## NASA Contractor Report 178400

## REPULSIVE FORCE SUPPORT SYSTEM

## FEASIBILITY STUDY

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## SUMMARY

A magnetic suspension and positioning system consistent with the original requirements is shown to be feasible. Satisfactory conceptual designs include: superconductive and permanent magnet model cores and support solenoids of either superconductive, cryogenic or room temperature windings. The selected system consists of a model with a permanent magnet core or a superconductive core which is positioned by five superconductive support solenoids.

Conceptual design and trade-off analyses lead to meeting the original requirements, except as noted in Part $I$, Requirements.

## I. REQUIREMENTS

The original requirement goals are listed in (parenthesis) along with the actual achieved specifications, all in the language of the statement of work.

1. The candidate magnetic suspension and positioning system has (shall have) the suspension coils mounted in the floor of the facility. All coils do (must) reside in a space 96 in. dia. by 40 in. deep (not to exceed 8 ft. square by 8 ft. deep).
2. The minimum gap between centerline of the suspended model and the top of the suspension coil housing is (shall be) 36 in. free and clear.
3. The maximum model weight is (shall be) 15 pounds plus the on-board core magnet system.
4. The core magnet is (shall be) 12 in. long by 4 in. diameter.
5. The $x$ axis (roll), the $y$ axis (pitch) and the $z$ axis (yaw) are as specified.
6. The candidate system shall be capable of suspension through an angle of 360 degrees ( 40 degrees) yaw angle with an accuracy of $\pm 0.04$ degrees $( \pm 0.002$ degrees). An improvement on the yaw angle accuracy would require improvement in sensor position location and in magnet turn location maintenance.
7. The system shall provide accuracies of $\pm 0.04$ degrees $( \pm 0.002$ degrees) in pitch and yaw and $\pm 0.003( \pm 0.001)$ inches in translation.
8. The operation cycle can be continuous (one hour per day required).

Table I-1 summarizes the above specifications.

TABLE I-1

REPULSIVE FORCE SUPPORT SYSTEM
SPECIFICATIONS

- MODEL MAGNET $4^{\prime \prime} O D \times 12^{n}$ L CYLINDER
- MINIMUM CLEARANCE OF 36" BELOW MODEL
- $\quad 360^{\circ}$ YAW POSITIONING ANGLE
- MODEL CONTROL ACCURACY $\pm 0.04$ DEGREE IN PITCH AND YAW, $\pm 0.003^{\prime \prime}$ IN TRANSLATION
- MODEL WEIGHS 15 ib . + MAGNET WEIGHT
- POSITIONING AND CONTROL SYSTEM IS WITHIN A 96"OD x $40^{\prime \prime}$ DEEP CYLINDER

The model core choices in Chapter III are between a superconductive coil with an optimized Q/M $\cong 1350 \mathrm{Am} / \mathrm{kg}$ and a permanent magnet core at $Q / M \cong 335 \mathrm{Am} / \mathrm{kg}$, where $M$ is the total magnet system mass. These cores are suitable for an external magnet levitation system, such as shown in Fig. II-1. The sketch is for a superconducting core solenoid and five superconducting support coils.

The system options in Chapter IV are between superconducting or permanent magnet model cores and the five support coils of superconductive or copper turns. The copper coils could be water cooled or liquid nitrogen cooled. There appears to be no value to the water cooled option.

The coil design in Chapter $V$ for the five support system coils is dominated by the ac losses generated during semicontinuous corrective pulsing. In Chapter VI a simulation system is used to demonstrate the angle and position history of the model coil subject to a corrective positioning sequence that is set to cancel calculated approximate momenta. The calculated momenta are based on the discrimination of the sensing system. The accuracies in Table $I-1$ are the result of the simulation studies.


Figure II-1. Superconducting (S/C) Model--S/C Magnet Support system at $5 \mathrm{kA} / \mathrm{cm}^{2}$.
III. SYSTEM CONFIGURATION
III.1. Model Core.

The model core size envelope is $30.48 \mathrm{~cm}(12 ")$ long and $10.16 \mathrm{~cm}\left(4^{\prime \prime}\right) \mathrm{OD}$. The model core can be either a permanent magnet of 1.2 tesla average remnant magnetism or a superconducting coil with or without a holmium core, in a liquid helium dewar.
III.1.1. Superconducting Option.

Epoxy-impregnated coils with current densities in excess of $20 \mathrm{kA} / \mathrm{cm}^{2}$ at fields of 6-9 tesla may be used. Such coils do not contain much copper or cooled surfaces, and their ability to tolerate disturbances is limited to the adiabatic heat capacity of the conductor material. However, the absence of large amounts of copper and helium in the windings allows such coils to operate at current densities up to ten times as large as those for cryostable coils, which is needed for model cores.

Higher values of magnetic moments may be achieved through using holmium core if space permits. Holmium has superior magnetic properties at 4.2 K with a saturation magnetic moment of 3.9 tesla. Table III-1 lists the magnetization of holmium at 4. 2 K [1,2].

Table III-1.
Holmium Magnetization vs. Applied Field at 4.2 K .

| Magnetization force (T) | 0 | 0.1 | 0.52 | 1.0 | 1.5 | 2.5 | 3.5 | 4.5 | 6.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Magnetization (T) | 0 | 1.6 | 2.48 | 2.9 | 2.98 | 3.12 | 3.25 | 3.35 | 3.7 |

The total magnetic pole strength per unit mass vs. design maximum field, $B$, and operating current density, $J$, are listed in Tables III-2 and III-3 with and without holmium core. As shown, the presence of holmium does not add to the values of $Q / M$ significantly, since there is limited space in the core. Table III-4 lists the specifications of the model coil design. A gross current density of $30 \mathrm{kA} / \mathrm{cm}^{2}$ at 6 T field with no holmium core is selected.

Table III-2.
Model Core Magnetic Pole Strength per Unit Mass Q/M vs. Design Maximum Field, $B$, and Operating Current Density, J. All cases have $O D=0.09 \mathrm{~m}$, $I D \geq 0.05 \mathrm{~m}$, and mandrel thickness $=1.27 \mathrm{~mm}$. MH is the holmium magnetization, $R I$ is the winding inner radius, $Q M$ is the winding pole strength, $Q H$ is the holmium pole strength, and $Q$ is the sum of $Q M+Q H$. The mass $M$ is the mass of the winding and holmium in addition to 10 kg for the model, dewar, and helium mass.

$B(T) \quad J(A / m * * 2) M H(T) \quad R I(m) \quad Q H \quad Q M \quad Q(A m) \quad Q / M(A m / k g)$

| 4.00 | $0.20 \mathrm{E}+09$ | 3.23 | $0.29 \mathrm{E}-01$ | $0.12 \mathrm{E}+04$ | $0.14 \mathrm{E}+05$ | $0.151 \mathrm{E}+05$ | $0.925 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4.00 | $0.30 \mathrm{E}+09$ | 3.23 | $0.34 \mathrm{E}-01$ | $0.38 \mathrm{E}+04$ | $0.16 \mathrm{E}+05$ | $0.197 \mathrm{E}+05$ | $0.118 \mathrm{E}+04$ |
| 4.00 | $0.40 \mathrm{E}+09$ | 3.23 | $0.37 \mathrm{E}-01$ | $0.53 \mathrm{E}+04$ | $0.17 \mathrm{E}+05$ | $0.222 \mathrm{E}+05$ | $0.132 \mathrm{E}+04$ |
| 4.00 | $0.50 \mathrm{E}+09$ | 3.23 | $0.39 \mathrm{E}-01$ | $0.62 \mathrm{E}+04$ | $0.18 \mathrm{E}+05$ | $0.237 \mathrm{E}+05$ | $0.140 \mathrm{E}+04$ |
| 4.00 | $0.60 \mathrm{E}+09$ | 3.23 | $0.40 \mathrm{E}-01$ | $0.69 \mathrm{E}+04$ | $0.18 \mathrm{E}+05$ | $0.248 \mathrm{E}+05$ | $0.146 \mathrm{E}+04$ |


| 5.00 | $0.20 \mathrm{E}+09$ | 3.46 | $0.25 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.16 \mathrm{E}+05$ | $0.158 \mathrm{E}+05$ | $0.974 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5.00 | $0.30 \mathrm{E}+09$ | 3.46 | $0.32 \mathrm{E}-01$ | $0.26 \mathrm{E}+04$ | $0.19 \mathrm{E}+05$ | $0.212 \mathrm{E}+05$ | $0.129 \mathrm{E}+04$ |
| 5.00 | $0.40 \mathrm{E}+09$ | 3.46 | $0.35 \mathrm{E}-01$ | $0.45 \mathrm{E}+04$ | $0.20 \mathrm{E}+05$ | $0.246 \mathrm{E}+05$ | $0.147 \mathrm{E}+04$ |
| 5.00 | $0.50 \mathrm{E}+09$ | 3.46 | $0.37 \mathrm{E}-01$ | $0.57 \mathrm{E}+04$ | $0.21 \mathrm{E}+05$ | $0.268 \mathrm{E}+05$ | $0.159 \mathrm{E}+04$ |
| 5.00 | $0.60 \mathrm{E}+09$ | 3.46 | $0.38 \mathrm{E}-01$ | $0.65 \mathrm{E}+04$ | $0.22 \mathrm{E}+05$ | $0.283 \mathrm{E}+05$ | $0.167 \mathrm{E}+04$ |


| 6.00 | $0.20 \mathrm{E}+09$ | 3.63 | $0.21 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.17 \mathrm{E}+05$ | $0.171 \mathrm{E}+05$ | $0.107 \mathrm{E}+04$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6.00 | $0.30 \mathrm{E}+09$ | 3.63 | $0.29 \mathrm{E}-01$ | $0.13 \mathrm{E}+04$ | $0.21 \mathrm{E}+05$ | $0.222 \mathrm{E}+05$ | $0.136 \mathrm{E}+04$ |
| 6.00 | $0.40 \mathrm{E}+09$ | 3.63 | $0.33 \mathrm{E}-01$ | $0.35 \mathrm{E}+04$ | $0.23 \mathrm{E}+05$ | $0.265 \mathrm{E}+05$ | $0.160 \mathrm{E}+04$ |
| 6.00 | $0.50 \mathrm{E}+09$ | 3.63 | $0.35 \mathrm{E}-01$ | $0.49 \mathrm{E}+04$ | $0.24 \mathrm{E}+05$ | $0.293 \mathrm{E}+05$ | $0.175 \mathrm{E}+04$ |
| 6.00 | $0.60 \mathrm{E}+09$ | 3.63 | $0.37 \mathrm{E}-01$ | $0.59 \mathrm{E}+04$ | $0.25 \mathrm{E}+05$ | $0.313 \mathrm{E}+05$ | $0.186 \mathrm{E}+04$ |


| 7.00 | $0.20 \mathrm{E}+09$ | 3.75 | $0.17 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.18 \mathrm{E}+05$ | $0.180 \mathrm{E}+05$ | $0.113 \mathrm{E}+04$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7.00 | $0.30 \mathrm{E}+09$ | 3.75 | $0.26 \mathrm{E}-01$ | $0.76 \mathrm{E}+02$ | $0.23 \mathrm{E}+05$ | $0.229 \mathrm{E}+05$ | $0.141 \mathrm{E}+04$ |
| 7.00 | $0.40 \mathrm{E}+09$ | 3.75 | $0.31 \mathrm{E}-01$ | $0.25 \mathrm{E}+04$ | $0.26 \mathrm{E}+05$ | $0.281 \mathrm{E}+05$ | $0.171 \mathrm{E}+04$ |
| 7.00 | $0.50 \mathrm{E}+09$ | 3.75 | $0.34 \mathrm{E}-01$ | $0.41 \mathrm{E}+04$ | $0.27 \mathrm{E}+05$ | $0.315 \mathrm{E}+05$ | $0.190 \mathrm{E}+04$ |
| 7.00 | $0.60 \mathrm{E}+09$ | 3.75 | $0.36 \mathrm{E}-01$ | $0.53 \mathrm{E}+04$ | $0.29 \mathrm{E}+05$ | $0.339 \mathrm{E}+05$ | $0.203 \mathrm{E}+04$ |

$8.00 \quad 0.20 \mathrm{E}+09$
$8.00 \quad 0.30 \mathrm{E}+09$
$8.00 \quad 0.40 \mathrm{E}+09$
$8.00 \quad 0.50 \mathrm{E}+09$
$8.00 \quad 0.60 \mathrm{E}+09$
$3.82 \quad 0.13 \mathrm{E}-01 \quad 0.00 \mathrm{E}+00$
$3.820 .24 \mathrm{E}-01 \quad 0.00 \mathrm{E}+00002 \mathrm{E}+05$
$3.82 \quad 0.29 \mathrm{E}-01 \quad 0.14 \mathrm{E}+04 \quad 0.28 \mathrm{E}+05 \quad 0.293 \mathrm{E}+05 \quad 0.179 \mathrm{E}+04$
$3.820 .32 \mathrm{E}-01 \quad 0.32 \mathrm{E}+04 \quad 0.30 \mathrm{E}+05 \quad 0.333 \mathrm{E}+05 \quad 0.202 \mathrm{E}+04$
$3.820 .34 \mathrm{E}-01 \quad 0.45 \mathrm{E}+04 \quad 0.32 \mathrm{E}+05 \quad 0.362 \mathrm{E}+05 \quad 0.218 \mathrm{E}+04$

| 9.00 | $0.20 \mathrm{E}+09$ | 3.83 | $0.92 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.19 \mathrm{E}+05$ | $0.189 \mathrm{E}+05$ | $0.120 \mathrm{E}+04$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9.00 | $0.30 \mathrm{E}+09$ | 3.83 | $0.21 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.26 \mathrm{E}+05$ | $0.257 \mathrm{E}+05$ | $0.160 \mathrm{E}+04$ |
| 9.00 | $0.40 \mathrm{E}+09$ | 3.83 | $0.27 \mathrm{E}-01$ | $0.40 \mathrm{E}+03$ | $0.30 \mathrm{E}+05$ | $0.302 \mathrm{E}+05$ | $0.186 \mathrm{E}+04$ |
| 9.00 | $0.50 \mathrm{E}+09$ | 3.83 | $0.31 \mathrm{E}-01$ | $0.23 \mathrm{E}+04$ | $0.33 \mathrm{E}+05$ | $0.349 \mathrm{E}+05$ | $0.212 \mathrm{E}+04$ |
| 9.00 | $0.60 \mathrm{E}+09$ | 3.83 | $0.33 \mathrm{E}-01$ | $0.37 \mathrm{E}+04$ | $0.35 \mathrm{E}+05$ | $0.382 \mathrm{E}+05$ | $0.231 \mathrm{E}+04$ |

## Table III-3.

Model Core Magnetic Pole Strength, per Unit Mass Q/M vs. Design Maximum Field, B, and Operating Current Density J. All cases have $O D=0.09 \mathrm{~m}$. There is no holmium mandrel in the core. The mass $M$ is the mass of the winding in addition to 10 kg for the model, dewar and helium mass.
$B(T) J(A / m * * 2) M H(T) \quad R I(m) \quad Q H \quad Q M \quad Q(A m) \quad Q / M(A m / k g)$

| 4.00 | $0.20 \mathrm{E}+09$ | 3.23 | $0.29 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.14 \mathrm{E}+05$ | $0.139 \mathrm{E}+05$ | $0.899 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4.00 | $0.30 \mathrm{E}+09$ | 3.23 | $0.34 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.16 \mathrm{E}+05$ | $0.159 \mathrm{E}+05$ | $0.114 \mathrm{E}+04$ |
| 4.00 | $0.40 \mathrm{E}+09$ | 3.23 | $0.37 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.17 \mathrm{E}+05$ | $0.169 \mathrm{E}+05$ | $0.129 \mathrm{E}+04$ |
| 4.00 | $0.50 \mathrm{E}+09$ | 3.23 | $0.39 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.18 \mathrm{E}+05$ | $0.175 \mathrm{E}+05$ | $0.140 \mathrm{E}+04$ |
| 4.00 | $0.60 \mathrm{E}+09$ | 3.23 | $0.40 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.18 \mathrm{E}+05$ | $0.180 \mathrm{E}+05$ | $0.148 \mathrm{E}+04$ |


| 5.00 | $0.20 \mathrm{E}+09$ | 3.46 | $0.25 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.16 \mathrm{E}+05$ | $0.158 \mathrm{E}+05$ | $0.955 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5.00 | $0.30 \mathrm{E}+09$ | 3.46 | $0.32 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.19 \mathrm{E}+05$ | $0.186 \mathrm{E}+05$ | $0.126 \mathrm{E}+04$ |
| 5.00 | $0.40 \mathrm{E}+09$ | 3.46 | $0.35 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.20 \mathrm{E}+05$ | $0.201 \mathrm{E}+05$ | $0.147 \mathrm{E}+04$ |
| 5.00 | $0.50 \mathrm{E}+09$ | 3.46 | $0.37 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.21 \mathrm{E}+05$ | $0.211 \mathrm{E}+05$ | $0.162 \mathrm{E}+04$ |
| 5.00 | $0.60 \mathrm{E}+09$ | 3.46 | $0.38 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.22 \mathrm{E}+05$ | $0.218 \mathrm{E}+05$ | $0.173 \mathrm{E}+04$ |


| 6.00 | $0.20 \mathrm{E}+09$ | 3.63 | $0.21 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.17 \mathrm{E}+05$ | $0.171 \mathrm{E}+05$ | $0.985 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6.00 | $0.30 \mathrm{E}+09$ | 3.63 | $0.29 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.21 \mathrm{E}+05$ | $0.209 \mathrm{E}+05$ | $0.135 \mathrm{E}+04$ |
| 6.00 | $0.40 \mathrm{E}+09$ | 3.63 | $0.33 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.23 \mathrm{E}+05$ | $0.230 \mathrm{E}+05$ | $0.160 \mathrm{E}+04$ |
| 6.00 | $0.50 \mathrm{E}+09$ | 3.63 | $0.35 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.24 \mathrm{E}+05$ | $0.244 \mathrm{E}+05$ | $0.179 \mathrm{E}+04$ |
| 6.00 | $0.60 \mathrm{E}+09$ | 3.63 | $0.37 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.25 \mathrm{E}+05$ | $0.253 \mathrm{E}+05$ | $0.194 \mathrm{E}+04$ |


| 7.00 | $0.20 \mathrm{E}+09$ | 3.75 | $0.17 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.18 \mathrm{E}+05$ | $0.180 \mathrm{E}+05$ | $0.997 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7.00 | $0.30 \mathrm{E}+09$ | 3.75 | $0.26 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.23 \mathrm{E}+05$ | $0.228 \mathrm{E}+05$ | $0.141 \mathrm{E}+04$ |
| 7.00 | $0.40 \mathrm{E}+09$ | 3.75 | $0.31 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.26 \mathrm{E}+05$ | $0.256 \mathrm{E}+05$ | $0.171 \mathrm{E}+04$ |
| 7.00 | $0.50 \mathrm{E}+09$ | 3.75 | $0.34 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.27 \mathrm{E}+05$ | $0.274 \mathrm{E}+05$ | $0.194 \mathrm{E}+04$ |
| 7.00 | $0.60 \mathrm{E}+09$ | 3.75 | $0.36 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.29 \mathrm{E}+05$ | $0.286 \mathrm{E}+05$ | $0.212 \mathrm{E}+04$ |


| 8.00 | $0.20 \mathrm{E}+09$ | 3.82 | $0.13 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.19 \mathrm{E}+05$ | $0.186 \mathrm{E}+05$ | $0.998 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8.00 | $0.30 \mathrm{E}+09$ | 3.82 | $0.24 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.24 \mathrm{E}+05$ | $0.244 \mathrm{E}+05$ | $0.145 \mathrm{E}+04$ |
| 8.00 | $0.40 \mathrm{E}+09$ | 3.82 | $0.29 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.28 \mathrm{E}+05$ | $0.279 \mathrm{E}+05$ | $0.180 \mathrm{E}+04$ |
| 8.00 | $0.50 \mathrm{E}+09$ | 3.82 | $0.32 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.30 \mathrm{E}+05$ | $0.301 \mathrm{E}+05$ | $0.206 \mathrm{E}+04$ |
| 8.00 | $0.60 \mathrm{E}+09$ | 3.82 | $0.34 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.32 \mathrm{E}+05$ | $0.317 \mathrm{E}+05$ | $0.228 \mathrm{E}+04$ |


| 9.00 | $0.20 \mathrm{E}+09$ | 3.83 | $0.92 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.19 \mathrm{E}+05$ | $0.189 \mathrm{E}+05$ | $0.993 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9.00 | $0.30 \mathrm{E}+09$ | 3.83 | $0.21 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.26 \mathrm{E}+05$ | $0.257 \mathrm{E}+05$ | $0.148 \mathrm{E}+04$ |
| 9.00 | $0.40 \mathrm{E}+09$ | 3.83 | $0.27 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.30 \mathrm{E}+05$ | $0.298 \mathrm{E}+05$ | $0.186 \mathrm{E}+04$ |
| 9.00 | $0.50 \mathrm{E}+09$ | 3.83 | $0.31 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.33 \mathrm{E}+05$ | $0.326 \mathrm{E}+05$ | $0.216 \mathrm{E}+04$ |
| 9.00 | $0.60 \mathrm{E}+09$ | 3.83 | $0.33 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.35 \mathrm{E}+05$ | $0.345 \mathrm{E}+05$ | $0.241 \mathrm{E}+04$ |

Table III-4.
Model Core Coil Specifications (Superconducting Solenoid).

Winding outer radius $(\mathrm{cm}) \quad 4.5$

Winding inner radius (cm) 2.9
Mandrell thickness (cm) 0.127
Winding length (cm) 22.86
Winding current density (kA/ cm ${ }^{2}$ ) 30.00
Winding maximum field (T) 6.0
III.1.2. Permanent Magnet Material Option.

A new superior permanent magnet material $\mathrm{Nd}_{15} \mathrm{Fe}_{77^{\mathrm{B}} 8}$ is planned for the model core [3,4]. The magnetic properties are listed in Table III-5.

Table III-5.
Magnetic Properties of $\mathrm{ND}_{15} \mathrm{Fe}_{77} \mathrm{~B}_{8}$ Magnetic Material.

|  | $\begin{gathered} \mathrm{Br} \\ (\mathrm{~T}) \end{gathered}$ | $\begin{gathered} \mathrm{Hc} \\ (\mathrm{kA} / \mathrm{m}) \end{gathered}$ | $\begin{aligned} & (\mathrm{BH}) \mathrm{max} \\ & \left(\mathrm{~kJ} / \mathrm{m}^{3}\right) \end{aligned}$ | TC $(K)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Na}_{15} \mathrm{Fe}_{77} \mathrm{~B}_{8}$ | 1.23 | 960 | 290 | 585 |
| $\mathrm{Nd}_{15}\left(\mathrm{Fe}_{0.9} \mathrm{Co}_{0.1}\right)_{77} \mathrm{~B}_{8}$ | 1.23 | 800 | 290 | 670 |
| $\mathrm{Nd}_{15}\left(\mathrm{Fe}_{0.8} \mathrm{Co}_{0.2}\right)_{77}{ }^{\mathrm{B}} 8$ | 1.21 | 820 | 260 | 740 |

As shown in Fig. III-1, the new permanent magnet material has large values of $M_{I}$ (residual magnetism) and $H_{C}$ (demagnetization critical field). $M_{r}$ stays well above 1.2 tesla for most of the demagnetizing field and well over 1.15 tesla up to $H_{c}=960$ $\mathrm{kA} / \mathrm{m}$ (1.21 tesla). With $M_{r}=1.2$ tesla, the magnetic properties of the model core are listed in Table III-6.


Figure III-1. $\begin{aligned} & \text { Demagnetized curve of } \mathrm{Nd}_{13.5^{\mathrm{Dy}} \mathrm{Y}_{1.5} \mathrm{Fe}_{77^{\mathrm{B}}} \text { magnet }} \text { sintered }\end{aligned}$

Table III-6.
Model Core Coil Specifications (Permanent Magnet).


Remnant magnetization (T) .................................... 1.2
Core length (cm)
30.48

Core diameter (cm) ......................................... 10.16
Core mass (kg) ................................................ 16.41
Model mass (kg) .................................................... 6.7
Total mass, M (kg) ......................................... 23.11
Pole strength, Q (Am) ............................................... $7.742 \times 10^{3}$
Q/M ( $\mathrm{Am} / \mathrm{kg}$ ) ...................................................... 335.
III.2. Levitation Magnet System.

The system under study is to levitate, position and control a 15 lb . model. The model is to be suspended $36^{\prime \prime}$ above the cryostat top plate. The array of magnets will control the position of the model in 5 degrees of freedom, namely the $x, y, z$ displacements and the yaw and pitch rotations. Model rolling is controlled with eccentric weights.
III.2.1. Levitation Magnet System Configuration.

The system consists of " $n$ " vertical solenoids arranged around the system center. The tops of the magnets are located as close as possible to the table surface. Because the model is allowed to assume any position between 0 and 360 degrees in the yaw direction, it is reasonable to assume that the magnet system should be arranged symmetrically around the vertical z-axis. The magnets may be arranged in one or more rings. Furthermore, each ring may perform a separate function. For example, a magnet array in one circle may be responsible for levitation and positioning while another array may be responsible for control and stability.

The first object of this study is to find the currents in the magnet array that satisfy a required $\mathrm{F}_{\mathrm{z}}$ (lift), and control $F_{x}, F_{y}, T_{y}$ and $T_{z}$ at any position for the model. Since the number of forces and torques is 5 , there is a need for at least 5 solenoids in the magnet array. A larger number of coils allows other constraints which depend on the nature of the system. For a superconducting magnet system, the minimum ampere-meters is
usually desired; while for resistive coils the criteria may be minimum ohmic heat losses in the coils. In the next section the procedure to optimize these two systems is analyzed.

## III.2.2. Magnet System Optimization.

For both superconductive and resistive systems the optimization problem may be divided into two parts: first to find the optimum magnet dimensions and secondly to find the optimum current distribution in the magnet array that satisfies the force constraints. First, an approximate formula is used to derive closed form expressions for optimum magnet dimensions. Second, using these expressions for the dimensions, an "exact" approach is used to calculate the optimum current distribution in the magnet array.

## III.2.2.1. Superconducting Magnets Optimum Size.

The main function of the magnet array is to produce a lift force on the model. The lift force on the model due to one magnet in the array may be approximated by

$$
\begin{equation*}
F_{z}=K Q_{1}(\pi / 3) \quad \mathrm{Jb}^{3}(1-\alpha) \quad\left[H^{-3}-(H+L)^{-3}\right] \tag{1}
\end{equation*}
$$

where
$K=$ constant for given locations of the magnet and the model
$Q_{1}=$ magnetic pole strength of the model
$J=$ current density in the magnet
$b=$ outer radius of the magnet
$\alpha=$ inner to outer diameter ratio of the coil
$H=$ vertical distance between top of coil and center line of model, and
$L=$ length of coil.

From Equation 1, we may define $F_{0}$ to be:

$$
\begin{aligned}
\mathrm{F}_{\mathrm{o}} \equiv\left(\mathrm{~F}_{\mathrm{z}} \mathrm{H}^{3} / \mathrm{KQ} Q_{1}\right) & =(\pi / 3) \mathrm{Jb}^{3}\left(1-\alpha^{3}\right) \mathrm{X}, \quad \text { where } \\
X & =1-(1+\mathrm{x})^{-3} \\
\mathrm{x} & =\mathrm{L} / \mathrm{H}
\end{aligned}
$$

The ampere-meters of the coil are

$$
A=\pi J b^{2} L\left(1-a^{2}\right)
$$

At any location the minimum of the ampere-meters $A$ subject to the constraint $F_{0}=$ constant is achieved when the following conditions are met:
b is as large as possible for this location, and
$\alpha=\sqrt{m+(1 / 2)}-1 / 2$, and
$m=2 /\left(x^{3}+4 x^{2}+6 x+1\right)$.
III.2.2.2. Superconducting Magnets Optimum Current Distribution.

The forces and torques acting on the model due to the magnet array system may be presented as

$$
\begin{equation*}
\left\{F_{i}\right\}=\left[S_{i j}\right]\left\{I_{j}\right\} \tag{5}
\end{equation*}
$$

where $\left\{F_{i}\right\}$ is the force vector, $i=1,5$
$\left[S_{i j}\right]$ is a pseudo-stiffness matrix whose elements $S_{i j}$ represent the force on the model in the $i^{\text {th }}$ direction due to a unit current in the $j^{\text {th }}$ coil.

The ampere-meters of the magnet array is

$$
\Sigma A_{j}=\Sigma\left|I_{j} \ell_{j}\right| \quad, \quad j=1, n
$$

where $I_{j}$ and $\ell_{j}$ are the current and conductor length of the $j^{\text {th }}$ coil.

For identical coils $\Sigma A_{j}=\ell \Sigma\left|I_{j}\right|$

Equation 6 shows that for fixed coil dimensions, minimum $\Sigma A_{j}$ occurs at minimum $\Sigma I^{2}{ }_{j}$ -

Thus it is required to minimize $\Sigma I^{2}{ }_{j}$ subject to the constraints of Equation 5.

Using Lagrange's approach, the problem reduces to minimizing an objective function $G$ defined as

$$
\begin{aligned}
& G=\Sigma I_{j}^{2}+\sum \lambda_{i} \sum S_{i j} I_{j}-\Sigma \lambda_{i} F_{i} \\
& \text { where } i=1 \text { to } 5 \\
& j=1 \text { to } n
\end{aligned}
$$

This function has an optimum value at the set of currents $I_{j}$ satisfied by the following ( $n+5$ ) simultaneous equations:

$$
\begin{array}{ll}
\left(\partial G / \partial I_{j}\right)=2 I_{j}+\Sigma \lambda_{i} \Sigma S_{i j}=0 & j=1, n \\
\left(\partial G / \partial \lambda_{i}\right)=\Sigma S_{i j} I_{j}-F_{i}=0 & i=1,5
\end{array}
$$

Arranging these equations in a matrix form we get
[B] $\{x\}=\{c\}$
where
$[B]=\left[\begin{array}{l:c}2[I]_{n \times n} & {[S]^{T}{ }_{n \times 5}} \\ \hdashline[S]_{5 \times n} & {[0]_{5 \times 5}}\end{array}\right]$
where [I] = identity matrix
$\{x\}=\left\{\begin{array}{c}I_{1} \\ : \\ I_{n} \\ \lambda_{1} \\ : \\ \lambda_{5}\end{array}\right\}$

$$
\{c\}=\left\{\begin{array}{c}
0 \\
: \\
0 \\
F_{1} \\
: \\
F_{5}
\end{array}\right\}
$$

Manipulating Equation 8 and solving for the current distribution, we get

$$
\begin{equation*}
\left\{I_{j}\right\}_{n \times 1}=[S]_{n \times 5}^{T}\left[S S^{T}\right]_{5 \times 5}^{-1}\left\{F_{i}\right\}_{5 \times 1} \tag{9}
\end{equation*}
$$

The elegance of equation (9) is that regardless of the number of magnets " $n$ ", the matrix to be inverted is always $5 \times 5$.

Solving Eq. (9) gives the current distribution in the magnet array that satisfies the force constraints and results in a minimum total ampere-meters in the coils.
III.2.2.3. Resistive Magnets Optimum Size

The power loss $P_{1}$ in one magnet in the array is
$P_{1}=I^{2} R=\rho \pi J^{2} b^{2} L\left(1-\alpha^{2}\right)$, where
$J=$ the gross current density, and
$\rho=$ the effective resistivity.
Minimizing the power dissipation subject to the force condition of Eq. (1) results in the following conditions:
b is as large as possible,
$\alpha=0.366$ and
$L=0.7373 \mathrm{H}$.

## III.2.2.4. Resistive Magnets Optimum Current Distribution

The total power dissipation in the coil array is

$$
P=\Sigma I_{j}{ }^{2} R_{j} \quad j=1, n
$$

For identical magnets the resistance $R_{j}$ is the same,

$$
\begin{equation*}
P=R \Sigma I_{j}^{2} \tag{13}
\end{equation*}
$$

where $R$ is the resistance of one coil.
Equation (13) shows that for an array of magnets with the same given dimensions, minimum power dissipation coincides with the minimum of $\Sigma I_{j}{ }^{2}$. This is the same condition for minimum ampere-meters. Consequently, the current distribution given by equation (9) results in a minimum power dissipation in the magnet array.

The six combinations of superconducting or permanent model magnets with superconducting or copper levitation magnets are listed in Table IV-1.

Table IV-1.
Model Core/Levitation Magnet Combinations.

Model Magnets Comment

| S/C | S/C | smallest magnets |
| :--- | :--- | :---: |
| P/M | S/C | simple model |
| S/C | Cu | $\left(\begin{array}{l}\text { LN } 2 \text { cooled } \\ \text { PM }\end{array}\right.$ |
| S/C | Cu | $\left(\mathrm{H}_{2} 0\right.$ cognets |
| PM | Cu | magnets $)$ |

> S/C $\equiv$ superconducting
> PM $\equiv$ permanent magnets
> $\mathrm{Cu} \equiv$ copper magnets
IV.1. Superconducting Levitation Coils.

The superconducting magnet array is optimized for minimum total ampere-meters. Five and six arrays are studied at varying location radii for current densities ranging from $50 \mathrm{MA} / \mathrm{m}^{2}$ to 300 MA/m². Table IV-2 lists magnet parameters for an $S / C$ model core.

The superconducting model in this table has a magnetic pole strength of $1350 \mathrm{Am} / \mathrm{kg}$ of total mass. The other parameters in this table are:
$N C=$ Number of coils in the array
$R=$ Location radius
L, RI, RO = Optimum coil dimensions ( $I=$ inner, $O=$ outer)

# Table IV-2. <br> Minimum Ampere-Meters S/C Magnet Arrays <br> for Levitation of an S/C Model. 

| MODEL $\mathrm{Q} / \mathrm{M}$ | $=1.350 \mathrm{E}+03 \mathrm{Am} / \mathrm{KG}$ | FX/W | FY/W | F2/ | TY/N | TN/W |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MOOEL LENGTH | $=3.000 E-01 \mathrm{~m}$ | . $000 \mathrm{E}+00$ | . $0008+00$ | 1.000E+00 | . $0008+00$ | . $0008+00$ |
| MIN OISTANCE | $=5.000 \mathrm{E}-02 \mathrm{~m}$ |  |  |  |  |  |


| NC | R | $L$ | RI | RO | JMaX | Bmax | ISTOT | $N *$ IS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | . 400 | . 924 | . 069 | . 210 | 1.500E+07 | 2.54 | 5.576E+06 | 8.575E+06 |
| 5 | . 450 | . 387 | . 120 | . 240 | 1.500E+07 | 1.76 | 2.552E+06 | $3.918 \mathrm{E}+06$ |
| 5 | . 500 | . 262 | . 160 | . 269 | 1.500E+07 | 1.34 | 1.882E+06 | $2.887 E+06$ |
| 5 | . 550 | . 204 | . 195 | . 298 | 1.500E+07 | 1.10 | 1.589E+06 | $2.435 E+06$ |
| 5 | . 600 | . 170 | . 229 | . 328 | 1.501E+07 | . 95 | $1.438 E+06$ | 2.202E+06 |
| 5 | . 650 | . 148 | . 260 | . 357 | 1.500E+07 | . 85 | $1.3588+06$ | $2.079 E+06$ |
| 5 | . 700 | . 133 | . 291 | . 386 | 1.501E+07 | . 79 | $1.323 E+06$ | $2.024 E+06$ |
| 5 | . 750 | . 122 | . 321 | . 416 | 1.501E+07 | . 74 | 1.317E+06 | $2.015 E+06$ |
| 5 | . 800 | . 114 | . 350 | . 445 | 1.500E+07 | . 70 | $1.334 E+06$ | $2.040 E+06$ |
| 5 | . 400 | . 459 | . 097 | . 210 | 2.000E+07 | 2.42 | 3.257E+06 | $5.0068+06$ |
| 5 | . 450 | . 273 | . 140 | . 240 | $2.000 E+07$ | 1.73 | $2.106 \mathrm{E}+06$ | 3.233E+06 |
| 5 | . 500 | . 200 | . 171 | . 269 | $2.001 E+07$ | 1.36 | 1.671E+06 | 2.572E+06 |
| 5 | . 550 | . 162 | . 212 | . 298 | $2.001 E+07$ | 1.14 | $1.464 \mathrm{E}+06$ | $2.243 E+06$ |
| 5 | . 600 | . 138 | . 244 | . 328 | 2.001E+07 | 1.00 | 1.350E+06 | $2.057 \mathrm{E}+06$ |
| 5 | . 650 | . 122 | . 275 | . 357 | $2.001 E+07$ | . 91 | $1.291 E+06$ | $1.975 E+06$ |
| 5 | . 700 | . 110 | . 308 | . 386 | 1.999E+07 | . 84 | 1.266E+06 | 1.936E+06 |
| 5 | . 750 | . 102 | . 335 | . 416 | $2.001 E+07$ | . 80 | 1.267E+06 | 1.938E+06 |
| 5 | . 800 | . 096 | . 364 | . 445 | $1.998 E+07^{\circ}$ | . 76 | $1.289 E+06$ | $1.972 E+06$ |
| 6 | . 450 | . 758 | . 072 | . 200 | 1.500E+07 | 2.27 | 4.939E+06 | 7.459E+06 |
| 6 | . 500 | . 393 | . 112 | . 225 | 1.500E+07 | 1.70 | $2.793 E+06$ | 4.227E+06 |
| 6 | . 550 | . 280 | . 145 | . 250 | 1.500E+07 | 1.36 | $2.168 \mathrm{E}+06$ | 3.288E+06 |
| 6 | . 600 | . 223 | . 174 | . 275 | $1.500 E+07$ | 1.16 | 1.881E+06 | $2.857 \varepsilon+06$ |
| 6 | . 650 | . 189 | . 202 | . 300 | 1.500E+07 | 1.02 | $1.736 \mathrm{E}+06$ | $2.639 E+06$ |
| 6 | . 700 | . 167 | . 228 | . 325 | $1.501 E+07$ | . 93 | 1.667E+06 | $2.537 E+06$ |
| 6 | . 750 | . 152 | . 253 | . 350 | 1.500E+07 | . 87 | 1.643E+06 | 2.503E+06 |
| 6 | . 800 | . 141 | . 278 | . 375 | $1.500 E+07$ | . 83 | $1.658 \mathrm{E}+06$ | $2.528 E+06$ |
| 6 | . 400 | 1.349 | . 049 | . 175 | 2.000E+07 | 3.13 | 9.523E+06 | 1.436E+07 |
| 6 | . 450 | . 423 | . 096 | . 200 | $2.000 E+07$ | 2.20 | 3.245E+06 | 4.906E+06 |
| 6 | . 500 | . 277 | . 131 | . 225 | $2.000 E+07$ | 1.69 | $2.313 E+06$ | 3.503E+05 |
| 6 | . 550 | . 212 | . 161 | . 250 | $2.0008+07$ | 1.38 | 1.919E+06 | $2.912 E+06$ |
| $\delta$ | . 600 | . 175 | . 190 | . 275 | $2.001 E+07$ | 1.19 | 1.721E+06 | 2.615E+06 |
| 6 | . 650 | . 152 | . 217 | . 300 | $2.0008+07$ | 1.07 | $1.616 E+06$ | $2.459 E+06$ |
| 6 | . 700 | . 136 | . 243 | . 325 | 2.000E+07 | . 99 | 1.572E+06 | 2.393E+06 |
| 6 | . 750 | . 125 | . 268 | . 350 | $2.0008+07$ | . 92 | 1.563E+06 | $2.382 E+06$ |
| 6 | . 800 | . 117 | . 293 | . 375 | $2.001 E+07$ | . 89 | 1.587E+06 | $2.420 E+06$ |

JMAX = Maximum current density in the magnet array
BMAX = Maximum field in the magnets
ISTOT = Operational ampere-meters at the specified model location

N*IS $=$ Total ampere-meters capacity of the system.
It is seen that increasing the allowable current density in the magnets has a small effect on the total ampere-meters capacity.

The total ampere-meters in five- and six-coil S/C arrays are plotted versus array location radius for an $S / C$ model (Fig. IV-1). The optimum ampere-meters is at a location radius of 70 cm . The five-coil array uses less ampere-meters than the sixcoil array. Table IV-3 lists parameters for the S/C arrays for a permanent magnet model (PM). In Fig. IV-2 the ampere-meters of the five-coil and six-coil S/C arrays are plotted versus magnet location radius. The optimum ampere-meters occurs around the 70 cm radius.

## IV.2. Resistive Levitation Coils.

The minimization of power consumption is the main goal for water-cooled or nitrogen-cooled coil designs. Water-cooled or cryocooled copper magnet arrays are shown to be feasible. Tables IV-4 and IV-5 list magnet array parameters for S/C and permanent magnet models. The significant parameters are:

Cu Ratio: ratio of copper in the windings
L, RI, RO: optimum magnet dimensions

Total ampere-meters of $\mathrm{s} / \mathrm{C}$ magnet array at current density of 50
$M A / \mathrm{m}^{2}$. Plots are for 5 -coil and 6 -coil arrays with 5 cm distance between adjacent coils. Optimum location radius is around 70 cm . magnet.
Figure IV-1.

Table IV-3.
Minimum Ampere-Meter S/C Magnet Arrays for Levitation of a SM Model.

| MODEL Q/M | $=3.350 E+02 \mathrm{Am} / \mathrm{KG}$ | $\mathrm{FX} / \mathrm{W}$ | $\mathrm{FY} / \mathrm{W}$ | $\mathrm{FL} / \mathrm{W}$ | $\mathrm{TY} / \mathrm{W}$ | TN/W |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| MODEL LENGTH | $=3.000 \mathrm{E}-01 \mathrm{~m}$ | $.000 \mathrm{E}+00$ | $.000 \mathrm{E}+00$ | $1.000 \mathrm{E}+00$ | $.000 \mathrm{E}+00$ | $.000 \mathrm{E}+00$ |


| NC | $R$ | L | RI | RO | Jmax | 8 max | ISTOT | N*IS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | . 600 | 1.397 | . 090 | . 328 | 1.500E+07 | 4.28 | 2.129E+07 | 3.268E+07 |
| 5 | . 650 | . 678 | . 136 | . 357 | 1.500E+07 | 3.45 | 1.135E+07 | 1.739E+07 |
| 5 | . 700 | . 493 | . 173 | . 386 | $1.500 E+07$ | 2.92 | 9.072E+06 | 1.390E+07 |
| 5 | . 750 | . 403 | . 205 | . 416 | 1.500E+07 | 2.57 | 8.117E+06 | 1.243E+07 |
| 5 | . 800 | . 348 | . 234 | . 445 | $1.500 E+07$ | 2.33 | $7.679 E+06$ | 1.175E+07 |
| 5 | . 550 | 1.168 | . 088 | . 298 | $2.000 \mathrm{E}+07$ | 5.00 | $1.941 \mathrm{E}+07$ | 2.980E+07 |
| 5 | . 600 | . 579 | . 135 | . 328 | $2.000 E+07$ | 3.91 | 1.057E+07 | $1.621 E+07$ |
| 5 | . 650 | . 418 | . 173 | . 357 | $2.000 \mathrm{E}+07$ | 3.24 | 8.385E+06 | 1.285E+07 |
| 5 | . 700 | . 339 | . 206 | . 386 | $2.000 E+07$ | 2.81 | $7.446 E+06$ | 1.140E+07 |
| 5 | . 750 | . 291 | . 237 | . 416 | $2.000 E+07$ | 2.52 | $6.996 E+06$ | 1.071E+07 |
| 5 | . 800 | . 260 | . 266 | . 445 | $2.0008+07$ | 2.33 | $6.812 \mathrm{E}+06$ | $1.042 \mathrm{E}+07$ |
| 6 | . 700 | . 980 | . 104 | . 325 | $1.500 E+07$ | 3.82 | $1.735 E+07$ | $2.629 E+07$ |
| $\delta$ | . 750 | . 663 | . 135 | . 350 | $1.500 ¢+07$ | 3.35 | 1.289E+07 | 1.956E+07 |
| 6 | . 800 | . 530 | . 162 | . 375 | $1.500 E+07$ | 3.02 | 1.128E+07 | $1.714 \mathrm{E}+07$ |
| 6 | . 600 | 2.124 | . 066 | . 275 | $2.000 \mathrm{E}+07$ | 5.18 | $3.773 E+07$ | 5.707E+07 |
| 5 | . 650 | . 745 | . 109 | . 300 | $2.000 \mathrm{E}+07$ | 4.23 | 1.446E+07 | 2.191E+07 |
| 6 | . 700 | . 525 | . 141 | . 325 | $2.000 E+07$ | 3.64 | 1.118E+07 | $1.697 E+07$ |
| 6 | . 750 | . 423 | . 168 | . 350 | $2.000 E+07$ | 3.23 | 9.874E+06 | 1.500E+07 |
| 6 | . 800 | . 364 | . 193 | . 375 | $2.000 E+07$ | 2.95 | 9.298E+06 | $1.414 \varepsilon+07$ |



Table IV-4.
Minimum Power Copper Magnet Arrays
for Levitation of an S/C Model.


| NC | $R$ | $L$ | RI | RO | JMAX | BMAX | CU-WT | N*IS | PTOT* | $N * P M A X *$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | . 300 | . 737 | . 055 | . 151 | $4.7798+07$ | 5.54 | $1.544 E+03$ | $1.0988+07$ | $7.0298+06$ | 1.399E+07 |
| 5 | . 350 | . 737 | . 066 | . 181 | $2.620 E+07$ | 3.58 | $2.201 E+03$ | $8.583 E+06$ | $3.016 E+06$ | 5.997E+06 |
| 5 | . 400 | . 737 | . 077 | . 210 | $1.604 E+07$ | 2.50 | $2.976 E+03$ | $7.104 E+06$ | 1.530E+06 | 3.039E+06 |
| 5 | . 450 | . 737 | . 088 | . 240 | $1.068 E+07$ | 1.87 | $3.866 \mathrm{E}+03$ | $6.143 E+06$ | 8.817E+05 | 1.749E+06 |
| 5 | . 500 | . 737 | . 098 | . 269 | $7.584 E+06$ | 1.46 | 4.873E+03 | $5.500 E+06$ | $5.612 E+05$ | $1.112 E+06$ |
| 5 | . 550 | . 737 | . 109 | . 298 | 5.675E+06 | 1.19 | 5.997E+03 | $5.064 E+06$ | $3.870 E+05$ | 1.664E+05 |
| 5 | . 600 | . 737 | . 120 | . 328 | 4.431E+06 | 1.00 | 1.237E+03 | 4.772E+06 | $2.849 E+05$ | 5.638E+05 |
| 5 | . 650 | . 737 | . 131 | . 357 | $3.584 E+06$ | . 86 | 8.593E+03 | 4.583E+06 | $2.215 E+05$ | 4.380E+05 |
| 5 | . 700 | . 737 | . 141 | . 386 | $2.985 E+06$ | . 76 | 1.007E+04 | 4.473E+06 | 1.802E+05 | $3.562 E+05$ |
| 5 | . 750 | . 737 | . 152 | . 416 | $2.552 E+06$ | . 69 | 1.166E+04 | 4.426E+06 | $1.524 E+05$ | $3.011 E+05$ |
| 5 | . 800 | . 737 | . 163 | . 445 | 2.228E+06 | . 63 | 1.336E+04 | 4.429E+06 | 1.332E+05 | $2.631 E+05$ |
| 5 | . 850 | . 737 | . 174 | . 475 | $1.981 \mathrm{E}+06$ | . 59 | $1.518 \mathrm{E}+04$ | 4.477E+06 | $1.198 \mathrm{E}+05$ | $2.365 E+05$ |
| 8 | . 300 | . 737 | . 046 | . 125 | $7.036 E+07$ | 6.82 | $1.264 E+03$ | $1.323 E+07$ | 1.235E+07 | $2.483 E+07$ |
| 6 | . 350 | . 737 | . 055 | . 150 | $3.7978+07$ | 4.37 | 1.820E+03 | $1.028 E+07$ | $5.171 E+06$ | $1.041 \mathrm{E}+07$ |
| 6 | . 400 | . 737 | . 064 | . 175 | 2.297E+07 | 3.05 | $2.477 E+03$ | 8.469E+06 | $2.574 E+06$ | 5.188E+06 |
| 6 | . 450 | . 737 | . 073 | . 200 | $1.515 \mathrm{E}+07$ | 2.26 | $3.235 E+03$ | 7.295E+06 | 1.460E+06 | $2.948 E+06$ |
| 6 | . 500 | . 737 | . 082 | . 225 | $1.069 E+07$ | 1.77 | 4.095E+03 | $6.511 E+06$ | 9.182E+05 | 1.956E+06 |
| 6 | . 550 | . 737 | . 091 | . 250 | $7.952 E+06$ | 1.44 | 5.055E+03 | $5.982 E+06$ | $6.269 E+05$ | 1.268E+06 |
| 6 | . 600 | . 737 | . 101 | . 275 | $6.182 E+06$ | 1.21 | 6.117E+03 | 5.627E+06 | 4.580E+05 | 9.275E+05 |
| 6 | . 650 | . 737 | . 110 | . 300 | 4.984E+06 | 1.05 | 7.279E+03 | $5.399 \mathrm{E}+08$ | 3.539E+05 | -7.175E+05 |
| 6 | . 700 | . 737 | . 119 | . 325 | $4.143 E+06$ | . 93 | 8.543E+03 | $5.267 E+06$ | $2.868 E+05$ | $5.818 E+05$ |
| 6 | . 750 | . 737 | . 128 | . 350 | 3.534E+06 | . 84 | 9.908E+03 | $5.211 \mathrm{E}+06$ | 2.419E+05 | 4.911E+05 |
| 6 | . 800 | . 737 | . 137 | . 375 | 3.083E+06 | . 77 | 1.137E+04 | $5.2198+06$ | $2.112 E+05$ | $4.291 E+05$ |
| 6 | . 850 | . 737 | . 146 | . 400 | 2.742E+06 | . 72 | $1.294 \mathrm{E}+04$ | $5.281 E+06$ | 1.900E+05 | $3.862 E+05$ |
| 7 | . 300 | . 737 | . 038 | . 105 | 1.007E+08 | 8.27 | $1.044 \varepsilon+03$ | $1.564 E+07$ | $2.089 E+07$ | 4.199E+07 |
| 7 | . 350 | . 737 | . 046 | . 127 | 5.339E+07 | 5.25 | $1.519 E+03$ | $1.207 E+07$ | 8.533E+06 | $1.718 \mathrm{E}+07$ |
| 7 | . 400 | . 737 | . 054 | . 149 | $3.190 E+07$ | 3.64 | $2.082 E+03$ | 9.885E+06 | 4.171E+06 | 8.409E+06 |
| 7 | . 450 | . 737 | . 062 | . 170 | $2.083 E+07$ | 2.69 | $2.735 \mathrm{E}+03$ | 8.479E+06 | $2.333 E+06$ | 4.710E+06 |
| 7 | . 500 | . 737 | . 070 | . 192 | $1.4585+07$ | 2.10 | $3.476 E+03$ | $7.542 E+06$ | $1.450 \varepsilon+06$ | $2.932 E+06$ |
| 7 | . 550 | . 737 | . 078 | . 214 | $1.078 E+07$ | 1.71 | 4.307E+03 | $6.910 E+06$ | $9.815 \mathrm{E}+05$ | $1.987 E+06$ |
| 7 | . 600 | . 737 | . 086 | . 235 | $8.341 E+00^{\circ}$ | 1.44 | $5.226 E+03$ | $6.486 E+06$ | 1.120E+05 | $1.443 \mathrm{E}+06$ |
| 7 | . 650 | . 737 | . 094 | . 257 | 6.698E+08 | 1.24 | 6.234E+03 | $6.214 E+06$ | $5.473 E+05$ | 1.110E+06 |
| 7 | . 700 | . 737 | . 102 | . 279 | 5.552E+06 | 1.10 | $7.330 E+03$ | $6.056 E+06$ | 4.417E+05 | 8.965E+05 |
| 7 | . 750 | . 737 | . 110 | . 300 | 4.726E+06 | 1.00 | $8.516 E+03$ | 5.989E+06 | $3.716 E+05$ | $7.548 E+05$ |
| 7 | . 800 | . 737 | . 118 | . 322 | 4.117E+08 | . 92 | $9.7908+03$ | $5.998 E+06$ | $3.239 E+05$ | $6.585 E+05$ |
| 7 | . 850 | . 737 | . 128 | . 344 | 3.659E+06 | . 86 | $1.115 \mathrm{E}+04$ | $6.072 E+06$ | 2.913E+05 | 5.924E+05 |

*PTOT \& N*PMAX are for water-cooled copper magnets at $60^{\circ} \mathrm{C}$. For liquid nitrogen-cooled magnets, divide by a factor of 10.

JMAX: maximum overall current density in the array
BMAX: maximum field
PTOT: optimum total power consumption at this configuration N*PMAX: number of coils times the maximum power consumption of any of them (reflects the size of the power supplies and serves as an upper bound on the power requirements) .

From Tables IV-4 and IV-5 it is seen that the six-coil array requires $55 \%$ more power than the five-coil array; and in either case, the power consumption is not prohibitive. Figure IV-3 shows the upper bound for the power requirements versus the location radius for the five- and six-coil arrays. It is clear from this sketch that the larger the location radius of the magnets, the less power consumption of the system.

Table IV-6 lists comparisons between the six different options discussed earlier. Ampere-meters in the magnets relate to forces on the model which may be a $20,000 \mathrm{Am}$ pole strength $\mathrm{S} / \mathrm{C}$ model or a 7,740 Am pole strength PM model. The S/C magnets are optimized for minimum ampere-meters and the copper magnets are optimized for minimum power. The power supply is required to charge (for $S / C$ coils) and to maintain $I^{2} R$ losses (for Cu coils).

Water-cooled copper magnets seem to offer no benefits. $\mathrm{LN}_{2}$ cooled copper appears interesting with S/C models. The S/C + S/C and $S / C+P M$ models seem attractive, particularly for extrapolation to larger systems.

Table IV-5.
Minimum Power Copper Magnet Arrays for Levitation of a PM Model.

| MODEL Q/M | $=3.350 E+02 \mathrm{Am} / \mathrm{kg}$ | $\begin{aligned} & F X / W \\ & .000 E+00 \end{aligned}$ | $\begin{gathered} F Y / W \\ .000 E+00 \end{gathered}$ | $\begin{gathered} F I / W \\ 1.000 E+00 \end{gathered}$ | $\begin{gathered} T Y / W \\ .000 E+00 \end{gathered}$ | $\begin{gathered} T N / W \\ .000 E+00 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MODEL LENGTH | $=3.000 \mathrm{E}-01 \mathrm{a}$ |  |  |  |  |  |
| CU RATIO | $=7.500 \mathrm{E}-01$ | $X$ | $Y$ | 2 | YAM | PITCH |
| MIN OISTANCE | $=5.000 \mathrm{E}-02 \mathrm{~m}$ | .000E+00 | . $000 \mathrm{E}+00$ | $1.000 E+00$ | . $000 \mathrm{E}+00$ | . OOOE +00 |


| NC | $R$ | L | RI | RO | JMAX | 8 Max | CU-WT | N*IS | PTOT* | N*PMAX |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | . 300 | . 737 | . 055 | . 151 | $1.926 E+08$ | 22.34 | $1.544 E+03$ | 4.424E+07 | 1.141E+08 | $2.272 E+08$ |
| 5 | . 350 | . 737 | . 066 | . 181 | $1.056 \mathrm{E}+08$ | 14.41 | $2.201 E+03$ | 3.459E+07 | 4.898E+07 | 9.739E+07 |
| 5 | . 400 | . 737 | . 077 | . 210 | $6.465 E+07$ | 10.09 | $2.976 E+03$ | $2.863 E+07$ | $2.485 E+07$ | 4.936E+07 |
| 5 | . 450 | . 737 | . 088 | . 240 | 4.303E+07 | 7.52 | $3.866 \mathrm{E}+03$ | $2.476 \mathrm{E}+07$ | $1.432 \mathrm{E}+07$ | 2.841E+07 |
| 5 | . 500 | . 737 | . 098 | . 269 | $3.056 E+07$ | 5.89 | 4.873E+03 | $2.216 E+07$ | 9.114E+06 | 1.806E+07 |
| 5 | . 550 | . 737 | . 109 | . 298 | $2.287 E+07$ | 4.79 | $5.997 E+03$ | $2.041 \mathrm{E}+07$ | 6.285E+06 | 1.245E+07 |
| 5 | . 600 | . 737 | . 120 | . 328 | 1.786E+07 | 4.03 | 7.237E+03 | 1.923E+07 | 4.627E+06 | $9.156 E+06$ |
| 5 | . 650 | . 737 | . 131 | . 357 | 1.44EE+07 | 3.48 | 8.593E+03 | $1.847 \mathrm{E}+07$ | 3.597E+06 | 7.114E+06 |
| 5 | . 700 | . 737 | . 141 | . 386 | 1.203E+07 | 3.08 | $1.007 E+04$ | $1.803 E+07$ | $2.926 E+06$ | $5.185 \varepsilon+06$ |
| 5 | . 750 | . $737^{\circ}$ | . 152 | . 416 | $1.028 E+07$ | 2.77 | $1.166 E+04$ | $1.783 E+07$ | $2.475 E+06$ | 4.890E+06 |
| 5 | . 800 | . 737 | . 163 | . 445 | 8.977E+06 | 2.54 | $1.336 E+04$ | $1.785 E+07$ | $2.163 E+06$ | 4.273E+06 |
| 5 | . 850 | . 737 | . 174 | . 475 | 7.985E+06 | 2.36 | $1.518 E+04$ | 1.804E+07 | $1.945 E+06$ | 3.841E+06 |
| 6 | . 300 | . 737 | . 046 | . 125 | $2.836 E+08$ | 27.49 | $1.264 E+03$ | $5.333 E+07$ | $2.006 E+08$ | $4.032 \varepsilon+08$ |
| 6 | . 350 | . 737 | . 055 | . 150 | 1.530E+08 | 17.60 | $1.820 \varepsilon+03$ | 4.143E+07 | 8.397E+07 | 1.690E+08 |
| 6 | . 400 | . 737 | . 064 | . 175 | 9.258E+07 | 12.27 | $2.477 E+03$ | $3.413 E+07$ | 4.180E+07 | 8.426E+07 |
| 6 | . 450 | . 737 | . 073 | . 200 | $6.105 E+07$ | 9.13 | $3.235 \mathrm{E}+03$ | $2.940 E+07$ | 2.372E+07 | 4.787E+07 |
| 6 | . 500 | . 737 | . 082 | . 225 | 4.306E+07 | 7.14 | 4.095E+03 | $2.624 E+07$ | 1.491E+07 | $3.013 E+07$ |
| 6 | . 550 | . 737 | . 091 | . 250 | $3.204 E+07$ | 5.81 | 5.055E+03 | $2.411 E+07$ | $1.018 E+07$ | $2.060 \varepsilon+07$ |
| 6 | . 600 | . 737 | . 101 | . 275 | $2.491 E+07$ | 4.89 | $6.117 E+03$ | $2.287 E+07$ | 7.437E+06 | 1.506E+07 |
| 6 | . 650 | . 737 | . 110 | . 300 | $2.008 E+07$ | 4.23 | 1.279E+03 | $2.176 \mathrm{E}+07$ | $5.748 \mathrm{E}+06$ | 1.165E+07 |
| 6 | . 700 | . 737 | . 119 | . 325 | 1.669E+07 | 3.75 | 8.543E+03 | 2.122E+07 | 4.657E+06 | 9.448E+06 |
| 6 | . 750 | . 737 | . 128 | . 350 | 1.424E+07 | 3.38 | 9.908E+03 | $2.100 E+07$ | $3.928 E+06$ | 7.975E+06 |
| 6 | . 800 | . 737 | . 137 | . 375 | 1.242E+07 | 3.11 | $1.1378+04$ | $2.103 E+07$ | $3.430 E+06$ | $6.968 E+06$ |
| 6 | . 850 | . 737 | . 146 | . 400 | 1.105E+07 | 2.90 | $1.294 E+04$ | $2.128 E+07$ | $3.085 E+06$ | $6.271 E+06$ |
| 7 | . 300 | . 737 | . 038 | . 105 | $4.058 \mathrm{E}+08$ | 33.34 | $1.044 E+03$ | $6.302 \varepsilon+07$ | $3.392 E+08$ | 6.819E+08 |
| 7 | . 350 | . 737 | . 046 | . 127 | 2.152E+08 | 21.15 | $1.519 E+03$ | $4.862 E+07$ | $1.386 \mathrm{E}+08$ | 2.790E+08 |
| 7 | . 400 | . 737 | . 054 | . 149 | 1.285E+08 | 14.66 | $2.082 E+03$ | $3.984 E+07$ | $6.173 E+07$ | 1.366E+08 |
| 7 | . 450 | . 737 | . 062 | . 170 | $8.395 E+07$ | 10.85 | $2.735 E+03$ | 3.417E+07 | $3.789 E+07$ | $7.650 E+07$ |
| 7 | . 500 | . 737 | . 070 | . 192 | 5.875E+07 | 8.46 | $3.476 \mathrm{E}+03$ | $3.039 E+07$ | $2.356 E+07$ | 4.762E+07 |
| 7 | . 550 | . 737 | . 078 | . 214 | 4.345E+07 | 6.88 . | 4.307E+03 | $2.784 E+07$ | $1.594 E+07$ | $3.226 E+07$ |
| 7 | . 600 | . 737 | . 086 | . 235 | 3.361E+07 | 5.79 | 5:226E+03 | $2.614 E+07$ | $1.156 E+07$ | $2.343 E+07$ |
| 7 | . 650 | . 737 | . 094 | . 257 | 2.699E+07 | 5.01 | $6.234 E+03$ | $2.504 E+07$ | 8.887E+06 | 1.802E+07 |
| 1 | . 700 | . 737 | . 102 | . 279 | 2.237E+07 | 4.44 | 1.330E+03 | 2.440E+07 | $7.173 \mathrm{E}+05$ | 1.456E+07 |
| 7 | . 750 | . 737 | . 110 | . 300 | 1.905E+07 | 4.02 | $8.516 E+03$ | $2.414 \mathrm{E}+07$ | $6.034 E+06$ | 1.226E+07 |
| 7 | . 800 | . 737 | . 118 | . 322 | $1.659 E+07$ | 3.70 | 9.790E+03 | $2.417 \varepsilon+07$ | $5.2618+06$ | 1.069E+07 |
| 7 | . 850 | . 737 | . 126 | . 344 | $1.474 E+07$ | 3.45 | $1.115 E+04$ | $2.447 E+07$ | 4.730E+06 | 9.621E+06 |

*PTOT \& N*PMAX are for water-cooled copper magnets at $60^{\circ} \mathrm{C}$. For liquid nitrogen-cooled magnets, divide by a factor of 10.
 plots are for 5 -coil and 6 -coil arrays with 5 cm distance between adjacent coils. The model is a superconducting magnet. - $\varepsilon-\Lambda I$ əxn6ṭa Maximum power requirements for an array of water-cooled copper magnets. The

IV-3. Levitation Coils Dimensions.
Three representative magnet cross-sections are sketched in Fig. IV-4 for the 5 levitation coils. The top sketch is for S/C model and $S / C$ magnets at $2 \mathrm{kA} / \mathrm{cm}^{2}$. The middle sketch is for a $P M$ model and $S / C$ magnets at $2 \mathrm{kA} / \mathrm{cm}^{2}$. The bottom sketch is for an S/C model with low current density copper magnets or for a PM model with higher current density copper magnets. Either copper magnet set could be cooled with water or liquid nitrogen.

Table IV-6.
System Comparison.

| Model Core <br> Levitation Magnets | $\begin{aligned} & s / C \\ & s / C \end{aligned}$ | $\begin{aligned} & \mathrm{PM} \\ & \mathrm{~S} / \mathrm{C} \end{aligned}$ | s/C CU <br> WATER | $\begin{gathered} \mathrm{PM} \\ \mathrm{CU} \\ \text { COOLED } \\ \hline \end{gathered}$ | $\begin{aligned} & \mathrm{S} / \mathrm{C} \\ & \mathrm{CU} \\ & \mathrm{~L} . \mathrm{N} \end{aligned}$ | $\begin{gathered} \text { PM } \\ \text { CU } \\ \text { IROGEN } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AMPERE-METERS (MAm) | 2.1 | 14.7 | 4.5 | 18.0 | 4.5 | 18.0 |
| CURRENT DENSITY ( $\mathrm{kA} / \mathrm{cm}^{2}$ ) | 1.5 | 1.5 | 0.3 | 1.2 | 0.3 | 1.2 |
| DC POWER SUPPLY <br> (kW) | 0.0 | 0.0 | 360.0 | 5800.0 | 36.0 | 580.0 |



S/C MODEL
S/C MAGNETS ${ }_{2}$ $\mathrm{J}=2.0 \mathrm{KA} / \mathrm{cm}^{2}$


PM MODEL $\mathrm{S} / \mathrm{C}$ MAGNETS
$\mathrm{J}=2.0 \mathrm{KA} / \mathrm{cm}^{2}$


SIC MODEL
CU MAGNETS $\mathrm{J}=0.3 \mathrm{KA} / \mathrm{cm}^{2}$

PM MODEL CU MAGNETS $\mathrm{J}=1.2 \mathrm{KA} / \mathrm{cm}^{2}$


Figure IV-4. Support Magnets for $S / C$ and PM Models.

## V. LEVITATION MAGNET SYSTEMS

Two options have been chosen for magnet system design. The first is superconducting coils and permanent magnet model. The second is nitrogen cooled coils and permanent magnet model.

## V.1. Superconducting Levitation Magnets

## Table V-1.

Specifications for the 5 Superconducting Levitation Magnets.



```
Location radius (m) ................................ 0.7
Magnet inner radius (m) ......................................0. 0.06
```




```
Magnet top to model distance (m) .................. 1.0
```



```
Maximum winding field (T) .....................................
Magnet current (A)
500.0
```

Table V-1 lists the specifications of the 5 superconducting solenoids used to levitate the permanent magnet core. The coils are optimized to have the least ampere-meters. The 500 A conductor chosen for this design is a one triplex of an 11 kA ac conductor used in our previous suspension designs. ${ }^{1,2}$ The 500 A triplex has a twist pitch of 2.2 cm . Each part of the triplex is a seven-strand conductor. The seven-strand conductor is six OFHC copper wires twisted around a superconducting center conductor and all soldered with Staybrite. Since the requirements of low ac losses and cryostability conflict with each other, the basic principle chosen for this conductor is to achieve cryostability
within the basic cable. Each superconducting strand has a diameter of 0.051 cm and contains 2041 filaments of $6.7 \mu \mathrm{~m}$ dia with a twist pitch of 1.27 cm . The copper-to-superconductor ratio for each superconducting strand is 1.8 . The reported losses on this conductor at a cycle that has $\dot{B}=9 \mathrm{~T} / \mathrm{s}$ during charge and discharge are summarized in Table V-2.

Table V-2.
ac Losses of the 500 A Triplex Conductor.

Eddy current losses (J/cycle/m) ....................................... 0.21
Hysteresis loss (J/cycle/m) ............................................... $10^{-5}$

Exact losses have not been calculated because finding the rms value of the correction currents is beyond the scope of this work. $|\Delta I / I|$ ranges between $10^{-4}$ to $10^{-2}$ depending on the yaw position and off-center signals. An estimate of ac losses for 500 A triplex conductor is about 200 W . The inductance matrix for the 5 levitation magnets is listed in Table V-3, as based on single turn coils. The maximum possible force between magnets is $18.5 \times 10^{4} \mathrm{~N}$.

Some details of the magnet support system and the cryostat are given in the Appendix. A distance of 9 cm from the top of the cryostat to the tip of the magnets is assumed for calculation purposes. It now appears that 13 cm is a more practical choice for final design.

Table V-3.
Inductance Matrix in $H$ for the 5 Superconducting Levitation Magnets Based on Single Turn Coils.

|  | Coil \#1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Coil \#1 | $2.99 \times 10^{-7}$ | -- | -- | -- | -- |
| 2 | $1.09 \times 10^{-8}$ | $2.99 \times 10^{-7}$ | -- | - | -- |
| 3 | $2.49 \times 10^{-9}$ | $1.09 \times 10^{-8}$ | $2.99 \times 10^{-7}$ | -- | -- |
| 4 | $2.49 \times 10^{-9}$ | $2.49 \times 10^{-9}$ | $1.09 \times 10^{-8}$ | $2.99 \times 10^{-7}$ | -- |
| 5 | $1.09 \times 10^{-9}$ | $2.49 \times 10^{-9}$ | $2.49 \times 10^{-9}$ | $1.09 \times 10^{-8}$ | $2.99 \times 10^{-7}$ |

## V. 2. Nitrogen Cooled Magnet System

Table V-4 lists the specifications for 5 nitrogen cooled copper solenoids used to levitate the permanent magnet model core. The coils are optimized for minimum ohmic heating. Each of the 5 levitation coils is a stack of pancakes of rectangular OFHC copper turns. The cooling surface is the surface between pancakes. For the optimized dimensions listed in Table V-4, the maximum heat flux $q$ in $W / \mathrm{cm}^{2}$ at the cooling surface for $N$ pancakes is

$$
q=14.2 / \mathrm{N}
$$

For $N=30$ the heat flux is less than $0.5 \mathrm{~W} / \mathrm{cm}^{2}$ (which is small). Each pancake is 12 turns of 6 kA square conductor 1.95 cm high. The separation between pancakes is 0.524 cm . There is 1 mm of insulation between turns. The maximum turn to turn ohmic voltage is 0.5 volt. The space between pancakes allows outward flow in the radial direction.

> Table $\mathrm{V}-4$. Specification of $\mathrm{LN}_{2}-$ Cooled Levitation Coils with PM Model Core

Number of magnets ..... 5
Location radius (m) ..... 0.7
Inner radius (m) ..... 0.141
Outer radius (m) ..... 0.386
Magnet height (m) ..... 0.737
Magnet top to model distance (m)
1.0
1.0
Maximum gross current density ( $A / \mathrm{m}^{2}$ ) ..... $1.203 \times 10^{7}$
\% Copper volume ..... 75
Total copper weight (kg) ..... $1.007 \times 10_{5}^{4}$
Total power for 5 magnets (w) ..... $2.926 \times 10_{5}^{5}$
Maximum power per magnet (w) ..... $1.157 \times 10^{5}$
Magnet current (kA) ..... 5

Each coil will be cooled as follows:

1. Variable flow along bore (single phase)
2. Constant radial outward flow (two phase)
3. Variable two phase flow along outer circumference. Using low pressure 2 atm . boiling nitrogen cooling the flow rate for each coil is 124 gallon/m (994 $\mathrm{ft}^{3} / \mathrm{hr}$ ) at $10 \%$ exit quality. The pressure drop along the central bore (1) is negligible compared to the pressure drop between pancakes (2) which is 0.05 psi. The pressure drop in the outer region (3) is 0.0077 psi/ft. This is so low compared to the 0.05 psi drop across pancakes that the flow will be close to uniform.

Preliminary design of the liquid nitrogen cryostat is aimed at determining minimum practical spacing from the top of the magnets to the upper surface of the vacuum jacket plate. Results of these calculations are presented in Table V-5.

| Magnet to Warm Surface Spacing for Liquid Nitrogen-Cooled Solenoids. |  |  |
| :---: | :---: | :---: |
| Item | space cm | Accumulated cm |
| Top of magnet | 0 | 0 |
| Flow space | 0.3 | 0.3 |
| Cold structure | 2.0 | 2.3 |
| Pressure deflection | 0.3 | 2.6 |
| Composite cold top plate | 2.5 | 5.1 |
| Insulation space | 2.5 | 7.6 |
| Top plate deflection | 0.8 | 8.4 |
| Composite top plate | 2.0 | 10.4 |
| Minimum total space from top of magnet |  | 10.4 |

Model control requirements are determined from a simulation study in which the exact motion of the model is calculated in response to sensor signals and restoring forces. Model motion is caused by magnetic forces and torques from the five control magnets. These forces are effectively always non-zero because the position and velocity are known only to the precision of the sensing system; furthermore, corrective forces applied every 50 ms will continuously change the positions and velocities. There are four main purposes of this study: (1) To determine achievable positioning specifications; (2) To find if the model can be held within allowed displacements in the five degrees of freedom ( $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{y}$ aw, and pitch); (3) To size the power supplies required to perform the control; and (4) To find or suggest future better positioning scenarios, especially for the yaw and pitch, as explained later.

The preliminary study is not complete; it does not account for all possible factors that affect precise positioning of the model. Not included in particular is the exact location of individual levitation coil windings after cooldown and their cumulative mechanical migration in response to magnetic forces over time. Nevertheless, this preliminary study shows that model positioning is attainable with the position sensing system provided by NASA.

The model and support magnet combination selected consists of a permanent magnet model and five superconducting support
magnets (Fig. VI-1). The specifications used for the simulation study are listed in Table VI-1.


Figure VI-1. Electromagnetic system used for levitation and control.

## Table VI-1.

Specifications for the PM Model Core and the Superconducting Levitation Coils.
A. Model

Total mass ................................ 23.11 kg
Total inertia ............................... $0.6 \mathrm{~kg} \mathrm{~m}^{2}$
PM pole strength ......................... $7.742 \times 10^{3} \mathrm{Am}$
PM length .................................. 12 " ( 30.48 cm )
B. Levitation Coils

No. of coils ............................... 5
Location radius ........................... 0.7 m
Inner radius ............................... 0.173 m
Outer radius ............................... 0.386 m
Coil height ............................... 0.493 m
No. of turns .............................. 3150
Maximum operating current ............... 512 A
Self-inductance .......................... 2.894 H

The assumptions used for the sensing and positioning system, Fig. VI-2, are:

1. The resolution for sensing positions of the model in the $x, y$, and $z$ directions is $0.001^{n}\left(2.54 \times 10^{-5} \mathrm{~m}\right)$.
2. The resolution for sensing angles of yaw and pitch is $0.002^{\circ}$.
3. Information about the above positions is available at 50 ms time intervals.
4. There is a 25 ms time lag between determining the model position and initiating power supply control current pulses.
5. The correction signals following the 25 ms time lag are in the form of $F_{m} \sin 2 \pi f t$ where $0<t<\tau e$, $f$ is the correction frequency, and $\tau_{e}=1 / 2 f$.
6. The sum of the time for operating the correction signals, $\tau_{e}$ plus the 25 ms time lag is equal or less than the 50 ms time interval discussed in (3.) above.


POSITIONING AND CONTROL SYSTEM

Figure VI-2. Feedback control.
7. Information for control of position, velocity and forces on the model is based on the resolution limits mentioned; for example, the $x$ location is known to be within a $0.001^{\prime \prime}$ region. The actual solution of the equation of motion in the 5 degrees of freedom is used to simulate information available from the sensors.

The forces and torques on the model when off central positions are functions of all 5 degrees of freedom. However, in the $x, y$, and $z$ directions the forces and torques are more linearly related to the displacements. This enables a prediction of drift time constants, $\tau=\sqrt{\mathrm{k} / \mathrm{m}}$. These time constants are typically 500 to 1000 ms for the $x, y$, and $z$ directions. In the yaw and pitch directions the torques are more related to $x, y$ and $z$ than to yaw and pitch angles. Unfortunately, such information about $x, y$, and $\mathbf{z}$ is not available on a continuous basis but rather on a discrete basis which sometimes makes the control system blind as far as the magnitude and direction of these torques is concerned. The small inertia of the model makes the problem even more difficult because the time constant for responding to the drifting torques is very small ( $<40 \mathrm{~ms}$ ) while the time interval between corrections is 50 ms . If the model acquires any significant momentum in roll or pitch during 50 ms , it would become impossible to position or control the model.

A computer program is constructed on the above-mentioned assumptions. The program solves the equation of motion of the model in 5 degrees of freedom yielding exact positions, velocities, and drifting forces and torques under equilibrium currents
that produce pure lift in the ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$, yaw and pitch)
$=(0,0,1,0,0)$ position. Forces and velocities supplied to the control system are based on the resolution of the sensors. Within these resolutions it is not expected that the computed values would correspond all the times with the values calculated from the sensed approximate positions. Many different scenarios and control decisions have been experimented with to control the model in the two difficult modes (yaw and pitch).

The structure of the simulation computer program is as

## follows:

1. For every period of time, $\tau=50 \mathrm{~ms}$, the five positions or angle zones are recorded. The resolution for determining these zones is discussed earlier in this chapter. The velocities in the 5 degrees of freedom are calculated based on the model position 50 ms earlier. The drifting forces are calculated assuming the 5 levitation coils are carrying the equilibrium currents.
2. 25 ms later, a correcting force is imposed on the model in the form of a half sine wave for a time period $\mathrm{T}_{\mathrm{e}}<50$ ms. The force is

$$
F_{c}=F_{\max } \sin \left(\pi t / \tau_{e}\right)
$$

for $0<t<\tau e$.
$F_{\text {max }}$ is related to the calculated drifting forces $\mathrm{F}_{\mathrm{d}}$ and the model velocities $V$ recorded 25 ms earlier according to

$$
\begin{equation*}
2 F_{\max } / \pi=\left\{\alpha F_{d}+\beta m V / T_{e}\right\} T / \tau e, \tag{2}
\end{equation*}
$$

where $m$ is the mass or inertia of the model depending on the degree of freedom and $\alpha$ and $\beta$ are multiplying factors to be evaluated.
3. The correction force $F_{c}$ is obtained from the current-force subroutine that finds the 5 current changes in the levitation coils needed to produce the correction forces. These $\pm \Delta I$ current pulses are added to the equilibrium currents.

Several combinations of $\alpha$ and $\beta$ are used. The combination of $\alpha=\beta=1$ results in a build-up oscillation in all degrees of freedom as shown in Table VI-2. Fig. VI-3 shows the effect of reducing the value of $\beta$ on this oscillation. The yaw direction is used as an example. There is no definite conclusion at the present time on the best combination of $\alpha$ and $\beta$. Fig. VI-4 and Table VI-3 show the position of the model in the 5 degrees of freedom over 4000 ms of operation for the case of $\alpha=1$ and $\beta=0.4$.

Tables VI-4 and VI-5 show the ratios of both the calculated forces and velocities compared to the exact forces and velocities computed from the equation of motion in the five degrees of freedom. As shown in these tables, quite often the calculated values do not agree with the computed ones in magnitude and direction. This is related to the discrete zone positioning from the sensors rather than exact positioning. Table VI-6 lists the correction currents and voltages of the power supplies as function of time. As shown, correction currents as small as $10^{-4}$ of
the equilibrium currents, e.g., are needed for corrections in the yaw or pitch directions.

We believe that it will be possible to improve the control system through more sophisticated programming based on the previous history of the position of the model rather than on the single previous position, as is the case in the current program. The use of $\alpha$ and $\beta$ as functions of the degree of freedom and position may result in better positioning as well. Such new, more advanced computer program is beyond the scope of the present effort.

$\beta=1$



$$
\beta=0.4
$$

Figure VI-3. Model position in yaw direction at time intervals 50 ms long for a value of $\alpha=1$ and $\beta=1,0.5$, and 0.4 respectively. As seen, better control can be achieved by using values of $\beta \leq 0.5$.

Table VI-2.
Calculated $x, y, z$, Yaw and Pitch Positions at 50 ms Correction Time Intervals.
Values of $\alpha=1.0$ and $\beta=1.0$ are used.

| Time (ms) | $\mathrm{X} / .001^{\prime \prime}$ | Y/. $001{ }^{\prime \prime}$ | 2/.001" | Yaw/.002 ${ }^{\circ}$ | Pitch/.002 ${ }^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 0 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 | 0 |
| 150 | 0 | 0 | 0 | -2 | 0 |
| 200 | 0 | 0 | 0 | -4 | 0 |
| 250 | 1 | 0 | 0 | -4 | -2 |
| 300 | 0 | 0 | 0 | -1 | -6 |
| 350 | 0 | 0 | 0 | 2 | -4 |
| 400 | -1 | 0 | 0 | 0 | 3 |
| 450 | -1 | 0 | 0 | -5 | 4 |
| 500 | 0 | 0 | 0 | -5 | -7 |
| 550 | 0 | 1 | 0 | 6 | -18 |
| 600 | -1 | 0 | 0 | 16 | -3 |
| 650 | -2 | 0 | 0 | 4 | 25 |
| 700 | -1 | 0 | 0 | -26 | 22 |
| 750 | 0 | 0 | 0 | -22 | -32 |
| 800 | -1 | 0 | 0 | 49 | -54 |
| 850 | -5 | 0 | 0 | 98 | 63 |
| 900 | -4 | 0 | 0 | 1 | 170 |
| 950 | 3 | 0 | 0 | -146 | 4 |
| 1000 | 3 | 0 | 0 | -24 | -356 |
| 1050 | -14 | 0 | 0 | 361 | -264 |
| 1100 | -19 | 0 | 0 | 256 | 638 |
| 1150 | 36 | 0 | 0 | -866 | 1070 |
| 1200 | 85 | 0 | -1 | -1490 | -413 |
| 1250 | -20 | 0 | -3 | 561 | -2368 |
| 1300 | -227 | 0 | -4 | 3639 | -591 |
| 1350 | -195 | 0 | -6 | 2618 | 4709 |
| 1400 | 171 | 0 | -11 | -2711 | 5274 |
| 1450 | 220 | 0 | -8 | -2432 | -8792 |
| 1500 | -831 | -2 | -25 | 12354 | ***** |



Fig. VI-4.a. Model position in $x$ and $y$ at time intervals 50 ms long. Values of $\alpha=1$ and $\beta=0.4$ are used for correction. The model position in $z$ is not shown except that it stays within $\pm 0.001$ in. all the time.


Fig. VI-4.b. Model position in yaw and pitch at 50 ms time intervals. Values of $\alpha=1$ and $\beta=0.4$ are used for correction.

Table VI-3.
Calculated $x, y, z$, Yaw and Pitch Positions at 50 ms Correction Time Intervals. $(\alpha=1.0$ and $\beta=0.4)$

| Time(ms) | X/.001" | Y/.001" | 2/.001 ${ }^{\text {n }}$ | Yaw/. 002 | Pitch/. 002 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 0 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 | 0 |
| 150 | 0 | 0 | 0 | -2 | 0 |
| 200 | 0 | 0 | 0 | -4 | 0 |
| 250 | 1 | 0 | 0 | -5 | -2 |
| 300 | 1 | 0 | 0 | -4 | -7 |
| 350 | 0 | 0 | 0 | -2 | -9 |
| 400 | 0 | 0 | 0 | 0 | -8 |
| 450 | 0 | 0 | 0 | 2 | -5 |
| 500 | -1 | 0 | 0 | 5 | -2 |
| 550 | -2 | 1 | 0 | 8 | 0 |
| 600 | -2 | 0 | 0 | 9 | 4 |
| 650 | -1 | 0 | 0 | 7 | 10 |
| 700 | -1 | 0 | 0 | 2 | 11 |
| 750 | -1 | 0 | 0 | 0 | 6 |
| 800 | -1 | 0 | 0 | 2 | 1 |
| 850 | -1 | 0 | 0 | 4 | 0 |
| 900 | -1 | 0 | 0 | 6 | 3 |
| 950 | -1 | 0 | 0 | 6 | 9 |
| 1000 | -1 | 0 | 0 | 4 | 13 |
| 1050 | -1 | 0 | 0 | 1 | 13 |
| 1100 | -1 | 0 | 0 | 0 | 9 |
| 1150 | -1 | 0 | 0 | 1 | 6 |
| 1200 | -1 | 0 | 0 | 1 | 5 |
| 1250 | -1 | 0 | 0 | 1 | 7 |
| 1300 | -2 | 0 | 0 | 1 | 11 |
| 1350 | -1 | 0 | 0 | 1 | 13 |
| 1400 | -1 | 0 | 0 | 0 | 13 |
| 1450 | -1 | 0 | 0 | -1 | 11 |
| 1500 | -2 | -1 | 0 | -4 | 8 |
| 1550 | -2 | -1 | 0 | -5 | 6 |
| 1600 | -2 | 0 | 0 | -5 | 4 |
| 1650 | -1 | 0 | 0 | -4 | 3 |
| 1700 | -1 | 0 | 0 | -3 | 4 |
| 1900 | -2 | -1 | 0 | -2 | 3 |
| 1950 | -2 | 0 | 0 | -1 | 3 |
| 2000 | -2 | 0 | 0 | -1 | 4 |

Table VI-4.
Ratio of Calculated to Computed Forces at 50 ms Correction Intervals.
( $\alpha=1.0$ and $B=0.4$ )



## Ratio of Calculated to Computed Velocities at 50 ms Correction Intervals. $(\alpha=1.0$ and $\beta=.4)$

| Time(ms) | $\mathrm{V}(\mathrm{x})$ | V ( y ) | V (z) | $V$ (yaw) | $V(\mathrm{pitch})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 50 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 100 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 150 | 0.00000 | 0.00000 | 0.00000 | 1.06338 | 0.00000 |
| 200 | 0.00000 | 0.00000 | 0.00000 | 1.19496 | 0.00000 |
| 250 | 4.32024 | 0.00000 | 0.00000 | 3.27362 | 0.49320 |
| 300 | 0.00000 | 0.00000 | 0.00000 | 0.57969 | 1.83462 |
| 350 | 1.34514 | 0.00000 | 0.00000 | 1.13477 | -1.48684 |
| 400 | 0.00000 | 0.00000 | 0.00000 | 1.88272 | 0.35124 |
| 450 | 0.00000 | 0.00000 | 0.00000 | 0.70975 | 1.03991 |
| 500 | 1.23397 | 0.00000 | 0.00000 | 1.22712 | 1.39766 |
| 550 | 2.24091 | 8.64438 | 0.00000 | 1.93011 | 0.92659 |
| 600 | 0.00000 | 1.19902 | 0.00000 | -0.54598 | 0.83505 |
| 650 | 1.36086 | 0.00000 | 0.00000 | 0.55985 | 2.14069 |
| 700 | 0.00000 | 0.00000 | 0.00000 | 1.71121 | -0.29596 |
| 750 | 0.00000 | 0.00000 | 0.00000 | -1.19816 | 0.84187 |
| 800 | 0.00000 | 0.00000 | 0.00000 | 0.81846 | 2.18153 |
| 850 | 0.00000 | 0.00000 | 0.00000 | 0.96895 | -1.10008 |
| 900 | 0.00000 | 0.00000 | 0.00000 | 2.59122 | 0.59701 |
| 950 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.16014 |
| 1000 | 0.00000 | 0.00000 | 0.00000 | 0.85458 | 24.62722 |
| 1050 | 0.00000 | 0.00000 | 0.00000 | 10.01179 | 0.00000 |
| 1100 | 0.00000 | 0.00000 | 0.00000 | -0.55009 | 0.98250 |
| 1150 | 0.00000 | 0.00000 | 0.00000 | 1.72844 | 1.73212 |
| 1200 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -0.92762 |
| 1250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.69196 |
| 1300 | 13.69375 | 0.00000 | 0.00000 | 0.00000 | 1.48081 |
| 1350 | 2.01157 | 0.00000 | 0.00000 | 0.00000 | 2.75307 |
| 1400 | 0.00000 | 0.00000 | 0.00000 | -4.38477 | 0.00000 |
| 1450 | 0.00000 | 0.00000 | 0.00000 | 0.52170 | 0.89381 |
| 1500 | 2.68454 | 7.10133 | 0.00000 | 1.59310 | 1.52508 |
| 1550 | 0.00000 | 0.00000 | 0.00000 | 3.45763 | 1.56762 |
| 1600 | 0.00000 | 2.28643 | 0.00000 | 0.00000 | 2.72046 |
| 1650 | 5.02479 | 0.00000 | 0.00000 | 0.76593 | -1.59664 |
| 1700 | 0.00000 | 0.00000 | 0.00000 | 1.43856 | 0.60910 |
| 1750 | 2.01368 | 0.00000 | 0.00000 | 0.00000 | 2.15759 |
| 1800 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 1850 | 0.00000 | 4.40873 | 0.00000 | 2.10720 | 1.46919 |
| 1900 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 3.07016 |
| 1950 | 0.00000 | 2. 37175 | 0.00000 | 3. 13842 | 0.00000 |
| 2000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.70467 |
| 2050 | 0.00000 | 1.89666 | 0.00000 | 1.46196 | 0.00000 |

Table VI-6.
Correction Currents and Voltages of the 5 Levitation Solenoids. Equilibrium Currents are: $I(1)=512.2 \mathrm{~A}, \mathrm{I}(2)=161.8 \mathrm{~A}$, $I(3)=-417.9 \mathrm{~A}, I(4)=-417.9 \mathrm{~A}$, and $I(5)=161.8 \mathrm{~A}$. $(\alpha=1$ and $\beta=0.4)$

| Tine(m) | I(1) | 1(2) | I(3) | I( ${ }^{\text {) }}$ | I(5) | $V(1)$ | $V(2)$ | $V(3)$ | $V(4)$ | $V(5)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 25 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 37 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 75 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 87 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 100 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 125 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 137 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 150 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 175 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -90.62 | 2.88 | -53.84 | -53.62 | 7.86 |
| 187 | -0.24870 | 0.02163 | -0.14720 | -0.14714 | 0.02158 | -5.69 | 0.49 | -3.37 | -3.37 | 0.49 |
| 200 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 90.62 | -7.88 | 53.64 | 53.61 | -7.86 |
| 225 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -90.62 | 7.88 | -53.64 | -53.61 | 7.86 |
| 237 | -0.24869 | 0.92164 | -0.14722 | -0.14713 | 0.02157 | -5.69 | 0.50 | -3.37 | -3.37 | 0.49 |
| 250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 90.62 | -7.89 | 53.64 | 53.61 | -7.86 |
| 275 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1189.94 | -1228.22 | 237.70 | 457.29 | -851.06 |
| 287 | 3.26555 | -3.37054 | 0.65224 | 1.28247 | -2.33560 | 74.72 | -77.12 | 14.92 | 29.34 | -53.44 |
| 300 | 0.00001 | -0.00001 | 0.00000 | 0.00000 | -0.00001 | -1189.95 | 1228.17 | -237.61 | -467.34 | 851.10 |
| 325 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -139.18 | -42.80 | -167.47 | 168.10 | 506.65 |
| 337 | -0.38195 | -0.11742 | -0.45966 | 0.46138 | 1.39037 | -8.74 | -2.69 | -10.52 | 10.56. | 31.81 |
| 350 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 139.18 | 42.80 | 167.49 | -168.09 | -506.65 |
| 375 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -986.95 | 924.86 | -296.69 | -151.17 | 1163.09 |
| 387 | -2.70851 | 2.53803 | -0.81416 | -0.41491 | 3.19199 | -61.97 | 58.07 | -18.63 | -9.49 | 73.03 |
| 400 | -0.00001 | 0.00001 | 0.00000 | 0.00000 | 0.00001 | 986.95 | -924.86 | 296.58 | 151.21 | -1163.12 |
| 425 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 90.61 | 28.61 | 75.90 | 31.36 | -44.36 |
| 437 | 0.24866 | 0.07853 | 0.20829 | 0.08608 | -0.12178 | 5.69 | 1.80 | 4.77 | 1.97 | -2.79 |
| 450 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -90.61 | -28.82 | -75.89 | -31.37 | 44.37 |
| 475 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 192.83 | 138.91 | 209.09 | 19.22 | -172.44 |
| 487 | 0.52919 | 0.38123 | 0.57375 | 0.05278 | -0.47327 | 12.11 | 8.12 | 13.13 | 1.21 | -10.83 |
| 500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -192.84 | -138.93 | -209.06 | -19.25 | 172.46 |
| 600 | -0.00001 | 0.00001 | -0.00001 | 0.00001 | 0.00000 | 1200.39 | -2037.31 | 1506.33 | -892.19 | -316.94 |
| 625 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 98.06 | -469.70 | 1474.75 | -1391.74 | 364.04 |
| 637 | 0.26907 | -1.28888 | 4.04702 | -3.81923 | 0.99898 | 6.16 | -29.19 | 92.50 | -87.39 | 22.86 |
| 650 | 0.00000 | 0.00000 | 0.00001 | -0.00001 | 0.00000 | -98.02 | 469.62 | -1474.76 | 1391.77 | -364.04 |
| 675 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1450.39 | -1041.74 | 602.47 | 139.55 | -1803.96 |
| 687 | 3.98034 | -2.85895 | 1.65344 | 0.38288 | -4.95052 | 91.07 | -65.41 | 37.83 | 8.76 | -143.27 |
| 700 | 0.00001 | -0.00001 | 0.00000 | 0.00000 | -0.00001 | -1450.45 | 1041.88 | -602.61 | -139.43 | 1803.88 |
| 725 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 84.44 | -302.75 | -49.70 | -142.38 | -468.22 |
| 737 | 0.23174 | -0.83085 | -0.1144 | -0.39071 | -1.28493 | 5.30 | -19.01 | -2.62 | -8.94 | -29.40 |
| 750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -84.46 | 302.76 | 41.69 | 142.42 | 468.21 |
| 775 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 146.74 | -375.55 | -92.12 | 130.27 | -10.50 |
| 787 | 0.40272 | -1.03058 | -0.25287 | 0.35757 | -0.02886 | 9.21 | -23.58 | -5.79 | 8.18 | -0.66 |
| 800 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -146.75 | 375.52 | 92.17 | -130.32 | 10.53 |
| 825 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 85.23 | -126.13 | -26.09 | 140.10 | 146.46 |
| 837 | 0.23390 | -0.34612 | -0.07154 | 0.38450 | 0.40193 | 5.35 | -7.92 | -1.64 | 8.80 | 9.20 |
| 850 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -85.23 | 126.13 | 26.10 | -140.11 | -146.46 |
| 875 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 53.57 | 15.99 | 30.89 | 75.41 | 88.97 |
| 887 | 0.14703 | 0.04389 | 0.08478 | 0.20695 | 0.24417 | 3.36 | 1.00 | 1.94 | 4.74 | 5.59 |
| 900 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -53.58 | -15.99 | -30.90 | -75.41 | -88.98 |
| 925 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 148.46 | 173.69 | 215.84 | -86.28 | -322.68 |

## VII. APPENDIX

## Cryostat and Cryogenic System Design

Preliminary design of the levitation magnet cryostat and cryogen system is illustrated on drawings MM-B-84, Fig. VII-1, and MM-B-85 (Fig. VII-2), and the cryostat sketch, Fig. VII-3. Most of the key features of the cryostat are shown in Fig. VII-1.

* The five magnets are each wound on 12.7 mm G-11 composite mandrels with 12.7 mm end plates. The lower portion of each winding mandrel is extended to provide vertical support from the bottom of the liquid container.
* Top and bottom of each magnet mandrel tube fits into precision bored holes in 19 mm G-11 structural plates. These plates hold the magnets in fixed relation to each other regardless of the magnitude and direction of resultant loads.
* The top and bottom plates are attached to an external 6.35 mm thick cylinder which provides rotational stiffness to the magnet structural assembly. The combination of heavy top and bottom plates and outer cylinder makes a rigid fixture for the assembly which is independent of the cryostat structure.


Figure VII-1.

Figure VII-2.

Figure VII-3.

* Additional structural stiffness is provided by five large re-entry pipes which house the vertical support legs. These ten inch schedule 5 pipes are welded to the bottom stainless steel cold plate and extend up through match bored holes in the magnet structural plates to provide added lateral rigidity.
* The inner cold assembly consists of composite bore tubes for each magnet, a 19 mm G-11 composite top plate, 0.61 m diameter stainless steel inner cylinder, 2.18 m diameter outer cylinder and a 16 mm stainless steel bottom plate. There is no metal in the magnet bores or in the space above them. Magnets are located some 30 cm above the bottom to reduce eddy current heating.
* All access to the cryostat is from the bottom in order to preserve a flat upper contour. Access requirements include helium liquid supply and vents, 5 pairs of power leads and instrumentation connections. Vapor cooled leads will operate from an interconnected dewar alongside the cryostat.
* The vacuum jacket consists of a 19 mm G-11 top plate, stainless steel inner and outer cylinders and G-11 warm bore tubes for each magnet. Multilayer insulation is used throughout the cryostat. Except for the magnet bore areas, there are separate 2 cm thick layers on either side of the copper vapor-cooled shield. Vertical portions of the shield
are conduction cooled from the bottom and are excluded from the magnet bores and area above them.
* The cryostat is supported from 5 long G-11 tubes plus three detachable radial supports which are only needed for shipping. The individual legs are 25.4 cm ID with 0.51 mm walls and 76 cm long. Attachment of the legs to the warm bottom plate is by pins which reduce cooldown bending moments and height adjustment is by means of shims.
* Warm bores are provided for each magnet and for the center of the array for structural purposes. These cold and warm tubes reduce the spans of the top plates which permits them to be considerably thinner than they would be otherwise. Through members for the inner container provide tension support for the 30 psia ( 0.207 MPa ) internal design pressure and the warm through members serve as posts to support $14.7 \mathrm{psia}(0.101 \mathrm{MPa})$ external atmospheric pressure.
* Figure VII-2 indicates location of components in the cryostat without attempting to show structural details. Specifically shown are the inside and outside diameter of each magnet and related warm bore tube, cold and warm inner and outer cylinders and the location of the five vertical support legs. Dimensions are preliminary and are subject to change. In particular, the outer cold cylinder fits very close to the magnet outside diameters and its radius may
need to be increased from 1 to 2 cm . In this case, a corresponding increase in the vacuum jacket radius is also required.

The cryogenic system schematic is shown in Fig. VII-3. This sketch shows a single magnet and one of ten leads and the related control systems. Controls include automatic liquid level maintenance with a dump valve to prevent warm vapor from entering the cryostat, shield cooling with a flow controller and a flow controller for each lead. These flow controllers are necessary to properly proportion flow before the remainder is vented. During magnet operation, eddy current heating creates more helium vapor than the leads and shield can utilize and the excess must be vented to avoid over-pressurizing the cryostat.

Thermal design of the cryogenic system is preliminary. Idling helium consumption is about 4 liters/hr for the cryostat plus 4.25 for the leads at zero current for a total of 8.25 liters per hour. Helium consumption increases by about 250 liters per hour at full power to a total of about 260 liters per hour. (Neither figure includes dewar or transfer line losses which should run less than $10 \%$ of actual consumption.) Operational planning has not been given serious thought, but it appears that the system can be supported by commercial 500 or 1000 liter dewars if an occasional interruption can be tolerated.
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| 16. Abstract <br> A new concept in magnetic levitation and control is introduced for levitation above a plane. A set of five vertical solenoid magnets mounted flush below the plane supports and controls the model in five degrees of freedom. The compact system of levitation coils is contained in a space 2.4 m ( 96 in .) diameter by 1 m ( 40 in .) deep with the top of the levitation system 0.9 m ( 36 in .) below the center line of the suspended model. The levitated model has a permanent magnet core held in position by the five parallel superconductive solenoids symmetrically located in a circle. The control and positioning system continuously corrects for model position in five dimensions using computed current pulses superimposed on the levitation coil base currents. The conceptual designs include: superconductive and $\mathrm{Nd}-\mathrm{Fe}-\mathrm{B}$ permanent magnet model cores and levitation solenoids of either superconductive, cryoresistive, or room temperature windings. |  |  |  |
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