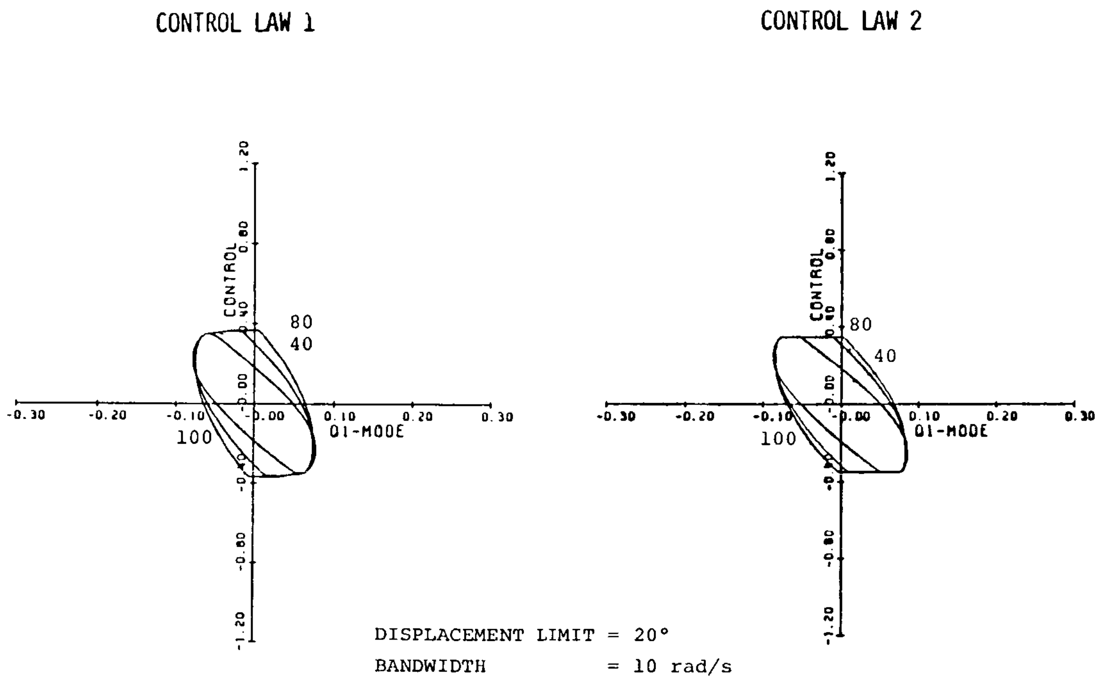
Increasing rate limits implies relaxing the constraints, and the rate saturation occurs at relatively larger values of the control and state variables. Therefore, the stability region sizes increase with rate limits.

EFFECTS OF RATE-LIMIT VARIATIONS (MCE GAIN)



Upon displacement saturation, the system is essentially open-loop and rate limit has no significance. The displacement limits restrain the area of closed-loop operation in the phase-plane, which causes drastic reduction in the sizes of stability regions. For this reason, the sizes of stability regions do not show significant size increases despite large increases in the rate limits.

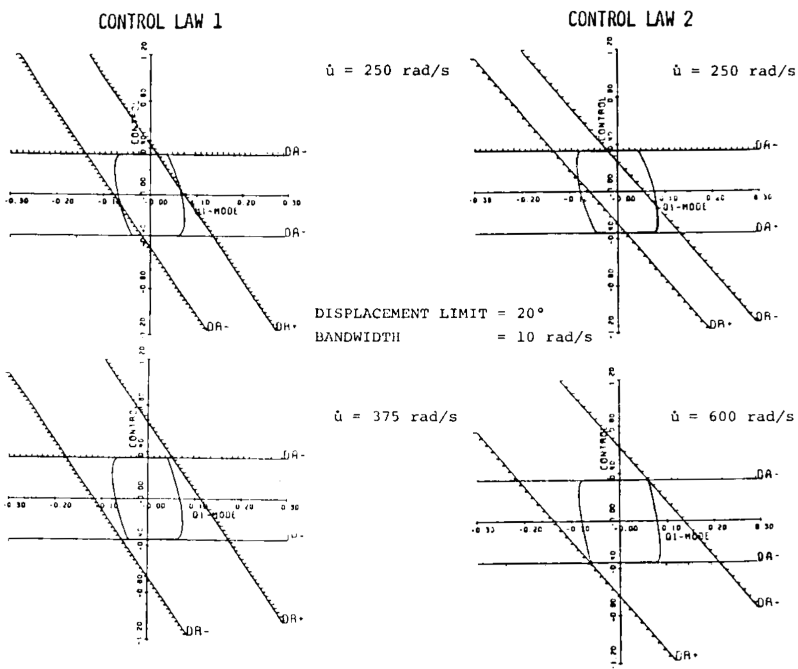
EFFECTS OF IMPOSING DISPLACEMENT LIMIT (MCE GAIN)



Marginal increases in the sizes of stability regions obtained for large increases in rate limits under displacement constraints suggest the possibility of invariance of stability regions with respect to the rate limits. Notice that a large part of the stability boundary lies within the rate-unsaturated region which is essentially sustained by the displacement limits. Increasing rate limits effectively bring in an even larger portion of the stability boundary within the rate-unsaturated region. Clearly, therefore there exists an upper bound on the rate limits beyond which the stability boundary is no longer influenced by the rate limits, as illustrated.

INVARIANCE OF STABILITY REGIONS WITH RATE LIMITS

(MCE GAIN CASE)

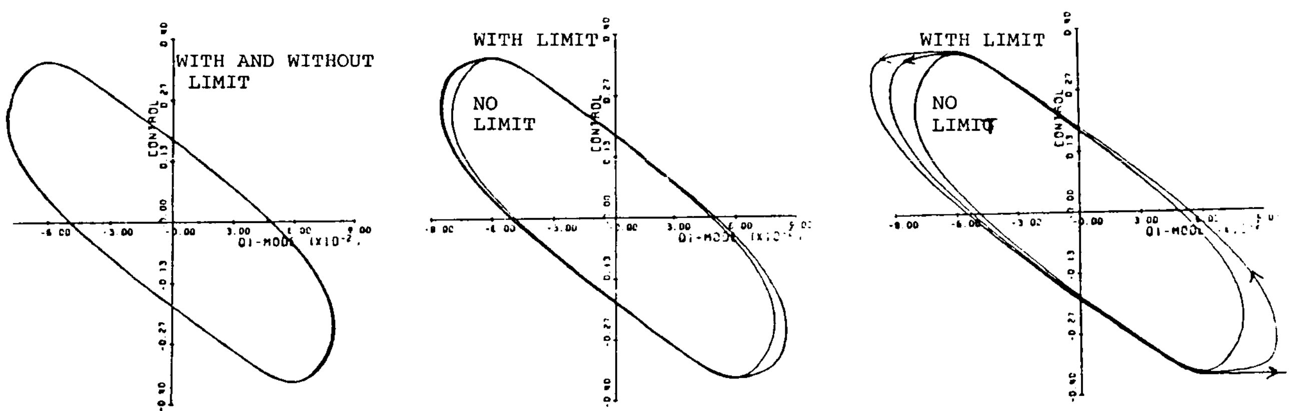


C-2

The large command limits do not affect the stability regions because the linear controller meets the demands required for closed-loop stability. Reduction in these limits deteriorates the controller capability. A small system divergence, due to the inability of the controller to meet the demanded control, causes a little expansion in the stability region; this divergence is overcome because of the reduced demand on the controller by a larger part of the stability boundary. As the command limit is reduced further, the system becomes unstable for relatively longer time placing even larger demands on the controller. The stability region vanishes because these demands cannot be satisfied by the controller.

EFFECTS OF COMMAND-LIMIT VARIATIONS (MCE GAIN)

CONTROL LAW 1



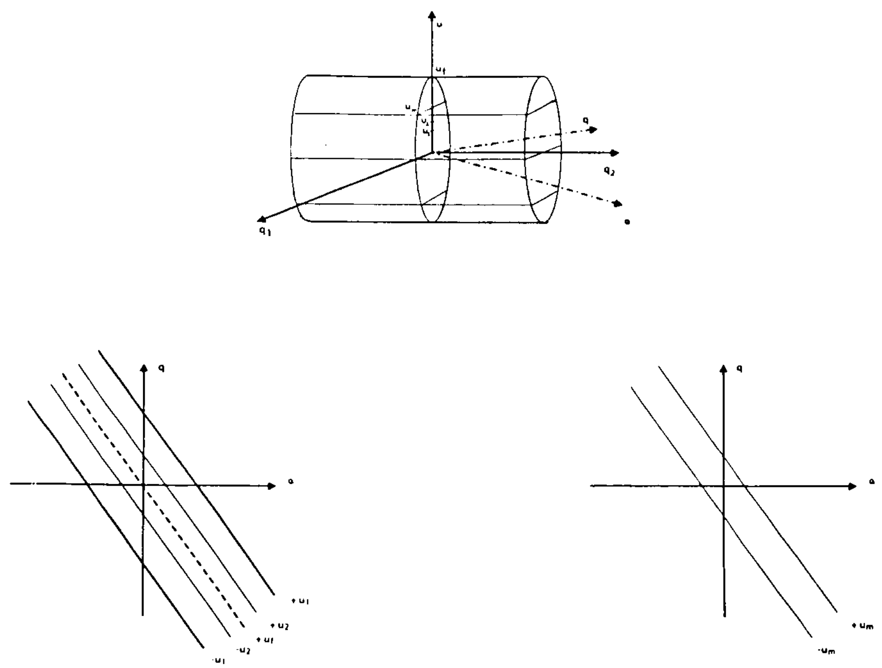
$|u_d|$ = 22 deg. $|u_d|$ = 21 deg. $|u_d|$ = 20.5 deg.

BANDWIDTH = 10 rad/s

RATE LIMIT = 40 rad/s

The limit cycle, representing the stability boundary in the phase-plane of unstable mode and control deflection, implies that a hypercylinder represents the overall stability region in the normal mode coordinates. With the state space coordinates located in the plane of normal modes, the stability regions clearly become functions of control deflection limits under rate constraints. Imposition of displacement limits implies that a chopped hypercylinder represents the stability region. Therefore, finite stability regions can exist in the state space coordinates only for those controls that satisfy the displacement saturation constraints.

INTERPRETATION OF RESULTS IN PHYSICAL COORDINATES



Under rate constraints, a hypercylinder is shown to represent the stability region in the normal mode coordinates when a saddle-point type system is controlled by feedback of the unstable mode and control deflection. Its intersection with the plane of unstable mode and control deflection is an unstable limit cycle. An increase in rate limits results in increased size of the stability regions, but imposing displacement constraints causes drastic reduction in their sizes. That is, the stability regions are largely dependent upon the displacement limits, and they can even be made independent of the rate limits. Feedback of control deflection effectively increases the region of rate-unsaturated operation resulting in increased sizes of stability regions. However, the sizes of stability regions are reduced with increased feedback gain due to early rate saturation. Unlike the case of joint rate and displacement constraints where the existence of a stability region is always guaranteed, in the case of joint rate and command constraints, the stability region can disappear when the command limit is reduced below a certain limit.

CONCLUSIONS

- LIMIT CYCLE REPRESENTS STABILITY BOUNDARY
- MARGINAL EFFECT OF ACTUATOR BANDWIDTH
- INCREASE IN STABILITY REGION SIZE WITH RATE LIMITS
- SIGNIFICANT SIZE REDUCTION UNDER CONSTRAINED DEFLECTION
- LARGER REGION WITH FEEDBACK OF CONTROL DEFLECTION
- REDUCED SIZES WITH INCREASED GAIN
- INTOLERANCE OF STABILITY REGIONS TO COMMAND LIMITS