IN-37-CR
104400
52P.

STUDY OF FREE-PISTON STIRLING ENGINE DRIVEN LINEAR ALTERNATORS

(NASA-CR-181425) STUDY OF FREE-PISTON STIRLING ENGINE DRIVEN LINEAR ALTERNATORS Final Report (Kentucky Univ.) 52 p Avail: NTIS HC A04/MF A01 CSCL 13I

N88-10355

Unclas
G3/37 0104400

S. A. Nasar

and

C. Chen

Electrical Engineering Department
University of Kentucky
Lexington, KY 40506-0046

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Lewis Research Center
Cleveland, Ohio
Under Grant No. NAG3-722

July 31, 1987

ACKNOWLEDGMENTS

We are indebted to NASA Lewis Research Center for Grant No. NAG3-722 which afforded the work reported here. We also wish to express our gratitude to Gene E. Schwarze, Technical Officer, for his guidance during the course of this research.

ABSTRACT

In this report, the analysis, design and operation of single-phase, single-slot tubular permanent magnet linear alternator is presented. The study includes the no-load and on-load magnetic field investigation, permanent magnet's leakage field analysis, parameter identification, design guidelines and an optimal design of a permanent magnet linear alternator. For analysis of the magnetic field, a simplified magnetic circuit is utilized. The analysis accounts for saturation, leakage and armature reaction.

List of Principal Symbols

B	Flux density, T
f_m	Frequency of the alternator, Hz
H	Magnetic field intensity, A/m
I	Demagnetizing component (of armature reaction) of stator (or primary) current, A
I_1	Stator (or primary) current, A
k	A coefficient
k_{fill}	Fill factor
$k_{\sigma m}$	Leakage coefficient of permanent magnet.
L_σ	Leakage inductance, H
L_m	Mutual inductance, H
L_{me}	Equivalent mutual inductance, H
R_a	Stator (or primary) winding resistance, Ω
R_L	Load resistance, Ω
W	Weight, lb.
W_1	Stator (or primary) winding turns, turn
Z_i	Iron loss, watt
Z_c	Copper loss, watt
ϕ	Flux, Wb.
λ	Flux linkage, Wb-turn
ω	Electrical angular frequency, R/S

Subscripts

gav	Average value in airgap
pch	Average value in the horizontal part of flux path in plunger back iron.
pcv	Average value in the vertical part of flux path in plunger back iron.
sp	Average value in stator pole shoes.
scv	Average value in the vertical part of stator back iron.
sch	Average value in the horizontal part of stator back iron.
g	airgap
p	plunger
s	stator
m	magnet
v	vertical
h	horizontal
c	core (back iron).

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Because it is a self-starting, quiet vibration-free and highly reliable device, the free-piston Stirling engine (FPSE) has recently received considerable attention. The FPSE is a strong candidate as a prime mover to drive a linear alternator in space power stations. The FPSE may use solar energy or nuclear energy as the source and can perform in any orientation—vertical, horizontal, inclined or upside down. For coupling with the FPSE, the linear alternator (LA) is most suited because of its simple structure and high efficiency of direct coupling. Fig. 1-1 shows a 5 kW free piston Stirling engine—linear alternator unit.

According to the method of excitation the linear alternators may be: (i) field-excited, with moving diodes; (ii) inductor or reluctance type, having both armature and field windings on the primary or (iii) permanent magnet (PM) excited. The PM linear alternator does not need any exciting winding and exciting power supply, so that it is simpler more reliable and lighter than the field-excited or reluctance-type linear alternator. Hence the PM linear alternator is considered more suitable for space power applications.

Past work on PM linear alternator is as follows:

(i) Primitive PM linear alternators are presented in [1], [2] and [3].

(ii) References [4] and [5] discuss the fundamental equations and basic design guidelines of a PM linear alternator.

(iii) Reference [6] gives one design of PM linear alternator, but the design is not optimal for either the ratio of power to weight of the alternator or the efficiency of the alternator.

It is fair to say that the above does not reflect adequate analysis and design of PM linear alternators. The present work consists of the field analysis, parameter identification, design guideline and optimal design of PM linear alternators.

The following discussion is for the single-phase, single-slot tubular linear alternator. But the results can be extended to multi-phase and multi-slot tubular linear alternators.

ORIGINAL PAGE IS OF POOR QUALITY

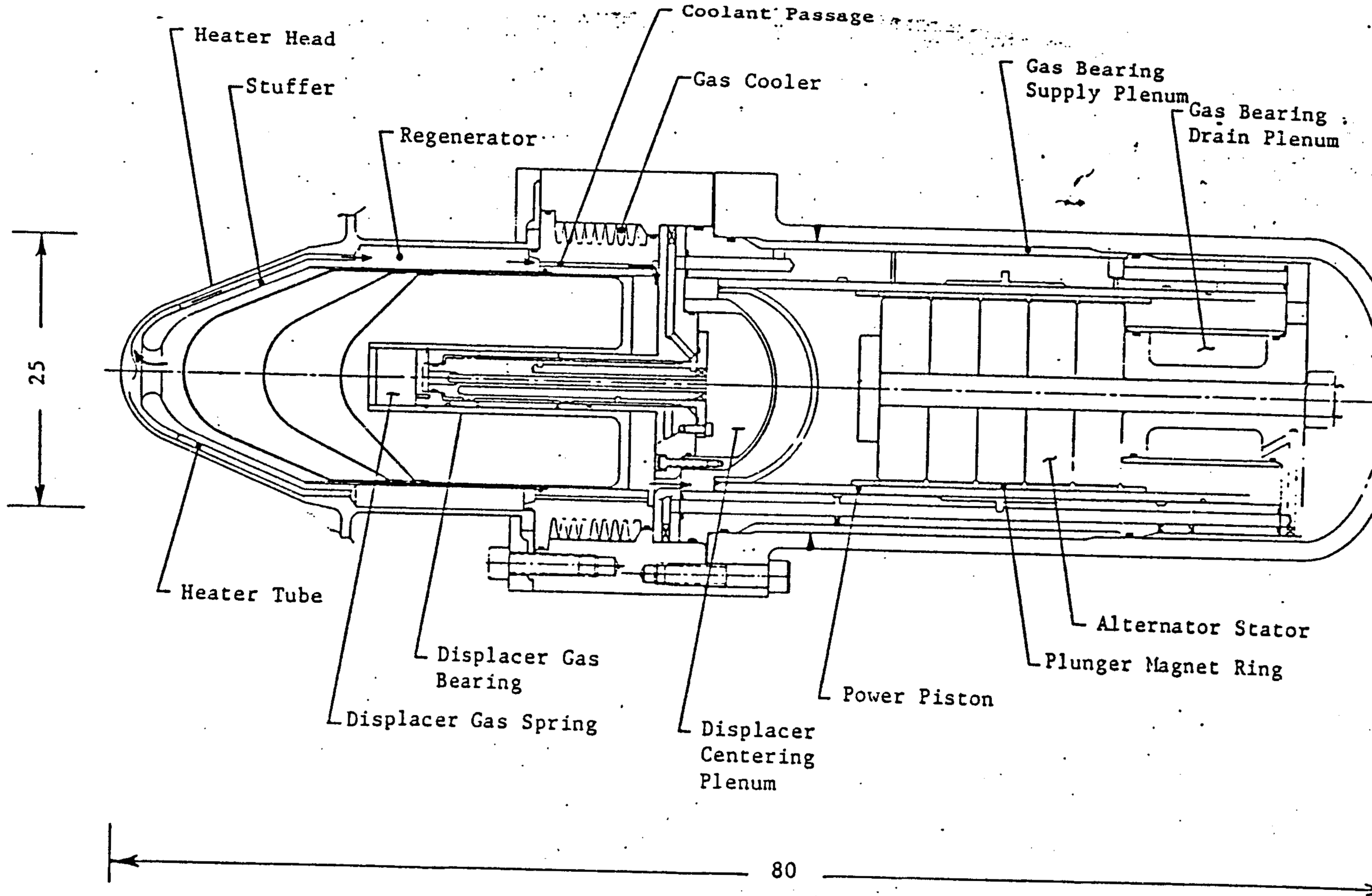


Figure 1-1 A five kW linear alternator (dimensions in cm).

CHAPTER 2

THE FIELD OF A TUBULAR PM LINEAR ALTERNATOR

2.1. Configuration of Single-Phase Single-Slot Tubular PM Linear Alternator

A single-phase single-slot tubular PM linear alternator is shown in Fig. 2-1, which has four pieces of magnets mounted on the plunger. The circular stator coils are embedded in stator slot. The plunger reciprocates with the stroke length:

$$\ell_{stroke} = \tau_m \tag{2-1}$$

The flux ϕ of permanent magnets in the airgap varies almost linearly with the position of the plunger, as shown in Fig. 2-2. If the plunger moves with a sinusoidal speed, a sinusoidal voltage will be induced in the stator winding. The flux of the permanent magnets has a maximum value at the ends of the plunger stroke.

2.2. Basic Equations of Magnetic Field and Demagnetization Curve of Permanent Magnet

Two very useful laws for electromagnetic field analysis are Ampere's Law and continuity of flux. Let there be n parts in a magnetic loop, then by the Ampere's Law:

$$WI = \sum_{i=1}^{n} H_i \ell_i \tag{2-2}$$

Continuity of flux requires that

$$\phi_1 = \phi_2 = \cdots = \phi_n \tag{2-3}$$

In (2-2) and (2-3) W is the coil turns linking the magnetic loop, I is the current in the coil, H_i, ϕ_i, ℓ_i are field intensity, flux and length of part i of the magnetic loop respectively, i=1,2,...n.

Let TASCORE 27 be the permanent magnet material used in the linear alternator for which the demagnetization curve is shown in Fig. 2-3. It may be seen that the demagnetization curve of TASCORE 27 is almost a linear curve and may be described approximately by:

$$H_m = H_c(1 - \frac{B_m}{B_r}) \tag{2-4}$$

or

$$B_m = B_r\,(1 - \frac{H_m}{H_c}) \tag{2-5}$$

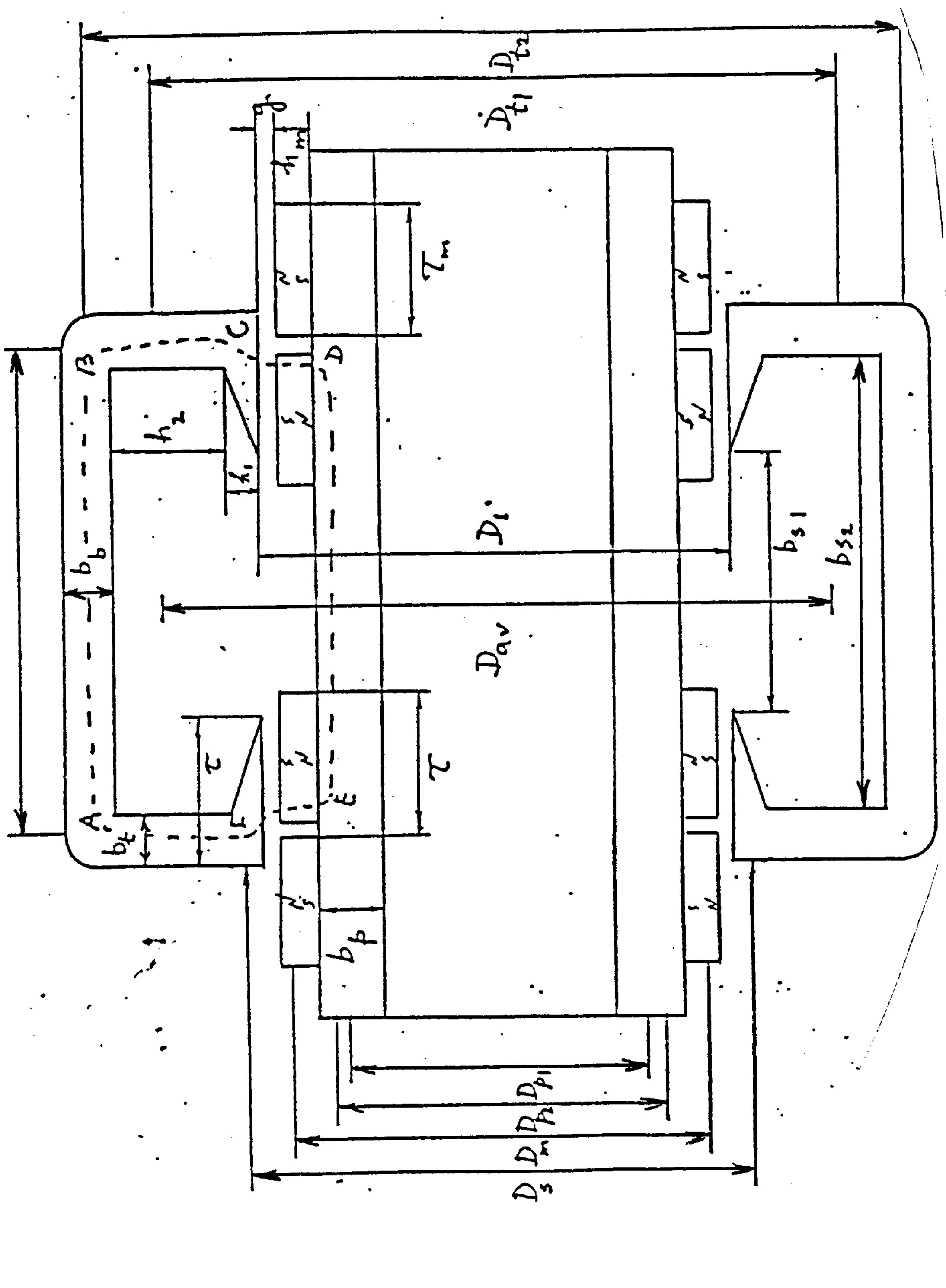


Figure 2-1 Linear alternator geometry.

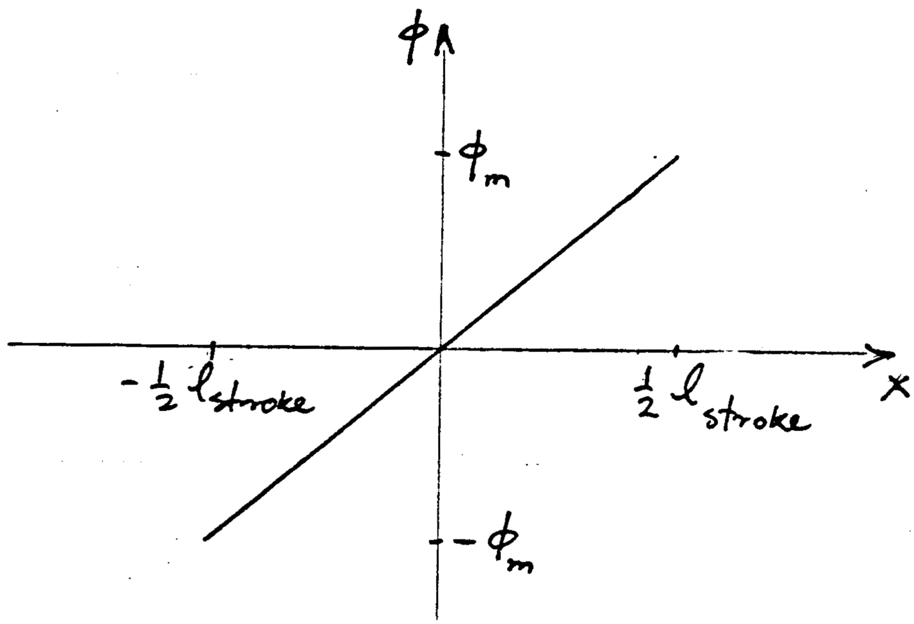


Figure 2-2 Flux variation with plunger position.

ORIGINAL PAGE IS
OF POOR QUALITY

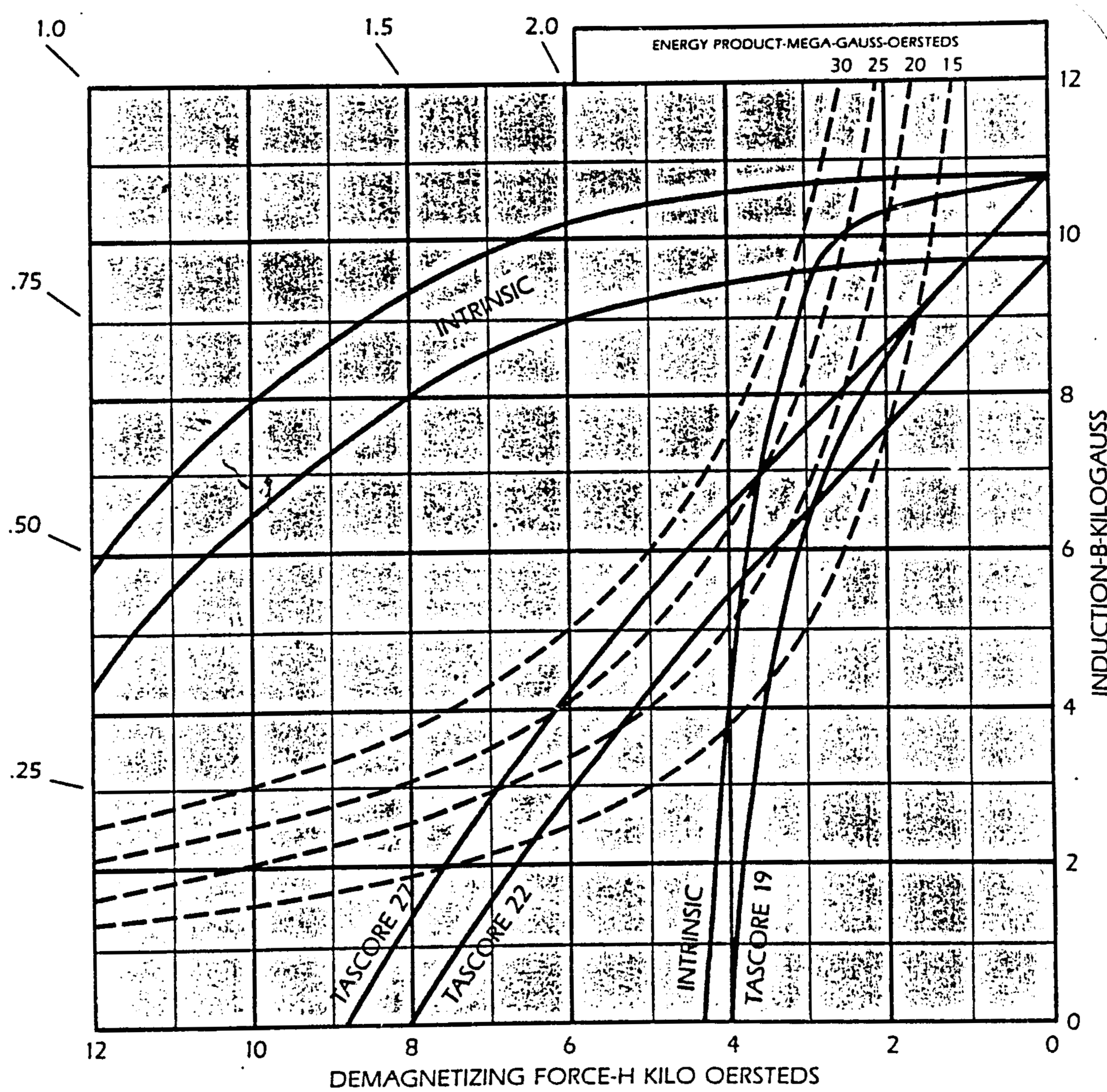


Figure 2-3 Demagnetizing curve.

2.3. Procedure to Calculate No Load Field

The dimensions shown in Fig. 2-1 have the following relationships:

$$D_m = D_i - 2g - h_m \quad (2\text{-}6)$$

$$D_{p1} = D_i - 2g - 2h_m - b_p \quad (2\text{-}7)$$

$$D_{p2} = D_i - 2g - 2h_m - \frac{1}{2}\, b_p \quad (2\text{-}8)$$

$$D_s = D_i + h_1 \quad (2\text{-}9)$$

$$D_{t1} = D_i + 2h_1 + h_2 + \frac{1}{2}\, b_b \quad (2\text{-}10)$$

$$D_{t2} = D_i + 2h_1 + 2h_2 + b_b \quad (2\text{-}11)$$

$$b = b_{s2} + b_t \quad (2\text{-}12)$$

The magnetic loop in Fig. 2-1 is divided into several sections. If it is assumed that the field is constant in each section, and by applying Ampere's Law to the path ABCDEFA, we obtain:

$$H_{sch} b + H_{scv}(2h_2+b_b) + H_{sp} 2h_1 + H_{gav}\, 2g + H_{pcv} b_p + H_{pch} b + H_m 2h_m = 0 \quad (2\text{-}13)$$

or

$$H_m = - \frac{1}{2h_m} \left[H_{sch} b + H_{scv}(2h_2 + b_b) + H_{sp} 2h_1 + H_{gav}\, 2g + H_{pcv}\, b_p + H_{pch}\, b\right] \quad (2\text{-}14)$$

It will be shown in the next chapter that the leakage flux of permanent magnet is not a constant and can be expressed as:

$$\phi_\sigma = -\, kH_m = -kH_c \,(1 - \frac{B_m}{B_r}) \quad (2\text{-}15)$$

where k is a constant determined by the dimensions of the PM alternator. Continuity of flux (Fig. 2-4) yields:

$$\tau_m D_m B_m + kH_c \,(1 - \frac{B_m}{B_r}) = \frac{\tau + \tau_m}{2}\,(D_i - g)\, B_{gav} \quad (2\text{-}16)$$

$$\frac{\tau + \tau_m}{2}\,(D_i - g)\, B_{gav} = \frac{1}{2}\,(\tau + b_t)\, D_s B_{sp} \quad (2\text{-}17)$$

$$\frac{1}{2}\,(\tau + b_t)\, D_s B_{sp} = b_t D_{t1}\, B_{scv} \quad (2\text{-}18)$$

$$b_t D_{t1} B_{scv} = b_t D_{t2} B_{sch} \quad (2\text{-}19)$$

$$D_m B_m = D_{p2} B_{pcv} \quad (2\text{-}20)$$

$$\tau_m D_{p2} B_{pcv} = b_p D_{p1} B_{pch} \quad (2\text{-}21)$$

The saturation curve of the core material (M-19) is shown in Fig. 2-5. The algorithm for the determination of the field of the permanent magnet is as follows:

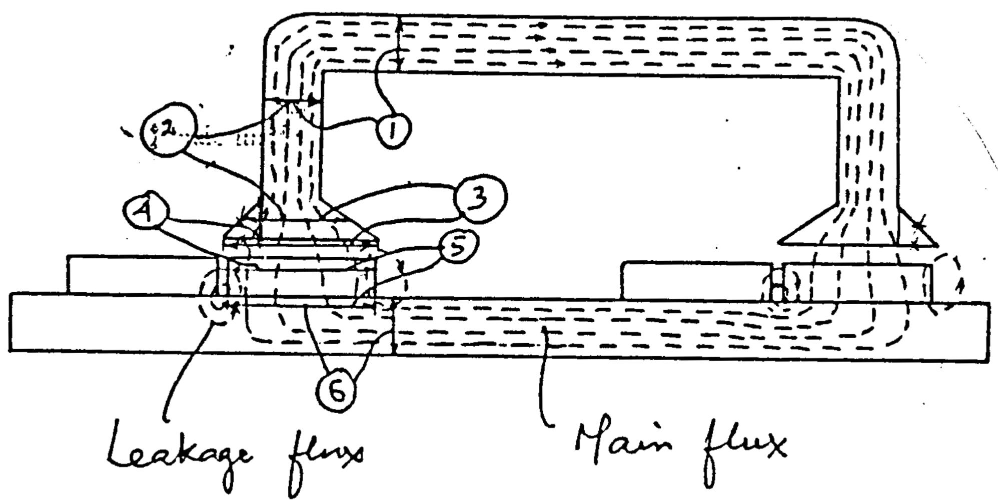


Figure 2-4 PM flux paths and continuity of flux.

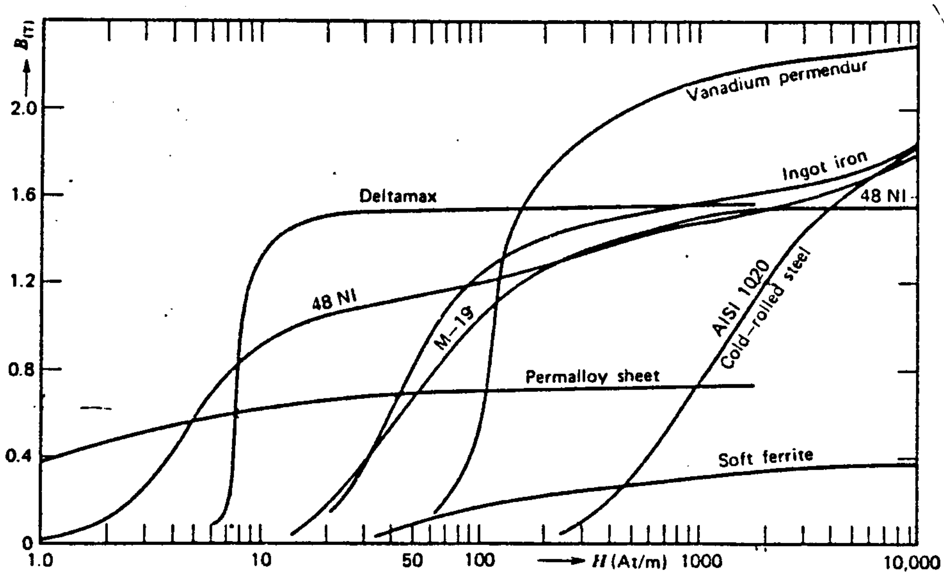


Figure 2-5 B-H curves of selected soft magnetic materials.

(i) Choose a value of H_{mo} on the demagnetization curve and an accuracy criterion ϵ.
(ii) Find B_{mo} from (2-5).
(iii) Find B in every part of the magnetic circuit with B_{mo} and (2-16) through (2-21).
(iv) Determine H for each B from the B-H curve (Fig. 2-5).
(v) Determine $H_{gav} = B_{gav}/\mu_o$ for the airgap.
(vi) Calculate H_{m1} from (2-14).
(vii) If $|H_{m0} - H_{m1}| \leq \epsilon_1$, stop calculations. The field due to this H_{mo} is the field of the permanent magnet.
(viii) If $|H_{mo} - H_{m1}| > \epsilon$, choose $\frac{1}{2}(H_{mo} + H_{m1})$ as a new H_{mo} and return to stop (ii).

2.4. Field of a Linear Alternator on Load

The flux distribution in a linear alternator is shown in Fig. 2-6, where ϕ_1 denotes the mutual flux due to the permanent magnet, ϕ_2 the mutual flux due to the stator mmf, $\phi = \phi_1 - \phi_2$ the net mutual flux and ϕ_σ the leakage flux due to permanent magnet. For ease of calculation, certain cross-sections are defined as follows. These cross-sections are denoted by S with subscripts,

$$S_{g1} = \pi (D_i - g) \frac{1}{2} (\tau + \tau_m) \tag{2-22}$$

$$S_{g11} = \pi (D_i - g)[\tau + 2(g + g_m)\frac{g/2}{g + h_m}] = \pi(D_i - g)(\tau + g) \tag{2-23}$$

$$S_{g2} = \pi (D_i - g - h_m)(\tau + g + h_m) \tag{2-24}$$

$$S_{g3} = \pi (D_i - 2g - h_m)(\tau + 2g + h_m) \tag{2-25}$$

$$S_{g3m} = \pi (D_i - 2g - h_m) \tau_m \tag{2-26}$$

$$S_{sp} = \pi (D_i + h_1) \frac{1}{2} (\tau + b_t) \tag{2-27}$$

$$S_{scv} = \pi (D_i + 2h_1 + h_2 + \frac{1}{2} b_b) b_t \tag{2-28}$$

$$S_{sch} = \pi (D_i + 2h_1 + 2h_2 + b_b) b_b \tag{2-29}$$

$$S_{pcv} = \pi (D_i - 2g - 2h_m - \frac{1}{2} b_p)(\tau + 2g + 2h_m) \tag{2-30}$$

$$S_{pch} = \pi (D_i - 2g - 2h_m - b_p)b_p \tag{2-31}$$

Let ϕ_m denote the flux due to the permanent magnet within the permanent magnets. Then:

$$\phi_m = \phi_\sigma + \phi_1 \tag{2-32}$$

and in the permanent magnet:

$$B_m = \frac{\phi_m}{S_{g3m}} - \frac{\phi_2}{S_{g3}} \tag{2-33}$$

In the airgap:

$$\frac{\phi_2}{S_{g2}\mu_o} (g + h_m) = - H_m h_m - (\frac{\phi_1}{s_{g1}} - \frac{\phi_2}{s_{g11}}) \frac{g}{\mu_o} \tag{2-34}$$

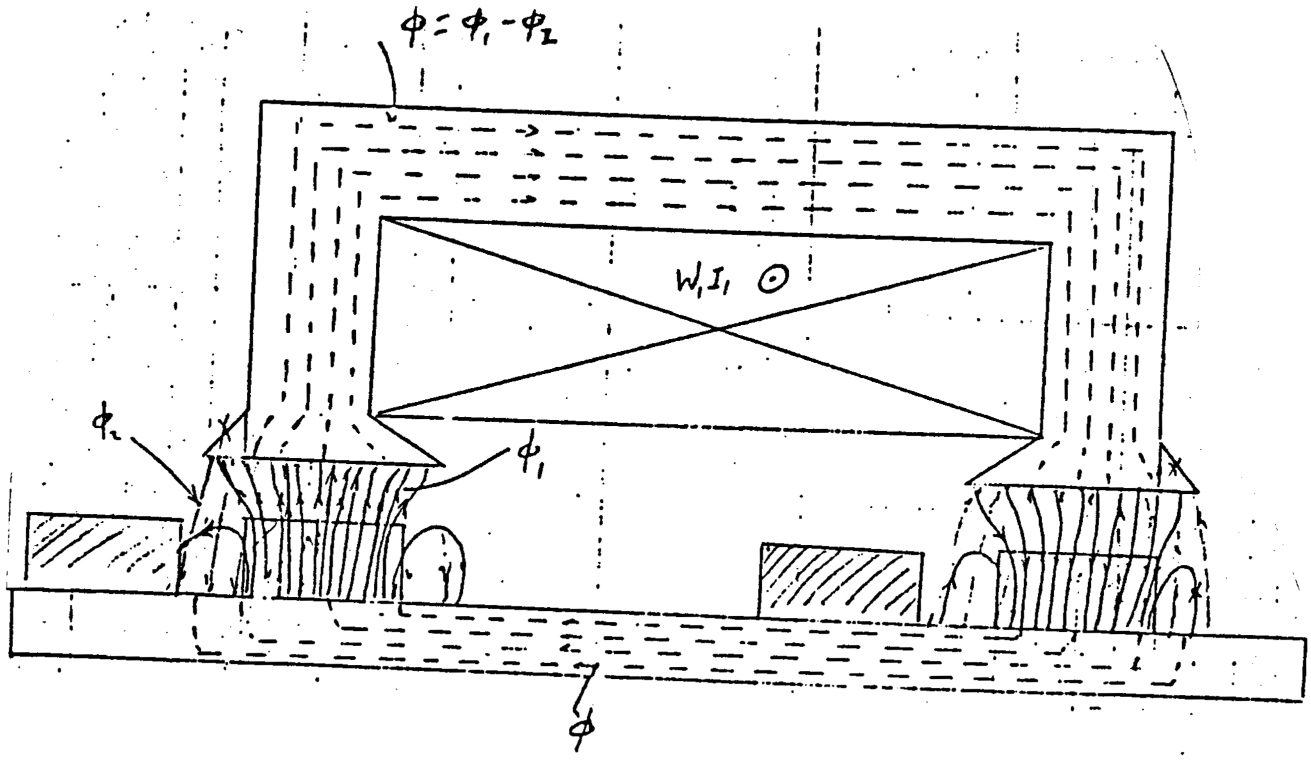


Figure 2-6 Fluxes in a LA.

In the iron portion of stator, total flux

$$\phi = \phi_1 - \phi_2 \tag{2-35}$$

$$B_{sp} = \frac{\phi}{S_{sp}} \tag{2-36}$$

$$B_{scv} = \frac{\phi}{S_{scv}} \tag{2-37}$$

$$B_{sch} = \frac{\phi}{S_{sch}} \tag{2-38}$$

In the iron portion of the plunger:

$$B_{pcv} = \frac{\phi_m - \phi_2}{S_{pcv}} \tag{2-39}$$

$$B_{pch} = \frac{\phi_m - \phi_2}{S_{pch}} \tag{2-40}$$

If ϕ_1, ϕ_2 and ϕ_σ are known, flux densities in every portion of the iron can be obtained from (2-32) and (2-35)-(2-40). For the corresponding flux density, the field intensity can be found from the magnetization curve of the core material.

Let:

$$F_m = 2H_{sp}h_1 + 2H_{scv}\,(h_2 + \frac{1}{2}\,b_b) + H_{sch}b + 2H_{pcv}\,(\frac{1}{2}\,b_p) + H_{pch}b \tag{2-41}$$

By Ampere's Law:

$$\frac{\phi_2}{\phi_1 - \phi_2}\,F_m + \frac{2\phi_2}{S_{g2}\mu_o}\,(g + h_m) = W_1 I \tag{2-42}$$

From Eq. (2-4), (2-15), (2-32) and (2-33):

$$H_m = H_c - \frac{H_c}{B_r}\,[\frac{\phi_1 - kH_m}{S_{g3m}} - \frac{\phi_2}{S_{g3}}] \tag{2-43}$$

or,

$$[1 - \frac{kH_c}{B_r S_{g3m}}]\,H_m = H_c - \frac{H_c\phi_1}{B_r S_{g3m}} + \frac{H_c\phi_2}{B_r S_{g3}} \tag{2-44}$$

Let

$$d = 1 - \frac{kH_c}{B_r S_{g3m}} \tag{2-45}$$

Equation (2-44) becomes:

$$dH_m = H_c - \frac{H_c\phi_1}{B_r S_{g3m}} + \frac{H_c\phi_2}{B_r S_{g3}} \tag{2-46}$$

and

$$\phi_1 = [(H_c - dH_m)\,\frac{B_r}{H_c} + \frac{\phi_2}{S_{g3}}]\,S_{g3m} \tag{2-47}$$

Substituting (2-47) into (2-34) yields:

$$\frac{\phi_2}{s_{g2}\mu_o}(g + h_m) = -H_m h_m - [(H_c - dH_m)\frac{B_r}{H_c} + \frac{\phi_2}{S_{g3}}]\frac{S_{g3m}g}{S_{g1}\mu_o} + \frac{\phi_2 g}{S_{g11}\mu_o} \qquad (2\text{-}48)$$

Or,

$$\phi_2 = \frac{-\dfrac{B_r S_{g3m} g}{S_{g1}}}{\dfrac{g + h_m}{S_{g2}} + \dfrac{S_{g3m} g}{S_{g1} S_{g3}} - \dfrac{g}{S_{g11}}} - \frac{\mu_o h_m - \dfrac{dB_r S_{g3m} g}{H_c S_{g1}}}{\dfrac{g + h_m}{S_{g2}} + \dfrac{S_{g3m} g}{S_{g1} S_{g3}} - \dfrac{g}{S_{g11}}} H_m \qquad (2\text{-}49)$$

Let:

$$a = \frac{-\dfrac{B_r S_{g3m} g}{S_{g1}}}{\dfrac{g + h_m}{S_{g2}} + \dfrac{S_{g3m} g}{S_{g1} S_{g3}} - \dfrac{g}{S_{g11}}} \qquad (2\text{-}50)$$

and

$$c = -\frac{\mu_o h_m - \dfrac{dB_r S_{g3m} g}{H_c S_{g1}}}{\dfrac{g + h_m}{S_{g2}} + \dfrac{S_{g3m} g}{S_{g1} S_{g3}} - \dfrac{g}{S_{g11}}} \qquad (2\text{-}51)$$

Now

$$\phi_2 = a + cH_m \qquad (2\text{-}52)$$

Substituting (2-52) into (2-42) gives:

$$\frac{\phi_2}{\phi_1 - \phi_2} F_m + \frac{2(g + h_m)}{S_{g2}\mu_o}(a + cH_m) = W_1 I \qquad (2\text{-}53)$$

$$H_m = \frac{1}{c}[-\frac{\phi_2}{\phi_1 - \phi_2} F_m + W_1 I]\frac{S_{g2}\mu_o}{2(g + h_m)} - a] \qquad (2\text{-}54)$$

To find the field of the linear alternator, choose (or guess) an initial value H_{mo} (of H_m). From (2-52), (2-47), (2-15), (2-32) and (2-35)-(2-40), the ϕ_2, ϕ_1, ϕ_σ and the flux densities in all parts of iron corresponding to H_{mo} can be found. Another value of H_m is obtained by the magnetization curve of iron and Eq. (2-41) and (2-54). After a few iterations exact values of H_m and the fluxes are obtained.

2.5. An Example

A single-slot single-phase tubular PM linear alternator has the configuration shown in Fig. 2-1 and has the following dimensions and parameters:

$\tau = 1.2''$; $\tau_m = 1.0''$; $h_1 = 0.2''$; $h_2 = 0.8''$; $g = 0.036''$;

$h_m = 0.46''$; $b_t = 0.5''$; $b_b = 0.45''$; $b_p = 0.7''$; $b_{s2} = 3.44''$;

$D_i = 11''$; $B_r = 1.07\ T$; $H_c = -770.305\ kA/m$; $W_1 = 21\ turns$

By the calculating algorithm in section 2.3, the no load field of the alternator is calculated. The B and H in every part of the machine are:

$B_m = 0.97242$ T $\qquad H_m = $ -70251.93000 A/m

$B_{sp} = 0.93418$ T $H_{sp} = 81.77248$ A/m
$B_{scv} = 1.43153$ T $H_{scv} = 973.84080$ A/m
$B_{sch} = 1.46938$ T $H_{sch} = 1698.5440$ A/m
$B_{pcv} = 1.05397$ T $H_{pcv} = 100.79380$ A/m
$B_{pch} = 1.56229$ T $H_{pch} = 3477.81500$ A/m
$B_{gav} = 0.73740$ T $H_{gav} = 585241.40000$ A/m

The mutual flux produced by PM is:

$$\phi = 0.01817 \ Wb$$

The leakage coefficient of PM is:

$$k_{\sigma m} = 1 + \frac{\phi_\sigma}{\phi} = 1.1446$$

When the stator current $I_1 = 209.3\ A$ and $\theta = 45^\circ$ (shown in Fig. 2-7). The demagnetizing component of the stator current is:

$$I = I_1 \sin\theta = 209.3 \times \sin 45^\circ = 148\ A$$

By the formulas given in section 2.4, the field is:

$B_m = 0.85001$ T $H_m = -158371.40000$ A/m
$B_{sp} = 0.50827$ T $H_{sp} = 40.57893$ A/m
$B_{scv} = 0.77887$ T $H_{scv} = 63.09837$ A/m
$B_{sch} = 0.79946$ T $H_{sch} = 64.95149$ A/m
$B_{pcv} = 0.80115$ T $H_{pcv} = 65.14345$ A/m
$B_{pch} = 1.18753$ T $H_{pch} = 127.50650$ A/m

The mutual flux due to PM is:

$$\phi_1 = 0.01549 \ Wb$$

The mutual flux due to the current I is:

$$\phi_2 = 0.00561 \ Wb$$

Net mutual flux:

$$\phi = \phi_1 - \phi_2 = 0.00988 \ Wb$$

The leakage coefficient of PM is:

$$k_{\sigma m} = 1 + \frac{\phi_\sigma}{\sigma_1} = 1.3822$$

The coefficient $k_{\sigma m}$ on load is larger than that for no load. The reason is that the magnitude of H_m increases with an increase in I.

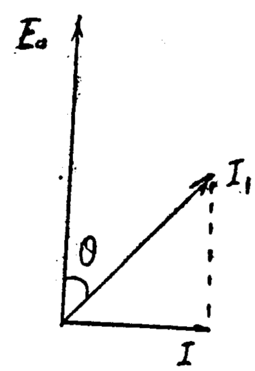


Figure 2-7 The phase relation between E_o and I_1.

CHAPTER 3

THE LEAKAGE FIELD OF PERMANENT MAGNETS IN A PM LINEAR ALTERNATOR

The leakage of the permanent magnets in a tubular PM linear alternator is shown in Fig. 3-1. Here the symmetrical line of a permanent magnet coincides with that of the stator teeth. For other positions of the permanent magnets the leakage field may be found in a way similar to the following. The field shown in Fig. 3-1 is only in the vicinity of a stator pole. The entire region of the field is divided into a number of sub-regions as shown. Flux calculations are made for these sub-regions.

In the flux calculations, the following conditions are assumed:

(a) In the sub-region ABCDA, the fluxes are perpendicular to AB and CD.

(b) In the sub-region BCGFEB, the flux pathes are semi-circles with different diameters and the center of those circles is the middle point between B and C.

(c) In the sub-region HIJKH, the flux pathes are quarter circles with center at point H.

(d) In the sub-region IJKPOMLI, the flux pathes are composed by quarter circles with center at point H and quarter circles with center at point K.

With the above assumption, in sub-region ABCD:

$$F_m = -H_m h \tag{3-1}$$

$$d\phi_{\sigma 1} = \frac{F_m}{\dfrac{g_m/2}{\mu_o \pi (D_i - 2g - 2h_m + 2h)dh}} = \frac{-2H_m \mu_o \pi (D_i - 2g - 2h_m + 2h)h}{g_m}\, dh \tag{3-2}$$

$$\phi_{\sigma 1} = \int_0^{h_m} d\phi_{\sigma 1} = \int_0^{h_m} \frac{-2H_m \mu_o \pi (D_i - 2g - 2h_m + 2h)h}{g_m}\, dh = -\frac{H_m \mu_o \pi}{g_m} [(D_i - 2g - 2h_m)\, h_m^2 + \frac{4}{3}\, h_m^3] \tag{3-3}$$

In sub-region BCGFEB:

$$r_1 = \sqrt{g^2 + [\frac{1}{2}\,(\tau_m + g_m - \tau)]^2} \tag{3-4}$$

$$F_m = -\,2H_m h_m \tag{3-5}$$

The reluctance on a flux path with width dr is:

$$dR_2 = \frac{\pi r}{\pi (D_i - g)\mu_o dr} = \frac{r}{\mu_o (D_i - g) dr} \tag{3-6}$$

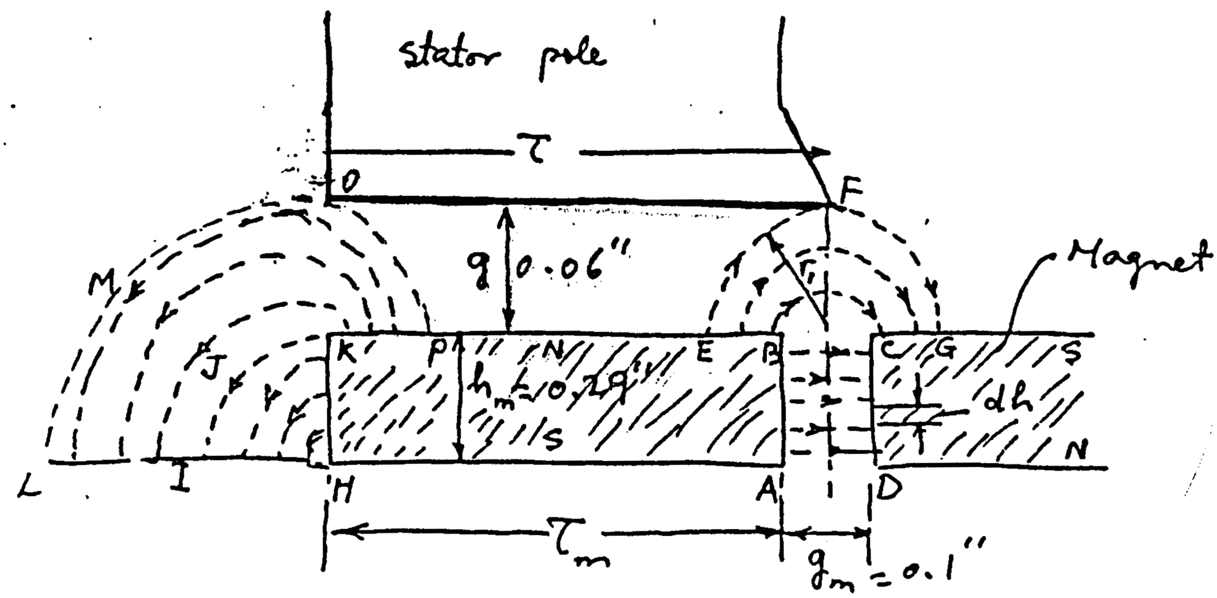


Figure 3-1 Leakage field of the permanent magnets.

where $D_i - g$ is the average diameter of the cross-section of the flux path.

The infinitesimal flux over the path is:

$$d\phi_{\sigma 2} = \frac{F_m}{dR_2} = -2\,\mu_o H_m h_m\,(D_i - g)\,\frac{dr}{r} \tag{3-7}$$

$$\phi_{\sigma 2} = \int_{g_m/2}^{r_1} d\phi_{\sigma 2}$$

$$= -2H_m \mu_o h_m (D_i - g) \ln \frac{2r_1}{g_m} \tag{3-8}$$

In the sub-region HIJKH:

$$F_m = -H_m\,h \tag{3-9}$$

$$dR_3 = \frac{\frac{\pi h}{2}}{\pi(D_i - 2g - h_m)\,\mu_o dh} = \frac{h}{2(D_i - 2g - h_m)\,\mu_o\,dh} \tag{3-10}$$

where $D_i - 2g - h_m$ is the average diameter of cross-section of the flux path.

$$d\phi_{\sigma 3} = \frac{F_m}{dR_3} = -\,2H_m \mu_o\,(D_i - 2g - h_m)\,dh \tag{3-11}$$

or

$$\phi_{\sigma 3} = \int_0^{h_m} d\phi_{\sigma 3}$$

$$= -2H_m \mu_o (D_i - 2g - h_m)\,h_m \tag{3-12}$$

In the sub-region of LMOPKJIL:

$$F_m = -H_m\,h_m$$

$$dR_4 = \frac{\frac{1}{2}\,\pi\,(h_m + h) + \frac{1}{2}\pi h}{\pi(D_i - g - h_m)\mu_o dh}$$

$$= \frac{h_m + 2h}{2(D_i - g - h_m)\mu_o\,dh} \tag{3-13}$$

$$d\phi_{\sigma 4} = \frac{F_m}{dR_4} = \frac{-2H_m \mu_o (D_i - g - h_m) h_m}{h_m + 2h}\,dh \tag{3-14}$$

$$\phi_{\sigma 4} = \int_0^{g} d\phi_{\sigma 4}$$

$$= -H_m\,\mu_o\,(D_i - g - h_m) \ln\left(\frac{h_m + 2g}{h_m}\right) h_m \tag{3-15}$$

Total leakage flux:

$$\phi_\sigma = \phi_{\sigma 1} + \phi_{\sigma 1} + \phi_{\sigma 3} + \phi_{\sigma 4}$$

$$= -H_m\mu_o \left\{ \frac{\pi}{g_m} \left[D_i - 2g - 2h_m)h_m + \frac{4}{3}\, h_m^2 \right] + 2(D_i - g)\ln\, \frac{2r_1}{g_m} \right.$$

$$\left. + 2(D_i - 2g - h_m) + (D_i - g - h_m)\ln\,(\frac{h_m + 2g}{h_m}) \right\} h_m \qquad (3\text{-}16)$$

where r_1 is given in (3-4).

The Eq. (3-16) may be rewritten as:

$$\phi_\sigma = -KH_m \qquad (3\text{-}17)$$

where

$$K = \mu_o \left\{ \frac{\pi}{g_m} \left[(D_i - 2g - 2h_m)h_m + \frac{4}{3}\, h_m^2 \right] + 2(D_i - g)\ln\, \frac{2r_1}{g_m} \right.$$

$$\left. + 2(D_i - 2g - h_m) + (D_i - g - h_m)\ \ln\ \left(\frac{h_m + 2g}{h_m} \right) \right\} h_m \qquad (3\text{-}18)$$

It is obvious that K is dependent on the configuration of the alternator. The flux ϕ_m in the permanent magnet is given by:

$$\phi_m = \pi\,(D_i - 2g - h_m)\ \tau_m B_m \qquad (3\text{-}19)$$

Combining (3-17), (3-19) and (2-4) yields:

$$\phi_\sigma = -K(H_c - \frac{H_c}{B_r}\, B_m)$$

$$= -K\ H_c + K\,\frac{H_c}{B_r}\ \frac{1}{\pi(D_i - 2g - h_m)\ \tau_m}\ \phi_m \qquad (3\text{-}20)$$

Equation (3-20) shows that the leakage flux decreases linearly with B_m (or ϕ_m). This happens because ϕ_σ is directly proportional to H_m, which decreases linearly with B_m. The conclusion is that after the dimensions of the linear alternator have been decided, to reduce leakage flux B_m has to be chosen as high as possible.

The above analysis may be used to calculate the leakage coefficient of the linear alternator. By definition the leakage coefficient is:

$$k_{\sigma m} = \frac{\phi_m}{\phi_m - \phi_\sigma} \qquad (3\text{-}21)$$

From (3-20):

$$k_{\sigma m} = \frac{\phi_m}{\phi_m + KH_c - K\,\dfrac{H_c}{B_r}\ \dfrac{1}{\pi(D_i - 2g - h_m)\tau_m}\ \phi_m}$$

$$= \frac{1}{1 + K\ H_c\ (\dfrac{1}{\phi_m} - \dfrac{1}{B_r\pi\ (D_i - 2g - h_m)\tau_m})} \qquad (3\text{-}22)$$

Defining:

$$k_B = \frac{B_m}{B_r} \tag{3-23}$$

and substituting (3-19), (3-23) into (3-22) yields:

$$k_{\sigma m} = \frac{1}{1 + kH_c(1 - k_B)\,\dfrac{1}{\phi_m}} \tag{3-24}$$

Since H_c is negative, when B_m approaches B_r, i.e. k_B approaches 1, $k_{\sigma m}$ decreases to the limit 1. If B_m decreases, because ϕ_m will decrease with B_m, the second term in the denominator of (3-24) will increase rapidly. Thus $k_{\sigma m}$ will also increase rapidly.

CHAPTER 4

PARAMETERS OF PM LINEAR ALTERNATOR

The three important electrical parameters of a single-phase PM linear alternator are: leakage inductance, magnetizing inductance and resistance of the stator winding. The methods and formulas to find the three parameters are presented as follows.

4.1. Determination of Slot Leakage Inductance

The shape and the dimensions of the slot of a tubular single-phase single-slot tubular PM linear alternator are shown in Fig. 4-1. The winding in the slot has W_1 turns. The slot is divided into two regions—ABCF and CDEF as shown in Fig. 4-1. Let the current in the winding be I_1.

For calculating the slot leakage inductance, the following assumptions are made:

a) The current density in the coil is uniform.

b) Iron around the slot has a much larger permeability than that of air.

c) The leakage flux in the slot is parallel to the bottom of the slot.

In region ABCF, over a depth dx, the infinitesimal flux linkage is:

$$d\lambda_1 = (W_1 \frac{x}{h_2})^2 \pi(D_o - 2x) \frac{\mu_o I}{b_{s2}} dx \tag{4-1}$$

The total flux linkage in the region ABCF becomes:

$$\lambda_1 = \int_0^{h_2} d\lambda_1 = \frac{\mu_o \pi I_1 W_1^2}{b_{s2}} \int_0^{h_2} (\frac{x}{h_2})^2 (D - 2x)\, dx$$

$$= \frac{\mu_{oo} \pi I_1 W_1^2}{b_{s2}} (\frac{1}{3} D_o h_2 - \frac{1}{2} h_2^2) \tag{4-2}$$

For the region CDEF, the approximate equivalent slot width is $\frac{1}{2}(b_{s1} + b_{s2})$. Thus:

$$d\lambda_2 = \mu_o \pi I_1 W_1^2 (\frac{2}{b_{s1} + b_{s2}}) (D_i + 2y)\, dy \tag{4-3}$$

$$\lambda_2 = \int_0^{h_1} d\lambda 2 = \mu_o \pi I_1 W_1^2 \frac{2}{b_{s1} + b_{s2}} \int_0^{h_1} (D_i + 2y)\, dy$$

$$= \mu_o \pi I_1 W_1^2 \frac{2}{b_{s1} + b_{s2}} (D_i h_1 + h_1^2) \tag{4-4}$$

Total slot leakage flux linkage is:

$$\lambda_{s\sigma} = \lambda_1 + \lambda_2 \tag{4-5}$$

Hence, from (4-2), (4-4) and (4-5), the slot leakage inductance is:

$$L_{s\sigma} = \frac{\lambda_{s\sigma}}{I_1} = \mu_o \pi W_1^2 (\frac{D_o h_2}{3b_{s2}} - \frac{h_2^2}{2b_{s2}} + \frac{2D_i h_1}{b_{s1} + b_{s2}} + \frac{2h_1^2}{b_{s1} + b_{s2}}) \tag{4-6}$$

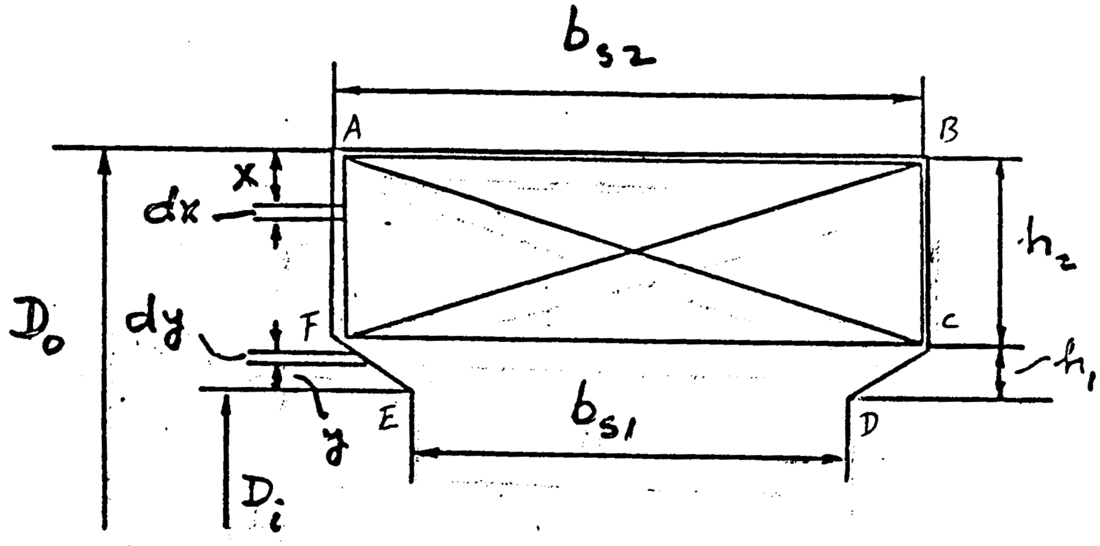


Figure 4-1 Slot geometry

where

$$D_o = D_i + 2h_1 + 2h_2 \tag{4-7}$$

Similar expressions may be obtained for other slot geometries.

For a traditional rotating alternator, with same shape of slots as shown in Fig. 4-1, the leakage inductance of one slot is:

$$L_{s\sigma} = \mu_o W_1^2 \, l_{ef} \left(\frac{h_2}{3b_{s2}} + \frac{2h_1}{b_{s1} + b_{s2}}\right) \tag{4-8}$$

where l_{ef} is the length of slot.

Comparing Eq. (4-6) and (4-8), there are two additional terms—term 2 and term 4—in (4-6). The reason is that unlike a traditional rotating alternator, the slot length of a tubular alternator is not the same for different diameters in the slot.

4.2. Determination of the Magnetizing Inductance

Applying Ampere's Law to the path ABCDEFA in Fig. 4-2, with a primary coil current I_1, we obtain:

$$\begin{aligned} &H_{sch}(b) + H_{scv}(2h_2 + b_b) + H_{sp}(2h_1) + H_{gav}2(g + h_m) \\ &\quad + H_{pcv}(b_p) + H_{pch}(b) = F = W_1 I_1 \end{aligned} \tag{4-9}$$

In Fig. 4-2, the flux distribution is given. With the dimensions defined in (2-6)-(2-12) and applying the continuity of flux condition across the five set of cross-sections in Fig. 4-2, we have:

$$(\tau + g + h_m)(D_m + g)\, B_{gav} = \frac{1}{2}(\tau + b_t)\, D_s B_{sp} \tag{4-10}$$

$$\frac{1}{2}(\tau + b_t) D_s \, B_{sp} = b_t D_{t1} \, B_{scv} \tag{4-11}$$

$$b_t D_{t1} B_{scv} = b_t D_{t2} B_{sch} \tag{4-12}$$

$$(\tau + g + h_m)(D_m + g)\, B_{gav} = (\tau + 2g + 2h_m)\, D_{p2} B_{pcv} \tag{4-13}$$

$$(\tau + 2g + 2h_m) D_{p2} B_{pcv} = b_p D_{p1} B_{pch} \tag{4-14}$$

For a certain value of B_{gav}, the flux density in other parts of the magnetic loop of the linear alternator can be obtained from Eq. (4-10) through (4-14). The corresponding field intensities are obtained from the magnetization curve of the core material. For air:

$$H_{gav} = \frac{1}{\mu_o} B_{gav} \tag{4-15}$$

After the field intensities in every part of the magnetic loop have been found, the magnetomotive force F needed to maintain the value of B_{gav} could be calculated by Eq. (4-9). The flux linkage, λ_m, is given by:

$$\lambda_m = W_1 \phi = W_1 B_{gav} \pi \, (D_m + g)(\tau + g + h_m) \tag{4-16}$$

Hence, the magnetizing inductance, L_m, may be expressed as:

$$L_m(I_1) = \frac{\lambda_m}{I_1} = \frac{W_1^2 B_{gav} \pi \, (D_m + g)(\tau + g + h_m)}{F} \tag{4-17}$$

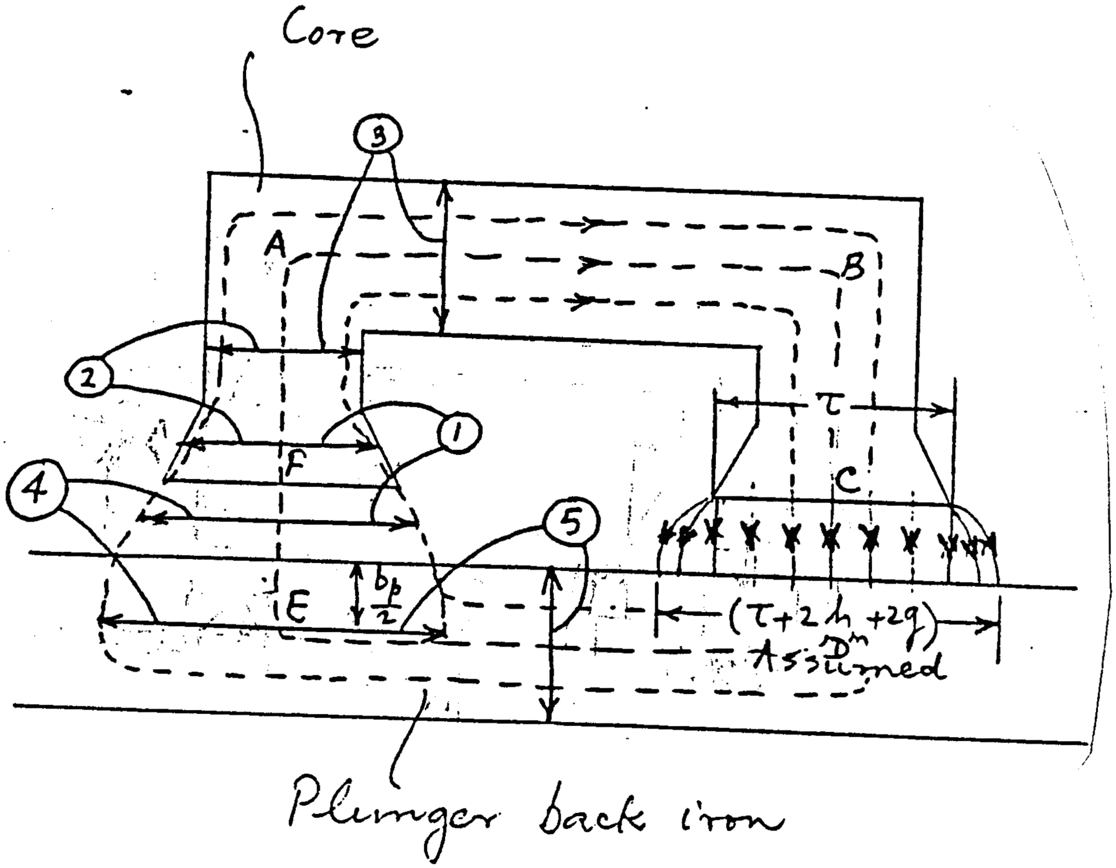


Figure 4-2 Flux path and continuity of flux.

Notice that this inductance is a function of the primary current I_1. Thus, the effect of core saturation is taken into account globally and L_m depends on the value of B_{gav}. Fig. 4-3 shows the variation of the L_m with B_{gav} in a tubular linear alternator.

4.3. The Equivalent Magnetizing Inductance of PM Linear Alternator

The method discussed in the preceding section is useful in finding the magnetizing inductance of a tubular linear alternator, such as self-excited, reluctance as well as the PM type. But for a PM linear alternator, the magnet's working point changes with the variation of load. Thus, the induced voltage will be reduced not only by the current reaction but also by the variation of the working point of the magnet when the load increases. For design and analysis of PM alternators, sometime it is convenient to define an equivalent magnetizing inductance as follows:

$$L_{me}(I_1) = \frac{E_o - E}{WI_1 \sin\theta} \tag{4-18}$$

where

E_o = no load induced voltage

E = airgap induced voltage corresponding to stator current I_1 and angle θ shown in Fig. 4-4.

ω = electrical angular frequency.

Now, the equation of circuit of Fig. 4-4 is:

$$\bar{E}_o = j\omega L_{me}\bar{I}_1 + j\omega L_{s\sigma}\bar{I}_1 + R_a\bar{I}_1 + \bar{V} \tag{4-19}$$

Consequently, $L_{me}(I_1)$ is called *an equivalent magnetizing inductance.*

4.4. Resistance of Stator Winding

The resistance of stator winding is given by:

$$R_a = \frac{\rho}{A_c} W_1 \pi (D_i + 2h_1 + h_2) \tag{4-20}$$

where

ρ = the resistivity of stator winding material

A_c = cross-section of stator winding wires

$D_i + 2h_1 + h_2$ = average diameter of stator winding

W_1 = turns of stator winding

4.5 An Example

For the PM tubular linear alternator, with the dimensions given in section 2.5, the leakage inductance of the slot is:

$$L_{s\sigma} = \pi W_1^2 \mu_o \left[\frac{(D_i + 2h_1 + 2h_2)h_2}{3\ b_{s2}} - \frac{h_2^2}{2\ b_{s2}} + \frac{2D_i h_1}{b_{s1} + b_{s2}} + \frac{2h_1^2}{b_{s1} + b_{s2}} \right]$$

$$= \pi \times 21^2 \times 1.26 \times 10^{-6} \left[\frac{(11 + 2 \times 0.2 + 2 \times 0.8) \times 0.8}{3 \times 3.44} - \frac{0.8^2}{2 \times 3.44} \right.$$

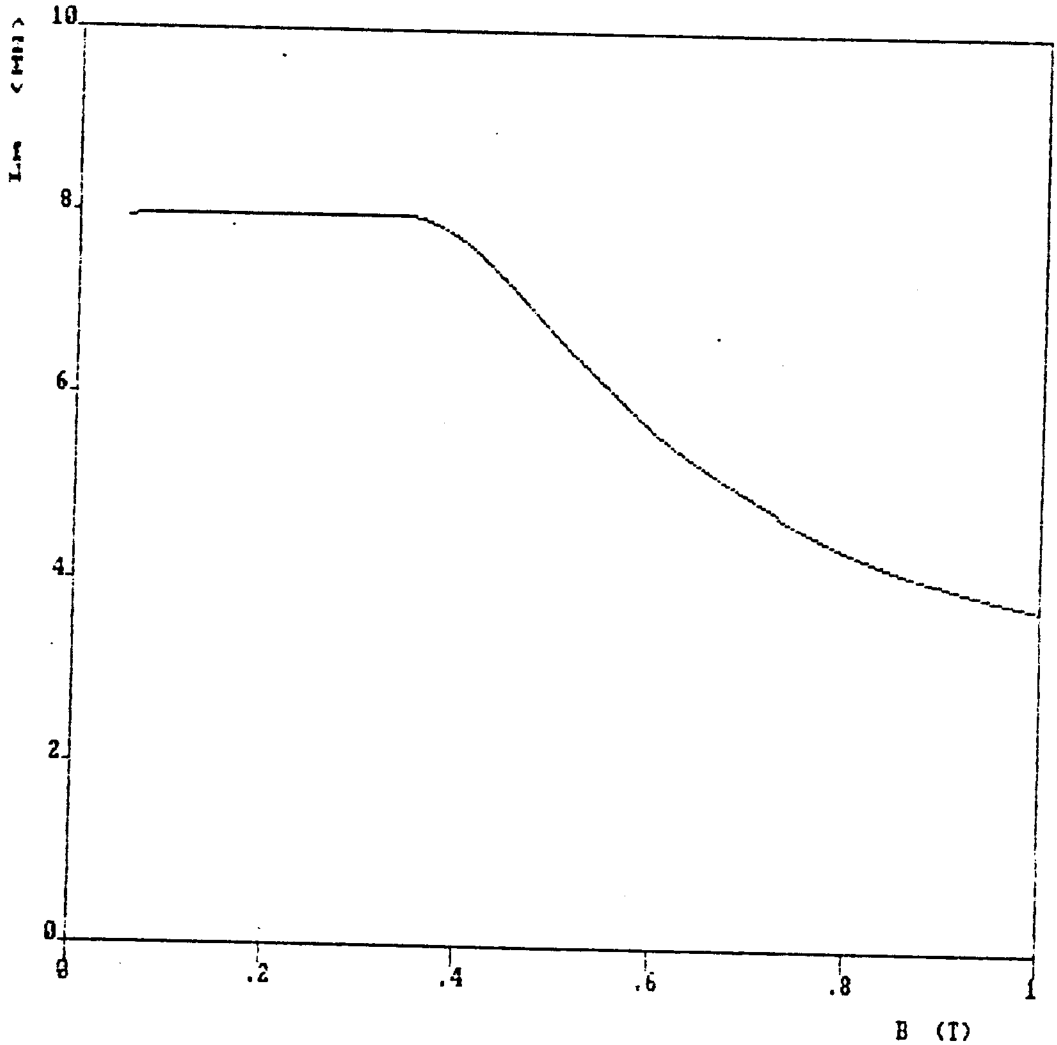


Figure 4-3 The magnetizing inductance versus flux density

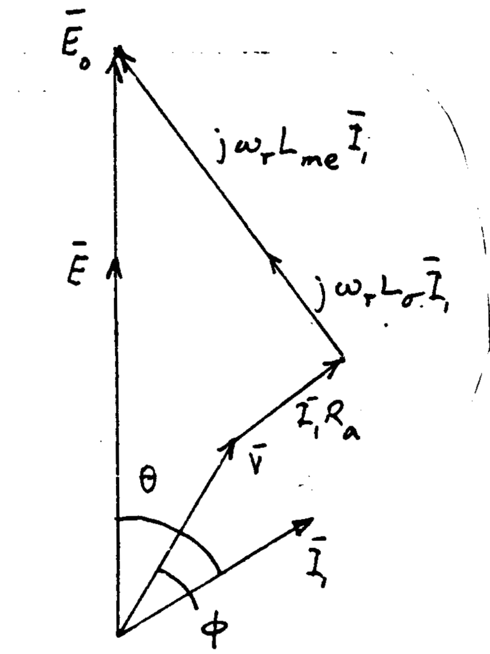


Figure 4-4 Phase diagram of a single phase alternator.

$$= + \frac{2 \times 11 \times 0.2}{2.04 + 3.44} + \frac{2 \times 0.2^2}{2.04 + 3.44} \Bigg] \times 0.0255$$

$$= 0.0771 \ mH$$

For $I_1 = 210\ A$ and $\theta = 45°$:

Using the method introduced in section 4.2, the magnetizing inductance is:

$$L_m = 0.822 \ mH$$

By the definition given in Section 4.3, the equivalent magnetizing inductance is:

$$L_{me} = 0.832 \ mH$$

The constant k, which is calculated by (3-18), is:

$$k = 0.3738 \times 10^{-7} \ Wb \ m/A \ turn$$

From this example, it can be seen that the value of L_m is close to that of L_{me}. But if k is much larger than the above value for designed alternator, the difference between L_m and L_{me} will be large.

CHAPTER 5

DESIGN OF A TUBULAR PM LINEAR ALTERNATOR

In Reference 5, the basic design guide of PM linear alternators with tuning capacitor is given. The design procedure of PM linear alternators *without tuning capacitor* is presented as follows with a design example. The configuration and dimension symbols of the designed alternator are the same as those shown in Fig. 2-1.

5.1. Given Data

Rated output power (s) = 25 kVA

Frequency (f_m) = 100 Hz

Plunger travel length (ℓ_{stroke}) = 1.0 inch

Assumptions and Estimates

Length of pole shoe of stator $\tau = 1.2\ in$

Pole length of plunger, $\tau_m = 1.0\ in$

Airgap $g = 0.036$ in

Full-load output voltage $V = 120$ V (rms)

Induced (or internal) voltage $E = \sqrt{2}(120)$ V (rms)

Maximum induced voltage $E_m = 2(120) = 240$ V (max)

Output current $I = \dfrac{S}{V} = \dfrac{25000}{120} = 208.33\ A$

Magnet material = Tascore 27 H $B_r = 1.07\ T$, $H_c = 770.305\ KA/m$

5.3. Design Procedure

Let $B_m = 0.97$ T (satisfied after iterations)

$$H_m = H_c\ (1 - \frac{B_m}{B_r}) = -\ 770.305\ (1 - \frac{0.97}{1.07}) = -\ 71.99\ kA/m \tag{5-1}$$

$$B_{gav} = \frac{B_m \tau_m}{\frac{1}{2}\ (\tau_m + \tau) K_{\sigma m}} = \frac{0.97 \times 1}{1.1 \times 1.05} = 0.7688\ T \tag{5-2}$$

where $k_{\sigma m}$ = leakage coefficient = 1.05 (assumed)

$$H_m h_m = \frac{B_{gav}}{\mu_o}\ g\ k_s = H_{gav}\ g\ k_s \tag{5-3}$$

where k_s = no-load saturation factor = 1.5 (assumed)

From (5-3):

$$h_m = \frac{B_{gav}}{\mu_o H_m}\ g k_s = \frac{0.7668 \times 0.036 \times 0.0254 \times 1.5}{1.26 \times 10^{-6} \times 71.99 \times 10^3} = 0.456\ in$$

The emf equation is:

$$E_m = B_{gav}\ W_1 \omega_r \ell_{stroke} \pi D_i \tag{5-4}$$

where W_1 is the number of turns on stator. Thus, from (5-4) with $W_1 = 21$ turns.

$$D_i = \frac{E_m}{\pi B_{gav} W_1 \omega_1 \ell_{stroke}} = \frac{240}{\pi \times 0.7668 \times 21 \times 628 \times 1 \times 0.0254} \tag{5-5}$$

$$= 0.2963\ m = 11.62\ in$$

5.4. Summary of Design Data

$D_i = 11\ in = 0.2805\ in$

$h_m = 0.46\ in$

$g = 0.036\ in$

$\tau = 1.2\ in$

$\tau_m = 1\ in$

$g_m = \tau - \tau_m = 0.2\ in$

5.5. Further Calculations

$$K = \left\{ \frac{\pi}{g_m} \left[(D_i - 2g - h_m)h_m + \frac{4}{3}\ h_m^2 \right] + 2(D_i - 2g - h_m) \right.$$

$$\left. + (D_i - g - h_m)\ln \left(\frac{h_m + 2g}{h_m} \right) \right\} h_m \mu_o$$

Leakage flux of permanent magnet is:

$$\phi_\sigma = -KH_m = -KH_c\ (1 - \frac{B_m}{B_r}) \tag{5-7}$$

The flux linking with the stator coil is:

$$\phi = \pi(D_i - 2g - h_m)\tau_m\ B_m - \phi_\sigma \tag{5-8}$$

which is related to the induced voltage by:

$$E = \sqrt{2}\ \pi\ W_1\ f_m\ \phi \tag{5-9}$$

With the current density chosen as $J_{co} = 5A/mm^2$, the wire cross-section area is given by:

$$A_{co} = \frac{I}{J_{co}} = \frac{208.33}{5} = 41.66\ mm^2 = 0.0641\ in^2$$

To obtain this area of cross-section, 4 No. 7AWG wires, each having a diameter of 0.1443 in, may be used in parallel. Thus:

$$A_{co} = 4 \times \frac{\pi}{4} (0.1443 \times 25.5)^2 = 42.54\ mm^2$$

Let the slot fill-factor $k_{fill} = 0.5$. Then slot area:

$$A = \frac{W_1 A_{co}}{k_{fill}} = \frac{21 \times 42.54}{0.5} = 1786.68\ mm^2 = 2.75\ in^2 \quad (5\text{-}10)$$

Let $b_t = 0.5$ in, $b_b = 0.45$ in, $h_1 = 0.2$ in, $b_p = 0.7$ in, $h_2 = 0.8$ in, and $b_{s2} = 3.44$ in. Then:

$$b_{s2} h_2 = 3.44 \times 0.8 = 2.753\ in^2 = A \quad (5\text{-}11)$$

which is adequate slot area.

By the computer calculation for above design, the following no-load data are obtained:

$$E = 169.505\ V$$

$$\phi = 0.01817\ W_b$$

$$k_{\sigma m} = 1.1446$$

$$B_m = 0.9724\ T$$

$$H_m = -70.252\ kA/m$$

Average diameter of the winding:

$$D_{av} = D_i + 2h_1 + h_2 = 11 + 0.4 + 0.8 = 12.2\ in \quad (5\text{-}12)$$

Stator winding resistance:

$$R_a = \rho \frac{W_1 \pi D_{av}}{A_{co}} = 2.1 \times 10^{-8} \frac{21 \times \pi \times 12.2 \times 0.0254}{42.54 \times 10^{-6}} = 0.0103\ \Omega \quad (5\text{-}13)$$

Stator leakage inductance:

$$L_\sigma = \mu_o \pi W_1^2 \left[\frac{(D_i + 2h_1 + 2h_2) h_2}{3 b_{s2}} + \frac{2 D_i h_1}{b_{s1} + b_{s2}} - \frac{h_2^2}{2 b_{s2}} + \frac{2 h_1^2}{b_{s1} + b_{s2}} \right]$$

$$= 1.26 \times 10^{-6} \times \pi \times 21^2 \left[\frac{13 \times 0.8}{3 \times 3.44} + \frac{2 \times 11 \times 0.2}{2.04 + 3.44} - \frac{0.8^2}{2 \times 3.44} + \frac{2 \times 0.2^2}{2.04 + 3.44} \right] \times 0.0254$$

$$= 0.0771\ mH \quad (5\text{-}14)$$

For $I = 208.33$ A and $\theta = 45^{\circ}$, equivalent magnetizing inductance $L_{me} = 0.832$ mH, as calculated by computer. This inductance accounts for armature reaction as well as the effect of change of the magnet leakage flux. Hence, the total synchronous reactance of the linear alternator becomes:

$$X_s = \omega_r (L_{me} + L_\sigma) = 628(0.832 + 0.0771) \times 10^{-3}$$

$$= 0.5709\ \Omega \quad (5\text{-}15)$$

5.6. Maximum Power Condition

Suppose that the load of the linear alternator is purely resistive and armature resistance is negligible, the phasor diagram of the linear alternator is shown in Fig. 5-1.

$$\text{Output power: } S = VI \tag{5-16}$$

From Fig. 5-1,

$$I X_s = E \sin \theta \tag{5-17}$$

or

$$I = \frac{E}{X_s} \sin\theta \tag{5-18}$$

And

$$V = E \cos \theta \tag{5-19}$$

Thus,

$$S = \frac{E^2}{X_s} \sin \theta \cos \theta = \frac{E^2}{2X_s} \sin 2\theta \tag{5-20}$$

which is maximum when $2\theta = 90°$ or $\theta = 45°$. Under this condition,

$$I = \frac{E}{\sqrt{2}\, X_s} = \frac{169.505}{\sqrt{2} \times 0.5709} = 209.95\ A \tag{5-21}$$

Rated Load: $R_L = X_s - R_a = 0.5709 - 0.0101 = 0.5608\ \Omega$

Output Voltage: $V = I\, R_L = 209.95 \times 0.5608 = 117.74\ V$

Output kVA $= 117.74 \times 209.95 \times 10^{-3} = 24.72$ kVA

5.7. Weight-to-Power Ratio

$$(Vol)_{stator} = \frac{\pi}{4} \left[(D_i + 2h_1 + 2h_2 + 2b_b)^2 - (D_i + 2h_1 + 2h_2)^2\right](2b_t + b_{s2})$$

$$+ \frac{\pi}{4} \left[(D_i + 2h_1 + 2h_2)^2 - (D_i + 2h_1)^2\right] 2b_t$$

$$+ \frac{2\pi}{4} \left[(D_i + 2h) - D_i^2\right]\left[\frac{1}{2} (b_t + \tau)\right]$$

$$= \frac{\pi}{4} (13.9^2 - 13^2) \times 3.54 + \frac{\pi}{4} (13^2 - 11.4^2) \times 1.0 + \frac{\pi}{4} (11.4^2 - 11^2) \times 0.85$$

$$= 109.936\ in^3 = 0.001823\ m^3 \tag{5-25}$$

$$(Vol)_{magnet} = 4\left[(D_i - 2g)^2 - (D_i - 2g - 2h_m)^2\right] \frac{\pi}{4}\, \tau_m$$

$$= (10.94^2 - 10.02^2) \times \pi \times 1$$

$$= 60.58\ in^3 = 0.001005\ m^3 \tag{5-26}$$

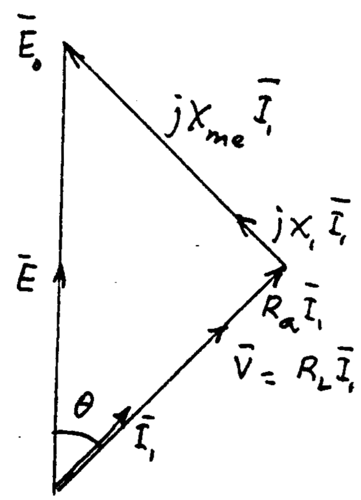


Figure 5-1 Phasor diagram of the alternator.

$$(Vol)_{plunger} = \frac{\pi}{4}\left[(D_i - 2g - 2h_m)^2 - (D_i - 2g - 2h_m - 2b_p)^2\right](b_{s2} + 2\tau)$$

$$= \frac{\pi}{4}(10.02^2 - 8.62)^2(3.44 + 2.4)$$

$$= 119.695\ in^3 = 0.001985\ m^3 \qquad (5\text{-}27)$$

$$(Vol)_{copper} = \pi\ D_{av}\ A_{co}\ W_1$$

$$= \pi \times 12.2 \times 0.06542 \times 21 = 52.655\ in^3 = 0.000873\ m^3 \qquad (5\text{-}28)$$

Assuming a mean mass density of 7.8×10^3 kg/m^3, the total weight of the linear alternator becomes:

$$W = (0.001823 + 0.001005 + 0.001985 + 0.000873) \times 7.8 \times 10^3$$

$$= 44.3508\ kg = 97.572\ lb. \qquad (5\text{-}29)$$

$$\frac{\text{weight}}{\text{power}} = \frac{44.3808}{24.72} = 1.794\text{kg/kW} = 3.947\ \text{lb/kW} \qquad (5\text{-}30)$$

5.8. Losses and Electrical Efficiency

Copper loss:

$$I^2R_a = 209.95^2 \times 0.0101 = 445.198\ W \qquad (5\text{-}31)$$

Core loss:

First we determine the weight of iron:

$$W_{sp} = 2 \times \frac{\pi}{4}\left[(D_i + h_1)^2 - D_i^2\right]\frac{1}{2}(b_t + \tau) \times 7.8 \times 0.254^3$$

$$= 1.5473\ kg = 3.404\ lb \qquad (5\text{-}32)$$

$$W_{scv} = \frac{\pi}{4}\left[(D_i + 2h_1 + 2h_2)^2 - (D_i + 2h_1)^2\right]2b_t \times 7.8 \times 0.254^3$$

$$= 3.9656\ kg = 8.724\ lb \qquad (5\text{-}33)$$

$$W_{sch} = \frac{\pi}{4}\left[(D_i + 2h_1 + 2h_2 + 2b_b)^2 - \right.$$

$$\left. - (D_i + 2h_1 + 2h_2)^2\right](b_{s2} + 2b_t) \times 7.8 \times 0.254^3$$

$$= 8.7056\ kg = 19.153\ lb \qquad (5\text{-}34)$$

$$W_p = 1.985 \times 7.8 = 15.483\ kg = 34.063\ lb \qquad (5\text{-}35)$$

From the flux density of every port of iron and core loss curve shown in Fig. 5-2, the core loss coefficients are found as follows:

$$C_{sp} = 0.304\left(\frac{100}{60}\right)^{1.3},\ C_{scv} = 0.587\left(\frac{100}{60}\right)^{1.3},\ C_{sch} = 0.609\left(\frac{100}{60}\right)^{1.3}\ \text{and}\ C_p = 0.957\left(\frac{100}{60}\right)^{1.3}$$

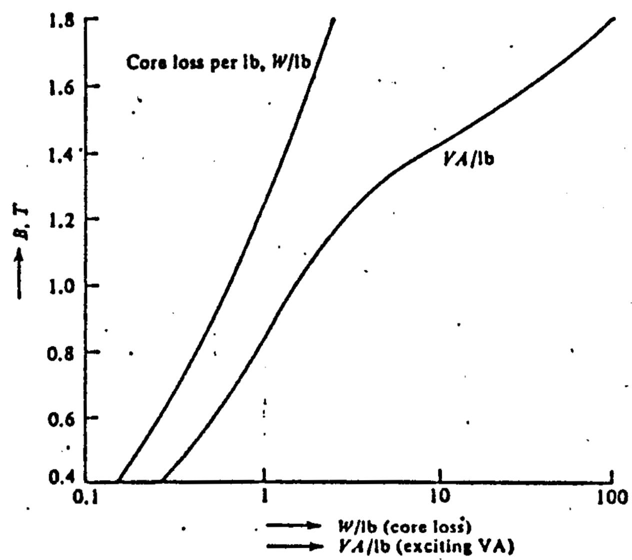


Figure 5-2 Core loss and exciting VA for M-19 steel at 60 Hz.

The corresponding losses are:

$$(loss)_{sp} = W_{sp} C_{sp} = 3.404 \times 0.304 \left(\frac{100}{60}\right)^{1.3} = 2.0103 \ W \quad (5\text{-}36)$$

$$(loss)_{scv} = W_{scv} C_{scv} = 8.724 \times 0.587 \left(\frac{100}{60}\right)^{1.3} = 9.949 \ W \quad (5\text{-}37)$$

$$(loss)_{sch} = W_{sch} C_{sch} = 19.153 \times 0.609 \left(\frac{100}{60}\right)^{1.3} = 22.66 \ W \quad (5\text{-}38)$$

$$(loss)_{p} = W_{p} C_{p} = 34.063 \times 0.957 \left(\frac{100}{60}\right)^{1.3} = 63.329 \ W \quad (5\text{-}39)$$

$$(loss)_{total\ core} = (loss)_{sp} + (loss)_{scv} + (loss)_{sch} + (loss)_{p} = 97.948 \ W \quad (5\text{-}40)$$

$$\text{copper loss} + \text{core loss} = 445.198 + 97.948 = 543.146 \ W \quad (5\text{-}41)$$

$$\text{Electrical efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{24.72}{24.72 + 0.543} = 97.85\% \quad (5\text{-}42)$$

CHAPTER 6

OPTIMAL DESIGN OF A PM LINEAR ALTERNATOR

6.1. Concept of Optimal Design

For a specified rating (kVA) and output voltage (or current), a PM linear alternator can have numerous possible shapes and dimensions. The objective of optimal design is to design the machine which has the best pre-determined characteristics and satisfies specified constraints such as output voltage and alternator rating. In our design, we will attempt to achieve a minimum power-to-weight ratio and a maximum electrical efficiency by applying a mathematical optimization technique.

In an optimal design, we choose design variables and an objective function, which is a function of design variables and describes the characteristics to be optimized. We then find the set of values of the design variables, which correspond to the maximum or minimum value of the objective function with some equality or inequality constraints, which are the required conditions that the design must satisfy. Mathematically, the optimization problem may be stated as follows:

For the design variables $x_1, x_2, ..., x_n = X$, find

$$\min F = f(x_1, x_2, ..., x_n) = f(X) \tag{6-1}$$

subject to:

$$g_j(X) \leq 0 \qquad j = 1,2,...,J \tag{6-2}$$

$$h_k(X) = 0 \qquad k = 1,2,...,K \tag{6-3}$$

6.2. Penalty Function Methods

The penalty function methods convert a constrained optimization problem into an unconstrainted optimization problem. These methods are algorithms which generate a sequence of points from $X^{(0)}$ to $X^{(T)}$, where $X^{(0)}$ is the initial point, $X^{(t)}$ the generic point and $X^{(T)}$ the limit point, and the best estimate of the optimal point X^* is produced by the algorithm. The points $X^{(t)}$, t=1,2,...,T are stationary points of the penalty function.

Consider the penalty function:

$$P(X,R) = f(X) + \Omega\left[R, g(X), h(X)\right] \tag{6-4}$$

where R is a set of penalty parameters, and Ω the penalty term. Then $P(X,R)$ is a function of R and the constraint functions. Penalty function methods are classified according to the procedure employed for handling inequality constraints. These methods are referred to as interior or exterior point method according as the sequence $X^{(t)}$ contains feasible or infeasible points, respectively. If the sequence of stationary point contains both feasible and infeasible points, the method is said to be mixed.

The interior penalty function technique arguments the objective function with a penalty term which is small at points away from the constraints in the feasible region, but which "blow up" as the constraints are approached.

The penalty function is of the form:

$$P(F,R_t) = f(X^{(t)}) - \sum_{j=1}^{J} \frac{R_t}{g_j[X^{(t)}]} + \frac{1}{\sqrt{R_t}} \sum_{k=1}^{K} h_k[X^{(t)}]^2 \qquad (6\text{-}5)$$

with $R_1 > R_2 > \cdots > R_t > R_{t+1} > \cdots > 0$

and $\lim_{t\to\infty} R_t = 0$

Therefore, when $t \to \infty$, $\frac{1}{\sqrt{R_t}} \to \infty$, and it forces $h_k[X^{(t)}]^2$, k=1,2,...,K, approach zero, when minimum of $P(F,R_t)$ is approached. Inequality constraints are satisfied since $X^{(t)}$, t=1,2,...,T, are feasible points of inequality constraints. When $t\to\infty$, $P(F,R_t)$ has the same external as $F = f(X)$ has.

The exterior penalty function technique modifies the objective function by adding a penalty whenever a constraint is violated. The penalty function takes the form:

$$P(F,R_t) = f[X^{(t)}] + R_t \sum_{j=1}^{J} < g_j[X^{(t)}] >^2 + \sum_{k=1}^{K} h_k[X^{(t)}]^2 \qquad (6\text{-}6)$$

with $0 < R_1 < R_2 ... < R_t < R_{t+1} < ....$

$$\lim_{t\to\infty} R_t = +\infty$$

and

$$<g_i[X^{(t)}]> = \begin{cases} 0 & \text{if } g_i[X^{(t)}] \le 0 \\ g_i[X^{(t)}] & \text{if } g_i[X^{(t)}] > 0 \end{cases}$$

With t increasing, the minimum of $P(F,R_t)$ will conjugate to the minimum of $F = f(X)$ and the constraints of $g_j(X)$ will be satisfied for j=1,2,...,J. Also the $h_k[X^{(t)}]^2$ will be force to approach zero when a minimum of $P(F,R_t)$ is approached for k=1,2,...,K.

6.3. Optimal Design of PM Linear Alternator

For the PM linear alternator shown in Fig. 2-1, we choose: $\tau_m = 1.0$ in, $h_1 = 0.2$ in. Let: $X_1 = g$; $X_2 = h_m$; $X_3 = D_i$; $X_4 = \tau$; $X_5 = b_b$; $X_6 = b_p$; $X_7 = h_2$; $X_8 = b_{s2}$; $X_9 = W_1$ (number of turns of the coil); and $X_{10} = b_t$ are the design variables, which change during the optimization procedure. Other dimensions can be obtained from τ_m, h_1 and the design variables. The cross-section of the wire used for the coil is given by:

$$A_{co} = \frac{h_2 b_{s2} k_{fill}}{W_1} \qquad (6\text{-}7)$$

where k_{fill} is the slot fill factor = 0.5 (assumed).

The objective function in this design is the sum of the ratio of weight to output power (lb/kVA) and the ratio of iron and copper losses to output power, i.e.:

$$F = \frac{W}{S} + \frac{Z_i + Z_c}{1000S} \qquad (6\text{-}8)$$

where W - weight in lb; S = kVA rating; Z_i = iron loss in watts; and Z_c = copper loss in watts.

The goal of the optimal design is to find a set of design variables which make F a minimum and satisfy the following constraints:

output power $S = 25\ kVA$

output voltage $V \leq 500\ V$

current density $J_c \leq 5A/mm^2$

pole shoe length $\tau \leq 1.2\ in$

airgap $g \geq 0.03\ in$

The optimization subroutine package OPTLIB was used to obtain the optimal design, and the method used is the exterior penalty function. So, the optimal design problem reduces to obtaining the design variables which correspond to the minimum value of an unconstrained function $P(F,R)$, which is a function of F and the above constraints. The algorithm of optimal design is as follows:

(i) Choose initial values of design variables.

(ii) Calculate no-load electromagnetic field, no-load induced voltage E_o, coil resistance R_a and leakage inductance $L_{s\sigma}$.

(iii) Calculate electromagnetic field corresponding to $\theta = 45°$ and purely resistive load. Find the induced voltage drop due to armature reaction and the working point variation of the permanent magnet. Then find the equivalent mutual inductance L_{me}, output voltage V, stator current I_1, output power S, weight W and the losses Z_i and Z_c.

(iv) Check if $P(F,R)$ has a minimum value and satisfies accuracy
 yes: Go to (vi)
 no: Go to (v)

(v) Find new search direction and new design variables, go to step 2.

(vi) Stop.

The flow chart of optimal design is shown in Fig. 6-1.

The initial values of the design variables were chosen as follows:

$$x_1 = g = 0.03\ in$$

$$x_2 = h_m = 0.4\ in$$

$$x_3 = D_i = 16.0\ in$$

$$x_4 = \tau = 1.1\ in$$

$$x_5 = b_b = 0.8\ in$$

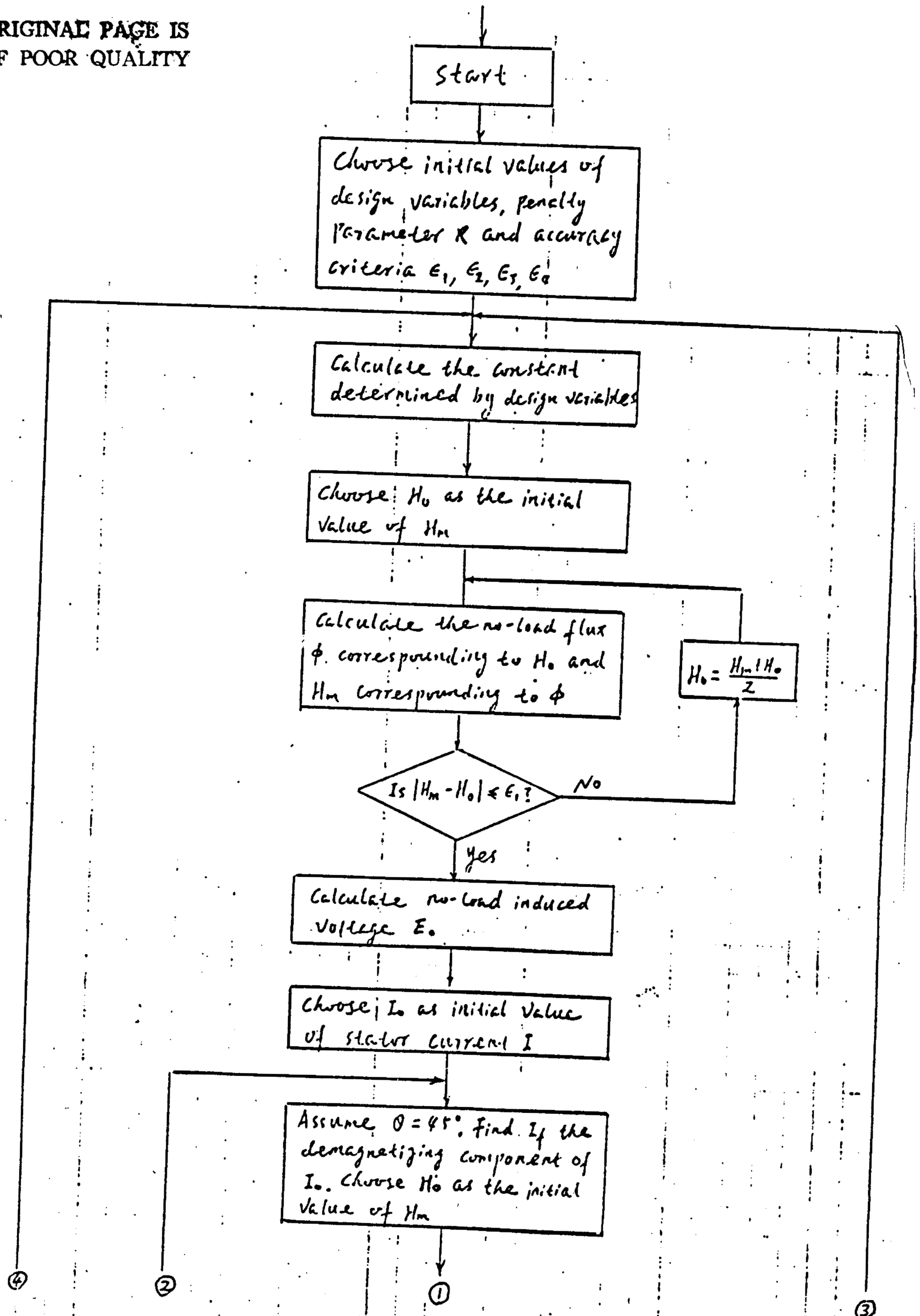
$$x_6 = b_p = 0.8\ in$$

$$x_7 = h_2 = 2.0\ in$$

$$x_8 = b_{s2} = 3.5\ in$$

$$x_9 = W_1 = 21\ turns$$

$$x_{10} = b_t = 0.8\ in$$


Start
Choose initial values of design variables, penalty parameter R and accuracy criteria ϵ1, ϵ2, ϵ3, ϵ4
Calculate the constant determined by design variables
Choose H0 as the initial value of Hm
Calculate the no-load flux ϕ corresponding to H0 and Hm corresponding to ϕ
Is |Hm - H0| ≤ ϵ1?
No
H0 = (Hm + H0)/2
Yes
Calculate no-load induced voltage E0
Choose I0 as initial value of stator current I
Assume θ = 45°, find Id the demagnetizing component of I0. Choose H0 as the initial value of Hm
④
②
①
③

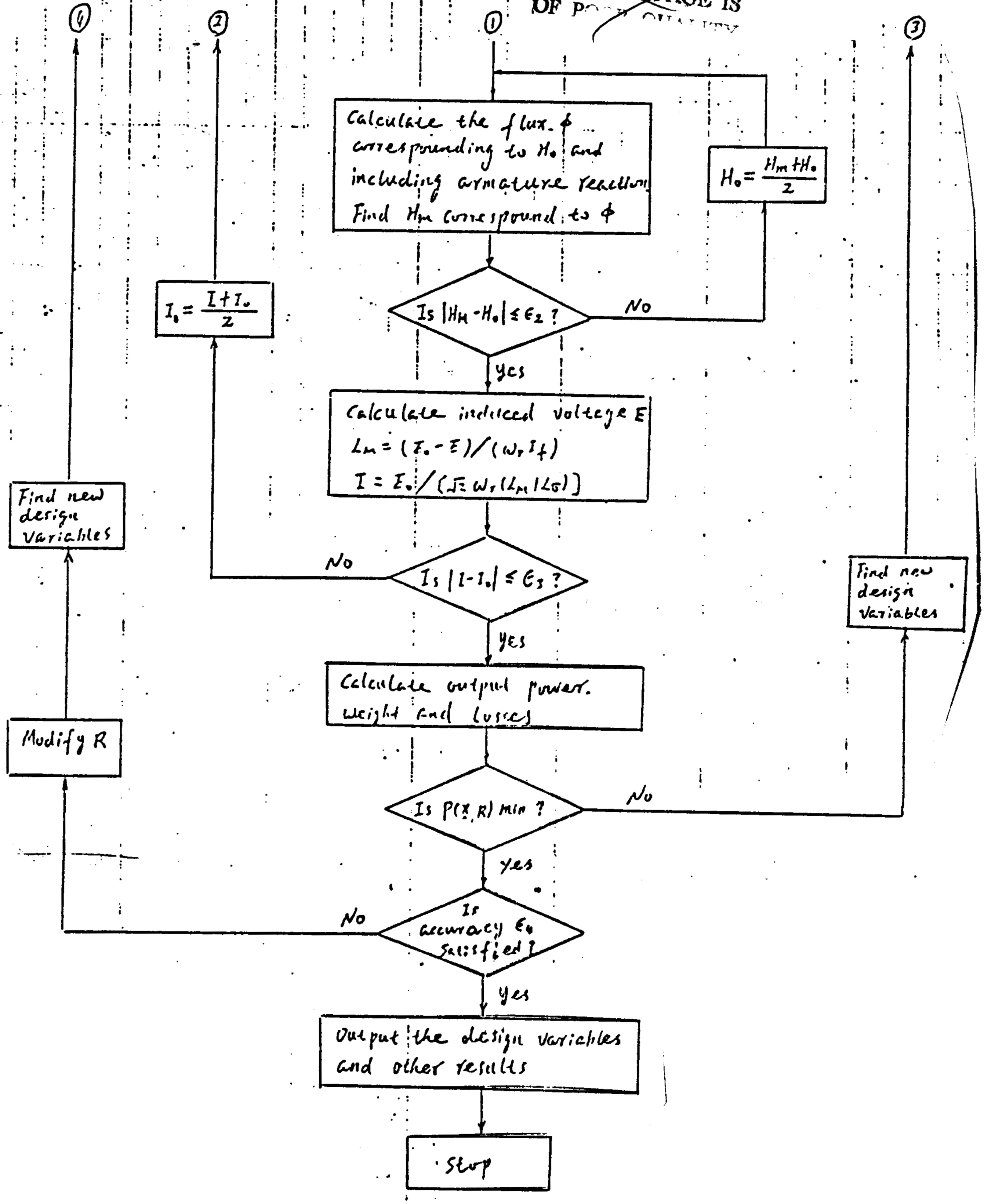


Figure 6-1 Flow chart of optimal design.

The optimal values of the design variables are:

$x_1 = g = 0.031\ in$

$x_2 = h_m = 0.44\ in$

$x_3 = D_i = 11.35\ in$

$x_4 = \tau = 1.2\ in$

$x_5 = b_b = 0.34\ in$

$x_6 = b_p = 0.50\ in$

$x_7 = h_2 = 0.91\ in$

$x_8 = b_{s2} = 3.63\ in$

$x_9 = W_1 = 85\ turns$

$x_{10} = b_t = 0.38\ in$

From the design the following characteristics of the linear alternator are obtained:

No-load induced voltage, $E_o = 571.58$ Volt

Output voltage, $V = 395.26$ Volt

Stator current, $I_1 = 62.74$ A

Stator resistance, $R_s = 0.142\ \Omega$

Stator leakage inductance, $L_{s\sigma} = 1.93$ mH

Stator mutual inductance, $L_m = 8.33$ mH

Cross-section of stator winding $A_{co} = 12.715\ mm^2$

Weight/power ratio, $\frac{W}{S} = 3.72\ lb/kVA$

Losses/output power $Z/1000S = 0.0233$

Output power, $S = 24.8\ kVA$.

Field of without load:

$B_m =$	0.89039 T	$H_m =$	-129306.30000 A/m
$B_{sp} =$	0.81040 T	$H_{sp} =$	66.29997 A/m
$B_{scv} =$	1.51587 T	$H_{scv} =$	2588.85600 A/m
$B_{sch} =$	1.58820 T	$H_{sch} =$	3974.03300 A/m
$B_{pcv} =$	0.95107 T	$H_{pcv} =$	83.88339 A/m
$B_{pch} =$	1.94321 T	$H_{pch} =$	15728.46000 A/m

Field for $I_1 = 62.74\ A$ and $\theta = 45^o$

$B_m =$	0.82138 T	$H_m =$	-178987.50000 A/m
$B_{sp} =$	0.48215 T	$H_{sp} =$	38.75073 A/m
$B_{scv} =$	0.90188 T	$H_{scv} =$	77.73460 A/m
$B_{sch} =$	0.94491 T	$H_{sch} =$	83.11403 A/m
$B_{pcv} =$	0.73906 T	$H_{pcv} =$	59.51506 A/m
$B_{pch} =$	1.51003 T	$H_{pch} =$	2477.14200 A/m

CHAPTER 7

CONCLUSIONS

In the preceding chapters certain important aspects of analysis, design and optimal design of single-phase single-slot tubular PM linear alternators have been investigated.

Methods developed in Chapters 2 and 3 are used to find the mutual magnetic field in every section of the linear alternator and the leakage magnetic field, with load or without load. Furthermore, the induced voltage may be obtained from the magnetic field, if the stator winding turns and frequency of the alternator are known.

The field of every section of the alternator, found by the methods developed in Chapter 2 and 3, is the average value of the field. However, it is good enough for finding the induced voltage. The advantage of these methods is that they make the calculation of the alternator's field much more easy and simple than by numerical method. The simplicity is significant for the optimal design of the alternators.

The methods and formulas presented in Chapter 4 are available for finding the parameters of the alternators, which are necessary for transient state and steady state analysis of the alternators.

The procedure discussed in Chapter 5 is the essential design guideline of tubular PM linear alternators. Combining the design guideline in Chapter 5 and mathematical optimization method, the technique presented in Chapter 6 ensures that the designed alternator has best specialized characteristics, which is defined by the optimal function, as shown in the design example.

The methods and formulas presented in this report and in reference [4] and [5] form the essential knowledge for analysis, design and operation of PM linear alternators.

REFERENCES

1. Roters, H. C., "Compensated reciprocating electrodynamic machine," US Patent 3,891,874, 1975.
2. Bhate, S. K., "Linear oscillating electric machine with permanent magnet excitation," US Patent 4,349,787, 1982.
3. Redlish, R. W., "Electromechanical transducer particularly suitable for a linear alternator driven by a free-piston Stirling engine," US Patent, 4,602,174, 1986.
4. Boldea, I. and Nasar, S. A., "Permanent magnet linear alternator: Part I fundamental equations," *IEEE Trans.*, Vol. AES-23, Jan 1987, pp. 73-78.
5. Boldea, I. and Nasar, S. A., "Permanent magnet linear alternator: Part II basic design guidelines," *IEEE Trans.*, Vol. AES-23, Jan 1987, pp. 79-82.
6. NASA Lewis Research (private communication).

APPENDIX I

```
      H0=0.5*HC                                                         CE 00
   10 CONTINUE                                                          CE 00
      BM=BR*(1.0-H0/HC)                                                 CE 00
      QL=-AK*H0                                                         CE 00
      Q=PI*(DI-2.0*GG-HSM)*TM*BM-QL                                     CE 00
      BSP=Q/SSP                                                         CE 00
      BSCV=Q/SSCV                                                       CE 00
      BSCH=Q/SSCH                                                       CE 00
      BPCV=(Q+QL)/SPCV                                                  CE 00
      BPCH=(Q+QL)/SPCH                                                  CE 00
      CALL BH(BSP,HSP)                                                  CE 00
      CALL BH(BSCV,HSCV)                                                CE 00
      CALL BH(BSCH,HSCH)                                                CE 00
      CALL BH(BPCV,HPCV)                                                CE 00
      CALL BH(BPCH,HPCH)                                                CE 00
      BGAV=Q/SG1                                                        CE 00
      HGAV=BGAV/U0                                                      CE 00
      FM=HSCH*B+HSCV*(2.0*H2+BB)+HSP*2.0*H1+HPCV*BP+HPCH*B+HGAV*2.0*GG  CE 00
      HM=-FM/2.0/HSM                                                    CE 00
      IF (ABS(HM-H0).LT.1.0) GO TO 50                                   CE 00
      H0=0.5*(HM+H0)                                                    CE 00
      GO TO 10                                                          CE 00
   50 HM=0.5*(HM+H0)                                                    CE 00
      BM=BR-BR/HC*HM                                                    CE 00
      AKM=(Q-AK*HM)/Q                                                   CE 00
      E0=SQRT(2.0)*PI*W1*FME*Q                                          CE 00
C                                                                       ODL00
      AS=H2*BS2                                                         ODL00
      AW=0.5*AS/W1                                                      ODL00
      RA=2.1E-8*W1*PI*(DI+2.0*H1+H2)/AW                                 ODL00
      BS=0.5*(BS1+BS2)                                                  ODL00
      ALL=PI*W1**2*U0*((DI+2.0*(H1+H2))*H2/3.0/BS2-H2**2/2.0/BS2+      ODL00
     * 2.0*(DI+H1)*H1/BS)                                               ODL00
      AIN=1.0E6*AW*5.0                                                  ODL00
      AI0=AIN                                                           ODL00
C                                                                       ODL00
      D=1.0-AK*HC/BR/SG3M                                               DE 00
      D1=BR*SG3M*GG/SG1                                                 DE 00
      D2=SG3M*GG/SG1/SG3                                                DE 00
      A=-D1/((GG+HSM)/SG2+D2-GG/SG11)                                   DE 00
      C=(-U0*HSM+D1/HC*D)/((GG+HSM)/SG2+D2-GG/SG11)                     DE 00
  105 AIF=AI0/SQRT(2.0)                                                 DE 00
      H0=0.5*HC                                                         DE 00
  110 CONTINUE                                                          DE 00
      Q2=A+C*H0                                                         DE 00
      Q1=((HC-D*H0)*BR/HC+Q2/SG3)*SG3M                                  DE 00
      Q=Q1-Q2                                                           DE 00
      BSP=Q/SSP                                                         DE 00
      BSCV=Q/SSCV                                                       DE 00
      BSCH=Q/SSCH                                                       DE 00
      BPCV=(Q1-AK*H0-Q2)/SPCV                                           DE 00
      BPCH=(Q1-AK*H0-Q2)/SPCH                                           DE 00
      CALL BH(BSP,HSP)                                                  DE 00
      CALL BH(BSCV,HSCV)                                                DE 00
      CALL BH(BSCH,HSCH)                                                DE 00
      CALL BH(BPCV,HPCV)                                                DE 00
      CALL BH(BPCH,HPCH)                                                DE 00
      FM=HSCH*B+HSCV*(2.0*H2+BB)+HSP*2.0*H1+HPCV*BP+HPCH*B              DE 00
      HM=((-Q2/(Q1-Q2)*FM+W1*AIF)*SG2*U0/2.0/(GG+HSM)-A)/C              DE 00
      IF (ABS(HM-H0).LT.1.0) GO TO 150                                  DE 00
```

```
      H0=0.5*(HM+H0)                                                    DE 00
      GO TO 110                                                         DE 00
  150 HM=0.5*(HM+H0)                                                    DE 00
      BM=BR-BR/HC*HM                                                    DE 00
      Q2=A+C*HM                                                         DE 00
      Q1=((HC-D*HM)*BR/HC+Q2/SG3)*SG3M                                  DE 00
      Q=Q1-Q2                                                           DE 00
      AKM=(Q1-AK*HM)/Q1                                                 DE 00
      E=SQRT(2.0)*PI*W1*FME*Q                                           DE 00
      AL=(E0-E)/AIF/628.0                                               DE 00
      AI=E0/SQRT(2.0)/(AL+ALL)/628.0                                    DE 00
      IF (ABS((AI-AI0)/AI0).LT.0.03) GO TO 152                          DE 00
      AI0=0.5*(AI+AI0)                                                  DE 00
      GO TO 105                                                         DE 00
  152 CONTINUE                                                          DE 00
C                                                                       ODL00
      D1=DI+2.0*(H1+H2+BB)                                              ODL00
      D2=DI+2.0*(H1+H2)                                                 ODL00
      D3=DI+2.0*H1                                                      ODL00
      D4=DI-2.0*GG                                                      ODL00
      D5=DI-2.0*(GG+HSM)                                                ODL00
      D6=DI-2.0*(GG+HSM+BP)                                             ODL00
      V1=PI/4.0*(D1**2-D2**2)*(BS2+2.0*BT)                              ODL00
      V2=PI/4.0*(D2**2-D3**2)*2.0*BT                                    ODL00
      V3=PI/4.0*(D3**2-DI**2)*0.5*(BT+T)*2.0                            ODL00
      VS=V1+V2+V3                                                       ODL00
      VM=PI*(D4**2-D5**2)*TM                                            ODL00
      VP=PI/4.0*(D5**2-D6**2)*(BS2+2.0*T)                               ODL00
      VC=PI*(DI+2.0*H1+H2)*AW*W1                                        ODL00
      W=7.8*(VS+VM+VP+VC)*1000.0*2.2                                    ODL00
      V=E0/SQRT(2.0)-AI*RA                                              ODL00
      S=V*AI                                                            ODL00
      ZC=AI**2*RA                                                       ODL00
      CALL CL(BSCH,FSCH)                                                ODL00
      CALL CL(BSCV,FSCV)                                                ODL00
      CALL CL(BSP,FSP)                                                  ODL00
      CALL CL(BPCH,FPCH)                                                ODL00
      FRA=(100.0/60.0)**1.3                                             ODL00
      ZSCH=2200.0*V1*FSCH*FRA                                           ODL00
      ZSCV=2200.0*V2*FSCV*FRA                                           ODL00
      ZSP=2200.0*V3*FSP*FRA                                             ODL00
      ZP=2200.0*VP*FPCH*FRA                                             ODL00
      ZR=ZSCH+ZSCV+ZSP+ZP                                               ODL00
      Z=ZC+ZR                                                           ODL00
      F=(1000.0*W+Z)/S*10.0                                             ODL00
      G(6)=(25000.0-S)/25000.0                                          ODL01
      G(3)=(AI-AIN)/10.0                                                ODL01
      G(2)=(0.3-X(1))*100.                                              ODL01
      G(1)=(X(4)-0.12)*10.                                              ODL01
      G(4)=(1.6*RR-BS1)*10                                              ODL01
      G(5)=V-500.0                                                      ODL01
      UF=F                                                              ODL01
      RETURN                                                            ODL01
      END                                                               ODL01
C                                                                       ODL01
      SUBROUTINE GRADU(X,S,SUM)                                         ODL01
      RETURN                                                            ODL01
      END                                                               ODL01
C                                                                       ODL01
      SUBROUTINE CL(B,F)                                                ODL01
```

```
      F=(B-0.4)*1.9/1.2+0.1                                             ODL01
      RETURN                                                            ODL01
      END                                                               ODL01
C                                                                       ODL01
C                                                                       BH 00
      SUBROUTINE BH(B,H)                                                BH 00
      DIMENSION BC(12),HC(12)                                           BH 00
      DATA BC/0.0,0.05,0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.75,1.8/        BH 00
      DATA HC/0.0,13.0,22.0,33.0,47.0,65.0,90.0,130.0,370.0,            BH 00
     *        4200.0,8000.0,10000.0/                                    BH 00
      DO 130 I=2,12                                                     BH 00
      IF (B.LT.BC(I).OR.B.GT.BC(12)) GO TO 110                          BH 00
      GO TO 130                                                         BH 00
  110 IF (B.GT.BC(12)) GO TO 120                                        BH 00
      L=I-1                                                             BH 00
      GO TO 140                                                         BH 00
  120 L=11                                                              BH 00
      GO TO 140                                                         BH 00
  130 CONTINUE                                                          BH 00
  140 H=HC(L)+(B-BC(L))*(HC(L+1)-HC(L))/(BC(L+1)-BC(L))                 BH 00
      RETURN                                                            BH 00
      END                                                               BH 00
C
..INC OPTLIB FORTRAN
C                                                                       ODL01
```