On the 'Glitches' in the Force Transmitted by an Electrodynamic Exciter to a Structure

Dantam K. Rao

Around resonance, the force transmitted by an exciter into a structure will be smaller or greater than a reference force generated by its coils due to electromechanical interaction. A simple analysis presented herein reveals how this phenomenon of force drop-off is controlled by three factors. The first factor, called Armature Mass Factor, describes a purely mechanical interaction between the structure and exciter. It signifies the value of armature-of-structure mass ratio relative to the modal loss factor. The electromechanical energy conversion and its interaction with the structure yields two additional factors, called Electrical Resistance and Electrical Inductance Factors. They describe the effects of coil resistance, inductance and magnetic field strength relative to structural damping and stiffness. Present analysis indicates that, under proper circumstances, more than 90% of the force drop-off can be eliminated if armature-to-structure mass ratio is smaller or equal to half of modal loss factor.

INTRODUCTION

Traditionally, in a typical measurement set-up, the force needed to vibrate a grounded structure is generated by an attached electrodynamic exciter. We usually assume that, in such set-up, a constant force is transmitted into the structure if a uniform sine voltage is inputted into the exiter via a power amplifier as shown in Fig. 1. But in reality, the amplitude and phase of transmitted force is substantially different from the force generated in the coils (around the resonance frequency) due to electromechanical interaction between exciter and structure, even if input voltage is constant. A _Force Glitch_ describes these local differences in the force transmitted into a structure around its resonance frequency. (In contrast, a Motion Glitch describes local variations in the table base-motion excitation of a free structure. We do not intend to study them here). These glitches can be smoothened by a compressor loop, but we assume that our measurement setup does not have such a loop.

As shown in Fig. 2, a glitch consists of a _Peak_ and a _Notch_ in the plot of transmitted force vs frequency around the structural resonance. At the Notch frequency, this force drops to the lowest level, while at the Peak it rises to its highest value. The Notch frequency equals the resonance of the entire vibrating system.

PRECEDING PAGE BLANK NOT FILMED

Our major interest lies in analyzing the factors causing <u>Force Drop-off</u> viz. drop in level of force transmitted (from that generated in the coils) to the Notch value. We review below some (but not all) literature dealing with the force drop-off.

Historically, many "mechanical" models have been used to explain the force drop-off. They account only for the mechanical parts; they also presume that armature coil generates a constant-amplitude force. Ewins [1] used a 1-degree model to explain how the transmitted force becomes small at the structural resonance frequency. Earlier, Granick and Stern [2] analyzed a 2-degree model to show that the Notch frequency equals the structural resonance, while Bangs [3] analyzed the effect of structural nonlinearity. Rao [4] described a 3-degree model to include a force transducer.

A few researchers have also employed an "electromechanical" model. This model accounts for all vibrating parts, including electrical and electromechanical conversions; they presume that the armature coil generates a force proportional to current flow. An earlier review by Rao [5] recorded some pertinent literature on equations for exciters; these equations are identical to Crandall et al [6]. Extensive work by Tomlinson [7,8] showed that the transmitted force can be distorted if the table vibrations are so large that nonlinear solenoid effects come into play.

Recently Olsen [9] established that a "smaller" armature-to-structural mass ratio, viz., lighter armature, is required to reduce the force drop-off. (Research prior to 60's showed [10] that the motion glitch can be smoothened by selecting a heavier armature, i.e., a larger armature-to-structural mass ratio.)

Thus we know that a "smaller" armature-to-structural mass ratio reduces force drop-off. But, a question of practical interest to the experimenter is, how "small" should this ratio be? Should it be 1/100 or 1-in-million? This paper attempts to quantify this ratio. Another major aim of this paper is to identify and investigate the effect of any "electromechanical" factors that reduce the force drop-off (in addition to mechanical factors).
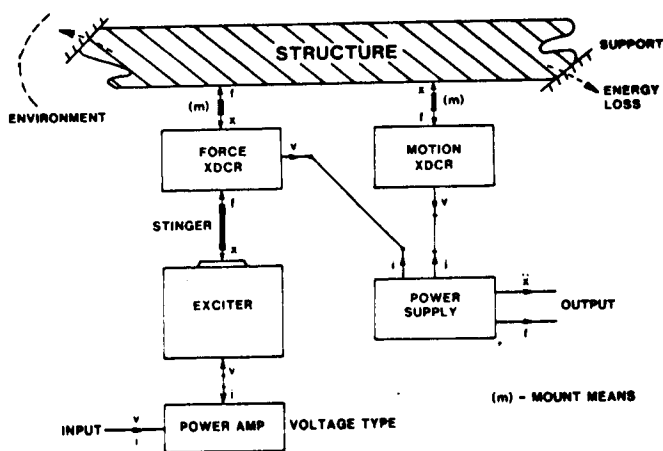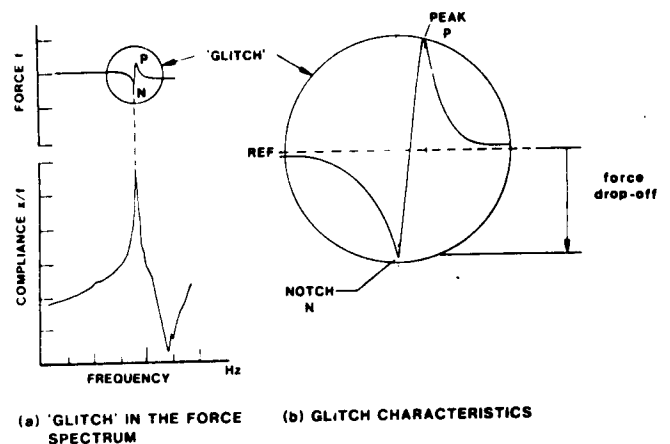
Fig. 1 Typical Setup for Measuring
Frequency Response

Fig. 2 Force Glitch Consisting of
a Peak and a Notch.

NOTATION

e = input sine voltage
i = current flowing in the coils
f_c = ampl. of force   ''   ''
f_d = force drop-off
f_o = ampl. of force transmitted
j = $\sqrt{-1}$
k_ℓ = force-to-current ratio
k^* = the complex modal stiffness
   = $k(1 + j\eta)$
k = real part of modal stiffness
L = self-inductance of the coil
m = the modal mass of structure +
   seismic part of force transducer
m_a = the eff. mass of armature +
   part of force xdcr + stinger
R = resistance of coil + source
x = displacement of str. + armature
η = the structural modal loss factor
ω = frequency of excitation
ω_N = natural frequency of structure $\sqrt{k/m}$
d_t = time derivative $d(\ )/dt$
sub o = amplitude (real or complex)

Factors Controlling the Force Drop-off

M = Armature Mass Factor   (eq. 4)
C = Electrical Resistance Factor   (eq. 9)
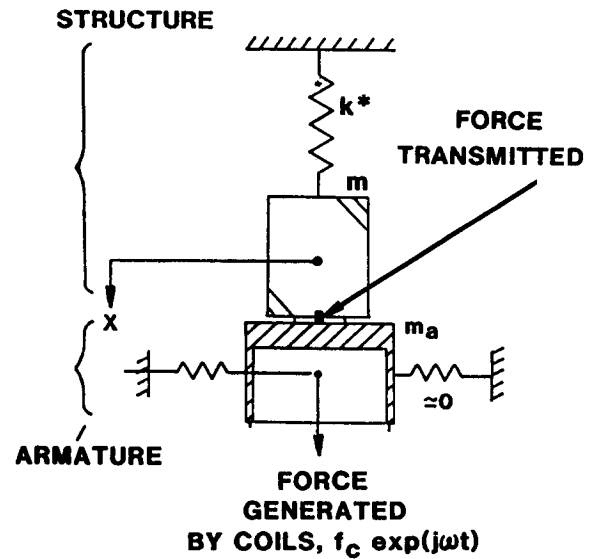K = Electrical Inductance Factor   (eq.10)

STRUCTURE

Fig. 3 "Mechanical" Model

"MECHANICAL" MODEL OF STRUCTURE ATTACHED TO AN EXCITER

Formula for Force Drop-off as a Function of Armature Mass

The equation of motion of a grounded structure attached rigidly to the armature of an exciter is (see Fig. 3 and Ref. [5] for assumptions)

$$(m + m_a)\ddot{x} + k^*x = f_c \exp(j\omega t) \qquad (1)$$

We rewrite this equation in the standard form $m\ddot{x} + k^*x = f_o \exp(j\omega t)$ where f_o, denoting the complex amplitude of force transmitted into the structure, is given by the difference [1] between the force generated in the coils and the inertial force needed to vibrate the armature,

$$f_o(\omega) = f_c + \omega^2 m_a x_o$$

$$= \frac{k^* - \omega^2 m}{k^* - \omega^2(m+m_a)} f_c \qquad (2)$$

247

where the complex amplitude of displacement x_o is obtained by solving (1). As shown in Fig. 2, we define the "force drop-off" f_d as the difference between the amplitude of reference force generated in the coils at zero-frequency, f_c , and the amplitude of the force transmitted into the structure at the natural frequency ω_N $= \sqrt{K/m}$. We use (2) to express the force drop-off in terms of a nondimensional factor M as given below:

$$f_d = f_o(0) - f_o(\omega_N)$$

$$= [1 - \frac{1}{\sqrt{\{1 + M^2\}}}] f_c \qquad (3)$$

where the <u>Armature Mass Factor</u> M controlling force drop off is

$$M = \frac{m_a/m}{\eta} = \frac{\text{armature-to-structure mass ratio}}{\text{str. loss factor}} \qquad (4)$$

Effect of Armature Mass on Amplitude and Phase of Transmitted Force

We display in Fig. 4 how armature mass influences transmitted force. This figure confirms the well known fact that a lighter armature beneficially reduces the force drop-off; but this also detrimentally reduces the frequency range between the Peak and Notch.

More significant is the additional phenomenon of <u>phase-drop</u> revealed by this Figure. The phase of the force signal (relative to that of force in the coils) drops to its lowest value at the Notch frequency and rises beyond it. This results in considerable fluctuations in the phase around the resonance frequency.
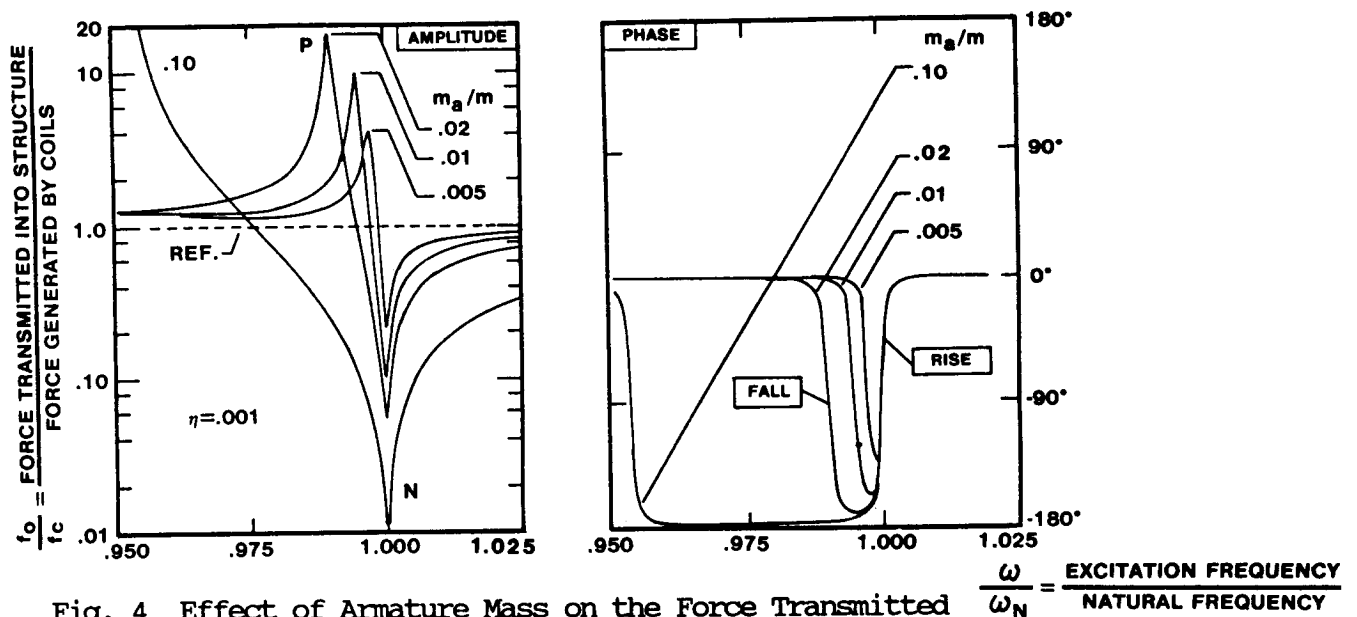

Fig. 4 Effect of Armature Mass on the Force Transmitted $\frac{\omega}{\omega_N} = \frac{\text{EXCITATION FREQUENCY}}{\text{NATURAL FREQUENCY}}$

For lighter armatures, this figure shows that the phase can fluctuate by as much as two full out-of-phase 180 deg. turns over a very narrow frequency range. The rate of rise in the Phase beyond Notch frequency appears, however, to be independent of armature mass. Hence although a lighter armature reduces the force drop-off and phase drop, we still need to use adequate frequency resolution to follow the sharp rise in the phase beyond the Notch frequency.

We display in Fig. 5 how the loss factor affects the force transmitted. This figure shows that heavier damping reduces the force drop off and widens the frequency range between the Peak and Notch. It also has the beneficial effect of reducing the phase drop; further the phase changes at a slower pace around the resonance.

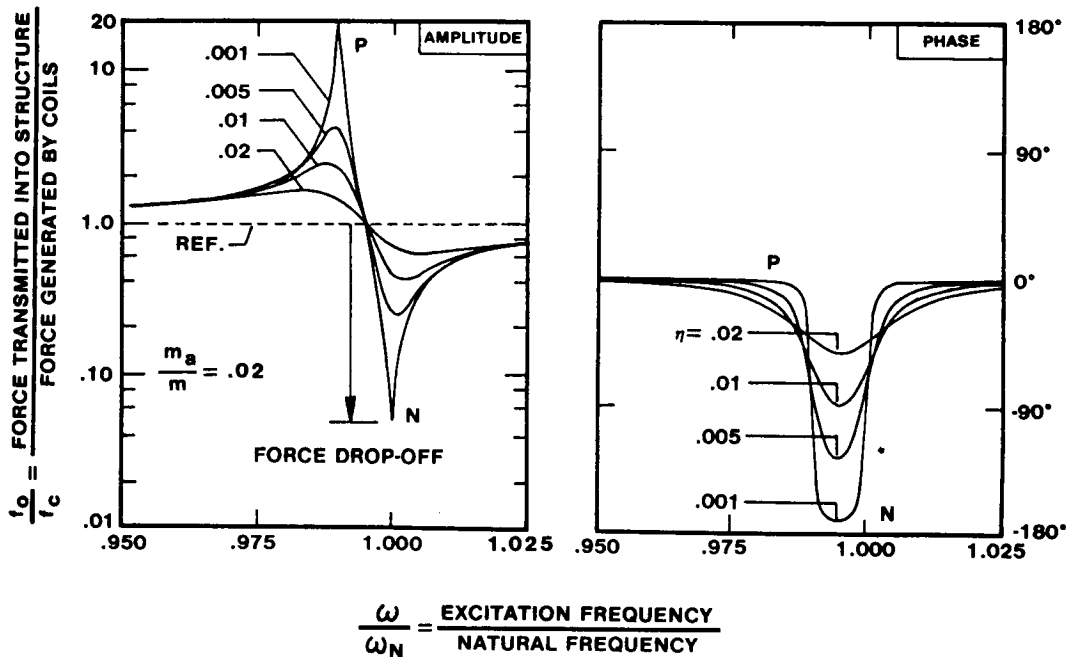$$\frac{\omega}{\omega_N} = \frac{\text{EXCITATION FREQUENCY}}{\text{NATURAL FREQUENCY}}$$

Fig. 5 Effect of Structural Loss Factor on Force Transmitted

Fig. 6 shows how the force drop-off is controlled by the Armature Mass Factor. From this figure, we conclude that 90% of coil-generated force can be transmitted into structure by choosing an armature-to-structure mass ratio that is less than half of the structural loss factor. This leads to a thumb rule, herein called the Half-Loss Factor Rule. Briefly stated, it recommends use of a light armature whose weight obeys the rule:

armature-to-structure mass ratio < half-of-loss factor   (5)

Then it is possible to transmit 90% of generated force into the structure at the frequency of resonance. For example, a structure with a modal mass = 10 kg and modal loss factor = 1/50 will require an armatur weighing 0.1 kg for the force drop-off to be 10%.

249

Formula for Force Drop-off Including Electromechanical Factors

Exciters work the principle of electromechanical conversion, an idealized version of which is shown in Fig. 7 as an conversion box. Ideal lossless electrical power inputted into it outputs mechanical force on a mass-less, frictionless push-rod. Fig. 8 shows how, in practical situations, the ideal electrical input is modified by the electrical resistance R and self-inductance L of the coil and the

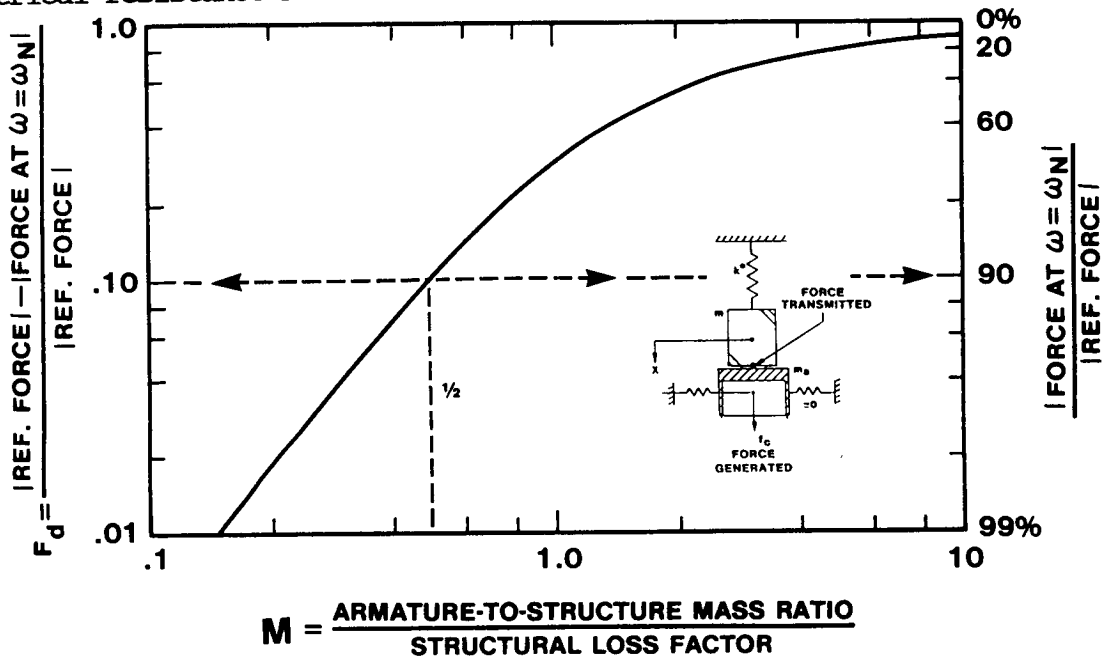$$M = \frac{\text{ARMATURE-TO-STRUCTURE MASS RATIO}}{\text{STRUCTURAL LOSS FACTOR}}$$

Fig. 6 Combined effect of Armature Mass and Str. Loss Factor on Force Drop-Off.

mechanical output by the mass of armature and structural properties. The equation of motion (1) thus modifies to (see [5] for details assumptions and derivation)

$$
\begin{array}{rcl}
(m + m_a)\ddot{x} + k^*x - k_B i &=& 0 \\
k_B x + (R+Ld_t)i &=& e_0 \exp(j\omega t)
\end{array}
\tag{6}
$$

We rewrite first of this equation in the standard form $m\ddot{x} + k^*x = f_0 \exp(j\omega t)$ where f_0 , denoting the complex amplitude of force transmitted into the structure, is given by the difference between the force generated in the coils (that is now proportional to the current) and the inertial force needed to vibrate the armature,

$$
\begin{aligned}
f_0(\omega) &= k_B i_0 + \omega^2 m_a x_0 \\
&= \frac{(k^* - \omega^2 m)R}{[k^* - \omega^2(m+m_a)][R+j\omega L] + j\omega k_B^2} f_c
\end{aligned}
\tag{7}
$$

250

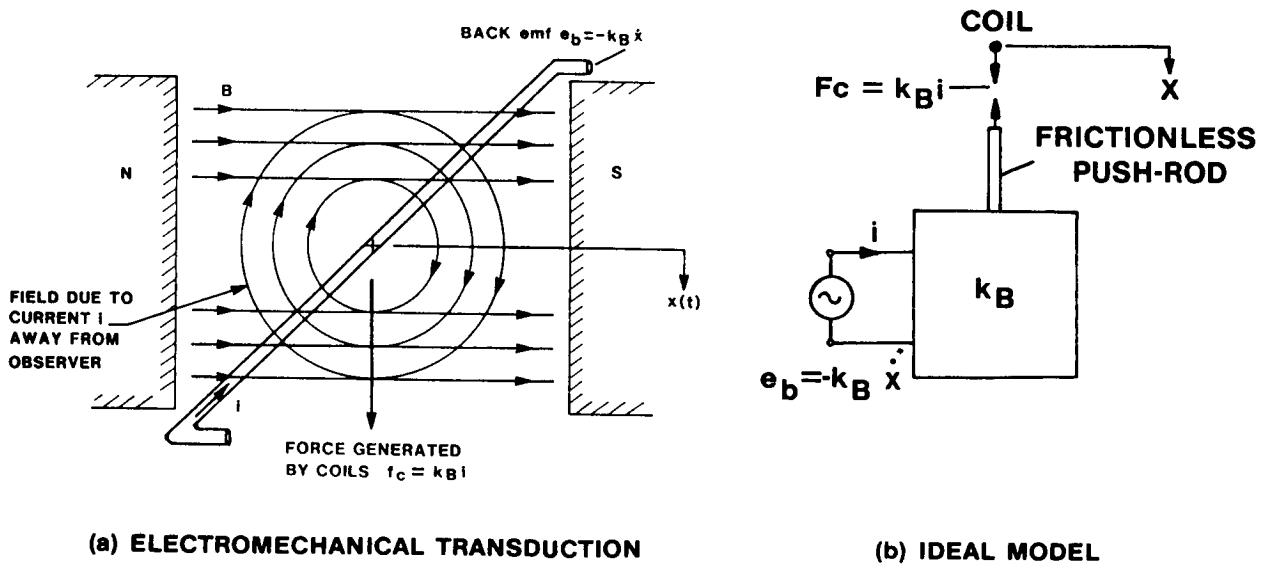(a) ELECTROMECHANICAL TRANSDUCTION   (b) IDEAL MODEL
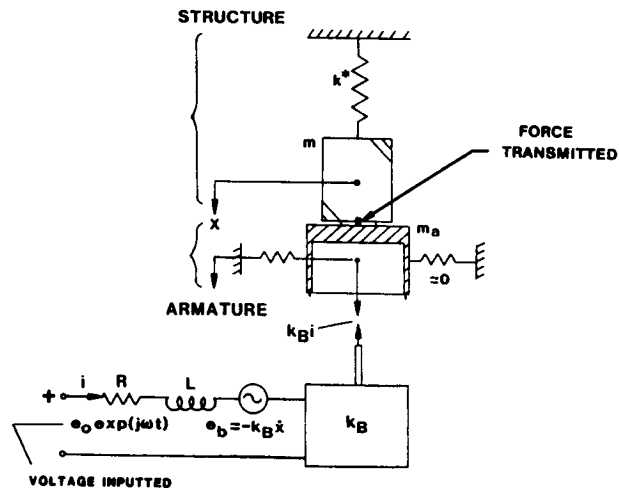
Fig. 7 Ideal Electromechanical Transducer

Fig. 8 Electromechanical Model of a Structure + Exciter

where x_o and i_o denote amplitudes of displacement and current that are obtained by solving (6) and f_c now denotes the force $k_B e_o/R$ transmitted into the structure at the zero frequency. The force drop-off now depends on two factors since

$$f_d = f_o(0) - f_o(\omega_N)$$

$$= [1 - \frac{1}{\sqrt{\{ (1-MC/K+C)^2 + (M+C/K)^2 \}}}] f_c \qquad (8)$$

251

Here C, called the <u>Electrical Resistance Factor</u>, quantifies nondimensionalized electrical damping whereas K, called the <u>Electrical Inductance Factor</u>, quantifies nondimensionalized electrical stiffness, both expressed relative to structural loss factor, and are defined by

$$C = \frac{(k_B{}^2/R)/\sqrt{km}}{\eta} = \frac{\text{elec. damping-to-str. crit. damping}}{\text{str. loss factor}} \quad (9)$$

$$K = \frac{(k_B{}^2/L)/k}{\eta} = \frac{\text{elec. stiffness-to-str. stiffness}}{\text{str. loss factor}} \quad (10)$$

Effect of Electromechanical Factors

Fig. 9 shows how the Electrical Resistance Factor C affects the force transmitted. It reveals that <u>lower</u> resistance can reduce the transmitted; it can also introduce unacceptable violent fluctuations in the phase. For example, for the parameters illustrated, the phase shows a <u>drop-rise-drop-rise</u> pattern over -180° to +180° between Peak and Notch. This is in contrast to the <u>drop-rise</u> pattern exhibited by the mechanical model as shown by Figs. 4 and 5.

Fig. 10 exhibits how electrical inductance factor K influences the force drop-off. This figure shows that a <u>larger</u> inductance can reduce the force transmitted and introduce unacceptable drop-rise-drop-rise fluctuations in the phase. These two figures re-emphasize the need for adequate frequency resolution to measure the phase of the force signal.

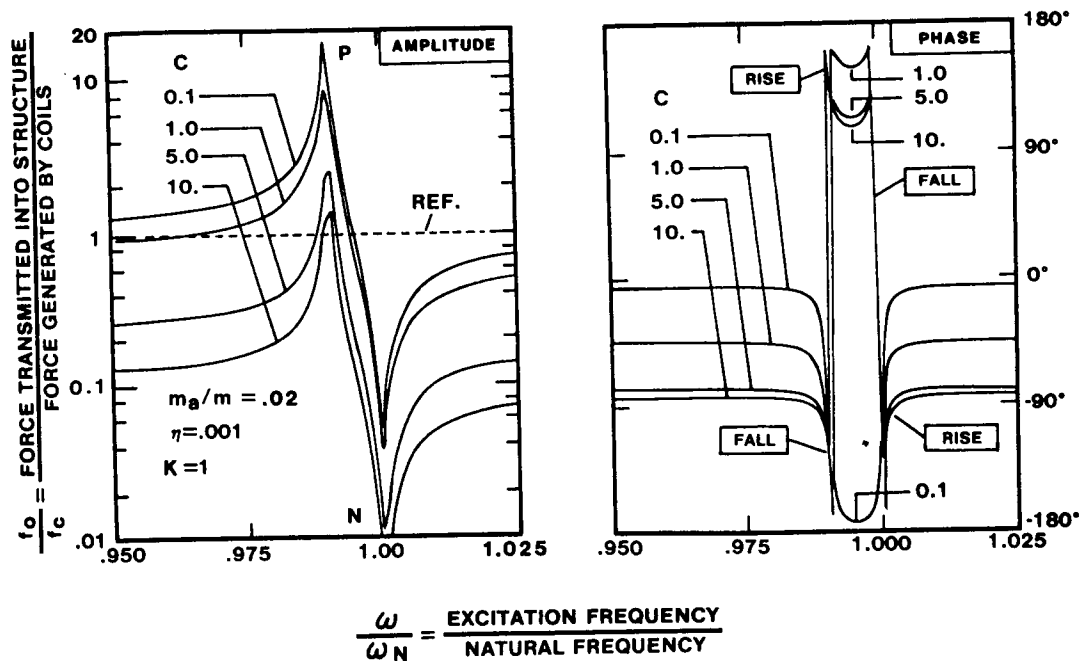$$\frac{\omega}{\omega_N} = \frac{\text{EXCITATION FREQUENCY}}{\text{NATURAL FREQUENCY}}$$

Fig. 9 Effect of Electrical Resistance Factor C on Force Transmitted
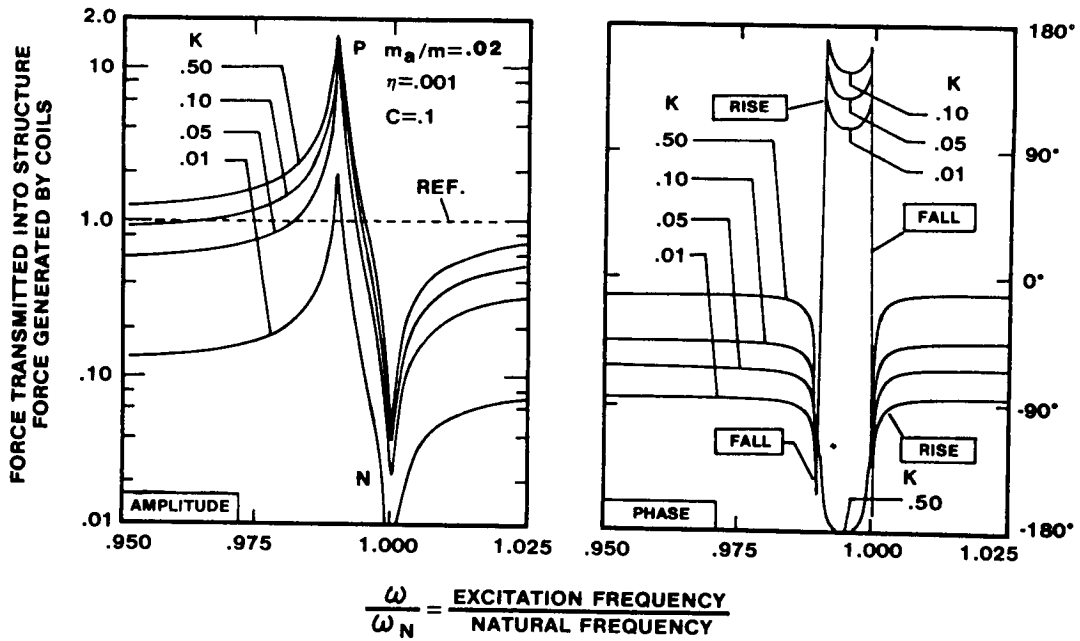
Fig. 10   Effect of Electrical Inductance Factor K on Force Transmitted.

The effect of Resistance Factor C on the force drop-off is revealed in Fig 11. This figure shows how a reduction in C value (i.e., increase in resistance) can eliminate the force drop-off.   Similar effect can be obtained by increasing the K value (i.e., reducing the inductance) as shown in Fig. 12.

Thus, by a judicious choice of M,C and K values, we can control the force drop-off observed at the resonance frequency of the structure.
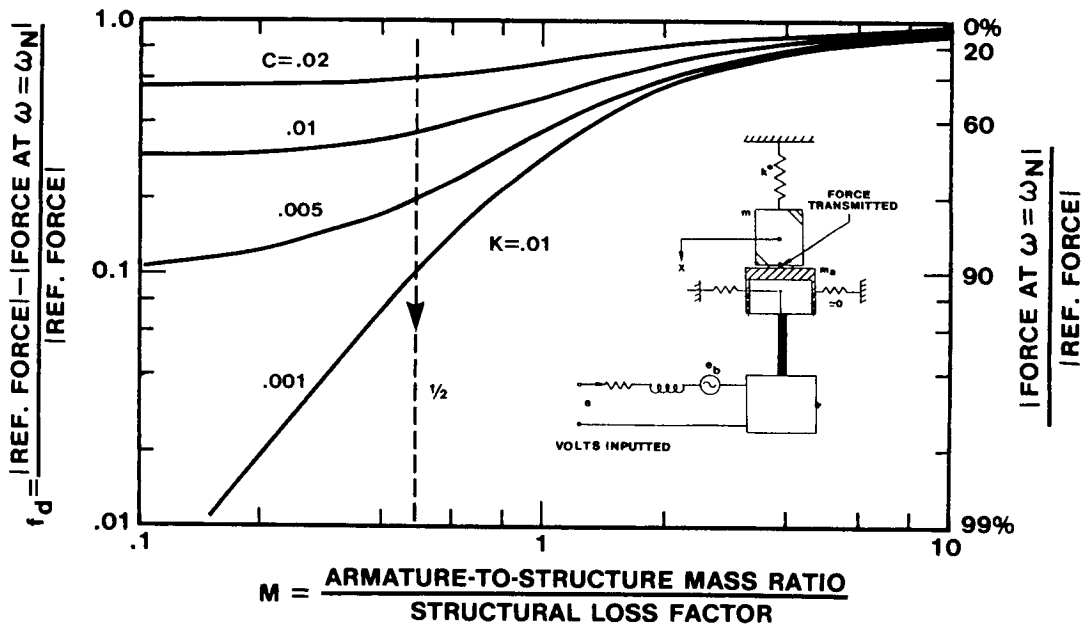

Fig. 11   Effect of Electrical Resistance Factor on Force Drop-off
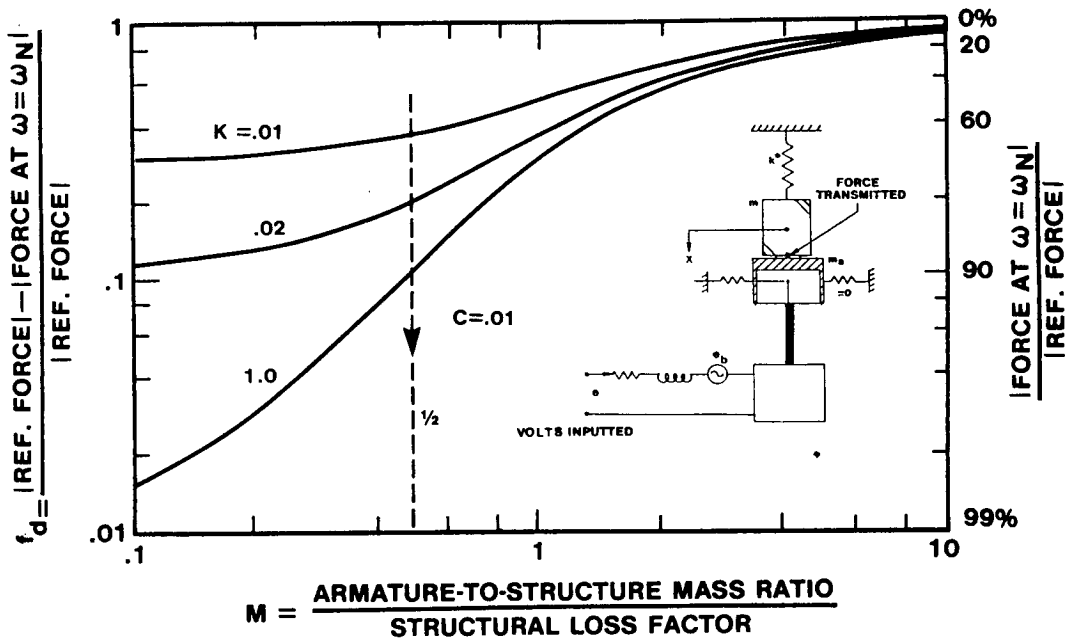
253

Fig. 12 Effect of Electrical Inductance Factor on Force Drop-off

CONCLUSIONS

The present paper identified three factors that affect the force transmitted by an electrodynamic exciter into a structure around the resonance frequency. This force transmitted is shown to depend on three factors. A purely mechanical factor, called Armature Mass Factor, describes the armature mass-to-structural mass ratio relative to the structural loss factor; it should be less than 1/2 to transmit more than 90% of force generated in the coils. The remaining two factors, called Electrical Resistance Factor C and Electrical Inductance Factor K describe the effect of coil resistance, inductance and magnetic field strength relative to structural damping and stiffness. Present analysis also revealed the phenomenon of phase-drop (in addition to the well-known phenomenon of force drop-off) that occurs around the resonance frequency. It also shows that the Electrical Resistance Factor should be decreased while Inductance Factor should be increased in order to reduce the force drop-off.

REFERENCES

1. D.J. Ewins, Modal Testing Theory and Practice. John Wiley Sons, 1984.

2. N. Granick and J.E. Stern, "Material Damping of Aluminum by a Resonant Dwell Technique," NASA TN D-2893, 1965.

3. W.F. Bangs, "Sinusoidal Vibration Testing of Nonlinear Spacecraft Structures," Shock, Vibration and Associated Environments, Bulletin No. 33, Part III, Mar. 1964, pp. 195-201.

4. D.K. Rao and D.I.G. Jones, "Investigation of Modes, Frequencies and Forced Response of a High Frequency Test System," Shock and Vibration Bulletin, No. 55, Part 2, pp. 27-38, June 1985.

5. D.K. Rao, "Electrodynamic Interaction Between a Resonating Structure and an Exciter," Proc. 5th Int. Conf. Modal Analysis, Apr. 1987, London.

6. S.H. Crandall, <u>Dynamics of Mechanical and Electromechanical Systems</u>, McGraw-Hill Book Co., 1968, p. 308.

7. G.R. Tomlinson, "A Simple Theoretical and Experimental Study for the Force Characteristics from Electrodynamic Exciters on Linear and Nonlinear Systems", Proc. 5th Int. Modal Anal. Conf., London, Vol. 2, pp. 1479-1486, Apr. 1987.

8. G.R. Tomlinson, "Force Distortion in Resonance Testing of Structures with electro-dynamic vibration Exciters", J. Sound and Vib., Vol. 63, No. 3, pp. 337-350, 1979.

9. N.L. Olsen, "Using and Understanding Electrodynamic Shakers in Modal Applications," Proc. 4th Int. Mod. Anal. Conf., Los Angeles, Vol. 2, pp. 1160-1167, Jan. 1986.

10. K. Unholtz, "Vibration Testing Machines", in Shock and Vibration Handbook, ed. by C.M. Harris and C.E. Crede, 1st ed., Vol. 2, pp. 25.1-25.74, McGraw-Hill Book Co., New York, 1961.