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**A COMPUTATIONAL PROCEDURE
FOR LARGE ROTATIONAL MOTIONS
IN MULTIBODY DYNAMICS**

by

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A Computational Procedure for Large Rotational Motions in Multibody Dynamics

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ABSTRACT

A computational procedure suitable for the solution of equations of motion for multibody systems is presented. The present procedure adopts a differential partitioning of the translational motions and the rotational motions. The translational equations of motion are then treated by either a conventional explicit or an implicit direct integration method. A principal feature of the present procedure is a nonlinearly implicit algorithm for updating rotations via the Euler four-parameter representation. The present procedure is applied to the rolling of a sphere through a specified trajectory, which indicates that the procedure yields robust solutions.

1. Introduction

The numerical solution of the equations of motion for multibody systems has been continuously challenging the dynamist. In general, computer simulation of multibody dynamical (MBD) systems requires a concerted integration of several computational aspects. These include a data structure for describing the system topology, computerized generation of the governing equations of motion, incorporation and accurate treatment of constraint conditions, implementation of suitable solution algorithms and easy interpretation of the simulation results.

In the past, the task of formulating equations of motion has been a dominant concern to many dynamists (see, e.g., [1-6]). From the computational viewpoint, it can be said that the differences in existing formulations lie principally in their ways of incorporating constraints [7-15] and in their resulting system topologies [2,16,17]. When the MBD systems become more complex, such as in the deployment of large space structures, streamlined accommodation of system topologies becomes a more important concern than elegance of formulation. This is because a flexible data structure can allow different modeling, different formulations and different solution techniques to be adapted to different subsystems. In a previous study [14-15], a new constraint treatment technique that can solve the constraint equations in a separate module from that for the translation and rotation variables was presented. A major feature of that study was to preserve the system topology for a variety of MBD systems.

In order to provide a complete set of solution modules, the constraint solution module must be interfaced with a solution module for the primary variables, viz, the translational and rotational variables. It is generally agreed that the solution of the translational motions can be treated either by a conventional explicit or an implicit direct integration method.

However, a wide spectrum of solution techniques has been proposed to integrate the rotational motions. Yet, along with constraint algorithms, many solution reliability and

efficiency issues in multibody simulation packages appear to hinge on how one solves the rotational degrees of freedom. This is especially true for flexible multibody systems wherein the higher frequency-response components are often due to rotational oscillatory motions. The objective of the present paper is thus to present a computational procedure for a robust and efficient treatment of rotational motions so that one can solve general MBD equations for a variety of system topologies.

2. Equations of Motion for Multibody Systems

The discrete equations of motion for flexible multibody systems can be expressed as [16]:

$$M\ddot{\mathbf{u}} + D(\dot{\mathbf{u}}) + \mathbf{S}(\mathbf{u}) + \mathbf{B}_N^T \boldsymbol{\lambda}_N + \mathbf{B}_H^T \boldsymbol{\lambda}_H = \mathbf{f}(t) \quad (2.1)$$

$$\Phi_N(\dot{\mathbf{u}}, t) = 0, \quad \Phi_H(\mathbf{u}, t) = 0 \quad (2.2)$$

where M is the mass matrix, $D(\cdot)$ is the generalized velocity-dependent force operator, $S(\cdot)$ is the internal force operator due to member flexibility, B_N and B_H are the gradients of the nonholonomic and holonomic constraints (2.2), λ_N and λ_H are the constraint forces, $\mathbf{f}(t)$ is the applied force, \mathbf{u} is the generalized displacement vector, (\cdot) denotes time differentiation and $(\)^T$ designates the matrix transposition.

The solution of (2.1) and (2.2) consists of two tasks: the satisfaction of the constraint conditions (2.2) to obtain $\boldsymbol{\lambda}$ and the computation of \mathbf{u} from (2.1). Procedures to obtain λ_N and λ_H by satisfying (2.2) were presented in [14,15] and will be adopted in Numerical Experiments. Hence, we will concentrate on the computation of \mathbf{u} .

3. Differential Partitioning of the MBD Equations

A basic difficulty in direct integration of (2.1) is that $\dot{\boldsymbol{\omega}}$ and $\boldsymbol{\omega}$ are not directly integrable, except for some special kinematic configurations. This motivates us to partition $\ddot{\mathbf{u}}$ into the

translational acceleration vector, $\ddot{\mathbf{d}}$, which is directly integrable and the angular acceleration vector, $\dot{\boldsymbol{\omega}}$, which is not, and treat them by a partitioned solution procedure [18-20], viz

$$\ddot{\mathbf{u}} = \begin{Bmatrix} \ddot{\mathbf{d}} \\ \dot{\boldsymbol{\omega}} \end{Bmatrix}, \quad \dot{\mathbf{u}} = \begin{Bmatrix} \dot{\mathbf{d}} \\ \boldsymbol{\omega} \end{Bmatrix} \quad (3 \cdot 1)$$

The equations of motion (2.1) can be partitioned according to the above acceleration partitioning:

$$\begin{bmatrix} M_d & 0 \\ 0 & M_\omega \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{d}} \\ \dot{\boldsymbol{\omega}} \end{Bmatrix} + \begin{Bmatrix} Q_d \\ Q_\omega \end{Bmatrix} = \begin{Bmatrix} f_d \\ f_\omega \end{Bmatrix} \quad (3 \cdot 2)$$

where

$$\begin{Bmatrix} Q_d \\ Q_\omega \end{Bmatrix} = \begin{Bmatrix} \mathbf{D}_d(\dot{\mathbf{u}}) + \mathbf{S}_d(\mathbf{u}) - \mathbf{B}_d^T \boldsymbol{\lambda} \\ \mathbf{D}_\omega(\dot{\mathbf{u}}) + \mathbf{S}_\omega(\mathbf{u}) - \mathbf{B}_\omega^T \boldsymbol{\lambda} \end{Bmatrix} \quad (3 \cdot 3)$$

in which

$$\mathbf{B} = \mathbf{B}_N + \mathbf{B}_H, \quad \boldsymbol{\lambda} = \boldsymbol{\lambda}_N + \boldsymbol{\lambda}_H \quad (3 \cdot 4)$$

To effect the node-by-node integration for the rotational degrees of freedom, we partition $\dot{\boldsymbol{\omega}}$ further into

$$\dot{\boldsymbol{\omega}} = [\dot{\omega}^1, \dot{\omega}^2, \dots, \dot{\omega}^P]^T \quad (3 \cdot 5)$$

where $\dot{\omega}^{(j)}$ is a (3×1) angular acceleration vector for the j-th node,

$$\dot{\omega}^{(j)} = [\omega_1^{(j)}, \omega_2^{(j)}, \omega_3^{(j)}] \quad (3 \cdot 6)$$

4. Review of Staggered Stabilized Procedure for Constraints

For simplicity, we rewrite (2.1) and (2.2) as

$$M\ddot{\mathbf{u}} + D(\dot{\mathbf{u}}) + S(\mathbf{u}) + B^T \boldsymbol{\lambda} = \mathbf{f} \quad (4.1)$$

$$\Phi(\mathbf{u}, \dot{\mathbf{u}}, t) = 0 \quad (4.2)$$

A penalty technique is introduced to obtain the constraints as follows

$$\boldsymbol{\lambda} = \frac{1}{\epsilon} \Phi(\mathbf{u}, \dot{\mathbf{u}}, t), \quad 0 < \epsilon \ll 1 \quad (4.3)$$

Differentiation of $\boldsymbol{\lambda}$ with respect to time yields

$$\epsilon \dot{\boldsymbol{\lambda}} = B\ddot{\mathbf{u}} + \dot{B}\dot{\mathbf{u}} + \frac{\partial \Phi}{\partial t} \quad (4.4)$$

Substituting $\ddot{\mathbf{u}}$ from (4.1) into (4.4), we obtain

$$\epsilon \dot{\boldsymbol{\lambda}} + BM^{-1}B^T \boldsymbol{\lambda} = BM^{-1}(\mathbf{f} - D(\dot{\mathbf{u}}) - S(\mathbf{u})) + \dot{B}\dot{\mathbf{u}} + \frac{\partial \Phi}{\partial t} = \mathbf{r}_\lambda \quad (4.5)$$

Hence, the solution of (4.1) and (4.2) has been replaced by (3.2) and (4.5).

5. Nonlinearly Implicit Procedure for Large Rotations

5.1 Euler Parameters and Angular Accelerations

The four-parameter Euler representation of the angular velocity for each node is expressed as (see, e.g., Wittenburg[21]):

$$\dot{\mathbf{q}} = \frac{1}{2} \begin{bmatrix} 0 & -\boldsymbol{\omega}^T \\ \boldsymbol{\omega} & -\tilde{\boldsymbol{\omega}} \end{bmatrix} \mathbf{q} = A(\boldsymbol{\omega}) \mathbf{q}, \quad \mathbf{q} = [q_0 \ q_1 \ q_2 \ q_3]^T \quad (5.1)$$

where

$$\tilde{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \quad \boldsymbol{\omega} = [\omega_1 \ \omega_2 \ \omega_3] \quad (5.2)$$

and the nodal-designating superscript is omitted for notational simplicity.

Time differentiation of (5.1) once more yields

$$\ddot{\mathbf{q}} = \mathbf{A}(\dot{\omega}) \cdot \mathbf{q} + \mathbf{A}(\omega) \cdot \dot{\mathbf{q}} \quad (5.3)$$

where $\mathbf{A}(\dot{\omega})$ is obtained by substituting $\dot{\omega}$ for ω in $\mathbf{A}(\omega)$.

Note that (5.3) contains the constraint condition

$$\mathbf{q}^T \mathbf{q} = 1 \quad (5.4)$$

in its second-derivative form:

$$\mathbf{q}^T \ddot{\mathbf{q}} + \dot{\mathbf{q}}^T \dot{\mathbf{q}} = 0 \quad (5.5)$$

5.2 Mid-Point Integration of Euler Parameters

Suppose that we know the state variables, $\dot{\omega}^k$ and ω^k , at the k -th time step and we want to solve for \mathbf{q}^{k+1} . Because of the special properties of $\mathbf{A}(\omega)$ and $\mathbf{A}(\dot{\omega})$, one can take advantage of the following set of mid-point rules:

$$\begin{cases} \mathbf{q}^{k+\frac{1}{2}} = \mathbf{q}^k + \delta \dot{\mathbf{q}}^{k+\frac{1}{2}} \\ \dot{\mathbf{q}}^{k+\frac{1}{2}} = \dot{\mathbf{q}}^k + \delta \ddot{\mathbf{q}}^{k+\frac{1}{2}} \\ \mathbf{q}^{k+1} = 2\mathbf{q}^{k+\frac{1}{2}} - \mathbf{q}^k, \quad \delta = \frac{h}{2} \end{cases} \quad (5.6)$$

where h is the stepsize.

Substituting (5.1) and (5.3) into (5.6), one obtains

$$\begin{cases} [I - \delta \mathbf{A}(\omega^{k+\frac{1}{2}}) - \delta^2 \mathbf{A}(\dot{\omega}^{k+\frac{1}{2}})] \mathbf{q}^{k+\frac{1}{2}} = [I - \delta \mathbf{A}(\omega^{k+\frac{1}{2}}) + \delta \mathbf{A}(\omega^k)] \mathbf{q}^k \\ [I - \delta \mathbf{A}(\omega^{k+\frac{1}{2}})] \dot{\mathbf{q}}^{k+\frac{1}{2}} = \dot{\mathbf{q}}^k + \delta \mathbf{A}(\omega^{k+\frac{1}{2}}) \mathbf{q}^{k+\frac{1}{2}}, \quad (\mathbf{q}^{k+\frac{1}{2}})^T \dot{\mathbf{q}}^{k+\frac{1}{2}} = 0 \end{cases} \quad (5.7)$$

Since $\mathbf{A}(\omega^{k+\frac{1}{2}})$ and $\mathbf{A}(\dot{\omega}^{k+\frac{1}{2}})$ are not available, we approximate them by

$$\mathbf{A}(\omega^{k+\frac{1}{2}}) \simeq \mathbf{A}(\omega^k) \quad , \quad \mathbf{A}(\dot{\omega}^{k+\frac{1}{2}}) \simeq \mathbf{A}(\dot{\omega}^k) \quad (5.8)$$

which are equivalent to a Newton-like tangent approximation. Hence, we have from (5.7a) and (5.8)

$$[I - \delta A(\omega^k) - \delta^2 A(\dot{\omega}^k)] \cdot q^{k+\frac{1}{2}} = q^k \quad (5.9)$$

Once $q^{k+\frac{1}{2}}$ is obtained from (5.9), we can obtain $\dot{q}^{k+\frac{1}{2}}$ from (5.7b) by

$$[I - \delta A(\omega^{k+\frac{1}{2}})] \dot{q}^{k+\frac{1}{2}} = \dot{q}^k + \delta A(\omega^{k+\frac{1}{2}}) q^{k+\frac{1}{2}} \quad (5.10)$$

Finally, one can update ω from the following formula:

$$\omega = 2(q_0 \dot{\mathbf{g}} - \tilde{\mathbf{g}} \dot{\mathbf{g}} - \dot{q}_0 \mathbf{g}), \quad \mathbf{g} = [q_1 \quad q_2 \quad q_3], \quad q_0^2 + \mathbf{g}^T \mathbf{g} = 1 \quad (5.11)$$

where $\tilde{\mathbf{g}}$ has the same form as $\tilde{\omega}$ as given by (5.2). Hence, if necessary, one can obtain $\dot{\omega}$ from the equations of motion and iterate on q and \dot{q} by (5.7).

The skew-symmetric solution matrices in the above difference equations can be explicitly inverted via the formula:

$$\begin{bmatrix} 1 & -a & -b & -c \\ a & 1 & c & -b \\ b & -c & 1 & a \\ c & b & -a & 1 \end{bmatrix}^{-1} = \frac{1}{1 + a^2 + b^2 + c^2} \begin{bmatrix} 1 & a & b & c \\ -a & 1 & -c & b \\ -b & c & 1 & -a \\ -c & -b & a & 1 \end{bmatrix} \quad (5.12)$$

Hence, the solution of (5.9) and (5.10) becomes straightforward.

5.3 Update of New Angular Orientation

Once q^{k+1} is computed from (5.9) and (5.6c), it is often required to compute the body-fixed basis vector, $\mathbf{b} = [b_1 \quad b_2 \quad b_3]^T$ in terms of the inertial basis vectors, $\mathbf{e} = [e_1 \quad e_2 \quad e_3]^T$. These two vectors are related by

$$\mathbf{b} = \mathbf{R} \mathbf{e} \quad (5.13)$$

where

$$\mathbf{R} = \begin{bmatrix} 2(q_0^2 + q_1^2) - 1 & 2(q_1 q_2 + q_0 q_3) & 2(q_1 q_3 - q_0 q_2) \\ 2(q_1 q_2 - q_0 q_3) & 2(q_0^2 + q_2^2) - 1 & 2(q_2 q_3 + q_0 q_1) \\ 2(q_1 q_3 + q_0 q_2) & 2(q_2 q_3 - q_0 q_1) & 2(q_0^2 + q_3^2) - 1 \end{bmatrix} \quad (5.14)$$

In order to satisfy the orthonormality of \mathbf{R} , it is crucial to satisfy the two constraints, viz, $\mathbf{q}^T \mathbf{q} = 1$ in computing \mathbf{q} and $\mathbf{q}^T \dot{\mathbf{q}} = 0$ in computing $\dot{\mathbf{q}}$, respectively. This can be accomplished for $\mathbf{q}^{k+\frac{1}{2}}$ by augmenting the constraint and solving the following equation by a Newton-like procedure:

$$\begin{bmatrix} \mathbf{E}_q & \mathbf{q} \\ \mathbf{q}^T & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{b}_q \\ 1 \end{Bmatrix} \quad (5.15)$$

where \mathbf{E}_q is the (4×4) solution matrix in the lefthand side of (5.7a) and \mathbf{b}_q is the righthand side vector of (5.7a), respectively.

Similarly, the constraint $\mathbf{q}^T \dot{\mathbf{q}} = 0$ can be satisfied by

$$\begin{bmatrix} \mathbf{E}_{\dot{q}} & \mathbf{q} \\ \mathbf{q}^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{b}_{\dot{q}} \\ 0 \end{Bmatrix} \quad (5.16)$$

where $\mathbf{E}_{\dot{q}}$ is the (4×4) solution matrix in the lefthand side of (5.10) and $\mathbf{b}_{\dot{q}}$ is the righthand side vector of (5.10), respectively.

In addition, after the solution has converged at the $(k + \frac{1}{2})$ -timestep, we must enforce the same two constraint conditions in updating \mathbf{q}^{k+1} and $\dot{\mathbf{q}}^{k+1}$. This is effected by the following simple procedures.

To maintain the constraint $\mathbf{q}^T \mathbf{q} = 1$ at the new $(k+1)$ -th timestep, we employ

$$\begin{bmatrix} \mathbf{I} & \mathbf{q} \\ \mathbf{q}^T & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} 2\mathbf{q}^{k+\frac{1}{2}} - \mathbf{q}^k \\ 1 \end{Bmatrix} \quad (5.17)$$

where \mathbf{I} is (4×4) identity matrix. Similarly, to enforce $\mathbf{q}^T \dot{\mathbf{q}} = 0$, we use

$$\begin{bmatrix} \mathbf{I} & \mathbf{q} \\ \mathbf{q}^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} 2\dot{\mathbf{q}}^{k+\frac{1}{2}} - \dot{\mathbf{q}}^k \\ 0 \end{Bmatrix} \quad (5.18)$$

This completes the present nonlinearly implicit procedure for large rotational motions.

6. Solution Procedures for MBD Equations

We recall (3.2) and (4.5) to summarize the present overall computational procedures

$$M_d \ddot{\mathbf{d}} = \mathbf{f}_d - D_d(\dot{\mathbf{u}}) - S_d(\mathbf{u}) - B_d^T \lambda \quad (6.1)$$

$$M_\omega \dot{\omega} = \mathbf{f}_\omega - D_\omega(\dot{\mathbf{u}}) - S_\omega(\mathbf{u}) - B_\omega^T \lambda \quad (6.2)$$

$$\epsilon \dot{\lambda} + BM^{-1}B^T \lambda = \mathbf{r}_\lambda = BM^{-1}(\mathbf{f} - D(\dot{\mathbf{u}}) - S(\mathbf{u})) + \dot{B}\dot{\mathbf{u}} + \frac{\partial \Phi}{\partial t} \quad (6.3)$$

The computational sequences are as follows.

$$\begin{cases} \dot{\mathbf{d}}^{n+\frac{1}{2}} = \mathbf{d}^{n-\frac{1}{2}} + h\ddot{\mathbf{d}}^n \\ \mathbf{d}^{n+1} = \mathbf{d}^n + h\dot{\mathbf{d}}^{n+\frac{1}{2}} \end{cases} \quad (6.4)$$

$$\begin{cases} \mathbf{A}(\omega^{k+\frac{1}{2}}) \simeq \mathbf{A}(\omega^k) & \mathbf{A}(\dot{\omega}^{k+\frac{1}{2}}) \simeq \mathbf{A}(\dot{\omega}^k) \\ [I - \delta \mathbf{A}(\omega^{k+\frac{1}{2}}) - \delta^2 \mathbf{A}(\dot{\omega}^{k+\frac{1}{2}})] \mathbf{q}^{k+\frac{1}{2}} = [I - \delta \mathbf{A}(\omega^{k+\frac{1}{2}}) + \delta \mathbf{A}(\omega^k)] \mathbf{q}^k \\ [I - \delta \mathbf{A}(\omega^{k+\frac{1}{2}})] \dot{\mathbf{q}}^{k+\frac{1}{2}} = \dot{\mathbf{q}}^k + \delta \mathbf{A}(\omega^{k+\frac{1}{2}}) \mathbf{q}^{k+\frac{1}{2}}, & (\mathbf{q}^{k+\frac{1}{2}})^T \dot{\mathbf{q}}^{k+\frac{1}{2}} = 0 \\ \mathbf{q}^{k+1} = 2\mathbf{q}^{k+\frac{1}{2}} - \mathbf{q}^k \\ \dot{\mathbf{q}}^{k+1} = 2\dot{\mathbf{q}}^{k+\frac{1}{2}} - \dot{\mathbf{q}}^k, & (\mathbf{q}^{k+1})^T \dot{\mathbf{q}}^{k+1} = 0 \\ \omega = 2(q_0 \dot{\mathbf{g}} - \ddot{\mathbf{g}} - \dot{q}_0 \mathbf{g}) \end{cases} \quad (6.5)$$

It should be mentioned that the above procedures require only (4×4) -matrices which can be inverted explicitly via (5.12).

$$(\epsilon \mathbf{I} + \delta \mathbf{B} \mathbf{M}^{-1} \mathbf{B}^T) \lambda^{k+\frac{1}{2}} = \delta \mathbf{r}_\lambda^{k+\frac{1}{2}} + \epsilon \lambda^k \quad (6.6)$$

We will now apply (6.4) - (6.6) together with (5.15) - (5.18) to some sample problems.

7. An Example—Dynamics of a Bowling Ball

This problem was investigated by Huston et al [22], whose equations do not involve the constraint force, λ . In the present analysis, we employ a formulation that incorporates the

constraint force as part of the system variables. Fig. 1 illustrates the ball with its radius a and an offset center r_0 that is to follow a sine curve

$$y = \sin x \quad (7.1)$$

The various matrices and vector quantities for (6.1)-(6.3) can be derived as

$$M = \begin{bmatrix} m & 0 & -mr_0\mathbf{e}_1 \cdot \mathbf{b}_2 & mr_0\mathbf{e}_1 \cdot \mathbf{b}_1 & 0 \\ 0 & m & -mr_0\mathbf{e}_2 \cdot \mathbf{b}_2 & mr_0\mathbf{e}_2 \cdot \mathbf{b}_1 & 0 \\ & & J_1 & 0 & 0 \\ \text{sym.} & & & J_2 & 0 \\ & & & & J_3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & -a\mathbf{b}_1 \cdot \mathbf{e}_2 & -a\mathbf{b}_2 \cdot \mathbf{e}_2 & -a\mathbf{b}_3 \cdot \mathbf{e}_2 \\ 0 & 1 & a\mathbf{b}_1 \cdot \mathbf{e}_1 & a\mathbf{b}_2 \cdot \mathbf{e}_1 & a\mathbf{b}_3 \cdot \mathbf{e}_1 \\ \cos x & -1 & 0 & 0 & 0 \end{bmatrix} \quad (7.2)$$

$$D_d(\dot{\mathbf{u}}) = -mr_0 \begin{Bmatrix} \omega_1\omega_3\mathbf{e}_1 \cdot \mathbf{b}_1 + \omega_2\omega_3\mathbf{e}_1 \cdot \mathbf{b}_2 - (\omega_1^2 + \omega_2^2)\mathbf{e}_1 \cdot \mathbf{b}_3 \\ \omega_1\omega_3\mathbf{e}_2 \cdot \mathbf{b}_1 + \omega_2\omega_3\mathbf{e}_2 \cdot \mathbf{b}_2 - (\omega_1^2 + \omega_2^2)\mathbf{e}_2 \cdot \mathbf{b}_3 \end{Bmatrix} \quad (7.3)$$

$$D_\omega(\dot{\mathbf{u}}) = - \begin{Bmatrix} \omega_2\omega_3(J_2 - J_3) \\ \omega_3\omega_1(J_3 - J_1) \\ \omega_1\omega_2(J_1 - J_2) \end{Bmatrix} \quad (7.4)$$

$$\mathbf{f}_d = \mathbf{0}, \quad \mathbf{f}_\omega = mgr_0 \begin{Bmatrix} \mathbf{e}_3 \cdot \mathbf{b}_2 \\ -\mathbf{e}_3 \cdot \mathbf{b}_1 \\ 0 \end{Bmatrix} \quad (7.5)$$

$$\mathbf{d} = [x, y]^T, \dot{\omega} = [\omega_1 \ \omega_2 \ \omega_3]^T, \lambda = [\lambda_1 \ \lambda_2 \ \lambda_3] \quad (7.6)$$

$$S(\mathbf{u}) = \mathbf{0}, \quad \mathbf{f} = \mathbf{0} \quad (7.7)$$

$$\mathbf{b} = \mathbf{Re} \quad (7.8)$$

$$\omega = 2(q_0\mathbf{g} - \tilde{\mathbf{g}}\dot{\mathbf{g}} - \dot{q}_0\mathbf{g}), \quad \mathbf{g} = [q_1 \ q_2 \ q_3]^T \quad (7.9)$$

There is a total of eight variables in the foregoing equations of motion as given by (7.6). However, in adopting the present solution procedure—viz, (6.4)-(6.6)—we solve for nine variables.

Numerical solutions of the rolling of sphere on a flat sinusoidal curve have been obtained with the data summarized in Table 1.

Table 1 Physical Dimensions and Initial Conditions for a Rolling Sphere

$$m = 71.32N, \quad a = 10.9cm, \quad r_0 = 0 \text{ or } 0.15cm$$

$$J_1 = J_2 = J_3 = 2/5ma^2, \epsilon = 10^{-6}$$

$$\dot{x}^0 = \dot{y}^0 = 0, \quad \{\omega_2^0 = -\omega_1^0 = 1, \omega_3^0 = 0\}$$

$$\dot{x}^0 = \dot{y}^0 = a\omega_1^0, \quad \{q_0^0 = 1, q_1^0 = q_2^0 = q_3^0 = 0\}$$

The ball track curve on the flat surface with time is shown in Fig. 2 for the case of no offset ($r_0 = 0$) and the corresponding angular velocities in Fig. 3. The time histories of the three constraint forces are shown in Fig. 4 wherein (λ_1, λ_2) correspond to the x and y - component of the constraint force to maintain the rolling contact condition, and λ_3 is to maintain the sinusoidal trajectory as imposed by (7.1). Hence, the first two constraints are indicative of skidding phenomenon and the third corresponds to the steering force required in the ball maneuvering. Notice that they exhibit highly nonlinear behavior while still periodic.

The ball track projected on the ball itself is shown in Figs. 5 through 8, the ($x - z$)-plane view in Fig. 5, the ($y - z$)-plane view in Fig. 6, the ($x - y$)-plane view in Fig. 7 and a three-dimensional trajectory in Fig. 8.

Convergence studies have been performed with increasing stepsizes and it has been found that the present procedure, viz, (6.4)-(6.6), maintains both the solution accuracy and stability for the stepsize up to $h \leq 0.15$. Although not described herein, an implicit algorithm was tried out to solve the translational motions, d , thus replacing (6.4) by a corresponding implicit procedure. It was determined that, while larger stepsizes were permitted, we not only had to iterate to convergence at each time step but also required more computations due to matrix solutions at each step and/or iteration. We hope that

further studies will illuminate how one can profit by a combined use of explicit and implicit algorithms to solve the translational part of the equations of motion.

Figures 9 - 11 show the y -direction surface traction of the ball with the offset center ($r_0 = 0.15a$), the angular velocities and the constraint forces. Note that, for the ball with no offset center, the average velocity in the y -direction is found from Fig. 2 to be about 1.2 unit/sec. On the other hand, the corresponding average velocity with the offset center is about 0.35 unit/sec. from Fig. 9 or about 1/3 of the no-offset case. A more dramatic variation with the offset center ball is illustrated in its angular velocities as shown in Fig. 10. Note that the angular velocities no longer exhibit periodic response, whereas they are periodic for the no-offset case (see Fig. 3). Likewise, the steering force to follow the sinusoidal curve ($y = \sin x$) becomes highly nonlinear, although nonlinearly periodic. The x and y -direction contact force to maintain the rolling contact condition between the ball and the surface, although bounded, manifests extremely nonlinear behavior.

Our experience with the example problem indicates that the present computational procedure for handling large rotational motions coupled with translational motions is robust and efficient. It is important to note that the present procedure traces not only the angular motions accurately but more importantly the constraint forces and the four Euler parameters (although these are not presented here). We hope to test the present computational procedure for larger and flexible structural systems in the coming months and report further results.

8. Concluding Remarks

A computational procedure for an accurate and efficient solution of large angular motions has been presented. The present procedure treats each nodal angular orientation separately in terms of the four-parameter Euler formula. Hence, one deals with only (4×4) -matrices whose inversions are done explicitly. It has been found from our numerical experiments

that it is essential to enforce the constraint condition on the Euler parameters (5.4) and its time derivatives at each integration step. The present procedure is thus well suited to a partitioned solution procedure wherein the translational motions are integrated separately from that of the rotations motions. In addition, the present procedure interfaces easily with the constraint force solution package as discussed in detail in Park and Chiou[14,15].

From a theoretical viewpoint, the present procedure can be considered a nonlinearly implicit procedure since the angular accelerations and angular velocities are used as arguments in the implicit solution matrices. The stability of the present procedure is at present difficult to assess and needs to be addressed in order for the procedure to take full advantage of its implicit characteristic. This stability question will require an involved analytical study and we hope to address this issue in the coming months.

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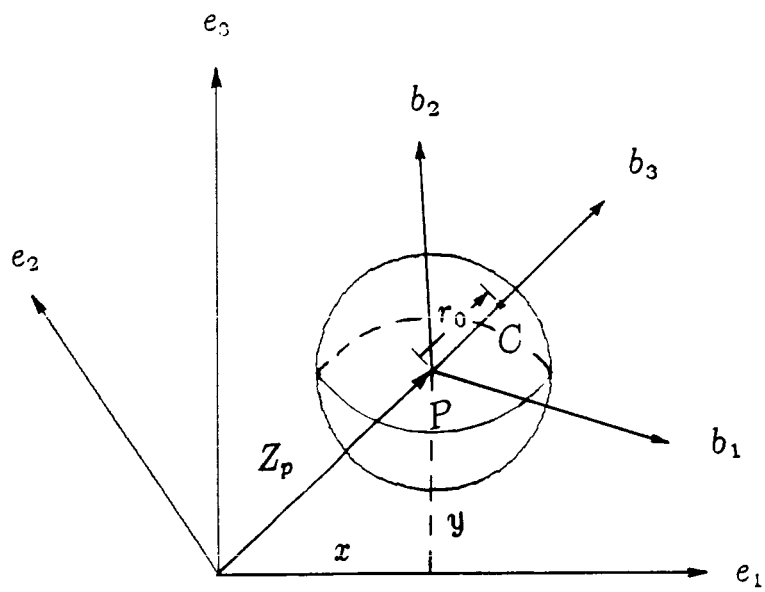


Fig. 1 Solid spherical ball rolling on a flat surface

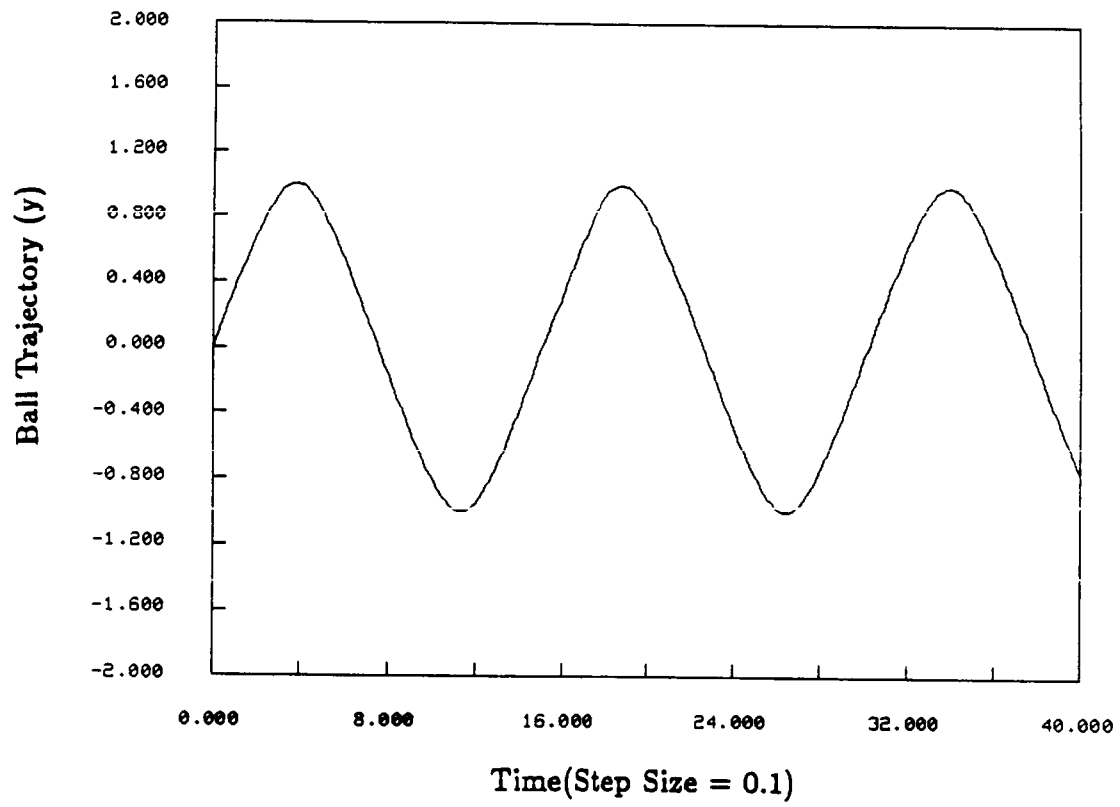


Fig. 2 Ball track on the flat surface with no offset

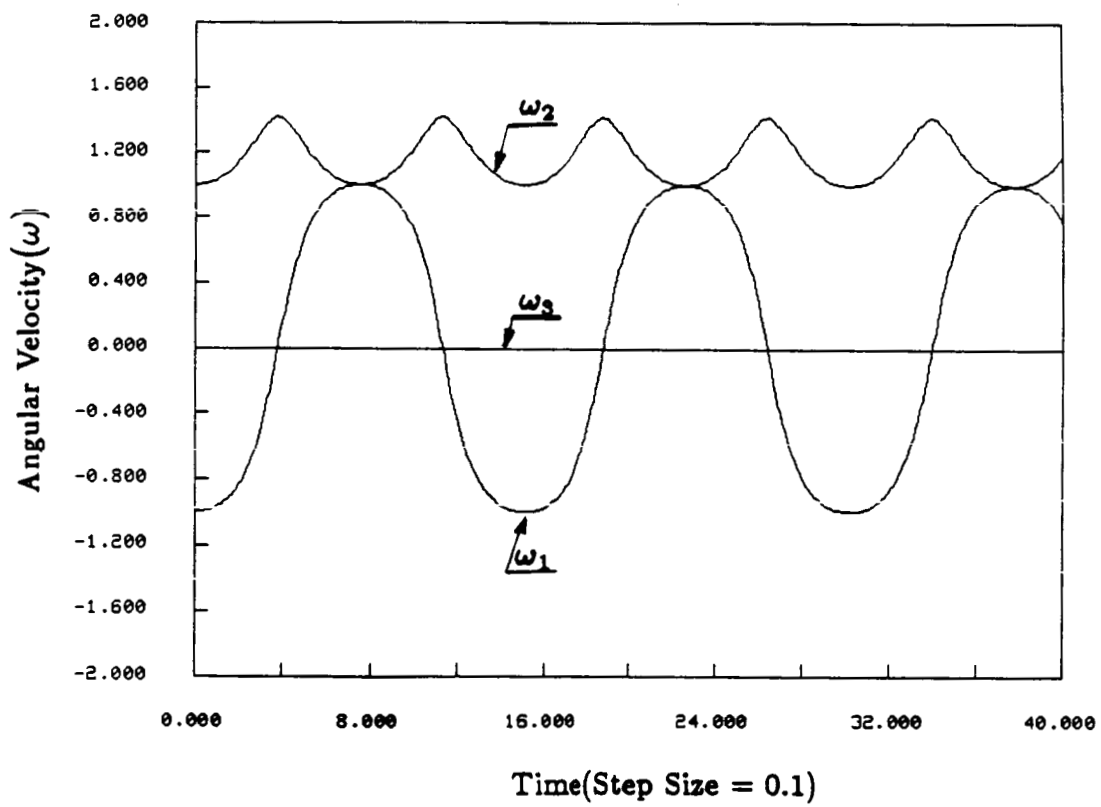


Fig. 3 Angular velocities of the sphere with no offset

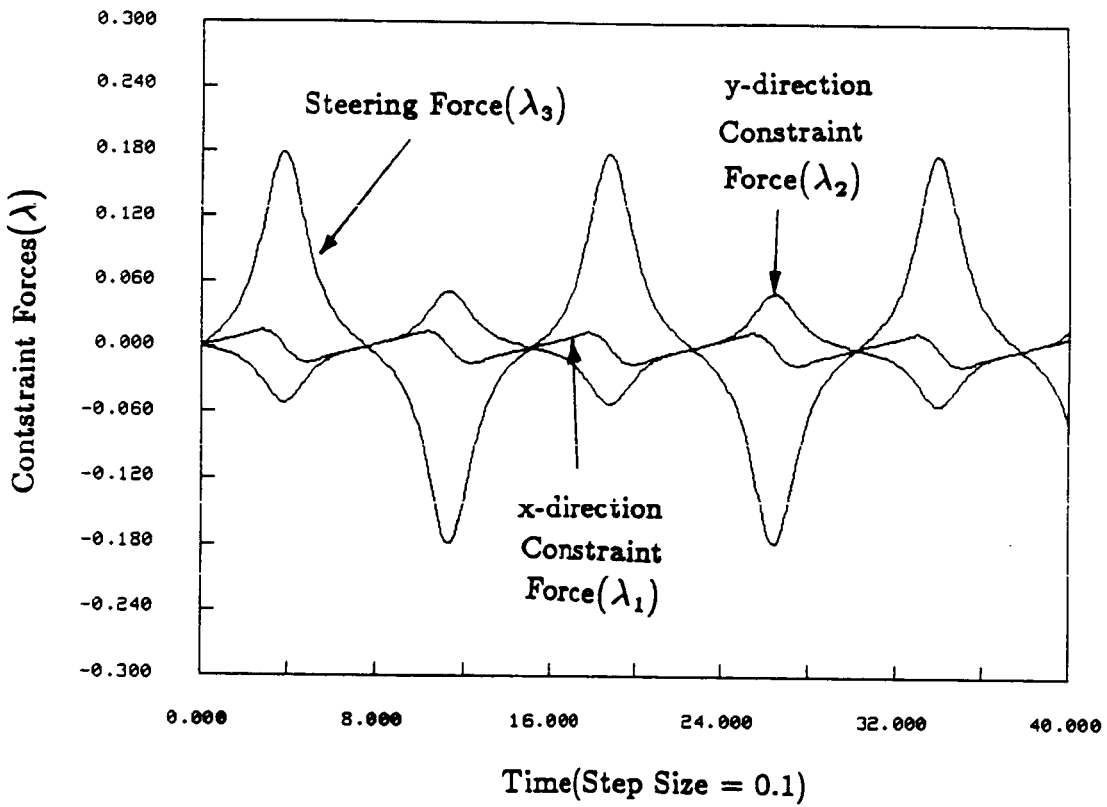


Fig. 4 Histories of three constraint forces on the rolling sphere

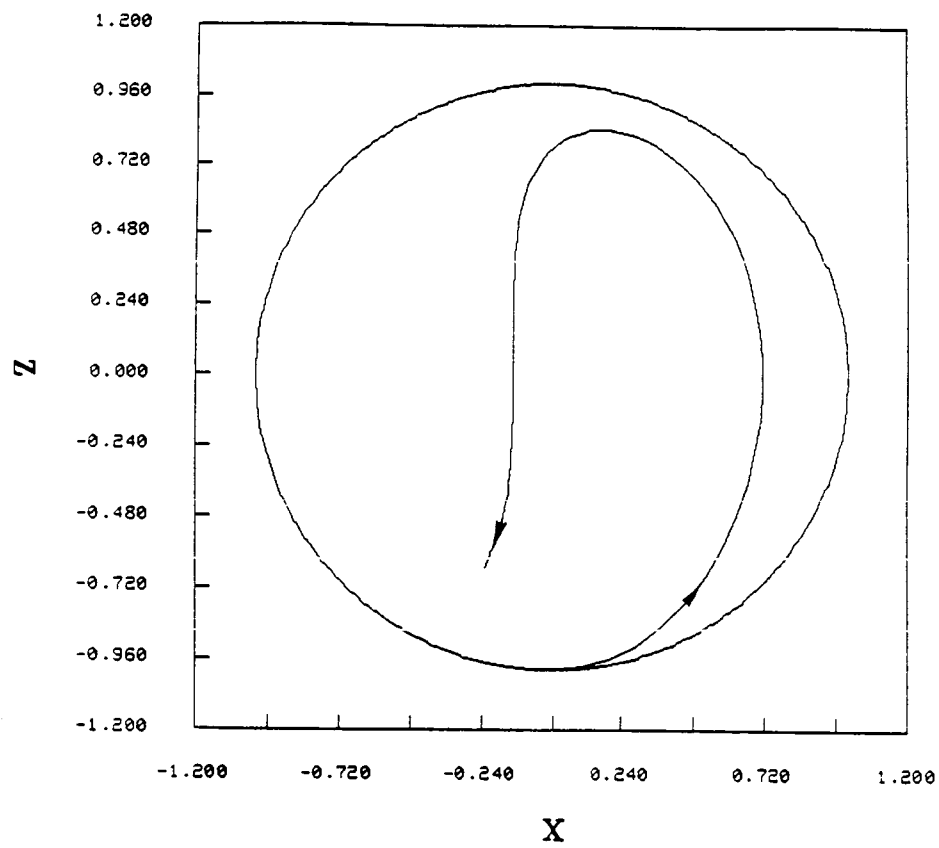


Fig. 5 Ball track projected on the X - Z plane of sphere

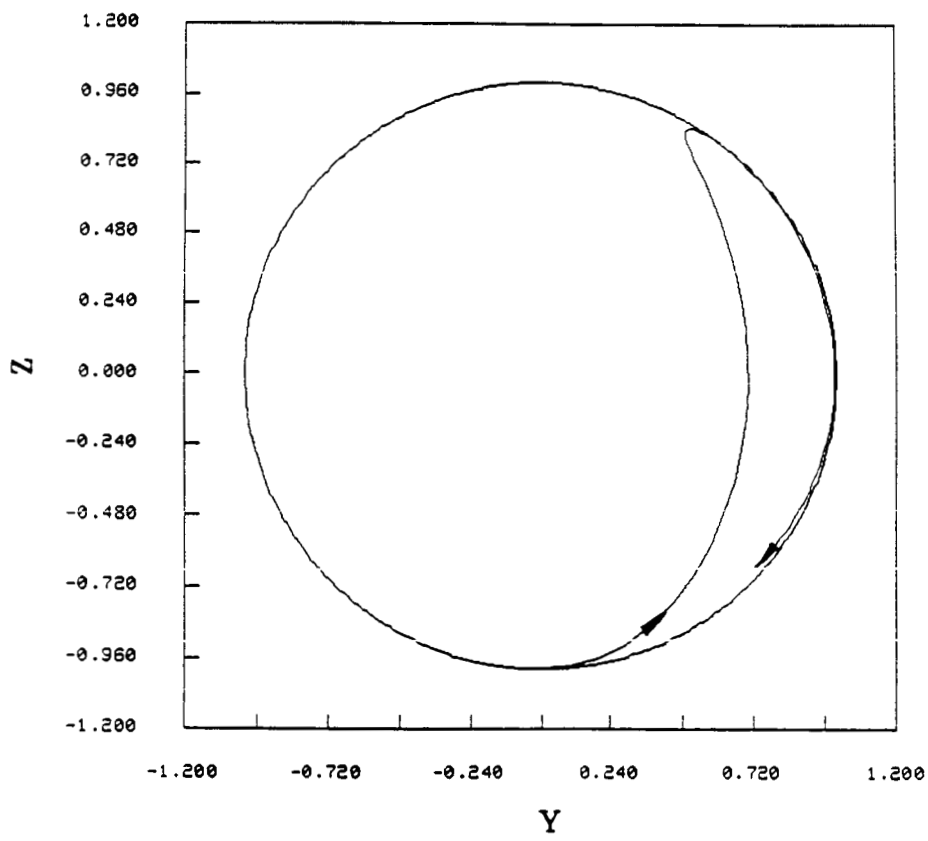


Fig. 6 Ball track projected on the Y - Z plane of sphere

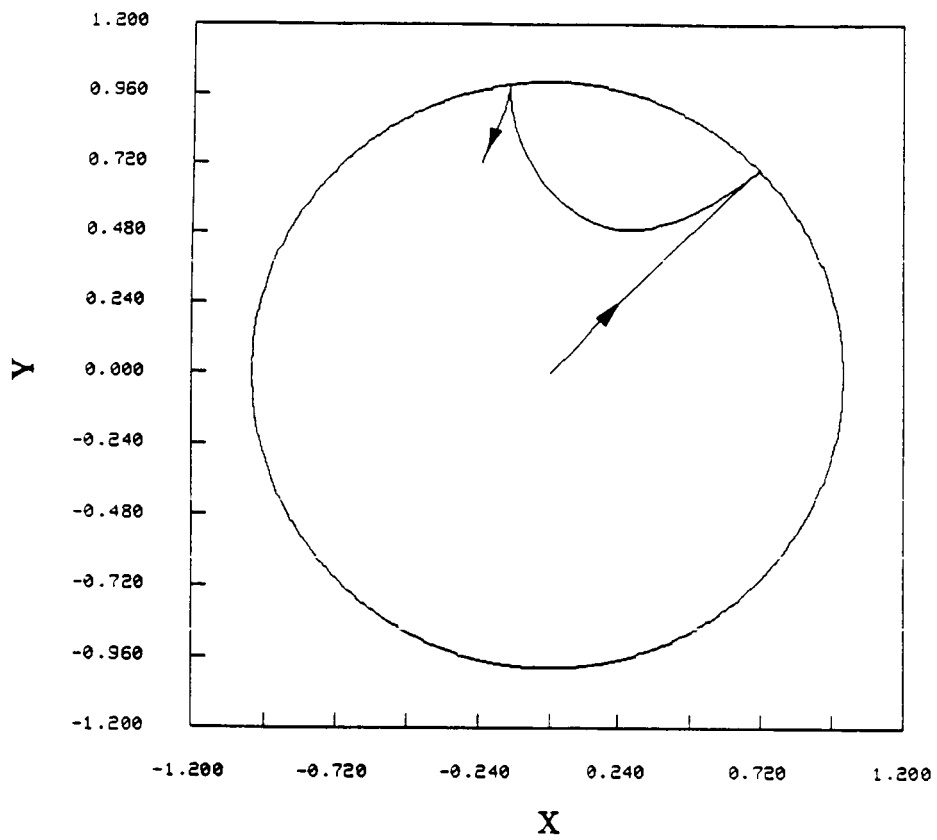


Fig. 7 Ball track projected on the X - Y plane of sphere

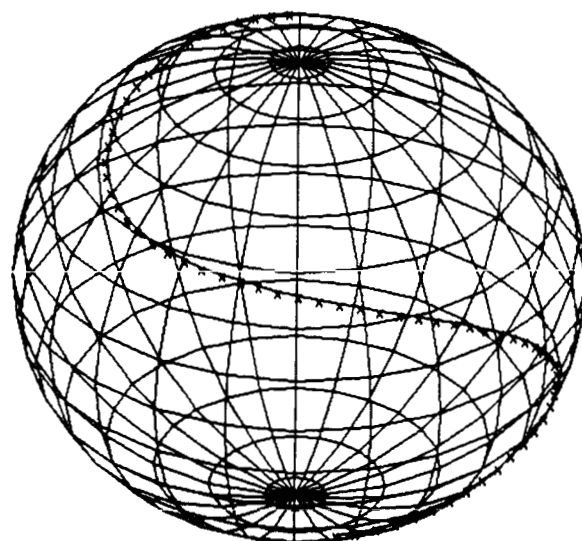


Fig. 8 Ball track projected on 3-D sphere surface

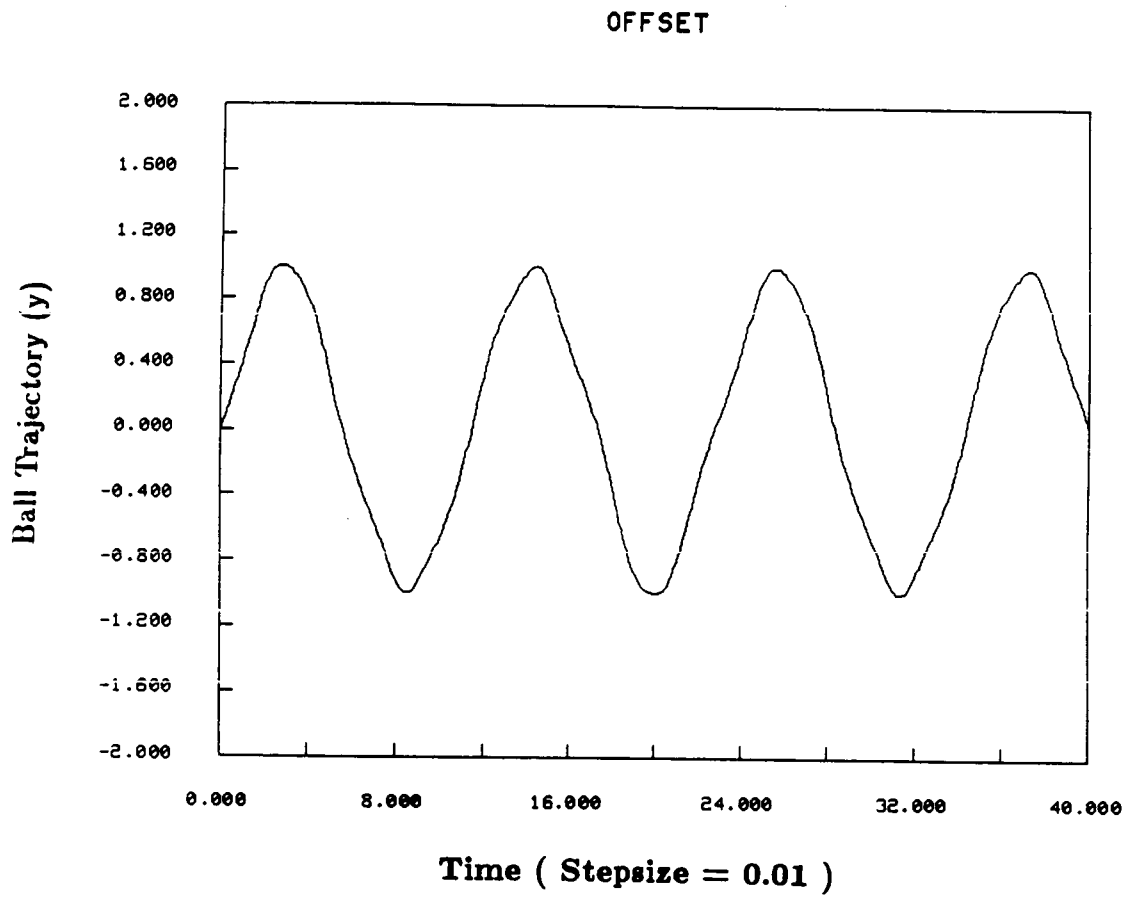


Fig. 9 Ball track on the flat surface with offset ($r_0 = 0.15a$)

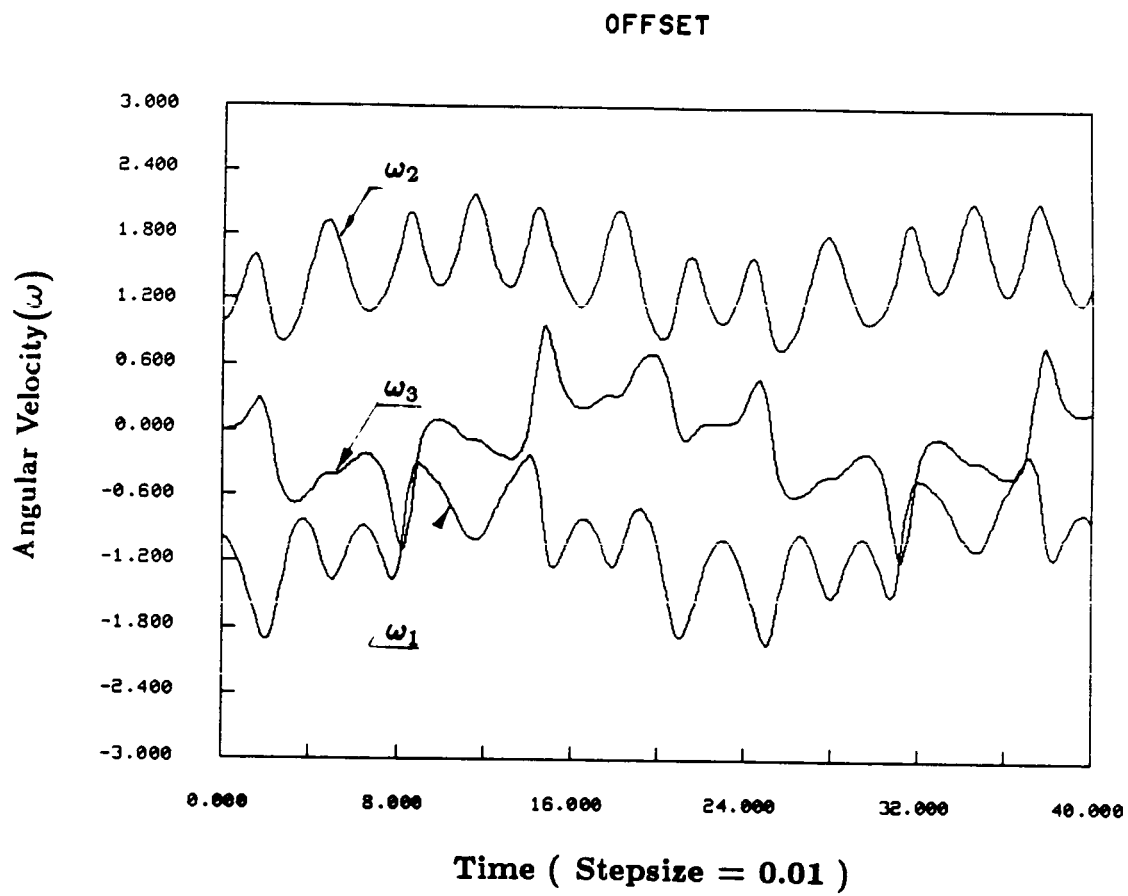


Fig. 10 Angular velocities of the sphere with offset center.

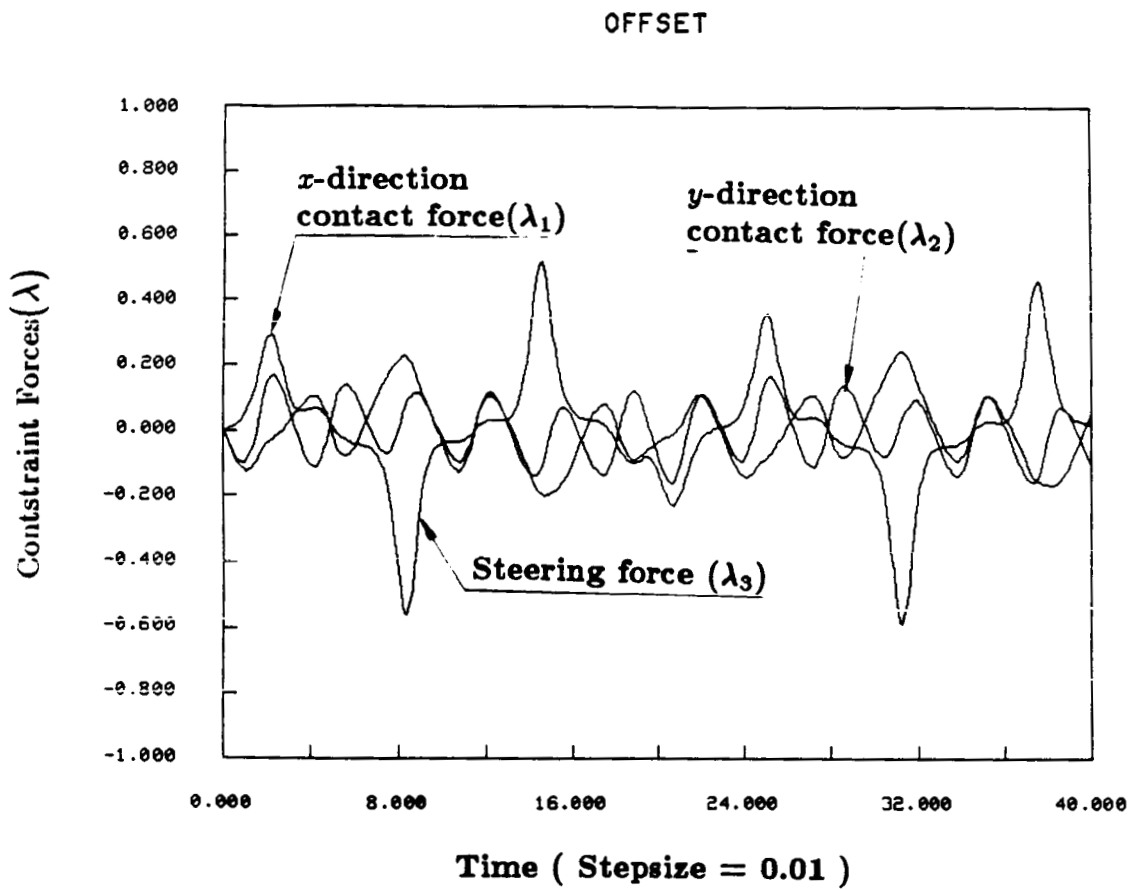


Fig. 11 Time histories of constraint forces with offset center