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Pressure Structure of Solar Coronal Loops

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Abstract

The steady state pressure structure of a coronal loop is discussed in terms of the MHD global invariants of an incompressible plasma. The steady state is represented by the superposition of two Chandrasekhar-Kendall functions corresponding to (n=m=0) and n=m=1) modes. The relative contribution of the two modes (ϵ) is found to depend on the surface pressure of the coronal loop which is also the pressure of the external medium. The mixed mode state does not exist for high values of the external pressure because ϵ becomes complex.

Steady State Model of Coronal Loops

The coronal loop is represented by a cylindrical column of plasma with periodic boundary conditions at the ends of the cylinder (z=0,L). The pressure (p) profile of an incompressible MHD plasma is given by

$$\nabla^2 p = \vec{\nabla}.[(\vec{\nabla} \times \vec{B}) \times \vec{B}] - \vec{\nabla}.[(\vec{V}.\vec{\nabla})\vec{V}] \qquad (1)$$

where $\vec{V}$, $\vec{B}$ are respectively the velocity and the magnetic field. The magnetic field B is defined in Alfven speed units i.e. $B \equiv B/\sqrt{4\pi\rho}$. The Chandrasekhar-Kendall representation of the velocity and magnetic fields is given in terms of B_{nm} where m is the azimuthal mode number and n is the axial mode number.

$$\vec{B}_{00} = \xi_0\lambda_0 c_0[\hat{e}_\theta\lambda_0 J_1(\gamma_0 r) + \hat{e}_z\lambda_0 J_0(\gamma_0 r)] \qquad (2)$$

$$\begin{aligned}\vec{B}_{11} = & -\xi_1 c_1\gamma_1[k_1 J_0(\gamma_1 r) + \frac{J_1(\gamma_1 r)}{\gamma_1 r}(\lambda_1 - k_1)]\sin(\theta + k_1 z)\hat{e}_r \\ & -\xi_1 c_1\gamma_1[\lambda_1 J_0(\gamma_1 r) - \frac{J_1(\gamma_1 r)}{\gamma_1 r}(\lambda_1 - k_1)]\cos(\theta + k_1 z)\hat{e}_\theta \\ & +\xi_1 c_1\gamma_1^2 J_1(\gamma_1 r)\cos(\theta + k_1 z)\hat{e}_z\end{aligned} \qquad (3)$$

$$\vec{V}_{00} = \frac{\eta_0}{\xi_0} \vec{B}_{00} \text{ etc} \; ; \; K_1 = \frac{2\pi}{L} \; , \; \lambda_{nm} = \pm (\gamma_{nm}^2 + k_n^2)^{\frac{1}{2}}$$

Substituting these fields in equation (1), one can solve for the pressure profile in various cases. λ_0 is determined from the constancy of the ratio of the toroidal invariant ψ_t to the poloidal one ψ_p as

$$\frac{\psi_t}{\psi_p} = + \frac{R}{L} \frac{J_1(\gamma_0 R)}{J_0(\gamma_0 R)} \tag{4}$$

where R is the radius of the loop. γ_1 is determined from the boundary conditions $B_r = 0$ at r=R. For a rigid perfectly conducting wall at r = R;

$$Rk_n \gamma_{nm} J_m'(\gamma_{nm} R) + M\lambda_{nm} J_m(\gamma_{nm} R) = 0 \tag{5}$$

The pressure profile in the state (00 + 11) is given as:

$$\frac{p - p_0}{p_t} = g_0 + 6 \epsilon g_1 \cos(\theta + k_1 z) - 2\epsilon^2 [g_2 \cos^2(\theta + k_1 z)$$

$$+ g_3 \cos(2\theta + 2k_1 z) \quad + g_4 \sin^2(\theta + k_1 z) - g_5] \tag{8}$$

$$p_t = \frac{\gamma_0^2 R^2}{8\pi^2 J_1^2(\gamma_0 R) R^4} \quad , \qquad \epsilon = \frac{c_1 \xi_1 \gamma_1^2}{c_0 \xi_0 \gamma_0^2}$$

$$\gamma_1 r = x_1 \quad , \quad \gamma_0 r = x_0$$

$$g_1(x_1) = \left[\frac{\lambda_1}{\gamma_1} J_0(x_1) J_1(x_0) - \frac{\lambda_1 - k_1}{\gamma_1} \frac{J_1(x_1) J_1(x_0)}{x_1} - J_0(x_0) J_1(x_1)\right]$$

$$g_2(x_1) = \left[\frac{(\lambda_1 - k_1)^2}{\gamma_1^2} \frac{J_1^2(x_1)}{x_1^2} + \left\{\frac{k_1 \lambda_1}{2\gamma_1^2} + \frac{(\lambda_1 - k_1)(3\lambda_1 + k_1)}{8\gamma_1^2} + \frac{1}{2}\frac{\lambda_1^2}{\gamma_1^2}\right.\right.$$

$$\left. + \frac{(\lambda_1 - k_1)^2}{8\gamma_1^2}\right\} J_0^2(x_1) + \left\{\frac{1}{2} + \frac{k_1(\lambda_1 - k_1)}{2\gamma_1^2} + \frac{(\lambda_1 - k_1)(3\lambda_1 + k_1)}{8\gamma_1^2}\right.$$

$$\left. + \frac{(\lambda_1 - k_1)^2}{8\gamma_1^2}\right\} J_1^2(x_1) - \left\{\frac{(\lambda_1 - k_1)(3\lambda_1 + k_1)}{4\gamma_1^2} + \frac{\lambda_1(\lambda_1 - k_1)}{\gamma_1^2} + \frac{(\lambda_1 - k_1)^2}{4\gamma_1^2}\right\}$$

$$\left. \frac{J_0(x_1) J_1(x_1)}{x_1} \right]$$

$$g_3(x_1) = \left[\left\{\frac{1}{4} - \frac{k_1^2}{4\gamma_1^2}\right\}J_0^2(x_1) + \left\{\frac{\lambda_1^2 - k_1^2}{4\gamma_1^2}\right\}J_1^2(x_1) - \left\{\frac{1}{2} + \frac{k_1(\lambda_1 - k_1)}{2\gamma_1^2}\right\}\right.$$

$$\left.\frac{J_0(x_1)J_1(x_1)}{x_1} - \frac{(\lambda_1 - k_1)^2}{4\gamma_1^2}\frac{J_1^2(x_1)}{x_1^2}\right]$$

$$g_4(x_1) = \frac{1}{2}\left[\frac{k_1}{\gamma_1}J_0(x_1) + \frac{(\lambda_1 - k_1)}{\gamma_1}\frac{J_1(x_1)}{x_1}\right]^2$$

$$g_5 = \frac{(\lambda_1 - k_1)^2}{16\gamma_1^2} + \frac{(\lambda_1 + k_1)^2}{8\gamma_1^2} + \frac{\lambda_1 k_1}{4\gamma_1^2} .$$

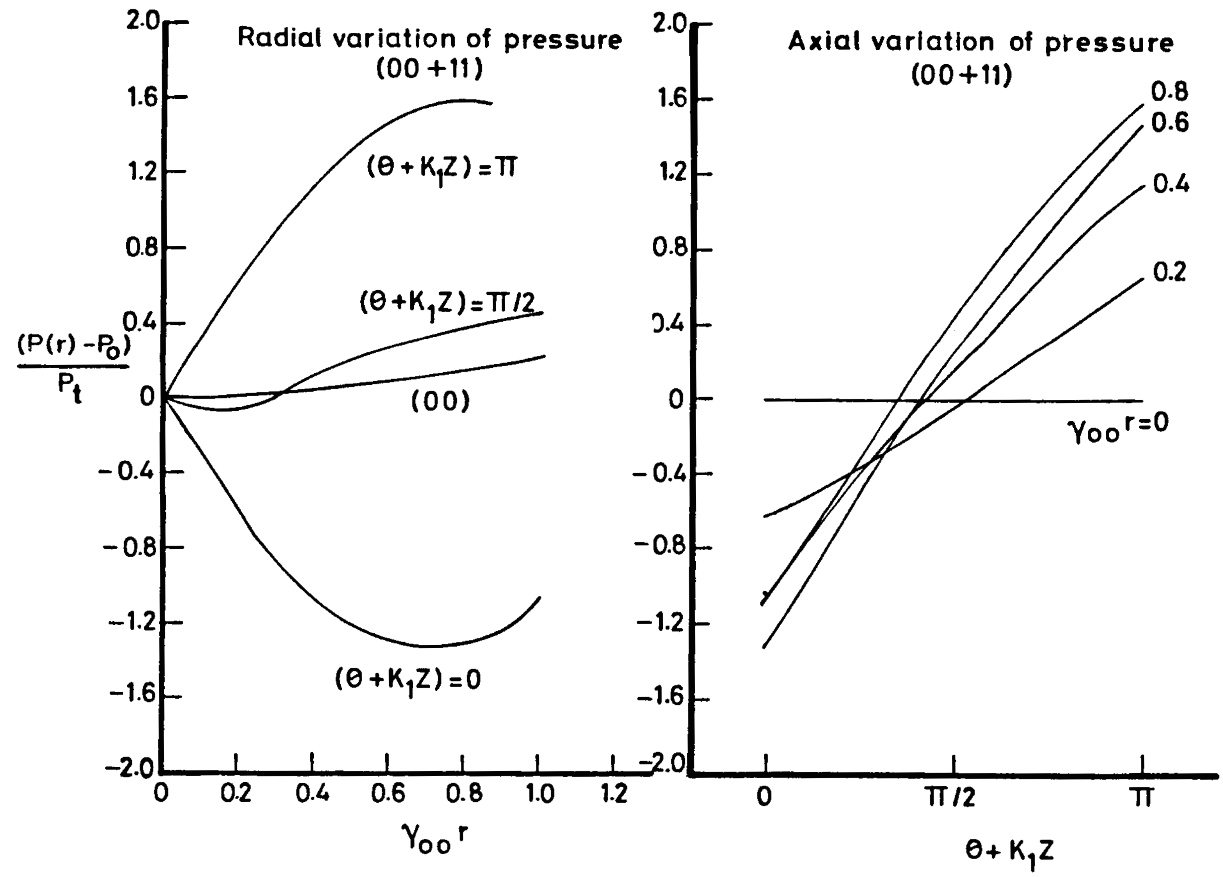
Here $p = p_0$ at $r = 0$, $z = 0$; c's are normalization constants.

An example

$$\frac{L}{R} = 5, \ \frac{\psi_t}{\psi_p} = 1 \ \text{gives} \ \gamma_0 R = 1$$

and

$$\lambda_1 R = 3.11, \ \gamma_1 R = 2.85, \ k_1 R = 1.25,$$



Total Energy, Magnetic Helicity and determination of ϵ.

The total energy $\tilde{W}$ for (00 + 11) state per unit length is

$$\tilde{W} = \frac{2(\gamma_0 R)\left(F_0 + \epsilon^2 \frac{\lambda_1}{\lambda_0} F_1\right)}{(F_0 + \epsilon^2 F_1)^2} \frac{\tilde{H}m^2}{\psi_t^2},$$

The magnetic helicity per unit length $\tilde{H}m$ is

$$\frac{\tilde{H}mR}{\psi_t^2} = F_0 + \epsilon^2 F_1$$

And the quantity,

$$I = \frac{(p_e(R) - p_0)\ \psi_t^6}{\tilde{H}m^4} = \frac{G_0 + \epsilon G_1 + \epsilon^2 G_2}{(F_0 + \epsilon^2 F_1)^4} .$$

For a fixed $\frac{R}{L}$, ψ_t/ψ_p, n, m. G_0, G_1, G_2, F_0 and F_1 are functions of $\gamma_0 R$ and $\gamma_1 R$ and completely determined. Therefore I is a function only of ϵ. ϵ can be determined from the following (approximate) equation:

$$\epsilon^2 = -\frac{(4F_1F_0^3 I - G_2) \pm [(4F_1F_0^3 I - G_2)^2 - H(F_0^4 I - G_0)\ 6F_1^2F_0^2 I]^{\frac{1}{2}}}{12IF_1^2F_0^2}$$

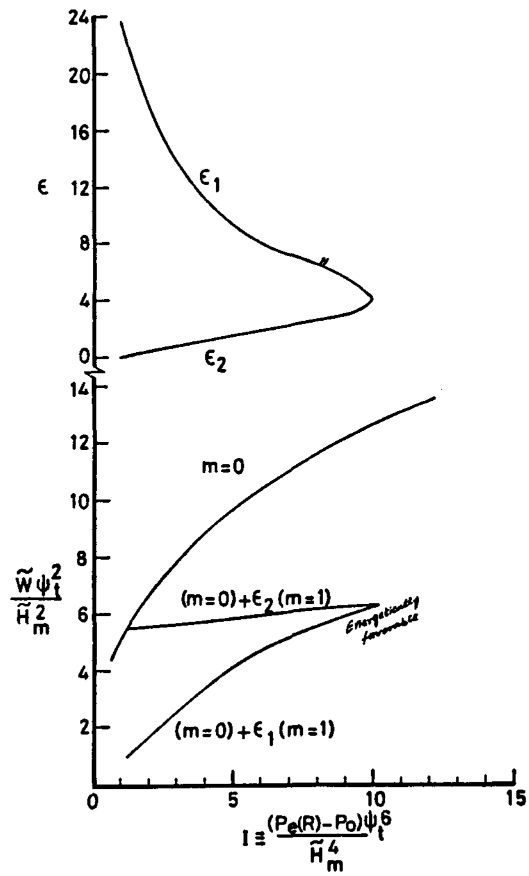
where

$$F_0 = \frac{\gamma_0^3 R^3}{2\pi J_1^2(\gamma_0 R)}\left[J_0^2(\gamma_0 R) + J_1^2(\gamma_0 R) - \frac{J_0(\gamma_0 R)J_1(\gamma_0 R)}{\gamma_0 R}\right]$$

$$F_1 = \frac{1}{4\pi^2\gamma_0 R\ J_1^2(\gamma_0 R)}\ \frac{\lambda_1}{\lambda_0}\ \frac{\gamma_0^4}{\gamma_1^4}\ \frac{1}{c_1^2}$$

$$G_0 = \frac{\gamma_0^2 R^2}{8\pi^2 J_1^2(\gamma_0 R)}\left(1 - J_0^2(\gamma_0 R) - J_1^2(\gamma_0 R)\right)$$

$$G_1 = \frac{3\gamma_0^2}{\gamma_1^2}\ \frac{\gamma_1^2 R^2}{4\pi^2 J_1^2(\gamma_0 R)}\left[\frac{\lambda_1}{\gamma_1} J_0(\gamma_1 R)J_1(\gamma_0 R) - \frac{\lambda_1 - k_1}{\gamma_1}\ \frac{J_1(\gamma_1 R)J_1(\gamma_0 R)}{\gamma_1 R} - J_0(\gamma_0 R)J_1(\gamma_1 R)\right]$$



A plot I vs ϵ is shown which shows that as the external pressure decreases, ϵ increases. For I $\geqslant$ 9.9, ϵ becomes complex.

$\frac{\tilde{W}\psi_t^2}{\tilde{H}m^2}$ is also plotted against I for the axisymmetric and the mixed mode state.

The m=0 curve is obtained by varying the ratio R/L and keeping ψ_t/ψ_p fixed. $\gamma_0 R$ increases as L/R increases and I also increases. For fixed L, I increases as R decreases, in other words smaller loops are in a medium of higher external pressure, which is as it should be. At a certain value of the external pressure the mixed mode state is energetically more favourable than the m=0 state. Thus there is a transition from m=0 state to (m=0 + m=1) state as the loop moves outwards in the corona in a medium of decreasing external pressure. The inner presure variation conforms to an increase of temperature along the axis from z=0, to z=L. The radial variation of temperature at the top of the loop confirms to the cool core and hot sheath model. Thus depending upon the position of the loops in the corona, one may observe them to be either in the m=0 state or (m=n=1) + (m=n=0) state and therefore the corresponding temperature variations are observed.