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CIRCUMSTELLAR SHELLS, THE FORMATION OF GRAINS, AND RADIATION TRANSFER

Jean Lefèvre

Advances in infrared astronomy during the last decade have firmly established the presence of dust around a large number of cold giant and supergiant stars. To describe the properties of stars and to understand their evolution, it is necessary to know the nature of the grains and their influence on stellar radiation. Two questions are considered in this chapter:

● How are grains formed around cold stars?

● How is the stellar radiation modified by shell?

These questions require a highly detailed description of grain properties. The development of high angular resolution measurements that permit the spatial analysis of scattered and emitted radiation and of molecular composition of the gas will increase the importance of studies on circumstellar shells.

GRAIN FORMATION

The formation and growth of grains in the shells of cold stars has recently been discussed by several authors. The most comprehensive works are those by Draine (1979, 1981), Deguchi (1980), Yamamoto and Hasegawa (1981), and Donn (1979). A critical review of this problem was also given by Czyzak et al. (1982). The formalism generally adopted is that of the theory of homogeneous nucleation. The basic ideas and the main results of the theory are recalled without mentioning the detailed calculations. The papers quoted above and references therein give a complete description of the theory. The particular circumstances encountered in circumstellar shells and in the neighborhood of variable stars will be considered with special attention.

Nucleation

In the theory of homogeneous nucleation, the solid clusters or aggregates grow or decay atom by atom (or molecule by molecule). For clusters of each size, there is a competition between gain and loss. The formation of solid grains really occurs only if the random process is able to produce a critical size beyond which the grain grows to its final size. For a cluster, the probability of growth by collision with a particle of the gas is proportional to its surface area: it is a slowly growing function of n, the number of atoms in the solid cluster. The probability of decay is related to the cohesion forces

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that keep the atoms together in the cluster. These forces are stronger when the number of atoms is higher (Gerlach, 1969). The probability of decay is then a decreasing function of n. When the partial pressure of condensing particles of the gas (monomers) equals the saturation pressure of the solid, gains and losses balance exactly for the bulk solid. When the partial pressure is lower than the saturation pressure, losses prevail over gains, whatever the size of the cluster. Finally, when the partial pressure is higher than the satuation pressure, a finite size exists for which losses and gains balance exactly. This equilibrium is unstable; any cluster can grow indefinitely if it becomes slightly larger than the critical size. It is then possible to define the critical nucleus as the smallest cluster, which, after growth by the addition of one atom, has a greater probability to grow than to decay. The rate of nucleation is the rate of the reaction leading from the critical nucleus to the next larger cluster. Thus, the existence of a critical size is a consequence of the variation of the free energy as a function of the number of atoms in the cluster. When this number is large (e.g., when the size of the cluster is about 10 Å ($n \sim 10^3$)), the variation of free energy can be related to the surface tension of the bulk material. If the critical nucleus is supposed to be spherical, its radius is then:

$$a^* = \frac{2 \sigma m}{\rho \, kT \ln S} \quad , \qquad (4\text{-}1)$$

where m is the mass of the monomer, p is the density of the solid, T is the temperature, and, S is the supersaturation ratio defined by:

$$S = \frac{P}{P_{sat}} \quad . \qquad (4\text{-}2)$$

P is the partial pressure of the monomer, and P_{sat} is the saturation pressure of the solid at temperature T.

Actually, the condensation is effective only when the critical nucleus is very small. It is then no longer possible to use the same value for the

surface tension or even to define the surface. This problem has been discussed in detail by Draine (1979). He was able to show, for graphite, that the number of atoms in the critical nucleus, n^*, is obtained with the value $\sigma/2$ when $n^* > 10$:

$$n^* = \text{integer part} \left\{ \frac{4\pi}{3} \cdot \left(\frac{m}{\rho} \right)^2 \cdot \left(\frac{\sigma}{kT \ln S} \right)^3 \right\} . \qquad (4\text{-}3)$$

The critical nucleus is sometimes defined as the smallest cluster for which the growth prevails. Equation (4-3) then corresponds to $n^* - 1$ (Draine, 1981; Deguchi, 1980).

The stationary nucleation rate in a dilute gas is obtained by multiplying the number of critical nuclei by the rate of collision of monomers with a nucleus:

$$J = \alpha \, \Gamma \left(\frac{m\sigma}{\pi\rho^2} \right)^{1/2} C_1^2 \exp \left\{ - \frac{n^* \ln S}{2} \right\} , \qquad (4\text{-}4)$$

where C_1 is the concentration of the monomer, and α is the sticking probability for an atom onto the critical nucleus. Usually known as the Lothe and Pound factor, Γ includes the translational, rotational, and vibrational partition functions of the atoms within the solid (Tabak et al., 1975; Deguchi, 1980). This point was discussed by Draine (1979), who concluded that the Lothe and Pound factor does not give the correct result when $n < 20$, at least for graphite. However, Czyzak et al. (1982) mention that a minor increase in surface energy would compensate for the effect of the Lothe and Pound factor on the nucleation rate. The question is open to discussion. Theoretically, the problem can be solved in each case by extrapolating the variation of the free energy as a function of n when n tends toward zero (Mutaftschiev, 1982).

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In any case, since n^* varies as $(\ln S)^{-3}$, the argument of the exponential in the nucleation rate varies as $-(\ln S)^{-2}$. It is generally small when S is lower than 6 or 7. When S is small, the critical nucleus is large and its appearance has a low probability. Finally, the uncertainty of the value of α must be kept in mind. $\alpha = 1$ is often used, and this gives an upper limit for J. The same formulation can be saved if a new definition of the supersaturation ratio is given (Lucy, 1976; Lefèvre, 1979; Draine, 1981):

$$ S = \frac{P}{P_{sat}(T_g)} \left(\frac{T_g}{T}\right)^{1/2}, \qquad (4\text{-}5) $$

where T_g is the temperature of the grains and T that of the gas. The modification may be important since P_{sat} varies rapidly with the temperature. Thus it is necessary to know the temperature of clusters containing a few atoms or several tens of atoms. A large gap exists between the quantum mechanical description of polyatomic molecules and the classical one of grains with several hundreds of atoms. The extrapolation of the results of the Mie theory toward very small dimensions is problematic. A reasonable limit probably corresponds to grains of radius greater than 10 Å, even when the modification of the refractive index due to the mean-free-path limitation of electrons is taken into account.

The attempt of Draine (1981) to solve this problem is interesting but rather risky. On one hand, the optical properties of grains of silicate are extrapolated to aggregate as small as 3 Å; on the other hand, the vibrational temperature of the molecule SiO is calculated with a simplified model. This molecule plays an important part in the kinetics of the formation of silicates. The value of the temperature obtained for the aggregates and for the molecules is the same with several gas densities. Thus, it can be hoped that the correct value is obtained in this way. The right fit, however, depends on the optical properties adopted for the silicate. Moreover, the same calculation done for small graphite grains would give a higher temperature. The fit would exist then only if the vibrational temperature of molecules C_2, C_3 ... were higher than that of SiO. The experimental and theoretical study of optical properties of very small aggregates is possible and would allow a complete description of the nucleation in a radiation field. The crux of the matter is the onset of collective phenomena when the number of atoms increases in the cluster.

What could be the influence of a chromosphere on the existence and the condensation of grains? As long as their equilibrium temperatures are radiative, grains can survive in a hot gas. For instance, near a star at 3000 K, the temperature of clean silicate grains reaches 1000 K at about $1.4\, R_*$. Could such grains form in the chromospheric environment? Two requirements must be satisfied. On one hand, the gas density must be sufficiently low so that the temperatures of the small aggregates are not affected by collisions. The answer again depends on the interaction of aggregates with the radiation field. If their temperatures can be obtained by extrapolation of Mie's calculations toward atomic dimensions, the preceding example requires $n < 10^{10}$ cm^{-3} if $T = 6000$ K is taken as a mean value for a chromosphere. On the other hand, the supersaturation ratio must be greater than one. Assuming in the most favorable case that Si is mainly in SiO, the nucleation of silicate near the star requires $n > 10^8$ cm^{-3}. Draine (1981) considers that the formation of clean silicate grains is possible in extended red-giant and supergiant chromospheres. A most favorable circumstance would be the presence of cold condensations in the hot gas as was proposed by Wright (1970) for 31 Cyg. Jennings and Dyck (1972) showed that the presence of circumstellar grains is accompanied by the disappearance of Ca II and Fe II chromospheric lines. Jennings (1973), studying the influence of grains on the structure of a chromosphere, found that the temperature is lowered and the chromospheric emission is quenched.

The temperature of the solid phase also plays an important part during the growth of

grains. As long as the partial pressure P is greater than $P_{sat}(T_g)\frac{T}{T_g}^{1/2}$, the radius of the grain grows as:

$$\frac{da}{dt} = \alpha \frac{P}{\rho} \left(\frac{m}{2\pi kT}\right)^{1/2} \quad . \quad (4\text{-}6)$$

The growth is faster when n_1, the density of the monomer, is higher, but also when the gas is hot, since the rate of growth varies as $n_1 T^{1/2}$.

Similarly, the decrease of the radius by evaporation is:

$$\frac{da}{dt} = -\alpha \frac{P_{sat}(T_g)}{\rho} \left(\frac{m}{2\pi kT_g}\right)^{1/2} \quad (4\text{-}7)$$

when the supersaturation ratio is much smaller than 1. The effective growth of the grain is given by the difference between the positive rate and the negative one. The equality between the two rates defines the critical radius and the final size of the grain is an asymptotic value. The number density of condensable atoms decreases due to the condensation itself and to the dilution in an expanding shell. The thermal evaporation is effective only when T_g is high (i.e., in the inner region of the shell). For instance, taking (Mukai and Mukai, 1973):

$$\text{Log} P_{sat} = 14.32 - \frac{38600}{T} \text{ for graphite },$$
$$\quad (4\text{-}8)$$
$$\text{Log} P_{sat} = 14.72 - \frac{26300}{T} \text{ for olivine },$$

(P_{sat} is in this case the partial pressure of SiO) and with $\alpha = 1$, the lifetime is about 10^7s for a grain of radius 100 Å when $T_g = 1750$ K for graphite and $T_g = 1150$ K for olivine. The lifetime is proportional to the radius. Such grains can survive in the stellar neighborhood and grow again during subsequent ejections of matter. The opacity effects in the shell are also important; grains formed close to the star attenuate the stellar radiation and allow further nucleation and growth.

Structure of the Grains: Amorphous or Crystalline

The interpretation of photometric measurements, particularly the profile of the silicate band at 10 μ and the flux curves in the far infrared from 10 μ to the millimetric waves has favored the introduction of amorphous grains in shell models. With the help of the theory of nucleation and growth, can we decide if the grains formed in circumstellar shells are amorphous rather than crystalline?

The structure of clusters containing few atoms or several tens of atoms is not known. It is only just possible, with the help of the molecular dynamics calculations, to compare the stability of different kinds of assemblies. For argon, Farges et al. (1977) found that small clusters ($n < 50$) do not possess the geometrical shape corresponding to the subsequent growth of the crystal. Their shape is close to that of the sphere. When n increases beyond 50, a structural transition occurs. On the other hand, Barker (1977) showed that groups of atoms with a definite pattern constitute the underlying structural unit of amorphous metals. In both cases, the icosahedron appears as a privileged structure. This polyhedron, limited by twenty equilateral triangles, corresponds to a stable arrangement of atoms. Hence, it may be quite difficult to make the distinction between amorphous and crystalline for very small aggregates. The difference is rather connected to the conditions of growth, particularly the variation of the temperature. Generally speaking, a high temperature, maintained long enough, leads to the stable phase (i.e., the crystal). For circumstellar grains, a theoretical determination of the structure requires the determination of the viscosity as a function of the temperature. Seki and Hasegawa (1981), considering that the temperature of condensation is low enough to give amorphous grains, studied their probability of crystallizing during their evolution in the

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shell. They found that around cold stars the grains are small ($a < 200$ Å) and have a low probability of crystallization. They must be mainly amorphous for graphite and for silicate as well. However, it is possible that the variation of the viscosity is not the same for different kinds of silicate.

Czyzak et al. (1982) critically discussed the arguments in favor of crystalline graphite. They concluded that crystals are "at best highly speculative" and emphasized the importance of a detailed description of the kinetics of growth throughout the shell. If the structure of graphite appears, order exists only at short scale, and it probably leads to highly disordered polycrystalline grains (i.e., aggregates of very small single crystals with different orientations).

Gail and Sedlmayr (1984a) give a clear analysis of the atomic processes involved in the growth of carbon grains. A grain grows as a crystal only when the sticking monomer is able to attain by migration a suitable lattice site. Then the comparison of the hopping time (i.e., the average time between two successive jumps of a monomer at the surface) and the average time between two successive captures at a given site is a good criterion. If the hopping time exceeds the capture time, the migration is not possible and the grain is amorphous. When the hopping time is just shorter than the capture time, the crystalline growth is not perfect, and a polycrystalline structure is obtained. In a cooling wind around a cold carbon star, Gail and Sedlmayr found that polycrystalline grains are produced only if their growth occurs when they are hotter than 1100 K.

The evolution of this problem has important implications in many astrophysical observations. At the moment, it is not possible to totally exclude one kind or another while the solid phase condenses. Amorphization can also be due to energetic collisions when eruptive phenomena or shock waves are produced at the stellar surface.

Time-Dependent Nucleation and Growth

The problem of the formation of grains can be solved only if the density and the tempera-

ture of the gas and the composition and the temperature of the clusters are known at every point in the shell and at each moment. The nucleation begins where the supersaturation ratio is significantly larger than 1. The grains then grow as long as $S > 1$. The saturation time scale governs the situation. The problem becomes even more difficult when the dynamical evolution of the shell is influenced by radiation pressure on the grains. In this case, the number and the size of the grains already formed determine the velocity of expansion of the shell. Conversely, the decrease in the density and temperature of the gas governs the growth of the grains. A more detailed description of these processes has been given by Deguchi (1980), Draine (1981), McCabe (1982), and Gail and Sedlmayr (1984b).

Deguchi studied shells of oxygen stars (C/O < 1) at 2000 and 2500 K, where the mass loss is caused by the radiation pressure on the grains. The shell is supposedly optically thin and $T \alpha r^{-1/2}$. The decrease of T follows the equilibrium temperature of a blackbody in a radiation field free of any adsorption and independent of the law of density of the gas in the shell. The coupled equations describing the motion of the gas and that of the grains are given without approximation, but are solved independently on each side of the sonic point. In these conditions, the nucleation of silicate (forsterite: $Mg_2 SiO_4$) occurs in a restricted part of the shell. Grains grow to several hundreds of angstroms if the mass loss is $10^{-6} M_\odot$ yr^{-1} and to several thousands of angstroms if it is $10^{-5} M_\odot$ yr^{-1}. Unfortunately, the temperature of the grains is not taken into account. For grains of forsterite, this temperature is lower than that of the gas. This must increase the nucleation, but can also limit the growth, due to the rapid depletion of the condensing gas.

Draine (1981) in the case of α Ori and McCabe (1982) for a carbon star take into account the temperature difference between the gas and the grains. The formation of grains is more efficient when it occurs near the star, where the density and the temperature of the

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gas are higher. This is possible only with materials which absorb the stellar radiation poorly and, on the contrary, easily emit their own thermal radiation in infrared bands. Then grains are cooler than the blackbody with the same shape, and an inverse greenhouse effect occurs. It happens with pure silicate ("clean" silicate) and silicon carbide. However, the infrared emission band at 10 μ observed around many cool oxygen stars can exist only if the silicate grains absorbed energy at shorter wavelengths. Around cold stars, the ultraviolet radiation is too low to heat the grains. Thus, silicates which are more or less absorbing in the visible and near infrared must be considered "dirty" silicates. The inverse greenhouse effect no longer exists for such grains; they cannot be formed as close to the star as clean silicates. That is why Draine considers an evolution of the quality of the grains throughout the shell. This possibility was already mentioned by Weymann (1977). Very close to the star, clean silicates nucleate and, when they are driven away, become condensation nuclei for absorbing more silicates. At every point in the shell, the quantity of absorbing material is limited by an increase in the temperature that it determines. Thus the temperature of the grains is nearly constant while they grow. The optical properties of such grains are not known. In the case of α Ori, the nucleation near the star requires an ejection velocity lower than that observed in the outer part of the shell.

The scenario proposed by McCabe (1982) to explain the formation of grains in the shell of a carbon star proceeds from the same idea. Near the star, graphite grains are much hotter than the blackbody and can nucleate only with a very high partial pressure of carbon. On the other hand, silicon carbide grains are cool enough to condense. Around a star at 2230 K (IRC + 10216), SiC grains form at 1.5 R$_*$. If their opacity is larger than 1, the attenuation of the stellar radiation is such that the nucleation of graphite grains becomes possible at about 5 R$_*$ and their growth up to 20 R$_*$. This happens only in optically thick shells. When the

shell is thin, clusters or grains of SiC could also serve as condensation nuclei for graphite.

The nucleation and growth in a supersonic wind driven by dust condensation was studied by Gail et al. (1984) and Gail and Sedlmayr (1984b) around heavily obscured carbon stars. The coupled equations describing the hydrodynamics, the dust formation, the grain growth, and the chemistry have been solved numerically. For a star with T_{eff} = 2300 K, M = 1 $M_\odot$, and L = 2.10^4 $L_\odot$, the nucleation of carbon grains occurs at 3 R_*, and the average radius of the final distribution is 2.10^{-6} cm (Figure 4-1). The sharp peak in the variation of the average radius is due to the fast growth of the first grains. Then the production of a
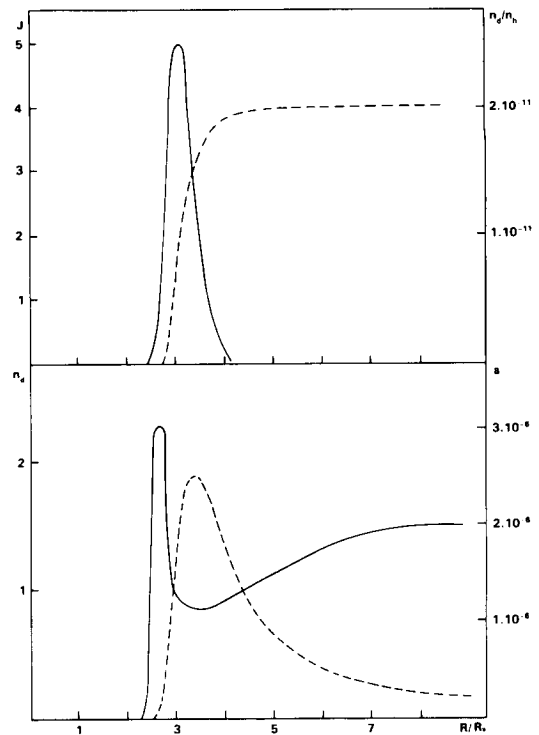

Figure 4-1. Nucleation and growth of carbon grains around a star with T_{eff} = 2300 K, M = 1 $M_\odot$, and L = 2.10^4 $L_\odot$ (from Gail and Sedlmayr, 1984b). The nucleation rate J (10^{-10} grains cm^{-3} s^{-1}) and the number of grains per hydrogen atom (top). The average grain radius a (cm) and the number density of grains (10^{-2} cm^{-3}) (bottom). These results correspond to a mass loss of 2.10^{-5} $M_\odot$ yr^{-1}.

larger number of very small grains (nucleation peak) makes the average size decrease. The grains are all formed within a thin shell.

Conclusions

The theory of homogeneous nucleation reasonably allows one to understand how grains are formed around cold stars and to locate the nucleation zone. The total number of grains, their final size, and their structure are more uncertain and are sensitive to various simplifications used in the models of the shell. The main uncertainties concern:

- The values of physical parameters of solids: the surface tension of the bulk material and of the small grains, the optical properties of very small aggregates, and the modifications of the refractive index due to impurities or defects

- Kinetics of the growth of grains in an expanding gas

The complexity of a real shell, compared with the extreme simplification of models, must be kept in mind. The gas contains a wide number of species with the ability to condense or deposit onto other grains. The outline of condensation theoretically obtained when the thermodynamical equilibrium is assumed, in absence of a strong radiation field, may be quite erroneous. The presence of molecules or radicals necessary for the nucleation works is not always obtained (Donn, 1976). The heteromolecular nucleation (i.e., the condensation of two or more different molecules) must also be considered. The nucleation of one specie is sometimes allowed only if another definite specie is present (nucleating agent); for instance, in the high terrestrial atmosphere, water nucleates in the presence of sulfuric acid.

A circumstellar shell is not an homogeneous medium. Local condensations or strong variations in the density permit the nucleation to begin, whereas the average density appears to be too low. The production of grains in definite sites makes possible an extension of the nucleation to regions of lower density or gives the entire shell condensation nuclei. Some of these complications were already considered by Salpeter (1974). When the number density of grains becomes high, coagulation works efficiently. Then the growth is faster and continues even when the supersaturation ratio is lower than 1. Coagulation is particularly efficient if every kind of turbulence is taken into account (Scalo, 1977). It could explain the existence of nonspherical grains such as those proposed by Svatos and Solc (1981) to obtain the polarization observed in Mira variable stars.

The propagation of shock waves in circumstellar shells probably influences the evolution of grains. Can they initiate the processes of growth by suddenly increasing the density or, on the contrary, destroy the grains by radiative heating? Many works exist on the propagation of shock waves in dusty gases. This mainly concerns the expansion of inhomogeneous gases in nozzle flows. Analytical methods perfected by hydrodynamicists will certainly be useful (Rudinger, 1973; Blythe and Shih, 1976). However, the problem must be stated in the frame of astrophysical conditions. The presence of a strong radiation field and unusual scale of time and dimension require special treatment.

The temperature of grains is calculated by using a well-defined photospheric stellar radius and the blackbody radiation at the effective temperature. In reality, this differs somewhat, mainly among the supergiants; the radiation at different wavelengths does not reach the same photospheric levels. The dilution factor of the radiation field should be a function of the wavelength. Schmid-Burgk and Scholz (1981) studied the possibility of forming grains inside the photosphere when the effects of sphericity are taken into account. The equilibrium temperature of grains is no longer purely radiative and is just higher than that of the gas. Schmid-Burgk and Scholz concluded that grains of Al_2O_3 can exist in the upper part of the atmosphere of low mass M giants.

Since the early fundamental works of Salpeter (1974) and Tabak et al. (1975), our

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knowledge of nucleation and growth of circumstellar grains has progressed under the pressure of the fast development of infrared photometry. Further progress will probably follow new findings on molecular abundances and chemistry in the shells. It is vital to know all aspects of the shell's interior—not only the density of monomers and molecules directly implied in the growth of the grains, such as C, C_2, C_3 .... or SiO and metallic oxides, but also the density of any molecules built with C, Si, or Mg.

RADIATIVE TRANSFER IN
CIRCUMSTELLAR DUST SHELLS

The absorption and the scattering of the stellar light in a circumstellar shell and the thermal infrared emission of the grains are obviously the physical consequence of the nucleation and growth of solid particles around cold giant and supergiant stars. However, it appears that most of the works on radiative transfer in circumstellar shells have been developed independently of those devoted to the formation of grains. Models of shells were built mainly to explain the results of photometry, particularly in the infrared. The constraints imposed by these results are indeed stronger, more numerous, and more accurate than those issued from the theoretical study of the nucleation and growth. For instance, the existence of silicate is indicated by their infrared bands and could not have been deduced for certain from the nucleation theory. Our knowledge on the nature and the size of the grains results from the best fit between models and observations. Studies on nucleation and growth most often attempt to justify a posteriori the existence of the grains proposed in the models. It must be hoped that this situation will evolve and that future works will include all aspects of the problem, including the dynamics of the gas and molecular equilibria.

The equation of radiative transfer must be solved in a spherical extended atmosphere—the shell—with a central source of radiation. Around cold stars, the ultraviolet radiation is too weak to ionize the gas in the shell; the grains

alone are responsible for absorption, scattering, and emission. At every point in the shell, the radiation field is anisotropic. The methods of treating the radiative transfer arise directly from those perfected for stellar atmospheres, with different approximations from one author to another. After the work of Leung (1976), the most comprehensive studies devoted to the shells of cold stars are those by Jones and Merrill (1976), Mitchell and Robinson (1981), and Rowan-Robinson and Harris (1982, 1983a, 1983b).

Generally speaking, flux curves are correctly fitted to spherical models for families of stars of the same spectral class, luminosity type, and chemical composition. Some difficulties exist at long wavelengths and in the absorption bands. Moreover, the right fit is often obtained by arbitrarily adjusting the optical properties of grains. The influence of different parameters will be examined first. Then the results obtained for oxygen stars and carbon stars will be summarized. Lastly, the influence of departures from spherical symmetry will be discussed.

Resolution of the Transfer Equation
in a Spherical Shell

The main goal is to obtain a correct description of the radiation field at every point in the shell. The range of wavelengths taken into consideration is necessarily very large to include the stellar radiation, the scattered light, and the thermal emission of grains. At short wavelengths, the scattering diagram of the grains, which fixes the angular repartition of the scattered light, must be carefully represented. High scattering orders—corresponding to photons scattered several times—cannot be neglected when the optical depth is larger than one. The simplifications most often concern this point; they usually give the correct result for quantities integrated over the entire spectrum when short wavelengths have a weak influence. On the other hand, the determination of the angular repartition of the radiation requires a precise description of scattering. Scattering by grains is never isotropic. When the

grains are much smaller than the wavelength, scattering is only symmetric (Rayleigh scattering). For submicronic grains, it is always possible to give a simple and accurate representation of the scattering diagram at every wavelength (White, 1979).

To define a model, it is necessary to fix the stellar characteristics (effective temperature and radius), the geometry of the shell (inner and outer radii and density), and the properties of grains (size and optical properties). The star is the only source of energy. We must know the flux it emits at every wavelength and its limb-darkening. The value of the effective temperature determines the temperature of the grains and the lower limit of the inner boundary of the shell. The photometric radius is usually used to normalize other lengths and does not appear explicitly. However, its definition is not evident for low-gravity stars. For instance, the angular diameter of Mira obtained by speckle interferometry (Bonneau et al., 1982) varies with the wavelength due to the extension of the photosphere and the wavelength dependence of the opacity.

The shell is limited by two spheres. The inner boundary of radius R_i may have different physical origins depending on other circumstances. It may be:

- The vaporization limit of the grains where they reach their maximum temperature. Closer to the star, they disappear in a time shorter than any other characteristic time of their evolution (expulsion, period of the star, etc.). For small graphite grains, it corresponds to about 1800 K, and for silicate to 1200 K.

- The region of nucleation and growth of the grains. The temperature of the hottest grains has practically the same value as the above; for a total gas density of 10^{-10} to 10^{-12} cm^3, the nucleation is possible and efficient for these values. In this situation, the density of the gas is implicitly assumed to be higher than the saturation limit. Mitchell and Robinson

(1980) pointed out that the concept of inner boundary is an abstraction: the numerical density and the size of grains vary over a finite distance and what does exist is rather an "inward tail." However, the nucleation and growth has maximal efficiency in a narrow region of an expanding shell mainly because of the decrease of the density of condensable material (Draine and Salpeter, 1977). Around a variable star, for different values of the phase, the inner boundary may be alternately defined by condensation and vaporization. The temperature of hottest grains is then nearly constant, but R_i varies.

- Finally, in an expanding shell without permanent nucleation, the inner radius is determined by the dynamical evolution and is larger than in the two preceding cases.

The outer limit is less precisely defined. For a shell that is expanding with a constant velocity, it corresponds to the time of expansion. It may also be due to the interaction with the diffuse interstellar medium. In actual fact, the shell must be bounded to keep its mass and far-infrared luminosity within limits fixed by the observations and the theory. The outer radius is most often taken to obtain the correct value of the flux at the longest measured wavelength or to give the right intensity ratio for the two silicate bands at 10 and 20 μ.

The law of density is most often taken as R^{-2}, corresponding to free expansion at constant velocity. If the formation and the ejection of dust are not continuous processes (Sanner, 1976; Bernat, 1981; Goldberg, 1983), the structure of the shell may be much more complicated. The model of grain used in the model is the main source of uncertainty. The distribution of dimensions, or the average dimension, have little influence on the equilibrium temperature for submicronic particles. On the contrary, optical properties have a strong influ-

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ence. Practically speaking, it was always necessary to fit the models to modify the refractive index measured on terrestrial materials. This point will be considered separately for graphite and silicates. Nevertheless, such an adaptation of an important parameter does not discredit models. It is not surprising that solid material built in circumstellar shells is somewhat different from pure species studied in the laboratory. This indicates an area for experimental and theoretical research, but it must be kept in mind that the optical properties adopted by each author are not the only possible choice.

Before models built for different kinds of stars are considered, some general results will be discussed.

Circumstellar Extinction Opacity Law. If $I(\lambda)$ is the intensity of the radiation emerging from the shell in a given direction and $I_o(\lambda)$ is the intensity emitted by the star, the effective opacity of the shell, $\tau(\lambda)$, can be defined by:

$$I(\lambda) = I_o(\lambda) \exp(-\tau(\lambda)) \quad . \quad (4-9)$$

The function $\tau(\lambda)$ must be known in order to infer the stellar properties from the observations. Along the radial direction, the extinction opacity is:

$$\tau_{ext}(\lambda) = \int_{R_{int}}^{R_{ext}} n_{(r)} \, \pi a^2 Q_{ext}(\lambda) \, dr \quad , \quad (4-10)$$

where $n_{(r)}$ is the number density of grains, a is their radius, and $Q_{ext}(\lambda)$ is their extinction efficiency. If a distribution of dimensions must be considered, a second integration is done over the radius. Similarly, τ_{sca} and τ_{abs} are defined with the scattering efficiency, Q_{sca}, and the absorption efficiency, Q_{abs}. These three quantities are proportional to the column density on the line of sight. They vary with λ exactly as Q's do. In the diffuse interstellar medium, any photon having an interaction with a grain (ab-

sorption of scattering) is lost for the observer. Then τ_{ext} computed along the direct trajectory from the star to the instrument gives exactly the effective opacity. The interstellar extinction curve has the shape of the variation of Q_{ext} with the wavelength. It is not the same with an unresolved circumstellar shell since photons scattered in the shell will be received by the observer. The shape of the circumstellar extinction law is consequently different from that of the interstellar extinction law at wavelengths where scattering is efficient, even if the grains have the same properties. However, τ_{ext} has a simple physical meaning: exp $(-\tau_{ext})$ is the probability for a stellar photon to go out through the shell without absorption or scattering. Thus, the direct stellar radiation is:

$$I_d(\lambda) = I_o(\lambda) \exp(-\tau_{ext}(\lambda)) \quad (4-11)$$

and necessarily $\tau(\lambda) < \tau_{ext}(\lambda)$. The equality occurs when the albedo is zero. When the shell is optically thin, a photon has a low probability to suffer more than one interaction. The only photons which cannot get out of the shell are those absorbed without previous scattering. The probability for a stellar photon to travel through the shell without absorption is exp $(-\tau_{abs})$. The emerging intensity is then in the thin case:

$$I(\lambda) = I_o(\lambda) \exp(-\tau_{abs}(\lambda)) \quad , \quad (4-12)$$

and $\tau = \tau_{abs}$. This is true only if τ_{sca} is low. When multiple scattering is significant, it effectively lengthens the path of the photon into the shell and increases its probability of absorption:

$$\tau_{abs} < \tau < \tau_{ext} \quad . \quad (4-13)$$

Moreover, τ is not a linear function of the number density of grains. The circumstellar law of opacity has a shape different from that of the interstellar one.

Advances in high angular resolution methods, particularly in interferometry, should bring important results. When the direct stellar

radiation, $I_d(\lambda)$, and the radiation scattered by the entire shell, $I_{sca}(\lambda)$, can be unambiguously separated, for a spherical thin shell:

$$I(\lambda) = I_d(\lambda) + I_{sca}(\lambda)$$

$$= I_o(\lambda) \exp(-\tau_{abs}(\lambda)) \quad ,$$

(4-14)

and

$$I_d(\lambda) = I_o(\lambda) \exp(-\tau_{ext}(\lambda)) \quad . \quad (4\text{-}15)$$

Then

$$\tau_{sca}(\lambda) = \ln(I(\lambda)/I_d(\lambda)) \quad . \quad (4\text{-}16)$$

Measurements at different wavelengths would give the shape of the variation of Q_{sca}. If the proper radiation of the star $I_o(\lambda)$ is known, the albedo is given by:

$$\gamma = \frac{\ln(I(\lambda)/I_d(\lambda))}{\ln(I_o(\lambda)/I_d(\lambda))} \quad . \quad (4\text{-}17)$$

The condition of validity of these relations must be emphasized; τ_{sca} must be low (i.e., $I_{sca}(\lambda)$ must be lower than $I_d(\lambda)$). If it is not true, the ratio gives a lower boundary for the albedo.

Intensity Profiles. Besides the flux curve, the more complete calculations give the angular repartition of the radiation at every wavelength. Mitchell and Robinson (1978) and Rowan-Robinson (1982) have pointed out the existence of a sharp maximum of the luminance in the direction of the inner boundary of the shell. This result was also found by Yorke and Shustov (1981) for protostellar shells and by Lefèvre et al. (1982) by numerical simulation. This bright ring corresponds to the maximum of the column density of grains reached when the impact parameter is equal to the internal radius of the shell. A maximum appears even if the variation of density is not as sharp as in the models. On the other hand, the central lowering of the intensity profile is reduced by the forward scattering of grains. The maximum

also exists for the scattered stellar light and the infrared thermal emission of grains. It is less and less pronounced when the opacity increases. The existence of a peak of luminance is important in the interpretation of high angular measurements. If the bright annulus can be detected, one limit of the shell is fixed with precision, and the radiative equilibrium of grains will be better understood. Preliminary results obtained by Roddier and Roddier (1983) for α Ori at $\lambda = 0.535 \, \mu$ indicate a concave bright rim at about 2.5 stellar radii. If it is confirmed and is clearly attributed to scattering by dust, this implies the existence of much less-absorbing grains than those proposed in models.

A maximum of the luminance must appear for every discontinuity of quick variation of the number density of grains; the detection of rings at different distances from the star could allow one to follow the history of successive ejections of matter when the rate of grain production is not constant.

Shells Around Oxygen Stars

The main features presented by the shells of oxygen stars are the broad emission bands at 10 and 20 μ attributed to silicates. Several works are devoted to the general properties of shells and give grids of models with adjustable parameters: Jones and Merrill (1976), Mitchell and Robinson (1981), and Rowan-Robinson and Harris (1982, 1983a). Some other works were developed for a particular object: Hagen (1978) and Tsuji (1978, 1979).

Jones and Merrill (1976) made the first quantitative attempt to explain the main characteristics of the flux curve for families of cold stars. The transfer equation is solved with the help of the Eddington approximation, and the angular repartition of the radiation is not calculated. The objects studied present a wide range of opacities. The flux curve and the profile of emission bands are obtained with good precision for stars with an effective temperature of 2400 K. A valuable result of this study shows the impossibility of obtaining a good fit with

clean silicates between 0.25 and 8 μ. The model of dirty silicate ($m = 1.55 - 0.1$ i for $\lambda < 8$ μ), widely used afterward, was proposed. Clean silicates (olivine or enstatite) absorb the ultraviolet radiation but are transparent for the visible and the near infrared. Cold stars do not radiate enough energy at short wavelengths to heat the grains. The absorption of clean silicates begins beyond 9 μ; such grains cannot reemit through their near-infrared band more than they absorb and are unable to give the observed emission. Grains must absorb in the visible or the near infrared. This absorption may be due to defects or impurities. However, no detailed model of the dirty silicate absorption was given.

The profile of the emission band at 10 μ is always wide and structureless. The silicates most often used in the models—enstatite (SiO_3Mg) and forsterite (SiO_4Mg_2)—present narrower bands with several peaks when crystallized. Thus, observations imply the existence of amorphous silicates. The transfer in the bands and the influence of various parameters were studied extensively by Mitchell and Robinson (1981). For small grains of "forsterite," particularly grains with $a = 0.1$ μ and $m = 1.55 - 0.01$ i for $\lambda < 8$ μ, around a star at 3000 K, they showed that the meaningful parameter is the opacity. The 10-μ band can appear either in emission or in absorption. When the opacity increases, the external regions of the shell are no longer heated by the stellar radiation. The cold grains absorb the radiation emitted by the hotter internal region. When the density varies as r^{-2}, an emission is observed at 10 μ if τ_{ext} (10 μ) < 2. If $2 < \tau_{ext}$ (10 μ) < 8, self-absorption appears at the center of the band. For instance, this is observed with NML Cyg. At last, if τ_{ext} (10 μ) > 8, the band is seen in absorption. The band at 20 μ is generally less intense and can also present this inversion. Mitchell and Robinson showed that, in every case, the determinant physical quantity is the temperature of the grains at the point where $\tau_{ext} = 1$, starting from outside.

The profile of the 10-μ band has recently been studied by Papoular and Pégourié (1983) around 23 giants and supergiants. The profile is the same for several shells and is attributed to amorphous forsterite grains with $a < 1$ μ. For some giants, the band is wider. This is not correlated to the spectral type of the star or the opacity, but rather to the galactic latitude. A possible explanation is a variation of the radius of grains, up to 4 μ. The increase of the size with the galactic longitude could be due to differences in the conditions of nucleation or modifications of erosion processes.

The most extended grid of models for M giants and supergiants was given by Rowan-Robinson and Harris (1982, 1983a). For 27 early M stars and 85 late M stars, a satisfactory concordance is obtained with the flux curves observed using small silicate grains similar to those of Jones and Merrill (1976) and with density varying r^{-2}. For each star, the opacity and the dimension of the shell are adjusted. The effective temperature is between 3000 and 3500 K for early-type stars and 2000 and 3000 K for late-type stars. In most of the cases, inner grains are hotter than 1000 K, but they are colder near the hottest stars when the opacity is low. Molecular bands do not sufficiently modify the total energy absorbed by the grains to have an effect on their equilibrium temperatures and the thermal radiation emitted by the shell. In most cases, scattering of an order greater than one can be safely neglected. This is the case, for instance, when the star is surrounded by small absorbing grains: most of the stellar radiation is in the near infrared where the albedo of such grains is very low. It is no longer true if the grains are large. On the whole, the observations are well explained by the grid of models with expanding spherical shells and more or less absorbing silicate grains. Moreover, VY CMa and NML Cyg must be considered separately owing to their asymmetry. For any of the objects studied, the model proposed by Rowan-Robinson and Harris is a good starting point for further adjustments.

The most often studied star is certainly α Ori. Models for its shell have been built by Hagen (1978) and Tsuji (1978, 1979). In a very detailed study, Tsuji used the method of Unno and Kondo (1976, 1977) to solve the transfer

equation. With dirty silicate grains ($k = 0.01$ and $k = 0.1$), angular dimensions compatible with the results of infrared interferometry (McCarthy and Low, 1977; Sutton et al., 1977) are obtained at different wavelengths. In this model, it is the inner boundary of the shell very close to the star: $R_i \simeq R_*$ to $3 R_*$. From the total flux and the angular diameter, Tsuji deduces an effective temperature of 3900 K. Unfortunately, dirty silicate grains as considered in the model cannot survive so close to such a star; they would be hotter than 2000 K. Rowan-Robinson and Harris used $T_{eff} = 3250$ K and obtained $R_i = 10 R_*$. The results of Roddier and Roddier (1983), already quoted, suggest $R_i \sim 2.5 R_*$. Thus, near the star, the grains must be less absorbing than those used in the models. The value adopted for the effective temperature is the determinant. Scargle and Strecker (1979) proposed $T_{eff} = 3580$ K, using to deredden α Ori a law having the same shape as the interstellar reddening law. Further determinations of the angular repartition of the light scattered around α Ori are needed in the visible and, when possible, at 10 μ. If the existence of grains close to the star is confirmed, the model of nucleation and growth proposed by Draine (1981), with clean silicate grains at the lower part of the shell and more absorbing ones outward, would not only be a solution for the thermodynamical problem of the condensation of grains, but would also explain the angular repartition of the radiation.

Shells Around Carbon Stars

The infrared emission of shells of carbon stars do not present strong bands. An emission feature at 12 μ, attributed to silicon carbide, is always weak. In all the models, the grains responsible for the absorption and the emission into the shell are carbon grains. Silicon carbide could not produce the emission observed at large wavelengths. The thermal emission is not always clearly distinct from the stellar continuum because carbon grains can survive close to the star and can emit efficiently in the near infrared. On the whole, it is possible to obtain a good representation of the flux curve with an expanding spherical shell and small carbon grains. Jones and Merrill (1976) mention that the emission peak moves to a larger wavelength when the opacity increases as the flux curve becomes narrower. This is due to the absorption of the stellar radiation and of the near-infrared radiation of hotter inner grains. Bergeat et al. (1976), analyzing the photometry of 29 carbon stars, concluded that the thermal emission between 1.25 and 8 μ is due to graphite grains of radius greater than 1 μ. In their sample, Miras have lower effective temperatures and thicker shells of cooler grains. Large grains efficiently scatter the near-infrared radiation, and their existence could be tested by high angular resolution measurements.

The results obtained in the far infrared beyond 20 μ and, for some stars, in the millimetric waves (Fazio et al., 1980) are much more difficult to explain with crystalline graphite grains. The extinction efficiency measured for amorphous carbon grains by Koike et al. (1980) decreases as λ^{-1} and allows a better fit.

Models of shells have been presented for 41 carbon stars by Rowan-Robinson and Harris (1983b). A simplified parametric representation of the extinction efficiency of carbon grains is used. The decrease of Q_{ext} at large wavelengths must be slow, and this excludes graphite. In these models, hotter grains are at 1000 to 1300 K. The authors mentioned that the temperature "tends to be lower for larger shell optical depths."

As for silicates, the optical properties adopted for carbon grains are sometimes arbitrary, and more attention must be paid to improve the coherence of models. Optical properties of graphite grains are not always correctly described. Graphite is strongly anisotropic. For spherical grains smaller than the wavelength, the extinction efficiency is:

$$Q_{ext}(\lambda) = \frac{2}{3} Q_{ext}^{\perp}(\lambda) + \frac{1}{3} Q_{ext}^{\parallel}(\lambda) , (4\text{-}18)$$

where $Q_{ext}^{\perp}$ is the extinction efficiency, the electric vector is perpendicular to the c-axis of the

crystal, and $Q_{ext}^{\parallel}$ is the extinction efficiency when it is parallel to the c-axis. The refractive index measured by Philipp (1977) allows the determination of $Q_{ext}^{\perp}$ and $Q_{ext}^{\parallel}$ is obtained from the results of Venghaus (1977). Beyond 10 μ, the variation of Q_{ext} is dominated by $Q_{ext}^{\parallel}$. This is clearly shown in the work by Mezger et al. (1982) on the origin of the diffuse galactic far-infrared and submillimetric emission. As a result, up to 100 μ Q_{ext} decreases more slowly than λ^{-2}. Moreover, a plasma resonance at 37 meV induces a secondary maximum of the extinction efficiency around 33 μ.

An excess of emission has been observed several times at 30 μ with carbon stars. Forrest et al. (1981) and Herter et al. (1982) obtained the profile of this emission for IRC + 10216. The profile of the band deduced by dividing the observed flux by a continuum varying by λ^{-1} (Herter et al.) presents a maximum at 30 μ, but this result, as mentioned by the authors, depends on the method of deconvolution. Conversely, if the observed flux is divided by the extinction efficiency of graphite grains, a smooth continuum is obtained. More precise determinations of Q_{ext} are needed for different kinds of carbon to test the validity of this hypothesis. Even if the emission band observed has nothing to do with carbon, the problem still remains: if graphite grains are responsible for the emission around carbon stars, "something" must appear at about 33 μ.

Several models have been published for the shell around IRC + 10216. However, the problem for this very interesting object is complicated. The nature of the central star is not known, and the shell is strongly asymmetrical. With a model of a spherical shell, Mitchell and Robinson (1980) nevertheless obtain a good fit with the flux curve between 1 and 100 μ. They give a very comprehensive review of the measurements and works published at this time. In their model, the shell extends from 20 to 5000 stellar radii, and the density varies by $r^{-1.3}$. The effective temperature of the star is 2000 K. The shell contains small graphite grains and a small fraction of spheroidal SiC grains neces-

sary for obtaining the correct flux between 10 and 14 μ. The hotter graphite grains are at 600 K. A fairly similar model was proposed by Keady (1982), but with amorphous carbon in an expanding shell which corresponds to a mass loss of 1.5 $\times$ $10^{-4} M_{\odot}$ yr^{-1}. The influence of the velocity field and the microturbulence on the profile of CO infrared lines at 2 and 4.6 μ is taken into account.

For the same object, Rowan-Robinson and Harris (1983b) found a shell extending from 5.5 to 550 stellar radii with a density proportional to r^{-2}. The hotter graphite grains are then at 850 K when T_{eff} = 2000 K.

Without solving the transfer equation, McCabe (1982) proposed a model to explain the molecular abundances observed throughout the shell and the nucleation and growth of grains near the star. His scenario has already been described: silicon carbide grains, responsible for the emission around 12 μ can condense at $r <$ 2 R_*. Shielding the stellar radiation at short wavelengths, they permit carbon grains to condense and grow between 5 and 20 R_*. This explains why the temperature of carbon grains is lower than the condensation temperature at the same place if the stellar radiation would not be partially absorbed. It would be interesting to confirm whether the quantity of SiC required is compatible with the strength of the emission at 12 μ.

Finally, the problems encountered in building models of shells around carbon stars are quite similar to those encountered with oxygen stars. The exact nature of the grains and the description of their optical properties require further investigation. The experimental study of amorphous carbon and polycrystalline grains, from the near infrared to the millimetric waves would be very useful.

Nonspherical Shells

Models of spherical shells necessarily give no explanation of the intrinsic polarization of stars. This polarization can be produced by elongated grains in a spherical shell, but a very

efficient mechanism of alignment is required to obtain a significant rate of polarization. The morphology of the shell is then probably also affected. The evidence of nonsphericity has been recognized in some cases; it is always associated with a strong intrinsic polarization at short wavelengths. The most striking examples are VY CMa, NML Cyg, and IRC + 10216. As for the latter, infrared interferometric measurements by McCarthy et al. (1980) indicate an asymmetry of the order of 3 to 1 for the principal directions. Models of nonspherical shells were at first mainly proposed to explain intrinsic polarization. The complete treatment of the transfer of the scattered stellar radiation in an ellipsoidal shell was realized without approximation by Daniel (1982), using numerical simulation. A shell with a constant density cannot produce a polarization higher than 12 percent. The rates of polarization observed for VY CMa—23 percent at $\lambda = 0.38$ μ (Serkowski, 1973)—or IRC + 10216—24 percent at $\lambda = 0.64$ μ (Dyck et al., 1971)—require a different repartition of the scattering material.

Another result highlighted by the computation is the persistence, when the opacity is modified, or a rotation of 90 degrees of the angle of polarization at a fixed wavelength. In fact, it is an individual property of grains which multiple scattering does not smooth out. Daniel (1982) obtains a relation between the index of refraction, the average radius of the grains, and the wavelength of rotation: this wavelength is shorter for smaller grains. The rotation disappears when the grains are strong absorbers, as, for instance, graphite ones. The steep variation of the angle of polarization is effectively observed for several objects, and shell models were built for ten strongly polarized stars. High rates can be obtained only if the direct stellar light is attenuated along the equatorial plane and is scattered mainly near the polar regions of the shell. For instance, the variation of the rate of polarization of VY CMa and the modification of the angle at 0.9 μ are well explained by an ellipsoidal shell with a high opacity along its equatorial plane. If the grains responsible for the scattering and the polarization are

poorly absorbing silicates, their mean size is 0.26 μ (Figure 4-2). This model of bipolar nebula was also proposed by Schmidt et al. (1980) and Staude et al. (1982). More recent models by Cohen and Schmidt (1982) for three carbon stars have a similar morphology: a torus or a disc of absorbing material obscures the stellar radiation, and the scattered light comes from the polar lobes. It is interesting to note that GL 1403 (CIT6) clearly has a rotation of polarization at 0.65 μ; it cannot be produced by graphite grains in an ellipsoidal shell of constant density. The rotation shows that the repartition of the materials is complex. It can also be due to silicon carbide grains.

The thermal equilibrium of grains and the transfer of infrared emitted radiation in an ellipsoidal shell has been studied by Lefèvre et al. (1983) by numerical simulation. Only the constant density case was considered. This model is certainly too simplified to represent reality, and the essential purpose was to evaluate the importance of the hypothesis of sphericity on several physical quantities. In realistic models, the law of repartition of density has a strong influence on the results and cannot be fixed arbitrarily. Density depends on the distance and at least one angular variable. The first problem to be solved is to understand the dynamics of the shell and to describe the physical mechanisms responsible for the loss of the spherical symmetry. Rotation and magnetic field are most often invoked. It is not certain that they work effectively for evolved stars with large dimensions; the rotation velocity and the magnetic field are probably low in this case. The shape of the shell can also be related to processes which induce the ejection of matter at the stellar surface, such as convection or shock waves.

In a shell of constant density, the temperature of the grains around a cold star maintains an almost spherical symmetry. As for silicate and graphite, the part taken by scattered light in the heating of grains is too weak to give significant asymmetry of the repartition of temperature. It would not be the same if the density changed according to the direction; the stellar
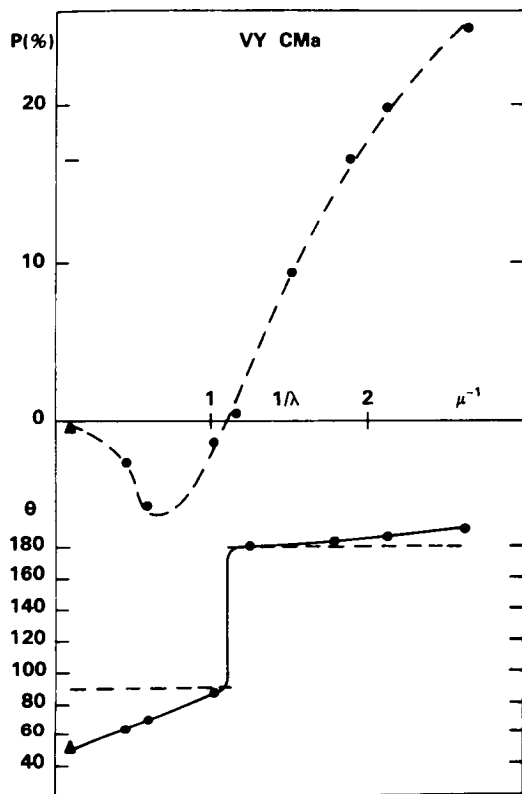
Figure 4-2. Variation of the rate of polarization p and of the angle of polarization θ for VY CMa. The dashed curves are the results obtained by numerical simulation by Daniel (1982). They are compared to the measurements of Serkowski (1973) (•) and Capps and Dyck (1972) (Δ). The negative values of p correspond to a polarization parallel to the polar axis of the shell and the positive ones to a polarization parallel to the equatorial plane. The variation of 90 degrees for θ corresponds to p = 0. The model is an ellipsoid with clean silicate grains of radius a = 0.26 μ.

radiation would be more attenuated along the equatorial plane, and the decrease in the temperature would be steeper. The radiation coming from the shell is very anisotropic due not only to the variation of opacity but also because it is strengthened by the scattered light in the polar direction. The anisotropy is low at large wavelengths and disappears when the optical depth is low in every direction. As a consequence, the flux curve depends on the shorten-ing angle: the stellar radiation is lowered when the observer is near the equatorial plane and band profiles are modified. Since observations are possible in one direction only, the interpretation of the results with a spherical model may lead to erroneous conclusions. The thermal energy emitted by the grains is equal to the energy lost by the stellar radiation only if all the directions of space are considered. Generally, this is not true for a single direction. For the same reason, it is not possible to guess the total flux emitted by the star and its shell when the intensity is known in only one direction. The effective temperature of the star deduced from an evaluation of the flux would be different if the shell is seen pole-on or equator-on. The number of adjustable parameters used in a spherical model almost always allows it to fit the observed flux curve. Photometric and polarimetric measurements with high angular resolution are needed to set additional constraints.

Numerical simulations have shown that the spatial repartition of flux is very asymmetric at every wavelength when the opacity is low. The shape of isophotes is mainly determined by the shortening angle and do not directly reflect the true dimensions of the shell. The bright annulus observed toward the inner boundary of spherical shells is again present when the extinction on the line of sight is not too high, but differences of opacity through the shell change it into a bright arc. The first results obtained by Roddier and Roddier (1983) for α Ori at λ = 0.535μ showed an asymmetric shell and a bright rim over a large sector of the image. In Figure 4-3, the visibility map of fringes produced by α Ori is shown. Their analysis allowed the image reconstruction of the shell. The results also displayed many fluctuations in the luminance: the shell is certainly complex. Moreover, frequent variations of the rate and angle of polarization reflect modifications of the structure of the shell at a rather small scale.

What developments can be expected in the near future? When the spherical symmetry is lost, the complexity of computations rapidly grows; most of the physical quantities explicitly
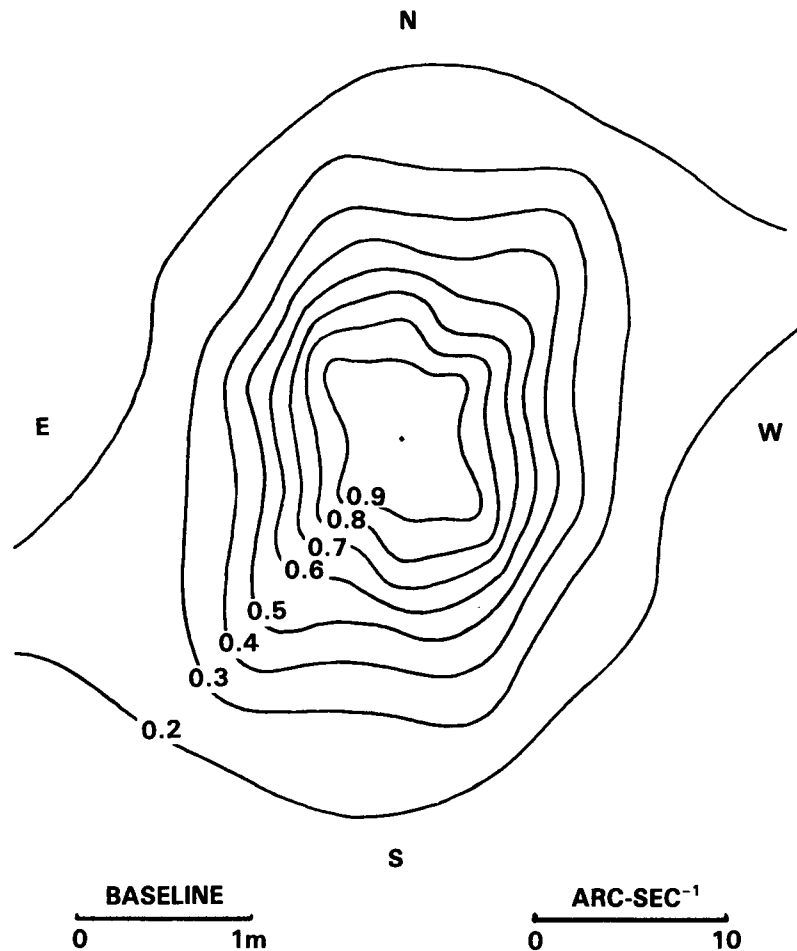
E   W

0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2

S

BASELINE

0   1m

ARC-SEC⁻¹

0   10

Figure 4-3. Isovisibility curves of the fringes produced by α Ori at λ = 5348 Å with the rotation shearing interferometer of Roddier and Roddier (1983). A departure from circular symmetry is clearly seen. Elongations in the visibility function are at 90 degrees from elongations in the image. Inner curves (low spatial frequencies) correspond to the shell.

involved in the resolution of the transfer equation depend on angular variables. It becomes difficult to systematically explore the influence of each parameter. Fairly simple repartitions of density can be studied by numerical simulation, but the increase of the computational time rapidly limits the possibilities. The problem of transfer must be tightly linked with the study of the dynamic evolution of the shell.

Theoretical progress in the problem of the formation of circumstellar grains and of the transfer of radiation in a shell accompany the advances of experimental methods. Spherical models of expanding homogeneous shells have thus far explained the flux curves obtained for nonresolved objects. The detailed spatial analysis of molecular abundances, polarization, luminence, etc. will consequently produce as a consequence a new generation of models, taking into account the inhomogeneities of the density, the effects of turbulence shock waves, and the temporal variations. One can but hope that the observations will not overemphasize exotic objects, but will also provide results for numerous shells with low opacity and slow variations. Significant progress in our knowledge

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of the properties of circumstellar grains will follow. These observations will in themselves permit the choice between candidate materials and guide experimental research.

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