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Simultaneous Structural and Control Optimization via Linear Quadratic Regulator Eigenstructure Assignment

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This paper presents a method for simultaneous optimal structural and control design of large flexible space structures (LFSS) to reduce vibration generated by disturbances. Desired natural frequencies and damping ratios for the closed-loop system are achieved by using a combination of linear quadratic regulator (LQR) synthesis and numerical optimization techniques. The state and control weighting matrices (Q and R) are expressed in terms of structural parameters such as mass and stiffness. The design parameters are selected by numerical optimization so as to minimize the weight of the structure and to achieve the desired closed-loop eigenvalues. An illustrative example of the design of a two bar truss is presented.

INTRODUCTION

Large structural systems in general and large space structures in particular present new challenges to the structural dynamicist and the control engineer as well. Indeed, such large systems may exhibit well over a thousand vibrational modes usually closely spaced and with little, if any, damping. Some form of active control is likely to be necessary in order to meet exacting stability and pointing requirements. In fact, structural requirements (primarily low mass) increase the need for active control. Some optimal trade off between structural and control criteria has to be achieved.

Until recently, the design of control systems for large structural systems was a two-step procedures: first the structure was designed based on structural criteria (primarily total weight); then in a second step a control system (satisfying some desired control objectives) was designed for the structure obtained in the first step. Inasmuch as a low weight (and thus low stiffness) structure will require high control energy, the design objectives of the two steps are to some extent contradictory so that an optimal control design for an optimally designed structure will not in general result in an overall control-structure optimal design. Both designs need to be carried out simultaneously.

LITERATURE REVIEW

The optimal structural and control design of large flexible space structures was recently investigated by several researchers. Venkayya and Tischler [1-2] have suggested that the performance index (PI) in optimal control of structural systems be a measure of the system total mechanical energy. By appropriately choosing the state and control weighting matrices, the PI can be expressed as the (weighted) sum of the kinetic, strain and potential (including control) energies. Khot and Venkayya [3-4] tackled the structural and control optimization problem by minimizing the weight of the structure with constraints on structural frequencies and the minimum Frobenious norm of the gain matrix. This process has to be carried out in an iterative fashion.

Becus and Lui [5] have proposed a general method to choose state and control weighting matrices in optimal control design so as to satisfy desired closed-loop eigenvalues. This was further extended by Becus and Sonmez [6] to allow for eigenvector assignment. In this paper we combine both ideas in order to obtain a method to carry out simultaneous optimal structural and control design.

Desired dynamic structural requirements (natural frequencies and damping ratios for example) can be expressed both in terms of desired closed-loop eigenstructure (eigenvalues and/or eigenvectors) and structural parameters (mass and stiffness for example). Using a PI of the form suggested in [1], the elements of the state and control weighting matrices (Q and R respectively) are also expressed in terms of structural parameters. Thus, when choosing the Q and R matrices (using the method of [5-6]) to satisfy a desired closed-loop eigenstructure (i.e. dynamic structural requirements), one in fact chooses new strucutral parameters and therefore carries out a simultaneous optimal control structure design.

In this paper a new design algorithm is developed so that a minimum weight structure with desired damping and natural frequency of the closed-loop system can be obtained. We compare the results with [3] in the last section.

SIMULTANEOUS STRUCTURAL AND CONTROL OPTIMIZATION

Consider a controlled structural dynamic system described by the discrete (finite element) model

$$M\ddot{r} + Kr = Du \tag{1}$$

where r is a vector of n physical displacements and the number of control inputs (forces) u is m. M, K and D are the mass, stiffness and applied load distribution matrices of appropriate dimensions respectively. Assume that M and K are positive definite.

The state space representation of Eq. (1) can be written as

$$\dot{x} = Ax + Bu \tag{2}$$

where $$x = [\dot{r}^T \quad r^T]^T \quad , \tag{3}$$

$$A = \left|\begin{array}{c|c} 0 & -M^{-1}K \\ \hline I & 0 \end{array}\right| = A(p) \quad , \tag{4}$$

and

$$B = \left|\begin{array}{c} M^{-1}D \\ \hline 0 \end{array}\right| = B(p) \quad , \tag{5}$$

where p is a vector of structural parameters of dimension l. The optimal steady-state control is a linear state feedback

$$\bar{u} = -Gx \quad . \tag{6}$$

The state feedback gain matrix G is obtained from LQR synthesis and the closed-loop system is given by

$$\dot{x} = (A - BG)x \quad . \tag{7}$$

LQR synthesis determines a control $\bar{u}$ which minimizes the quadratic performance index [1]

$$PI = \int_0^\infty \left[\Theta_m \dot{r}^T M \dot{r} + \Theta_k r^T K r + \Theta_r u^T D^T K^{-1} D u\right] dt \tag{8}$$

or in the state space coordinates

$$PI = \int_0^\infty \left[x^T Q x + u^T R u\right] dt \tag{9}$$

where

$$Q = \left|\begin{array}{c|c} \Theta_m M & 0 \\ \hline 0 & \Theta_k K \end{array}\right| = Q(p) \tag{10}$$

and

$$R = \left[\Theta_r D^T K^{-1} D\right] = R(p) \tag{11}$$

for positive scaling parameters Θ_m, Θ_k and Θ_r. In Eq. (8), PI is the absolute weighted sum of the kinetic, strain and potential energies.

The relationship between characteristic polynomial of the optimal system and weighting matrices is obtained as follows [7]

$$\det(sI - Z) = \left|\begin{array}{c|c} sI - A & BR^{-1}B^T \\ \hline Q & sI + A^T \end{array}\right| \tag{12}$$

or

$$\phi_c(s)\phi_c(-s) = \phi_0(s)\phi_0(-s)\det\left[I + R^{-1}H^T(-s)QH(s)\right] \tag{13}$$

where H(s) is the open-loop transfer function matrix

$$H(s) \triangleq (sI - A)^{-1}B \quad , \tag{14}$$

Z is the canonical system matrix, $\phi_c(s)$ and $\phi_o(s)$ are the closed-loop and open-loop characteristic polynomials respectively.

For a given desired closed-loop pole $s = s_d$ which is not an open-loop pole, the determinant in the right-hand side of Eq. (13) must equal zero when the weighting matrices Q and R take values which yield the desired closed-loop eigenvalues. In order to use numerical optimization techniques to solve Eq. (13) for Q and R, we, as in Ref. [8], set the objective function as

$$obj = \det\left[I + R^{-1}H^T(-s_d)QH(s_d)\right] = 0 \quad . \tag{15}$$

The desired characteristic equation corresponding to Eq. (15) is

$$\prod_{j=1}^{n} = (s - s_{d_j})(s + s_{d_j}) = 0 \tag{16}$$

where s_{d_j} is the j-th desired closed-loop eigenvalue.

Q and R are determined by equating coefficients of the terms involving equal powers of s in Eqs. (15) and (16). This yields

$$\begin{aligned} f_1(p) &= 0 \\ &\vdots \\ f_k(p) &= 0 \end{aligned} \tag{17}$$

where k is the number of equality constraints which involve equal powers of s in Eqs. (15) and (16).

The objective in structural and control optimization is to make the selection of design parameters so that the structure weight is a minimum and the specified closed-loop eigenvalues are satisfied. The optimization problem can be stated as

$$\begin{aligned} &\text{Minimize the weight} \quad && W = W(p) \\ &\text{subject to} && \text{Eq. (17)} \\ &\text{and} && P_s \geq \bar{P}_s, \; s = 1, \cdots, l, \end{aligned} \tag{18}$$

where $\bar{P}_s$ denote minimum allowable values of the structural design parameters.

ILLUSTRATIVE EXAMPLE

In order to illustrate the feasibility of the above algorithm, the structural two bar truss model shown in Fig. 1 was considered as a simple design example. For the geometry shown, the dynamical equations of motion (Eq. (1)) are

$$\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \begin{vmatrix} \ddot{r}_1 \\ \ddot{r}_2 \end{vmatrix} + k_1 \begin{vmatrix} (A_1+A_2) & 2(A_1-A_2) \\ 2(A_1-A_2) & 4(A_1+A_2) \end{vmatrix} \begin{vmatrix} r_1 \\ r_2 \end{vmatrix} = \begin{vmatrix} \cos(\theta) \\ \sin(\theta) \end{vmatrix} u \qquad (19)$$

$$\text{thus} \quad M = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \qquad (20)$$

and

$$K = k_1 \begin{vmatrix} (A_1 + A_2) & 2(A_1 - A_2) \\ 2(A_1 - A_2) & 4(A_1 + A_2) \end{vmatrix} \qquad (21)$$

are the optimal mass and stiffness of the structure respectively. In Eq. (19), A_1 and A_2 are the cross-sectional areas of the bars and $k_1 = E/(5L)$ is a stiffness coefficient, E representing the elastic modulus of the bars and L the length of the members. A control force u is located at the vertex with θ being the angle between its line of action and the horizontal. r_1 and r_2 are the horizontal and vertical displacements of the vertex respectively.

The dimensions of the structure were given in unspecified consistent units. The elastic modulus of the members was assumed to be 1 and the density ρ of the structural material was assumed to be 0.001. A nonstructural mass of 2 units was attached at node 2 and the structural mass of the members was ignored for simplicity (thus the mass matrix of Eq. (20)). The actuator and sensor were located in element 1 connecting node 1 and 2. The minimum cross-sectional area was set equal to 10 units for both members.

Once the choice of the material is fixed, the design variables are the cross-sectional areas of the members A_1 and A_2, the scaling parameters Θ_m, Θ_k and Θ_r, and the angle θ of the applied load with respect to the horizontal. The optimal closed-loop eigenvalues are specified as $s_{d_1} = -0.0228 \pm 1.17j$ and $s_{d_2} = -0.361 \pm 4.81j$. Arbitrary lower and upper values of θ were set at 30° and 60° respectively.

Analytical and numerical computations were carried out using MACSYMA™ [9] for symbolic algebraic manipulations, MATLAB [10] for matrix computations and LQR synthesis, and GRG2 [11] for numerical optimization. The numerical results for several representative optimal designs are listed in Table 1. A discussion of these results appears in the next section.

Table 1. Optimal Two Bar Truss Designs

DESIGN	A_1	A_2	WEIGHT	Θ_m	Θ_k	Θ_r	θ deg	ACHIEVED CLOSED-LOOP EIGENVALUES
1	101.98	889.88	22.18	1	1.12	2.78	60	$-0.5074 \pm 1.25j$ $-0.382 \pm 4.82j$
2	788.86	78.81	19.40	1	1.26	53.80	55	$-0.0702 \pm 1.17j$ $-0.361 \pm 4.81j$
3	531.74	53.02	13.08	1	1.87	73.68	60	$-0.0393 \pm 1.17j$ $-0.361 \pm 4.81j$
4	442.86	44.16	10.89	1	2.25	73.68	60	
5	311.18	31.03	7.65	1	3.20	73.68	60	
6	115.36	11.50	2.84	1	8.64	73.68	60	

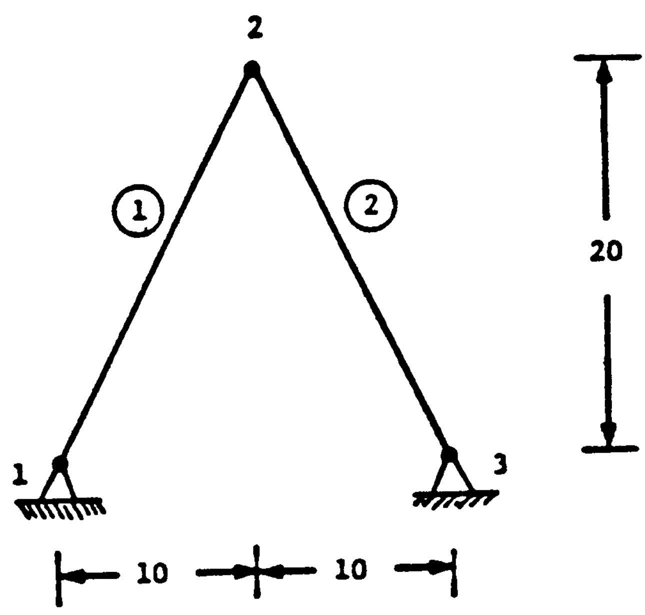

Fig. 1 Two Bar Truss

DISCUSSION OF RESULTS

For this simple example the six design variables were not independent. The scaling parameters Θ_m and Θ_k appeared in the constraint equations only as the combination $k_1\Theta_m/\Theta_k$. This combination was then used as one of five independent design variables. To obtain the values of Table 1, Θ_m was arbitrarily set equal to 1 then Θ_k was evaluated by multiplying the value obtained by numerical optimization by $k_1 = 0.0089$.

Since there are five independent design variables and only four equality constraints, there are many solutions to the optimization problem. In order to obtain a unique solution one could arbitrarily fix the value of one of the five independent design variables or equivalently introduce an additional constraint.

Of all designs presented in Table 1, Design 6 is the best since it leads to the lowest value for the weight. This "optimal" design leads to a weight of 2.84 which is less than half of the best design of Ref. [3] (6.417).

A closer examination of Table 1 leads to some interesting observations. Designs 3 through 6 have weights which are inversely proportional to Θ_k. In fact the product $\Theta_k \times$Weight is nearly constant for these four designs and equal to 24.45. In addition it can be seen that for these four designs the ratio A_1/A_2 is nearly constant and equal to 10. It is conjectured that many other designs could be obtained by choosing areas satisfying this relationship and calculating the corresponding Θ_k while keeping the other design variables constant.

Design 1 is representative of several designs for which the ratio A_1/A_2 is nearly constant and equal to 0.1 while Design 2 leads to an angle less than the upper bound value of 60°. For all designs obtained the product $\Theta_k \times$Weight was nearly constant and equal to 24.45.

Finally it must be noted that as more weight is given to the control effort the achieved closed-loop eigenvalues are closer to the desired eigenvalues. As more weight is being given to the strain energy cost the total weight decreases.

CONCLUSION

An algorithm for simultaneous structural and control optimization design of a minimum weight structure with desired closed-loop eigenvalues was proposed. It has been shown that structural and control designs can be obtained by LQR assignment. The design parameters were appropriately selected by numerical optimization so as to minimize the weight of the structure and to achieve desired natural frequencies and damping ratios. The feasibility of the algorithm was demonstrated by applying it to a simple example. Further work is needed to investigate the application of the algorithm to large-order systems.

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