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STOCHASTIC MODEL OF THE NASA/MSFC GROUND FACILITY FOR LARGE SPACE STRUCTURES WITH UNCERTAIN PARAMETERS - THE MAXIMUM ENTROPY APPROACH

REPORT

by

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1. INTRODUCTION

The National Aeronautics and Space Administration and the Department of Defense are actively involved in the development of a validated technology data base in the areas of control/structures inter-action, deployment dynamics and system performance for Large Space Structures (LSS). In the Control System Division of the System Dynamics Laboratory of the NASA/MSFC, a Ground Facility (GF), in which the dynamics and control system concepts being considered for LSS applications can be verified, has been designed and built under Dr. Henry Waites' supervision [8]. The viability and versatility of this MSFC LSS ground test facility was recognized by the U. S. Air Force Wright Aeronautical Laboratory as a site for their Vibration Control of Space Structures (VCOSS) testing.

One of the important aspects of the GF is to verify the analytical model for the control system design. The procedure is to describe the control system mathematically as well as possible, then to perform tests on the control system, and finally to factor those results into the mathematical model.

However, development of a "correct" mathematical model of a system is still an art. In constructing large order structural models, various errors, such as modelling errors, parameter errors, improperly modeled uncertainties, and errors due to linearization of non-linear effect, create a great challenging task of determining "best" models for a dynamic system. It is recognized that it is conceivable that better performance will be anticipated when uncertainties are modeled through stochastic multiplicative and additive noise terms. Optimal control strategies generated under all possible parameter variations will definitely create more robust control systems, under controllability and observability conditions, than those generated by the usual approaches [2]. To aviod ad hoc assumptions regarding "a priori" statistics, Hyland [2,3,4] used the maximum entropy principle to determine a priori probability assignment induced from available data. A main advantage of maximum entropy approach is that it sacrifies as little near-nominal performance as possible while securing performance insensitivity over the likely range of modelling errors.

In this report, we design a stochastic control model of the NASA/MSFC Ground Facility for LSS control verification through the maximum entropy principle adopted in Hyland's method [2,3,4]. Using ORACLS, a computer program is implemented for this purpose. Four models are then tested. Results are presented in this report.

2. MAXIMUM ENTROPY MODELLING

Consider a linear system :

$$\dot{X} = AX + BU + \omega_1$$
$$Y = CX + \omega_2 \qquad (1)$$

where

$$X \in R^n,\ U \in R^m,\ Y \in R^\ell,\ A \in R^{nxn},\ B \in R^{nxm},\ C \in R^{\ell xm},$$

and

$$SD(\omega_1, \omega_2) = (v_1, v_2).$$

We seek to determine a dynamic compensator

$$\dot{Z} = A_c Z + FY$$
$$U = -KZ \qquad (2)$$

where $Z \in R^n$, $A_c \in R^{nxn}$, $F \in R^{nx\ell}$ and $K \in R^{mxn}$ that minimizes the Quadratic Cost Function :

$$J = \int_0^\infty (X^T R_1 X + U^T R_2 U)\ dt \qquad (3)$$

where R_1 and R_2 are penalty matrices. The maximum entropy (ME) design approach [1,2,3,4,5] is used to minimize J in the presence of parameter uncertainties.

In most instances, the actual system dynamics differ from the nominal model by an error distribution matrix. The basic premise of ME error modelling is that the magnitude of the error is a white-noise process $\alpha(t)$. Assuming there are p uncorrelated error sources, the system dynamic matrices become :

$$A_{actual} = A + \sum_{i=1}^{p} \alpha_i(t) A_i \tag{4}$$

with the B_{actual} and C_{actual} matrices taking similar forms.

For the simplicity and in order to get a good inside look at the ME design technique, we assume there is only one error distribution matrix A_1 in the system. Under these assumption, the necessary conditions for optimality of the Quadratic Cost Function can be derived after the system dynamics are presented by means of stochastic differential equations. The resulting equations take the form of two Riccati equations and two Lyapunov equations, all coupled by the stochastic parameters [6]. That is, we need to solve four nonnegative-definite P, Q, $\hat{P}$ and $\hat{Q}$ such that

$$\begin{aligned}
&PA_s + A_s^T P + A_1^T PA_1 - P_s^T R_2^{-1} P_s + R_1 + A_1^T \hat{P} A_1 = 0 \\
&A_s Q + QA_s^T + A_1 QA_1^T - Q_s V_2^{-1} Q_s^T + V_1 + A_1 \hat{Q} A_1^T = 0 \\
&\hat{P} A_{Qs} + A_{Qs}^T \hat{P} + P_s^T R_2^{-1} P_s = 0 \\
&A_{ps} \hat{Q} + \hat{Q} A_{ps}^T + Q_s V_2^{-1} Q_s^T = 0
\end{aligned} \tag{5}$$

where

$$A_s \triangleq A + \tfrac{1}{2} A_1^2, \quad P_s \triangleq B^T P, \quad Q_s \triangleq QC^T,$$

$$A_{Qs} \triangleq A_s - Q_s V_s^{-1} C, \quad A_{ps} \triangleq A_s - BR_2^{-1} B^T P.$$

The compensator matrices then take on the following forms,

$$A_c = A_s - Q_s V_2^{-1} C - B R_2^{-1} P_s$$

$$F = Q_s V_2^{-1} \tag{6}$$

$$K = R_2^{-1} P_s.$$

Unfortunately, the covariance matrices V_1 and V_2 of the Wiener processes ω_1 and ω_2, respectively, in (1) are usually not known. However, we developed a method of estimating those two import matrices as follows.

Consider the system

$$\dot{X} = AX + BU + \omega_1. \tag{7}$$

(7) can be rewritten as

$$dX^i = (\sum_j A_j^i X^j + \sum_k B_k^i U^k)dt + d\omega_1^i, \qquad i = 1,\cdots,n.$$

Let $r_1^{ij} = E[X^i X^j]$ and $r_1^{ij} = E[X^i X^j]$. By Ito's rule, we have

$$d(X^i X^j) = (dX^i)X^j + X^i(dX^j) + (dX^i)(dX^j)$$

$$= (\sum_k A_k^i X^k X^j + \sum_\ell B_1^i U^\ell X^j)dt$$

$$+ (\sum_k A_k^j X^k X^i + \sum_\ell B_1^j U^\ell X^i)dt$$

$$+ X^j d\omega_1^i + X^i d\omega_1^j + (d\omega_1^i)(d\omega_2^j). \tag{8}$$

With the assumption that X and ω_1 are uncorrelated, (8) becomes

$$\dot{r}^{ij} = \sum_k A_k^i r^{kj} + \sum_k A_k^j r^{ki} + \sum_\ell B_1^i q^{\ell j} + \sum_\ell B_\ell^j q^{kj} + V_1^{ij}$$

or

$$V_1^{ij} = \sum_k A_k^i r^{kj} + \sum_k A_k^j r^{ki} + \sum_\ell B_\ell^i q^{1j} + \sum_\ell B_\ell^j q^{kj} - \dot{r}^{ij}, \quad (9)$$

where $V_1^{ij}dt = E[d\omega_1^i \, d\omega_1^j]$ and $q^{ij} = E[U^i X^j]$.

If we assume, in addition, that X and U are uncorrelated, then we can drop the terms involving q^{ij} in (9). And (9) becomes

$$V_1^{ij} = \sum_k A_k^i r^{kj} + \sum_k A_k^j r^{ki} - \dot{r}^{ij} \quad (10)$$

for $i, j = 1,2,\cdots,n$.

Therefore, if r and $\dot{r}$ can be estimated, then the covariance matrix V_1 can be estimated through (10).

Estimation of V_2 for ω_2 in the equation $Y = CX + \omega_2$ is a much easier job. We simply use the standard statistics technique to estimate V_2 by $E[(Y - CX)(Y - CX)^T]$.

3. COMPUTATION ALGORITHM

In this report, we treat all four equations in (5) together as a single Riccati equation. This approach is different from the one proposed by Gruzen [6] in which each iteration involves solving the first two equations of (5) as Riccati equations and then solving the last two equations of (5) as Lyapunov equations.

We can rewrite (5) as following:

$$
\begin{aligned}
&PA_s + A_s^T P - P^T B R_2^{-1} B^T P + A_1^T (P + (A_1^{-1})^T R_1 A_1^{-1} + \hat{P}) A_1 &&= 0 \\
&QA_s^T + A_s Q - QC^T V_2^{-1} CQ + A_1 (Q + A_1^{-1}\ V_1 (A_1^{-1})^T + \hat{Q}) A_1^T &&= 0 \\
&\hat{P} A_{Qs} + A_{Qs}^T \hat{P} - \hat{P}^T \theta R_2^{-1} \theta^T \hat{P} + P_s^T R_2^{-1} P_s &&= 0 \\
&\tilde{Q} A_{Ps}^T + A_{Ps} \hat{Q} - \hat{Q} \theta V_2^{-1} \theta^T \hat{Q} + Q_s V_1^{-1} Q_s^{-1} &&= 0
\end{aligned}
\qquad (11)
$$

where matrix θ indicates zero matrix with appropriate dimension. In a more concise form, we have:

$$P^* A^* + (A^*)^T P^* - (P^*)^T B^* (R^*)^{-1} (B^*)^T P^* + (H^*)^T Q^* H^* = 0 \qquad (12)$$

where

$$
P^* = \begin{bmatrix} P^T & & & 0 \\ & Q & & \\ & & \hat{P} & \\ 0 & & & \hat{Q} \end{bmatrix}, \qquad
R^* = \begin{bmatrix} R_2 & & & 0 \\ & V_2 & & \\ & & R_2 & \\ 0 & & & V_2 \end{bmatrix},
$$

$$
A^* = \begin{bmatrix} A_s & & & 0 \\ & A_s^T & & \\ & & A_s & \\ 0 & & & A_{Ps}^T \end{bmatrix}, \qquad
B^* = \begin{bmatrix} B & C^T & 0 \\ 0 & & \theta \end{bmatrix},
$$

$$H^* = \begin{bmatrix} A_1 & & & 0 \\ & A_1^T & & \\ & & P_s & \\ 0 & & & Q_s^T \end{bmatrix}$$

and

$$Q^* = \begin{bmatrix} P + (A_1^{-1})^T R_1 A_1^{-1} + \hat{P} & & & 0 \\ & Q + A_1^{-1} V_1 (A_1^{-1})^T + \hat{Q} & & \\ & & R_2^{-1} & \\ 0 & & & V_2^{-1} \end{bmatrix} .$$

Note that (12) does not exactly match the standard algebraic Riccati equation form:

$$PA + A^T P - PBR_2^{-1} B^T P + R_1 = 0. \qquad (13)$$

Because there are unknown parameters in the last term $H^{*T}Q^*H^*$ in (12). This character affects the iteration scheme significantly. The constant term of the Riccati equation (12) includes P^* matrix. Consequently, the equation must be iterated through several times, updating P^* solution in the constant term each time until it converges to a solution. The iteration strategy is illustrated in Figure 1.

The convergence criterion used in our program is when $\| P^{*(n)} - P^{*(n-1)} \| < \varepsilon$, where ε is a preset tolerance.

The software package ORACLS [7] provides a control system design and analysis environment. This package provides subroutines such as basic matrix manipulations (addition, subtraction, multiplication, transpose, etc.) and Riccati solver. The design algorithm is implemented in FORTRAN (see Appendix).

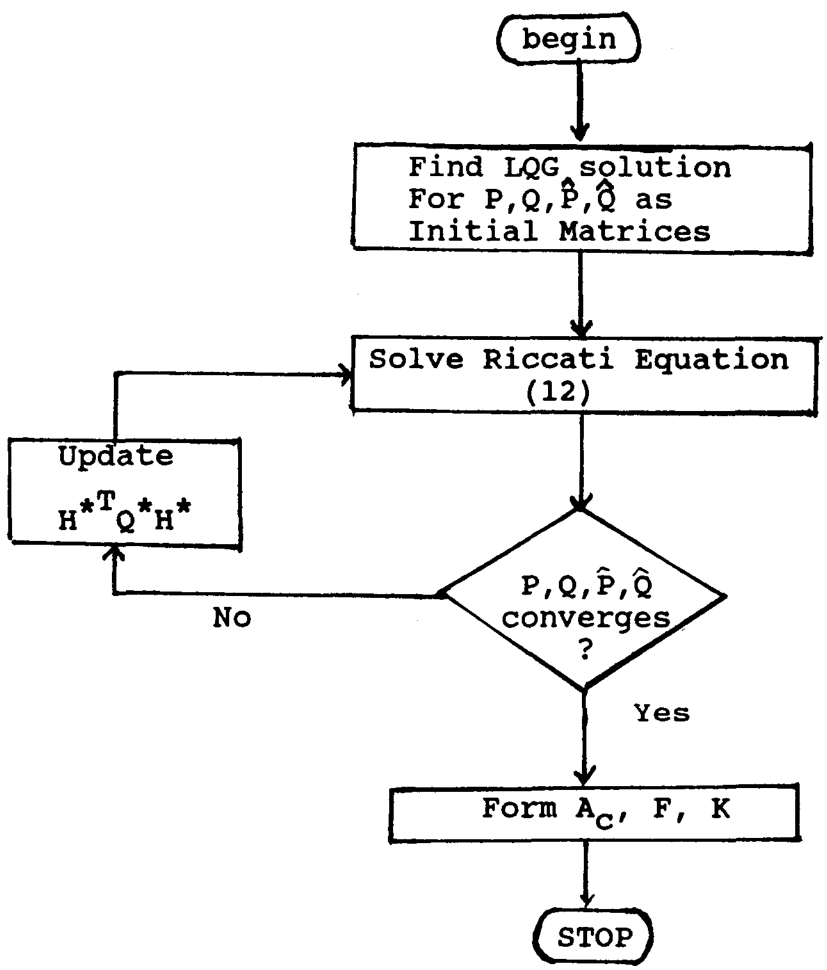

Figure 1

4. COMPUTATION RESULTS OF ME DESIGN

In this section, we applied the ME design algorithm to the MSFC Ground Test Facility in which dynamics and control system concepts being considered for LSS applications can be verified [8].

There are 50 modes in the system model. For the purpose of testing the algorithm and the FORTRAN program, we only consider stochastic models with only one mode. Mode 8 is chosen for this purpose. We also assume there is only one error distribution matrix of A in the system. Therefore, the stochastic model concerned in this section is

$$\dot{X}(t) = (A + \alpha_1(t)A_1)X(t) + BU(t) + \omega_1(t)$$
$$Y(t) = CX(t) + \omega_2(t) \qquad (14)$$

Data collected from an analytical model in four different settings have been provided by Dr. Henry Waites. Using those data, we designed four settings and through which we can determine corresponding compensator matrices. In all of those four settings, we choose the modal damping $\xi_8 = 0.5\%$,

$$R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \text{and} \qquad R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The dynamic matrices of those four models are:

Model 1 (EXP5VL Sept. 24. 1986): $\omega_8^2 = 17.44$

$$A = \begin{bmatrix} 0 & 1 \\ -17.44 & -0.04176 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -0.00154 & 0 & 0 & 0.01 & 0.000176 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \\ 0 & 0.000176 \end{bmatrix}.$$

Model 2 (EXP6UL Oct. 1. 1986): $\omega_8^2 = 17.44$

$$A = \begin{bmatrix} 0 & 1 \\ -17.44 & -0.04176 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.0016 & 0 & 0 & -0.01036 & -0.0004875 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & -0.01036 \\ 0 & -0.0004875 \end{bmatrix}.$$

Model 3 (EMVL Dec. 29. 1986): $\omega_8^2 = 19.41$

$$A = \begin{bmatrix} 0 & 1 \\ -19.41 & -0.04176 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.0001 & 0 & 0.002 & -0.0082 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0.002 \\ 0 & -0.082 \\ 0 & 0 \end{bmatrix}.$$

Model 4 (EMFVLL Jan. 20. 1987): $\omega_8^2 = 14.4$

$$A = \begin{bmatrix} 0 & 1 \\ -14.4 & -0.03796 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.00031890 & 0 & 0 & 0.001617 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0.0016171 \\ 0 & 0 \end{bmatrix}.$$

As pointed out by Gruzen [6], we can scale the position coordinate by the modal frequency ω_8, the first equation of (14) is transformed into an equivalent representation:

$$\frac{d\tilde{X}}{dt} = \begin{bmatrix} 0 & w_8 \\ -\omega_8 & -2\xi\omega_8 \end{bmatrix} \tilde{X} + BU, \qquad (16)$$

where $\tilde{X} = \begin{bmatrix} x\omega_8 \\ \dot{x} \end{bmatrix}$ with the transformation matrix $T = \begin{bmatrix} \omega_8 & 0 \\ 0 & 1 \end{bmatrix}$.

Therefore, we can assume the uncertainty distribution matrix for the last four models takes the following form:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -0.01 \end{bmatrix}.$$

The compensation matrices resulted from the algorithm are summerized as following.

Model	1	2
P	3.91 D+04 -2.50 D-03 -2.50 D-03 3.91 D+04	3.63 D+04 -2.50 D-03 -2.50 D-03 3.63 D+04
Q	4.01 D+02 -2.00 D+00 -2.00 D+00 4.01 D+02	3.72 D+04 -1.85 D+02 -1.86 D+02 3.72 D+04
P	1.47 D-06 4.88 D-09 4.83 D-09 1.95 D-11	1.58 D-06 5.26 D-09 5.26 D-09 2.10 D-11
Q	0 0 0 0	0 0 0 0
A_c	-5.0 D-01 2.39 D-03 1.07 D-06 -6.51 D+00	-5.0 D-01 2.39 D-03 1.15 D-06 -6.50 D+00
F	-6.02 D+01 0 6.88 D+00 0 3.91 D+02 7.05 D-02	5.81D+01 0 -1.77D+01 0 -3.76D+02 -1.81D+01
K	3.85 D-06 -6.02 D+01 0 0 0 0 -2.50 D-05 3.91 D+02 -4.40 D-07 6.88 D+00	-4.0 D-06 5.81 D+01 0 0 0 0 2.59 D-05 -3.76 D+02 1.22 D-06 -1.77 D+01

Model	3			4		
P	5.61 D+04	-2.50 D-03		1.47 D+06	-2.50 D-03	
	-2.50 D-03	5.61 D+04		-2.50 D-03	1.47 D+06	
Q	5.61 D+04	-2.81 D+02		1.53 D+06	-7.65 D+03	
	-2.81 D+02	5.61 D+04		-7.65 D+03	1.53 D+06	
$\tilde{P}$	5.61 D+04	-2.50 D-03		1.47 D+06	-2.50 D-03	
	-2.50 D-03	5.61 D+04		-2.50 D-03	1.47 D+06	
$\tilde{Q}$	0	0		0	0	
	0	0		0	0	
A_c	-5.0 D-01	2.27 D-03		-5.0 D-01	2.64 D-03	
	7.85 D-07	-6.50 D+00		2.58 D-08	1.47 D+00	
F	5.61 D+00	1.12 D+02	0	4.70 D+02	0	0
	0	-4.60 D+02	0	0	2.38 D+03	0
K	-2.50 D-07	5.61 D+00		-7.97 D-07	4.70 D+02	
	0	0		0	0	
	-5.00 D-06	1.12 D+02		0	0	
	2.05 D-05	-4.60 D+02		-4.04 D-06	2.38 D+03	
	0	0		0	0	

Figure 2

5. CONCLUSION

In general, the major issues relevant to the control of flexible space structures are "robustness" with respect to both parameter modelling errors and truncation of higher order modes. Several methods have been developed recently to deal with those problems. Among them, the maximum entropy and optimal projection (MEOP) method developed by Hyland and Bernstein specifically for the flexible structure control problems seems very promising.

In this report, we examined the ME portion of the design method. Using ORACLS, we implemented a computer program for ME method. Four small scaled models are then tested and the resulted compensation matrices are given.

The extension of this project, natually, would be to test the OP portion of the design method and then combine those two programs to have a complete MEOP design tool.

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APPENDIX 1

COMPUTER PROGRAM FOR ME DESIGN

```
C*********************************************************************
C*   DRIVER FOR THE MAXIMUM ENTROPY DESIGNER
C*      INPUTTAPE = 5      OUTPUTTAPE = 6
C*********************************************************************
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION P1(64),A1(64),B1(64),R1(64),Q1(64),H1(64),F1(64),
     1           DUMMY(2100),P(4),Q(4),PHAT(4),QHAT(4),E1(4)
       DIMENSION NP1(2),NA1(2),NB1(2),NR1(2),NQ1(2),NH1(2),NF1(2),
     1           IOP(3),NE1(2)
       DIMENSION P2(64),A2(64),B2(64),R2(64),Q2(64),H2(64),F2(64),
     1           D21(64),D22(64),D23(64),D24(64),D25(64),D26(64)
       DIMENSION NP2(2),NA2(2),NB2(2),NR2(2),NQ2(2),NH2(2),NF2(2),
     1           ND21(2),ND22(2),ND23(2),ND24(2),ND25(2),ND26(2)
       DIMENSION A5(4),A51(4),Q5(4),P5(4),Q55(4),P55(4),C5(6),B5(10),
     1           V51(4),V52(9),R51(4),R52(25),VI52(9),RI52(25),B3(256),
     1           R3(256),TH3(96),TH4(64),TH5(96),TH6(96),TH7(192),
     1           TH8(64),H3(256),TA51(4),TA5(4),A31(4),A32(4),TA6(6),
     1           TC5(6),A33(4),TB5(10),TA7(10),A34(4),Z1(12),Z2(8),
     1           Z5(128),X1(16),X21(8),X2(16),X31(12),X3(16),X5(32),
     1           X6(48),X7(64),X8(128),Q31(4),Q32(4),Z6(20),Z7(20),
     1           Z8(15),Z9(27),X9(24),X10(24),X11(60),X12(36),X13(45),
     1           X14(48),X15(108),X16(144),X17(48),Q3(256),F3(256),
     1           P3(256),X4(16),X18(192),A3(256),TP4(256),TP5(256),
     1           PP(4),PPH(4),QQ(4),QQH(4),R15(64),TM1(4),F(6),TM2(4),
     1           TM3(10),TM4(10),K(10),E2(4),ACC(4),TRN(4),TRNI(4),AC(4)
       DIMENSION NA5(2),NA51(2),NQ5(2),NP5(2),NQ55(2),NP55(2),NC5(2),
     1           NB5(2),NV51(2),NV52(2),NR51(2),NR52(2),NVI52(2),
     1           NRI52(2),NB3(2),NR3(2),NTH3(2),NTH4(2),NTH5(2),NTH6(2),
     1           NTH7(2),NTH8(2),NH3(2),NTA51(2),NTA5(2),NA31(2),NA32(2),
     1           NTA6(2),NTC5(2),NA33(2),NTB5(2),NTA7(2),NA34(2),NZ1(2),
     1           NZ2(2),NZ5(2),NX1(2),NX2(2),NX21(2),NX31(2),NX3(2),
     1           NX5(2),NX6(2),NX7(2),NX8(2),NQ31(2),NQ32(2),NZ6(2),
     1           NZ7(2),NZ8(2),NZ9(2),NX9(2),NX10(2),NX11(2),NX12(2),
     1           NX13(2),NX14(2),NX15(2),NX16(2),NX17(2),NX18(2),
     1           NQ3(2),NF3(2),NP3(2),NX4(2),NA3(2),NTP4(2),NTP5(2),
     1           NP(2),NQ(2),NPHAT(2),NQHAT(2),NPP(2),NPPH(2),NQQ(2),
     1           NQQH(2),NR15(2),NTM1(2),NF(2),NTM2(2),NTM3(2),NTM4(2),
     1           NK(2),NE2(2),NACC(2),NTRN(2),NTRNI(2),NAC(2)
       INTEGER   IERR,ITE
       REAL SCLE,EPSI
       LOGICAL IDENT,DISC,FNULL
C
C     INPUT HOLLERITH DATA FOR TITLE OF OUTPUT
        CALL RDTITL
C
C       TO OBTAIN INITIAL P AND Q BY SOLVING THE ASSOCIATED RCT EQA
C       FROM LQG METHOD
C
C          INPUT COEFFICIENT MATRICES FOR THE INITIAL LQG SYSTEM
C
      CALL READ(4,A5,NA5,A51,NA51,C5,NC5,B5,NB5)
      CALL READ(4,V51,NV51,V52,NV52,R51,NR51,R52,NR52)
      CALL READ(4,A1,NA1,B1,NB1,R1,NR1,Q1,NQ1)
      CALL READ(2,D22,ND22,R15,NR15)
      CALL READ(4,B3,NB3,R3,NR3,TH3,NTH3,TRN,NTRN)
      DO 10 I=1,2
      NVI52(I)=3
      NRI52(I)=5
      NH1(I)=8
      NQ2(I)=8
```

```
      ND23(I)=8
      NTH4(I)=8
      NP2(I)=8
      NA2(I)=8
      NB2(I)=8
      NH2(I)=8
      NF2(I)=8
      ND21(I)=8
      ND24(I)=8
      ND25(I)=8
      ND26(I)=8
      NP5(I)=2
      NP55(I)=2
      NQ5(I)=2
      NQ55(I)=2
      NTRNI(I)=2
 10   CONTINUE
      CALL UNITY(VI52,NVI52)
      CALL GAUSEL(3,3,V52,3,VI52,IERR)
      CALL UNITY(RI52,NRI52)
      CALL GAUSEL(5,5,R52,5,RI52,IERR)
      CALL EQUATE(R1,NR1,R2,NR2)
      CALL UNITY(Q2,NQ2)
      CALL GAUSEL(8,8,R1,8,Q2,IERR)
      CALL UNITY(D23,ND23)
      CALL GAUSEL(8,8,R15,8,D23,IERR)
      CALL PRNT(VI52,NVI52,4HVI52,1)
      CALL PRNT(RI52,NRI52,4HRI52,1)
      CALL PRNT(R2,NR2,4H  R2,1)
      CALL PRNT(Q2,NQ2,4H  Q2,1)
      CALL PRNT(D23,ND23,4H D23,1)
      CALL UNITY(TRNI,NTRNI)
      CALL GAUSEL(2,2,TRN,2,TRNI,IERR)
      CALL PRNT(TRNI,NTRNI,4HTRNI,1)
      EPSI=0.001
      DIFF=100.0
C
C         CHECK IF A IS ASYMPOTICALLY STABLE BY  CSTAB
C
          IOP(1)=0
          IOP(2)=0
          IOP(3)=0
          SCLE=1.0
          CALL CSTAB(A1,NA1,B1,NB1,F1,NF1,IOP,SCLE,DUMMY)
C
C      READY TO CALL SUBROUTINE   RICNWT
          IDENT=.TRUE.
          DISC=.FALSE.
          FNULL=.FALSE.
          DO 50 I=1,550
          DUMMY(I)=0.0
 50       CONTINUE
          CALL RICNWT(A1,NA1,B1,NB1,H1,NH1,Q1,NQ1,R1,NR1,F1,NF1,P1,
     1                NP1,IOP,IDENT,DISC,FNULL,DUMMY)
C
C     BEGINNING OF THE STEP 2
C
      PRINT *, '  '
C
C
```

```
C     CREATE MATRIX D21
C
      DO 20 I=1,64
      D21(I)=0.0
      D26(I)=0.0
20    CONTINUE
      D21(1)  =P1(19)
      D21(9)  =P1(27)
      D21(2)  =P1(20)
      D21(10) =P1(28)
      D21(19) =P1(1)
      D21(27) =P1(9)
      D21(20) =P1(2)
      D21(28) =P1(10)
C
C     CREAT MATRIX B2
C
      CALL NULL(B2,NB2)
C
C    CREATE MATRIX A2
C
      CALL TRANP(D22,ND22,D24,ND24)
      CALL MULT(D23,ND23,D24,ND24,D25,ND25)
      CALL MULT(D22,ND22,D25,ND25,D24,ND24)
      CALL MULT(D21,ND21,D24,ND24,D25,ND25)
      CALL SUBT(A1,NA1,D25,ND25,A2,NA2)
C
C    CREATE MATRIX H2)
C
      D26(1)=P1(1)
      D26(2)=P1(2)
      D26(9)=P1(9)
      D26(10)=P1(10)
      D26(22) =P1(19)
      D26(30) =P1(27)
      D26(23) =P1(20)
      D26(31) =P1(28)
      CALL TRANP(B1,NB1,D22,ND22)
      CALL MULT(D22,ND22,D26,ND26,H2,NH2)
C
C   CHECK IF A2 IS ASYMPTICALLY STABLE BY CSTAB
C
      CALL CSTAB(A2,NA2,B2,NB2,F2,NF2,IOP,SCLE,DUMMY)
C
C   READY TO CALL RICNWT TO FIND P2
C
      IDENT=.FALSE.
      DO 60 I=1,550
      DUMMY(I)=0.0
60    CONTINUE
      CALL RICNWT(A2,NA2,B2,NB2,H2,NH2,Q2,NQ2,R2,NR2,F2,NF2,P2,NP2,
     1IOP,IDENT,DISC,FNULL,DUMMY)
      ITE=0
C
C  END OF SEARCHING INITIAL MATRICES BY LQG
C
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, '*** STARTING LQG SOLUTIONS ARE :'
C
```

```
C    START ITERATIVE ALGORITHM
C
C
      P5(1)=P1(1)
      P5(2)=P1(2)
      P5(3)=P1(9)
      P5(4)=P1(10)
      P55(1)=P2(1)
      P55(2)=P2(2)
      P55(3)=P2(9)
      P55(4)=P2(10)
      Q5(1)=P1(19)
      Q5(2)=P1(20)
      Q5(3)=P1(27)
      Q5(4)=P1(28)
      Q55(1)=P2(19)
      Q55(2)=P2(20)
      Q55(3)=P2(27)
      Q55(4)=P2(28)
      CALL EQUATE(P5,NP5,P,NP)
      CALL EQUATE(P55,NP55,PHAT,NPHAT)
      CALL EQUATE(Q5,NQ5,Q,NQ)
      CALL EQUATE(Q55,NQ55,QHAT,NQHAT)
      CALL EQUATE(P5,NP5,PP,NPP)
      CALL EQUATE(P55,NP55,PPH,NPPH)
      CALL EQUATE(Q5,NQ5,QQ,NQQ)
      CALL EQUATE(Q55,NQ55,QQH,NQQH)
      PRINT *, ' '
      CALL PRNT(P,NP,4H   P,1)
      PRINT *, ' '
      CALL PRNT(Q,NQ,4H   Q,1)
      PRINT *, ' '
      CALL PRNT(PHAT,NPHAT,4HPHAT,1)
      PRINT *, ' '
      CALL PRNT(QHAT,NQHAT,4HQHAT,1)
C
C     CREATE  H*
C
      NTH4(1)=8
      NTH4(2)=8
      CALL UNITY(TH4,NTH4)
      TH4(37)=P1(1)
      TH4(38)=P1(2)
      TH4(45)=P1(9)
      TH4(46)=P1(10)
      TH4(55)=P1(19)
      TH4(56)=P1(20)
      TH4(63)=P1(27)
      TH4(64)=P1(28)
      GO TO 110
100   ITE=ITE+1
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, ' ************ THIS IS ITERATION  ',ITE,'  ******'
      PRINT *, ' '
      PRINT *, ' ***  NEW SOLUTIONS ARE **'
```

```
      CALL EQUATE(P5,NP5,P,NP)
      CALL EQUATE(P55,NP55,PHAT,NPHAT)
      CALL EQUATE(Q5,NQ5,Q,NQ)
      CALL EQUATE(Q55,NQ55,QHAT,NQHAT)
      PRINT *, ' '
      CALL PRNT(P,NP,4H   P,1)
      PRINT *, ' '
      CALL PRNT(PHAT,NPHAT,4HPHAT,1)
      PRINT *, ' '
      CALL PRNT(Q,NQ,4H   Q,1)
      PRINT *, ' '
      CALL PRNT(QHAT,NQHAT,4HQHAT,1)
         PRINT *, ' '
         PRINT *, ' '
         PRINT *, ' '
         PRINT *, ' *** THE L-2 NORM = ',DIFF,' *****'
         IF (DIFF .LT.EPSI)GO TO 130
      PRINT *, ' '
C
C     CREATE  H*
C
      NTH4(1)=8
      NTH4(2)=8
      CALL UNITY(TH4,NTH4)
      TH4(37)=P1(1)
      TH4(38)=P1(2)
      TH4(45)=P1(17)
      TH4(46)=P1(18)
      TH4(55)=P1(35)
      TH4(56)=P1(36)
      TH4(63)=P1(51)
      TH4(64)=P1(52)
 110  CALL MULT(TH3,NTH3,TH4,NTH4,TH5,NTH5)
      NTH6(1)=12
      NTH6(2)=8
      CALL NULL(TH6,NTH6)
      CALL JUXTC(TH5,NTH5,TH6,NTH6,TH7,NTH7)
      NTH8(1)=4
      NTH8(2)=16
      CALL JUXTR(TH7,NTH7,TH8,NTH8,H3,NH3)
C
C     CREATE MATRIX  A*
C
C       FIND A31
C
      CALL EQUATE(A51,NA51,TA51,NTA51)
      CALL MULT(A51,NA51,TA51,NTA51,TA5,NTA5)
      CALL SCALE(TA5,NTA5,TA51,NTA51,0.5)
      CALL ADD(A51,NA51,TA51,NTA51,A31,NA31)
      CALL EQUATE(A31,NA31,E1,NE1)
C
C       FIND A32
C
      CALL TRANP(A31,NA31,A32,NA32)
C
C       FIND A33
C
      CALL MULT(VI52,NVI52,C5,NC5,TA6,NTA6)
      CALL TRANP(C5,NC5,TC5,NTC5)
      CALL MULT(TC5,NTC5,TA6,NTA6,TA51,NTA51)
```

```
      CALL MULT(Q5,NQ5,TA51,NTA51,TA5,NTA5)
      CALL SUBT(A31,NA31,TA5,NTA5,A33,NA33)
C
C       FIND A34
C
      CALL TRANP(B5,NB5,TB5,NTB5)
      CALL MULT(TB5,NTB5,P5,NP5,TA7,NTA7)
      CALL MULT(RI52,NRI52,TA7,NTA7,TB5,NTB5)
      CALL MULT(B5,NB5,TB5,NTB5,TA51,NTA51)
      CALL SUBT(A31,NA31,TA51,NTA51,A34,NA34)
C
C       FIND A*
C
      NZ1(1)=2
      NZ1(2)=6
      NZ2(1)=2
      NZ2(2)=4
      NZ5(1)=8
      NZ5(2)=16
      CALL NULL(Z1,NZ1)
      CALL NULL(Z2,NZ2)
      CALL NULL(TA5,NTA5)
      CALL NULL(TH4,NTH4)
      CALL NULL(Z5,NZ5)
      CALL NULL(TH4,NTH4)
      CALL JUXTC(A31,NA31,Z1,NZ1,X1,NX1)
      CALL JUXTC(TA5,NTA5,A32,NA32,X21,NX21)
      CALL JUXTC(X21,NX21,Z2,NZ2,X2,NX2)
      CALL JUXTC(Z2,NZ2,A33,NA33,X31,NX31)
      CALL JUXTC(X31,NX31,TA5,NTA5,X3,NX3)
      CALL JUXTC(Z1,NZ1,A34,NA34,X4,NX4)
      CALL JUXTR(X1,NX1,X2,NX2,X5,NX5)
      CALL JUXTR(X5,NX5,X3,NX3,X6,NX6)
      CALL JUXTR(X6,NX6,X4,NX4,X7,NX7)
      CALL JUXTC(X7,NX7,TH4,NTH4,X8,NX8)
      CALL JUXTR(X8,NX8,Z5,NZ5,A3,NA3)
C
C    CREATE  Q*
C
C       FIND Q31
C
      CALL UNITY(A32,NA32)
      CALL GAUSEL(2,2,A51,2,A32,IERR)
      CALL TRANP(A32,NA32,TA5,NTA5)
      CALL MULT(R51,NR51,A32,NA32,TA51,NTA51)
      CALL MULT(TA5,NTA5,TA51,NTA51,A31,NA31)
      CALL ADD(P5,NP5,A31,NA31,A33,NA33)
      CALL ADD(A33,NA33,P55,NP55,Q31,NQ31)
C
C       FIND Q32
C
      CALL MULT(V51,NV51,TA5,NTA5,TA51,NTA51)
      CALL MULT(A32,NA32,TA51,NTA51,A31,NA31)
      CALL ADD(Q5,NQ5,A31,NA31,A33,NA33)
      CALL ADD(A33,NA33,Q55,NQ55,Q32,NQ32)
C
C       FIND Q*
C
      NZ6(1)=2
      NZ6(2)=10
```

```
      NZ7(1)=5
      NZ7(2)=4
      NZ8(1)=5
      NZ8(2)=3
      NZ9(1)=3
      NZ9(2)=9
      CALL NULL(TA51,NTA51)
      CALL NULL(X3,NX3)
      CALL JUXTC(Q31,NQ31,Z6,NZ6,X9,NX9)
      CALL JUXTC(TA51,NTA51,Q32,NQ32,X21,NX21)
      CALL JUXTC(X21,NX21,X3,NX3,X10,NX10)
      CALL JUXTC(Z7,NZ7,RI52,NRI52,X13,NX13)
      CALL JUXTC(X13,NX13,Z8,NZ8,X11,NX11)
      CALL JUXTC(Z9,NZ9,VI52,NVI52,X12,NX12)
      CALL JUXTR(X9,NX9,X10,NX10,X14,NX14)
      CALL JUXTR(X14,NX14,X11,NX11,X15,NX15)
      CALL JUXTR(X15,NX15,X12,NX12,X16,NX16)
      NX17(1)=12
      NX17(2)=4
      CALL NULL(X17,NX17)
      CALL NULL(TH8,NTH8)
      CALL JUXTC(X16,NX16,X17,NX17,X18,NX18)
      CALL JUXTR(X18,NX18,TH8,NTH8,Q3,NQ3)
C
C      CHECK IF A* IS ASYMPOTICALLY STABLE BY CSTAB
C
      DO 120 I=1,256
      P3(I)=0.0
      F3(I)=0.0
120   CONTINUE
      IOP(1)=0
      CALL CSTAB(A3,NA3,B3,NB3,F3,NF3,IOP,SCLE,DUMMY)
C
C      READY TO CALL RICNWT
C
      IOP(1)=0
      CALL RICNWT(A3,NA3,B3,NB3,H3,NH3,Q3,NQ3,R3,NR3,F3,NF3,P3,NP3,
     1IOP,IDENT,DISC,FNULL,DUMMY)
180   P5(1)=P3(1)
      P5(2)=P3(2)
      P5(3)=P3(17)
      P5(4)=P3(18)
      P55(1)=P3(69)
      P55(2)=P3(70)
      P55(3)=P3(85)
      P55(4)=P3(86)
      Q5(1)=P3(35)
      Q5(2)=P3(36)
      Q5(3)=P3(51)
      Q5(4)=P3(52)
      Q55(1)=P3(103)
      Q55(2)=P3(104)
      Q55(3)=P3(119)
      Q55(4)=P3(120)
      IF (ITE .GT. 1)GO TO 250
      DIFF=0.0
      DO 210 I=1,4
      DIFF=DIFF+(P5(I)-PP(I))**2+(P55(I)-PPH(I))**2
      DIFF=DIFF+(Q5(I)-QQ(I))**2+(Q55(I)-QQH(I))**2
210   CONTINUE
```

```
      GO TO 220
250   CALL SUBT(P3,NP3,TP4,NTP4,TP5,NTP5)
         DIFF =0.0
         DO 160 I =1,256
         DIFF=DIFF+TP5(1)**2
160      CONTINUE
220      DIFF=SQRT(DIFF)
         CALL EQUATE(P3,NP3,TP4,NTP4)
         IF(ITE .GT.50)GO TO 140
                GO TO 100
      CALL NORMS(256,16,16,TP5,1,RN1)
      CALL NORMS(256,16,16,TP5,1,RN2)
      CALL NORMS(256,16,16,TP5,1,RN3)
      DIFF=RN1
      IF (DIFF.LT. EPSI)GO TO 180
      DIFF=RN2
      IF (DIFF.LT. EPSI)GO TO 180
      DIFF=RN3
      IF (DIFF.LT. EPSI)GO TO 180
      IF(ITE .GT.15 )GO TO 140
      GO TO 100
130   PRINT *, ' '
      PRINT *, '  ****** CONVERGES WITH TORELANCE = ',EPSI
      GO TO 150
140   PRINT *, '  ***** DIVERGES WITH OVER  ',ITE,'  ITERATIONS.'
      GO TO 164
150   CALL TRANP(C5,NC5,TC5,NTC5)
      CALL MULT(Q,NQ,TC5,NTC5,TA6,NTA6)
      CALL MULT(TA6,NTA6,VI52,NVI52,F,NF)
      CALL MULT(F,NF,C5,NC5,TM2,NTM2)
      CALL TRANP(B5,NB5,TM3,NTM3)
      CALL MULT(RI52,NRI52,TM3,NTM3,TM4,NTM4)
      CALL MULT(TM4,NTM4,P,NP,K,NK)
      CALL MULT(B5,NB5,K,NK,TM1,NTM1)
      CALL SUBT(E1,NE1,TM2,NTM2,E2,NE2)
      CALL SUBT(E2,NE2,TM1,NTM1,ACC,NACC)
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, '******** COMPENSATOR MATRICES AFTER TRANSFORMATION'
      PRINT *, ' '
      CALL PRNT(ACC,NACC,4H ACC,1)
      CALL PRNT(F,NF,4H   F,1)
      CALL PRNT(K,NK,4H   K,1)
      PRINT *, ' '
      PRINT *, ' '
      PRINT *, '*****CPMPENSATOR MATRIX AC'
      CALL MULT(TRNI,NTRNI,ACC,NACC,A33,NA33)
      CALL MULT(A33,NA33,TRN,NTRN,AC,NAC)
      PRINT *, ' '
      CALL PRNT(AC,NAC,4H  AC,1)
164   STOP
      END
```