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# Optimizing the Antenna System of a Microwave Space Power Station: Implications for the Selection of Operating Power, Frequency, and Antenna Size

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OPTIMIZING THE ANTENNA SYSTEM OF A MICROWAVE SPACE POWER STATION:  
IMPLICATIONS FOR THE SELECTION OF OPERATING POWER, FREQUENCY,  
AND ANTENNA SIZE

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SUMMARY

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A design for a space power station that is to transmit power to the surface of a planet via high powered microwaves should commence with the optimum design of the transmitting and receiving antenna combination that is to be employed. Once one has assured that the desired amount of power has been transferred (which, after all, is the purpose of any power transmission system), one can, from the constraints imposed by such a design, tailor other parameters of the system such as antenna sizes and weights, power density in the planets atmosphere (e.g., to avoid electrical breakdown of the atmosphere), and frequency of operation. It is the purpose of this brief analysis to provide the working equations of such an optimized antenna system, and to give examples of their use. Related problems that should be analyzed in the future will then be discussed and a flow chart of the indicated order of priority will be presented. The analysis given here differs from previous work on this subject (ref. 1) in that the development given below will allow analytical expressions to be obtained for the relevant parameters. This is made possible by employing an approximation procedure that will be given during the exposition.

Let  $P(L)$  be the total power intercepted by the receiving antenna on the planet due to a wave field of power  $P_0$  transmitted by an orbiting microwave space power station. The antennas are taken to be circular and separated by a distance given by  $L$ . Defining the efficiency  $\eta$  of the transmitting/receiving antenna combination as

$$\eta = \frac{P(L)}{P_0} = \frac{\int_S |E(\rho, L)|^2 d\rho}{\int_{S_0} |E_0(r)|^2 dr} \quad (1)$$

where  $E(\rho, L)$  is the electric field at a point  $\rho$  on the surface of the receiving antenna aperture due to the electric field  $E_0(r)$  at a point  $r$  on the transmitting antenna aperture, one wishes to maximize the ratio given by equation (1) by specifying an optimal field  $E_0(r)$  to accomplish such a maximization. (The quantities  $S_0$  and  $S$  that appear under the integrals denote integration over the surfaces of, respectively, the transmitter aperture and receiving aperture.) This problem is completely defined when the received field  $E(\rho, L)$  is given in terms of the transmitted field  $E_0(r)$ . So long as one is not considering supergain antennas (ref. 1), one can use the Fresnel-Kirchhoff approximation to obtain

$$E(\rho, L) = -\frac{ik}{2\pi} \frac{e^{ikL}}{L} \int_{S_0} E_0(r) e^{ik(\rho - r)^2/2L} dr \quad (2)$$

where the wavenumber  $k$  is given by  $k = 2\pi/\lambda$  with  $\lambda$  being the wavelength. Substituting equation (2) into equation (1), converting the integrals to ones in plane polar coordinates and evaluating the angular parts (upon taking the E-fields to be azimuthally symmetric) one obtains

$$\eta = F^2 \frac{\int_0^1 \left| \int_0^1 E_0(r') e^{ik/2L R_T^2 r'^2} J_0(F\rho' r') r' dr' \right|^2 \rho' d\rho'}{\int_0^1 |E_0(r')|^2 r' dr'} \quad (3)$$

where

$$F = \frac{kR_T R_R}{L} \quad (4)$$

is the Fresnel number of the aperture system with  $R_T$  being the radius of the transmitting antenna and  $R_R$  that of the receiving antenna. Due to the symmetry assumed for the E-field,  $E_0(r) = E_0(r)$  is now a scalar function of distance from the origin of coordinates (taken to be at the center of the aperture).

The efficiency  $\eta$  is now a functional only of the field distribution in the transmitter aperture. It is desired to maximize  $\eta$  with respect to  $E_0(r)$ . Employing the well known variational theory (ref. 2) to equation (3), one finds that  $\eta$  is maximal when the functional combination  $E_0(r') \exp(ikR_T^2 r'^2/2L)$  is such that

$$E_0(r') e^{ikR_T^2 r'^2/2L} = g(F, r') \quad (5)$$

where the function  $g(F, r')$  is an eigenfunction of the equation

$$(\eta)^{1/2} g(F, r') = F \int_0^1 g(F, x) J_0(Fr'x) x dx \quad (6)$$

The desired eigenfunction must correspond to the largest eigenvalue, in this particular case, given by  $\eta$ . The functions that satisfy these constraints are hyperspheroidal functions (ref. 3). Working with such functions can be quite cumbersome. One can, however, obtain a useful analytical solution as will now be shown. Making the change of variables  $s = \sqrt{F} r'$  and  $y = \sqrt{F} x$ , one has from equation (6)

$$(\eta)^{1/2} g(F, s) = \int_0^{\sqrt{F}} g(F, y) J_0(ys) y dy \quad (7)$$

In the case of "large values" of  $F$ , one has seen that (eq. (4)) the produce  $R_T R_R$  is large, thus making  $\eta$  approach its maximum value of one. By the term "large value" is meant a value  $\sqrt{F} = U$  large enough such that the contributions to the integrand by the kernel  $J_0(ys)$  for values of  $y$  greater than  $U$  are negligible. If this is the case, one can formally let  $\eta \rightarrow 1$  and  $\sqrt{F} \rightarrow \infty$ . In this case, equation (7) can be approximately written

$$g(F,s) = \int_0^{\infty} g(F,y) J_0(ys) y dy \quad (8)$$

The function  $g(F,y)$  that can satisfy this relationship is that (ref. 4) where

$$g(F,s) = \exp\left(-\frac{s^2}{2}\right)$$

or, in terms of the original variables  $r' = s/\sqrt{F}$  and  $r = R_T r'$  (the latter identity stems from a change of variables made in eq. (3)).

$$g(F,r) = \exp\left(-\frac{Fr^2}{2R_T^2}\right) \quad (9)$$

Substituting this into equation (5), one obtains in this approximation (again reinstating the variable  $r = R_T r'$ )

$$E_O(r) = A(F) \exp\left(-\frac{Fr^2}{2R_T^2} - i \frac{kr^2}{2L}\right) \quad (10)$$

where the coefficient  $A(F)$  has been admitted to fix this approximate solution to other known quantities of the problem. Before this is done, however, it is interesting to note the form of the solution given by equation (10) to the variational problem; it specifies that in order to obtain an optimally efficient transmitter/receiver system, one must establish in the transmitter aperture a quadratic phase distribution with focusing on the receiver and a Gaussian amplitude distribution. A similar analysis culminating in the same results has been previously obtained (ref. 5).

In order to fix the coefficient  $A(F)$ , one can make use of the constraint that the total power  $P_O$  radiated by the transmitter is given by

$$P_O = \frac{c}{4} \int_0^{R_T} |E_O(r)|^2 r dr \quad (11)$$

where  $c$  is the speed of light. Substituting equation (10) into equation (11) and solving for  $A(F)$  gives

$$A(F) = \frac{1}{R_T} \left( \frac{8FP_0}{c(1 - e^{-F})} \right)^{1/2} \quad (12)$$

Equations (10) and (12) represent the solution (albeit, an approximate one) to the above variational problem.

Using equations (10) and (12) in equation (2) for the field at the receiver, one finds that

$$E(\rho, L) = - \frac{ikR_T^2 A(F)}{L} e^{ikL + (ik\rho^2/2L)} \int_0^1 e^{-Fr'^2/2} J_0 \left( \frac{F\rho r'}{R_R} \right) r' dr' \quad (13)$$

The ratio of the received field at any radial point  $\rho$  to that at the center of the receiver aperture (i.e., at  $\rho = 0$ ) is then simply given by

$$\frac{E(\rho, L)}{E(0, L)} = \frac{F}{1 - e^{-F/2}} \int_0^1 e^{-Fr'^2/2} J_0 \left( \frac{F\rho r'}{R_R} \right) r' dr' \quad (14)$$

Finally, using equations (10) and (12) in equation (3) gives for the optimized efficiency

$$\eta = \frac{2F^3}{1 - e^{-F}} \int_0^1 \left( \int_0^1 e^{-Fr'^2/2} J_0 \left( \frac{F\rho r'}{R_R} \right) r' dr' \right)^2 \rho' d\rho' \quad (15)$$

To facilitate the following calculations, a graph of equation (15) is given in figure 1 as  $\eta$  versus  $F$ . It is seen that  $\eta$  rapidly approaches one as  $F$  increases. Similarly, a graph of equation (14), for various values of  $F$ , appears in figure 2; here, the data is given in the form of the ratio  $E(\rho, L)/E(0, L)$  versus the ratio  $\rho/R_R$ . Figure 2 clearly shows the structure of the main lobe and side-lobes of the electric field distribution for distances  $L$  and for values of  $R_T$ ,  $R_R$ , and  $k$  such that  $F = kR_T R_R / L$  is equal to 1, 3, or 5. One sees that, in each instance, by requiring  $R_R$  to be such that  $\rho/R_R = 7$ , the receiver radius will subtend the first side-lobe; this is taken to be the case in the remainder of what follows. Hence, letting  $R_R \rightarrow \rho = 7R_R$  in equation (4) will secure this requirement:

$$F = \frac{7kR_T R_R}{L} \quad (\text{Assures that first side-lobe is subtended at receiver}) \quad (16)$$

Consider as examples efficiencies  $\eta = 0.6, 0.85, \text{ and } 0.99$ . From figure 1, corresponding to these values, one finds that, respectively,  $F = 2, 3, \text{ and } 5$ . In the case that  $L = 16,500 \text{ km}$  (the synchronous orbit for an orbiting microwave power station about Mars), one can use equation (16) to find corresponding relationships between  $k$  (and thus  $\lambda$ ) and the required product  $R_T R_R$  for the prevailing transmitter/receiver aperture sizes. Figure 3 shows plots of  $\sqrt{(R_R R_T)}$  versus frequency for an optimized antenna system operating at the efficiencies considered above. In the event that one can have  $R_R = R_T = R$ , one can immediately read-off the value for  $R$ .

It must be noted, however, that arbitrarily specifying  $R_T = R_R = R$  is not without its potential drawbacks. In particular, among other prevailing constraints, is that of the possibility of electrical breakdown due to ionization in the atmosphere of the planet (ref. 6). For this consideration, one needs to know the maximum power density of the received wave field. This can be easily found via equation (13); one has for the power density on the center of the beam,

$$S(0) = \frac{c}{8\pi} |E(0,L)|^2 = \frac{P_0 F (1 - e^{-F/2})^2}{\pi R_R^2 (1 - e^{-F})} \quad (17)$$

Taking  $\sqrt{(R_T R_R)} = R_R$ , equation (17) is used to plot the quantity  $S(0)/P_0$  versus frequency for the same set of parameters as before and also appears in figure 3. Hence, for a desired power  $P_0$ , one can determine the maximum power density for the particular situation and check to see if it approaches the breakdown density for the planetary atmosphere. This, as well as other problems remain to be considered for microwave power transmission onto the Martian surface.

A simplified breakdown of the interconnections of the entire problem of microwave power transmission from antenna system design considerations is given in figure 4. It is shown that the implications of antenna size not only have a direct bearing on the maximum power density but also has obvious mechanical implications, both of which can dictate a redesign in terms of antenna size or, if not solely satisfactory, a selection of a different operating frequency and via, e.g. (fig. 3), a new antenna size selections.

Finally, one should consider the actual implementation of the optimal field, as specified by equation (10), across the transmitter antenna aperture. For example, such a field distribution will not be continuous but discrete (due to finite sized radiators, etc.). A study should be conducted where the discrete realization of such a field be itself optimized with respect to e.g., side-lobe suppression and system maximum efficiency.

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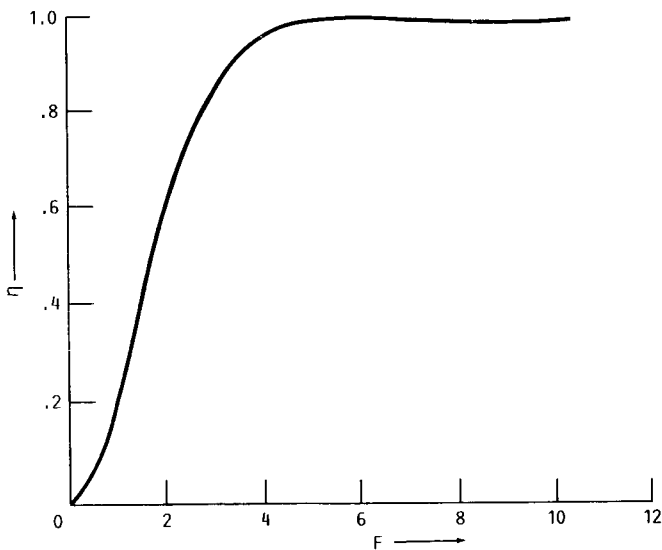


FIGURE 1. - PLOT OF ANTENNA SYSTEM EFFICIENCY  $\eta$  VERSUS SYSTEM FRESNEL NUMBER  $F$  AS OBTAINED FROM EQ. (15).

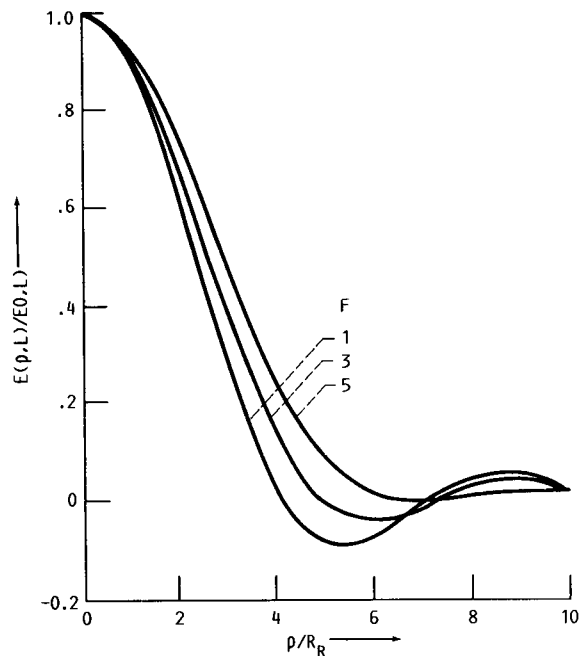


FIGURE 2. - PLOT OF RELATIVE ELECTRIC FIELD AMPLITUDE AT RECEIVER  $E(\rho,L)/E(0,L)$  VERSUS NORMALIZED RADIAL DISPLACEMENT FROM BEAM CENTER,  $\rho/R_R$ , OBTAINED FROM EQ. (14) FOR VARIOUS VALUES OF  $F$ .

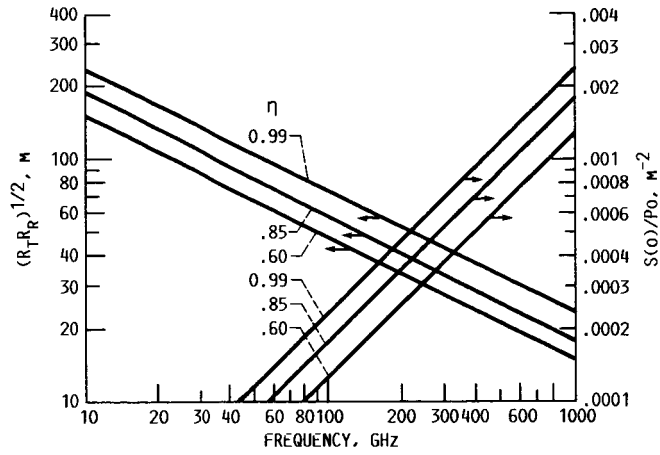


FIGURE 3. - PLOT OF  $(R_T R_R)^{1/2}$  AND CORRESPONDING NORMALIZED POWER DENSITY  $S(f)/P_0$  VERSUS FREQUENCY FOR VARIOUS VALUES OF  $\eta$ .

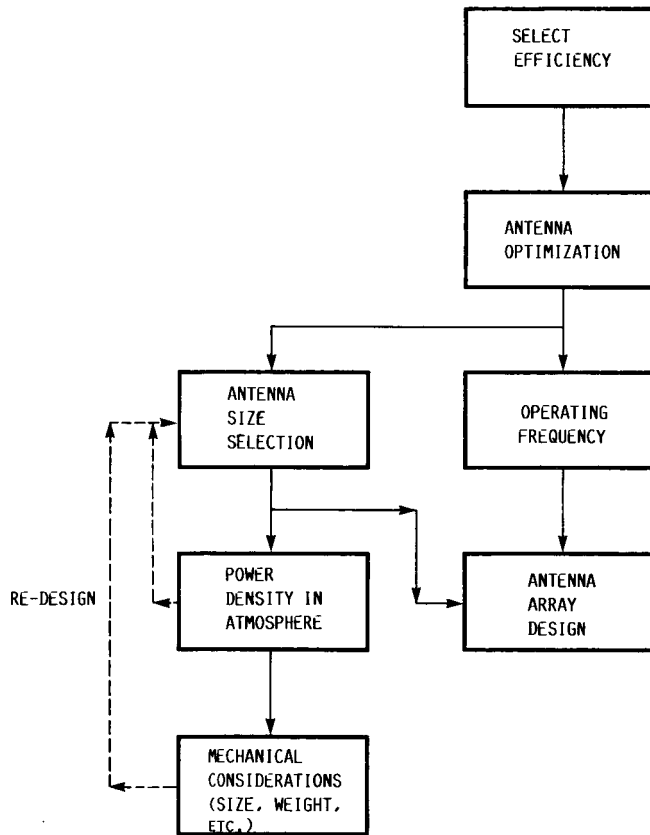


FIGURE 4. - HIERARCHIAL ARRANGEMENT OF THE SELECTION OF DESIGN FACTORS ENTERING INTO THE SELECTION OF ANTENNA SIZE.



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