Structural Frequency Functions for an Impulsive, Distributed Forcing Function

Vesta I. Bateman

The response of a penetrator structure to a spatially distributed mechanical impulse with a magnitude approaching field test force levels (1-2 Mlb) was measured. The frequency response function calculated from the response to this unique forcing function is compared to frequency response functions calculated from response to point forces of about two thousand pounds and a hundred thousand pounds. The results show that the strain gages installed on the penetrator case respond similarly to a point, axial force and to a spatially distributed, axial force. This result suggests that the distributed axial force generated in a penetration event may be reconstructed as a point axial force when the penetrator behaves in a linear manner.

INTRODUCTION

Structural system response measured for the calculation of frequency response functions is typically stimulated by a low-level force (100's of pounds) applied at a point by an instrumented hammer. Often, in their intended use environment, the systems encounter much higher service loads which are distributed over the structure, and linearity of their responses under these conditions must be assumed. This work describes the response of a structure to three different axial forcing functions: a low-level (about two thousand pounds), point force generated by an instrumented hammer; a high-level (about 100 klb), point force generated with a Reverse Hopkinson Bar technique; and a high-level (1-2 Mlb), distributed force generated with an explosive. The structure is an earth penetrator case whose design is typical of those at Sandia National Laboratories and is shown in Figure 1. The case is a hollow structure and does not include the internal components normally present for a field test. The case material is steel (Type 4340) which has a yield strength in excess of 170 ksi; the case was not noticeably deformed by any of the tests described in this paper. These tests were undertaken to characterize the case structural response to axial loads and to assess the effect of spatial distribution of the axial load over the ogival nose. There were three goals initially defined for this series of tests.

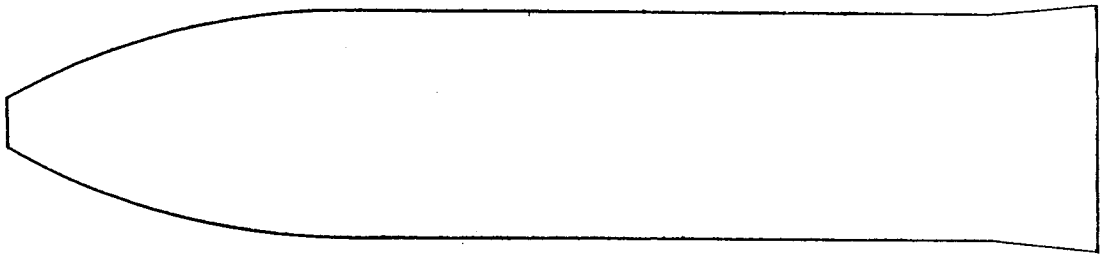

Figure 1: Penetrator Case Used for Structural Response Measurements with Three Axial Forcing Functions.

The first goal was to obtain good structural response measurements for the penetrator case by selection of optimum strain gage locations and by proper strain gage installation. Four axial locations, two on the interior and two on the exterior, were desired for the test series; each location had four gages spaced equidistantly around the circumference. Optimum locations were determined from the displacement mode shapes for one axial and three lateral modes which were identified in the 4096 Hz bandwidth for a modal analysis of the case performed by the Modal Testing Group at Sandia National Laboratories. The technique for inferring the strain mode shapes from modal data has recently been developed at Sandia [1] and will not be presented here. The locations chosen have good structural response for the axial and lateral modes below 4096 Hz and are: 7 in and 14 in from the case rear on the interior and 14 in and 24 in from the rear on the exterior. The gages installed on the case exterior required no special installation technique. The gages were installed on the interior with a fixture [2] which has been designed to insure consistent, accurate installation of four gages at an interior case location. The interior installation technique is also used to instrument penetrators for field test.

A second goal of these tests was to assess the structural response of the penetrator case to axial forcing functions spatially distributed on the penetrator ogival nose. The distributed forcing function was implemented with Deta Sheet explosive shaped into three different spatial distributions. The Deta Sheet configurations are described in "Explosive Loading Tests of Penetrator Unit" by Mr. John L. Cawlfield at this conference. Since the explosive forcing function time history can not be measured directly, the duration was limited to 20 μs so that the frequency content would be independent of the forcing function shape for frequencies 0-10 kHz. Consequently, only the force spatial distribution varied for the frequency range of interest, and the measured responses to the different force distributions could be compared over a frequency range of approximately 10 kHz. The multiple axial locations for the strain gages allowed an observation of how the structural response changes with axial location for a particular distributed force input.

The calculation of the structural frequency response functions for the penetrator case response at different locations was the third goal. Since the symmetric loading of the nose with the Deta Sheet configurations yields a net axial force (and resultant impulse), the structural frequency response functions for the distributed forcing function may be compared to the axial frequency response functions for the point axial forces at both high and low levels. The ultimate goal of this work is to reconstruct the force environment with a frequency response function and the structural response measured in a penetration event. The reconstructed force may be used to verify the computer models that are used for penetration design.

The method used for the structural frequency response function (FRF) calculation is the ratio of the response spectrum divided by the forcing function spectrum. A spectrum in this paper is the Fourier transform of a time history with a rectangular window. It is recognized that this results in leakage errors. The alternative was to use an exponential window which would reduce the leakage but increase the apparent damping of the structure. For this paper, the rectangular window was considered the better compromise.

The individual spectra for the force input and the measured response were combined to form the magnitude of the frequency response function which is defined as

$$H(j\omega) = \frac{O(j\omega)}{I(j\omega)}$$

where

$H(j\omega)$ = the structural frequency response function (FRF)
$O(j\omega)$ = the Fourier transform of the measured structural response
$I(j\omega)$ = the Fourier transform of the axial force input.

The division in the above equation is accomplished by a point-by-point complex division of the two spectra. This formulation represents a single input-single output linear system with no noise [3] and was chosen to facilitate the comparison of the transient responses to the three different transient forcing functions. Structural frequency response functions are often calculated with cross-spectrum and auto-spectrum functions from the average of many response measurements. Since multiple measurements with statistically meaningful characteristics were not possible for the high-level, point force and the distributive force, this simplified method for the FRF calculation was chosen.

All the FRFs in this paper are axial FRFs which means that four gage responses at the same distance from the rear of the penetrator case were combined to eliminate the lateral response. The lateral response for all the forcing functions was minimal and could not be distiguished from the noise level after it was separated from the axial response. For the purposes of force reconstruction, the FRF is not calculated beyond those frequencies for which the input forcing function spectrum has decreased by half its low frequency amplitude. Since the spectrum for the forcing function appears in the denominator of FRF calculation, the decreasing amplitude of the force spectrum will artificially amplify the higher frequencies of the structural response in the FRF.

A LOW-LEVEL, POINT FORCING FUNCTION TEST

The strain gage response to a point force of about 1900 lb was measured. This force was generated by an instrumented hammer with a metal tip and had a duration of about 300 μs. The strain gage response to the force was about $\pm$ 10 $\mu\epsilon$ which is a very low strain level. However, the response was sufficient to characterize the penetrator's structural response to the force. The measured force and strain gage response were used to calculate a frequency response function (FRF) shown in Figure 2. The FRF was calculated up to a frequency of 5400 Hz with a frequency resolution of 4 Hz and contains the first two axial modes of the penetrator case which are 2744 Hz and 5056 Hz. A third axial mode at 6976 Hz was determined from the spectrum of the strain gage time history. The FRF was not calculated for higher frequencies because the spectrum magnitude for the hammer impact had decreased by 50 percent at 5400 Hz. The FRF amplitude at frequencies above 5400 Hz was increasingly amplified by the decreased amplitude in the input force spectrum. As a consequence, both the second (since its frequency is so close to the upper frequency limit of 5400 Hz) and

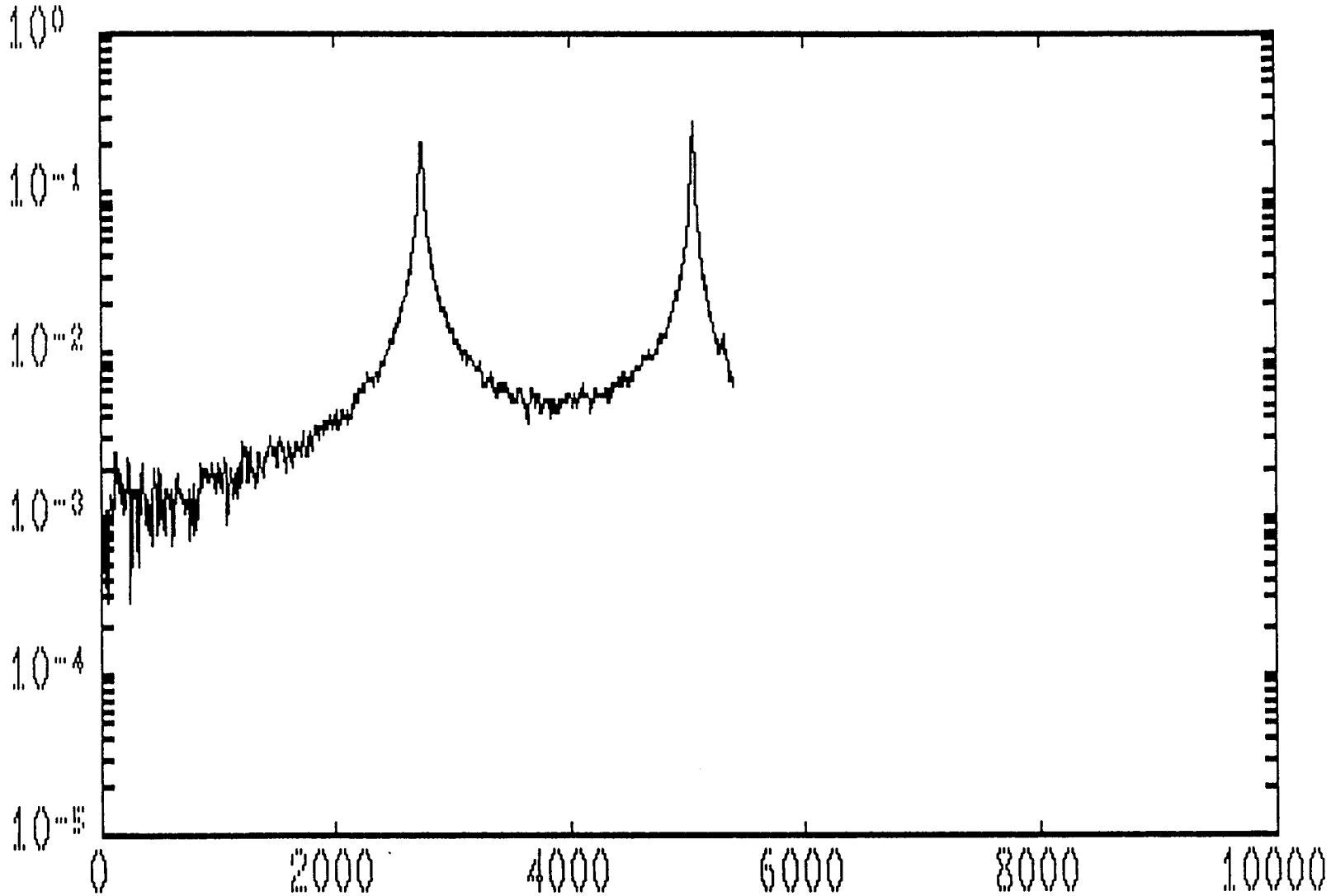

Figure 2: Structural Frequency Response Function for a Penetrator Case with a 1900 lb, 300μs, Point Forcing Function at Measurement Point 14 in. from Rear, Interior (4 Hz Frequency Resolution).

third axial modes have higher amplitude than is characteristic of the structure. Therefore, this FRF would be useful for force reconstruction of axial structural response for only the first axial mode.

This low-level force test was performed to verify the dynamic response of the strain gages and to determine the axial modes of the penetrator case. The strain gage response was consistent for all the locations on the penetrator case. Only one location is shown here because all locations had essentially the same response to this low-level force. The response exhibited by the penetrator case at this low-level provided a basis of comparison for the remaining two tests.

A HIGH-LEVEL, POINT FORCING FUNCTION TEST

The high-level, point force input was generated in the Shock and Climatic Division shock lab with a test technique called the Reverse Hopkinson Bar [4]. This technique was developed to simulate the shock environment experienced by a vehicle during water impact at velocities as high as 600 ft/s and allows the independent control of the shock pulse amplitude and pulse duration. The Reverse Hopkinson Bar test creates a square-shaped forcing function and is configured as shown in Figure 3. An air gun is used to propel a steel bar (1 in diameter, 10 in long) toward the test structure. The force generated at the interface of the steel bar and the test structure is measured with strain gages installed at 2 in from the point of impact. Although these strain gages measure the correct amplitude of the elastic wave created by the impact, the gages do not record the correct duration because they are not at the point of impact. A method has been developed to correct the pulse duration and the corresponding spectrum [5]. The corrected spectrum for the Reverse Hopkinson Bar forcing function was used to calculate the FRF for this high-level, point force.

The dimensions of the Hopkinson Bar were chosen to generate a square pulse whose duration was about 100 μs. There appeared to be some reflections in the pulse from the penetrator case which extended the duration somewhat so that the spectrum for this high-level, point force was useable to about 7 kHz. A time history of the response to this high-level, point force is shown in Figure 4; it contains 8192 response points sampled at 50 kHz (20 μs per point). It is evident that the response was not measured for a sufficiently long period because the response amplitude is significant, about 25% of the peak amplitude, at the end of the record. The FRFs for this forcing function are shown in Figures 5-8; the FRF for each strain gage location is shown separately.

The amplitude of the peaks for the three axial modes vary for each location which is to be expected. However, the general magnitude of the FRF's is consistent for the four locations. Additionally, the frequencies for the three axial modes agree with the low-level force test measurements to within the frequency resolutions of the two calculations.

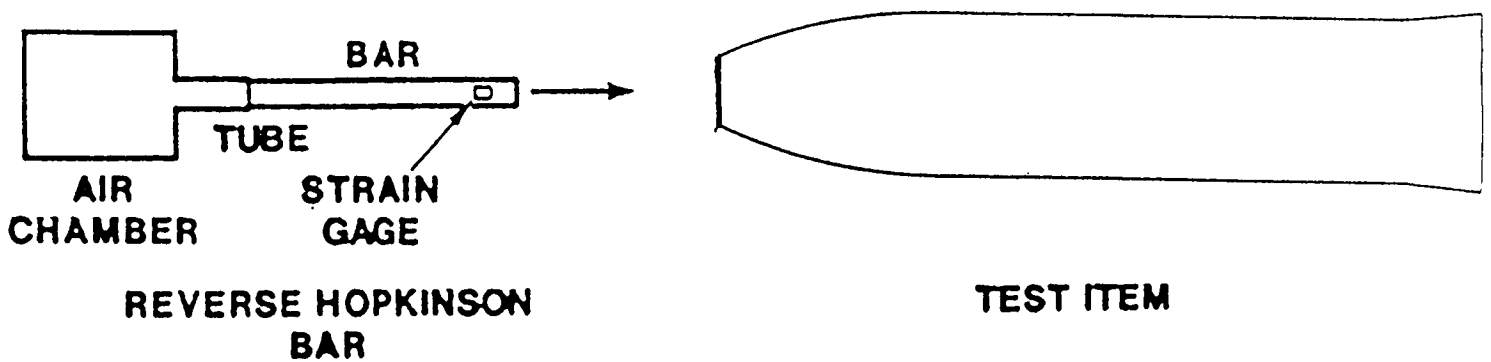

Figure 3: Reverse Hopkinson Bar Test Configuration for a Penetrator Case and a 1 in Diameter, 10 in Long Hopkinson Bar.

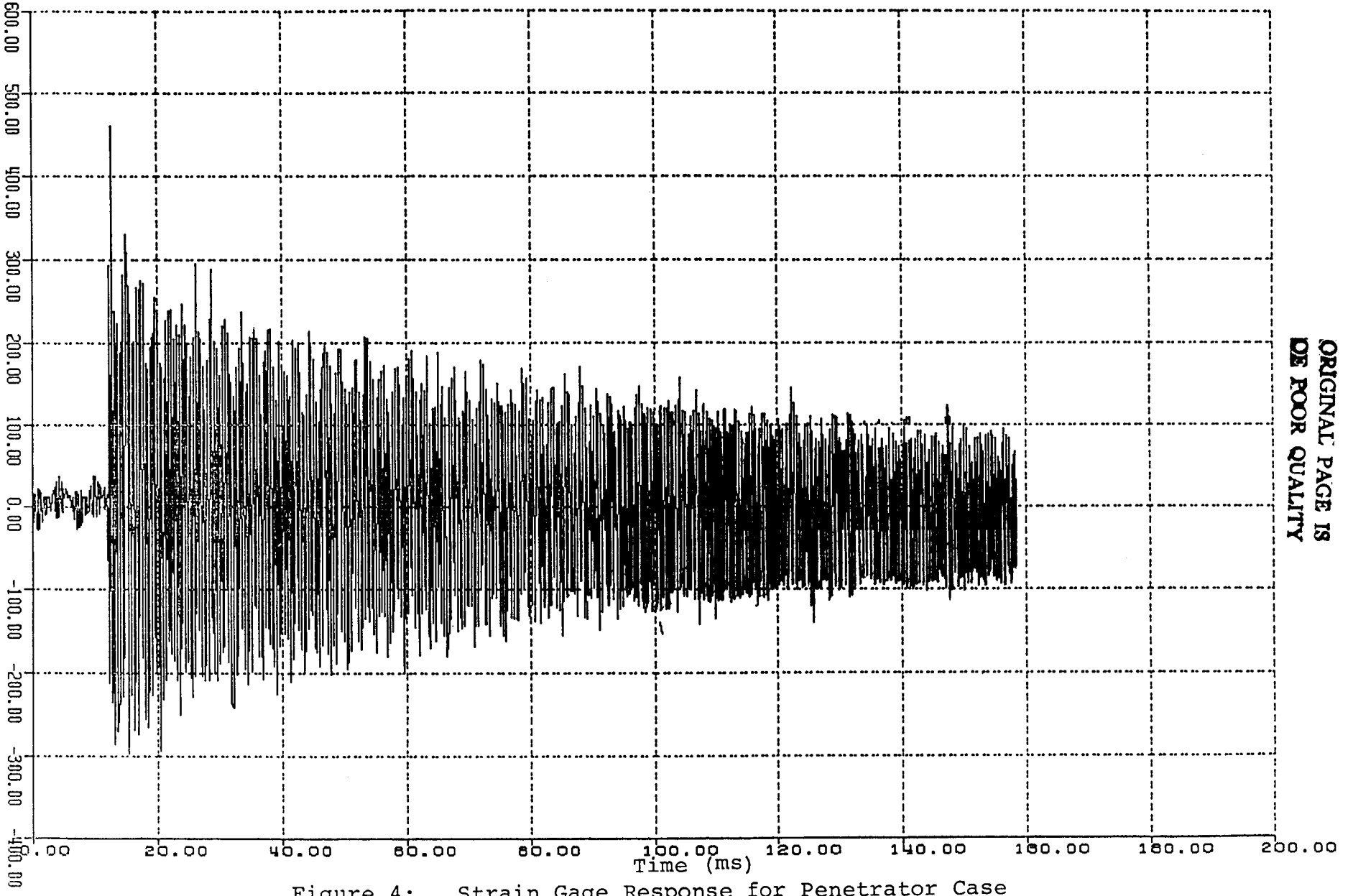

Figure 4: Strain Gage Response for Penetrator Case with 100 klb, 100 μs Point Forcing Function (8k Sample Points, 20 μs per Point).

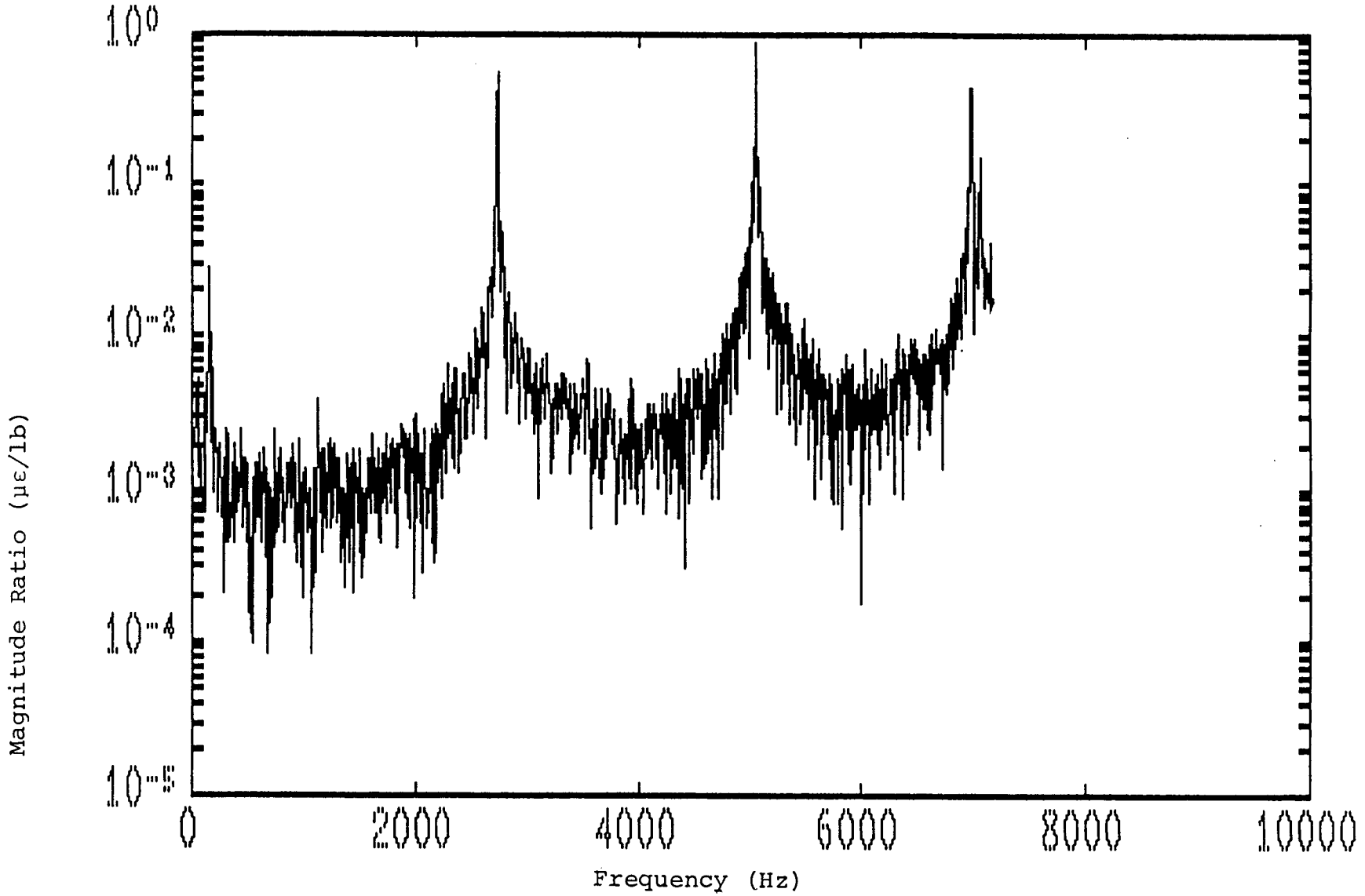

Figure 5: Structural Frequency Response Function for Penetrator Case with a 100 klb, 100 μs, Point Forcing Function at Measurement Point 7 in. from Rear, Interior (6 Hz Frequency Resolution).

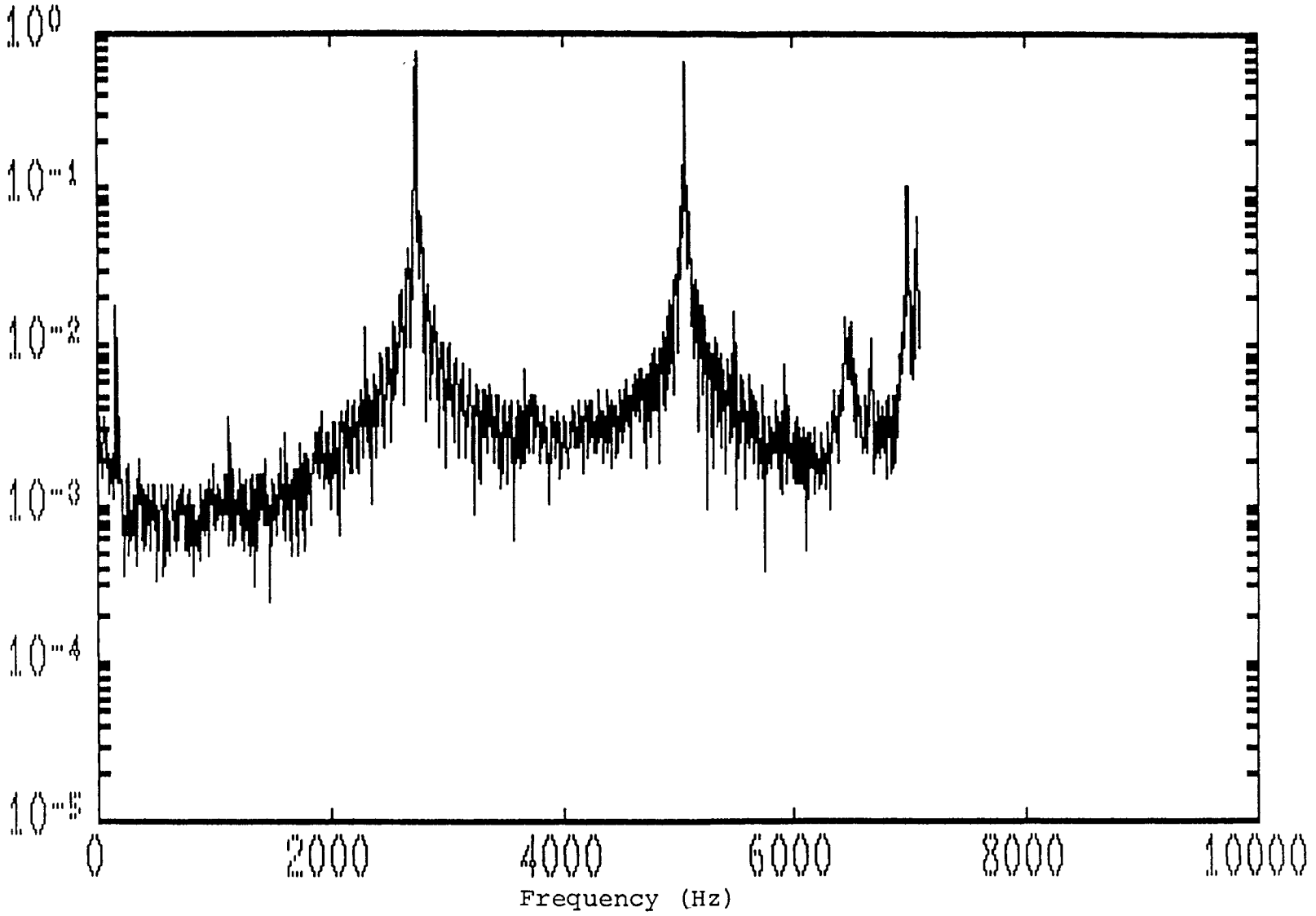

Figure 6: Structural Frequency Response Function for Penetrator Case with a 100 klb, 100 μs, Point Forcing Function at Measurement Point 14 in. from Rear, Interior (6 Hz Frequency Resolution).

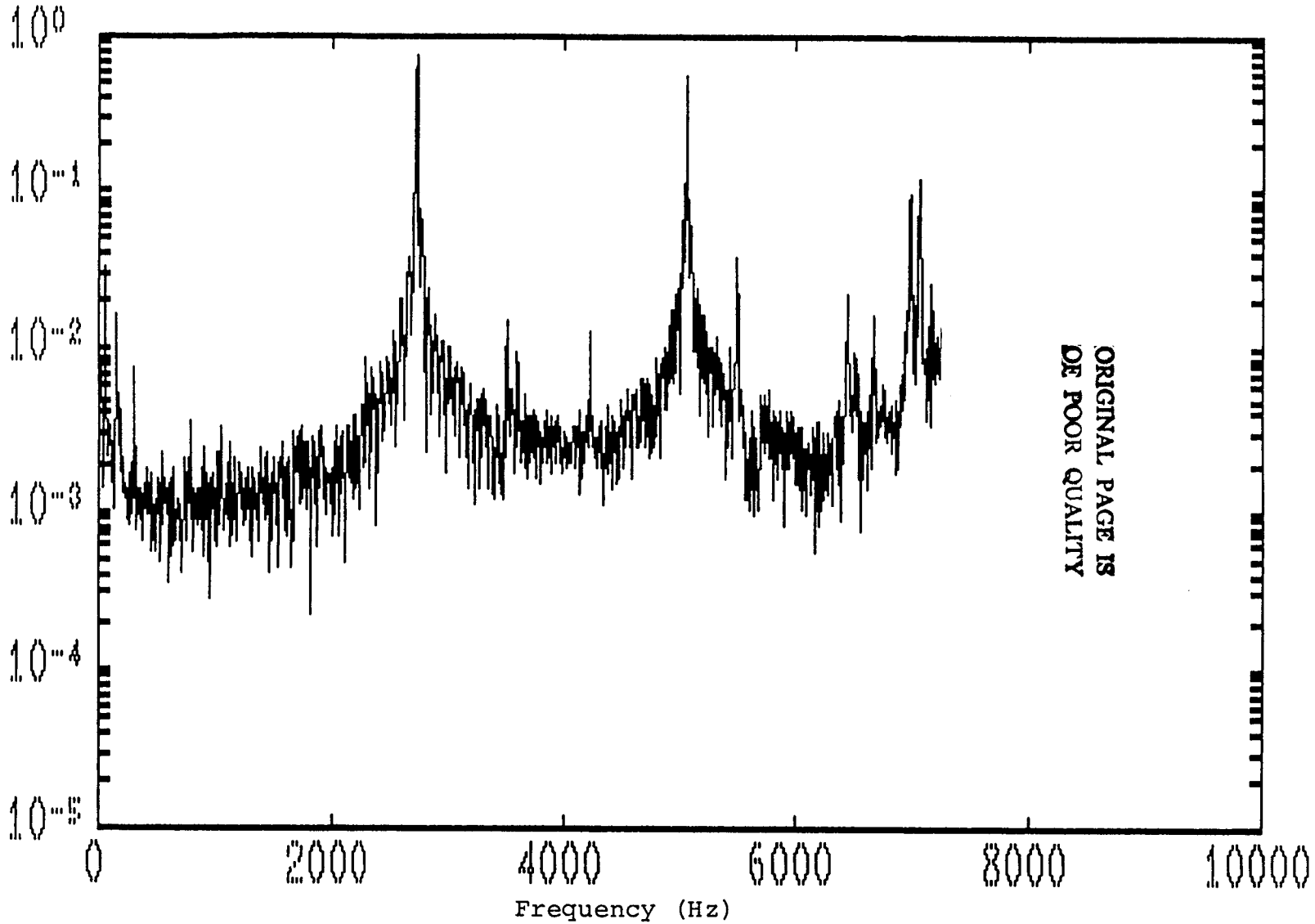

Figure 7: Structural Frequency Response Function for Penetrator Case with a 100 klb, 100 μs, Point Forcing Function at Measurement Point 14 in. from Rear, Exterior (6 Hz Frequency Resolution).

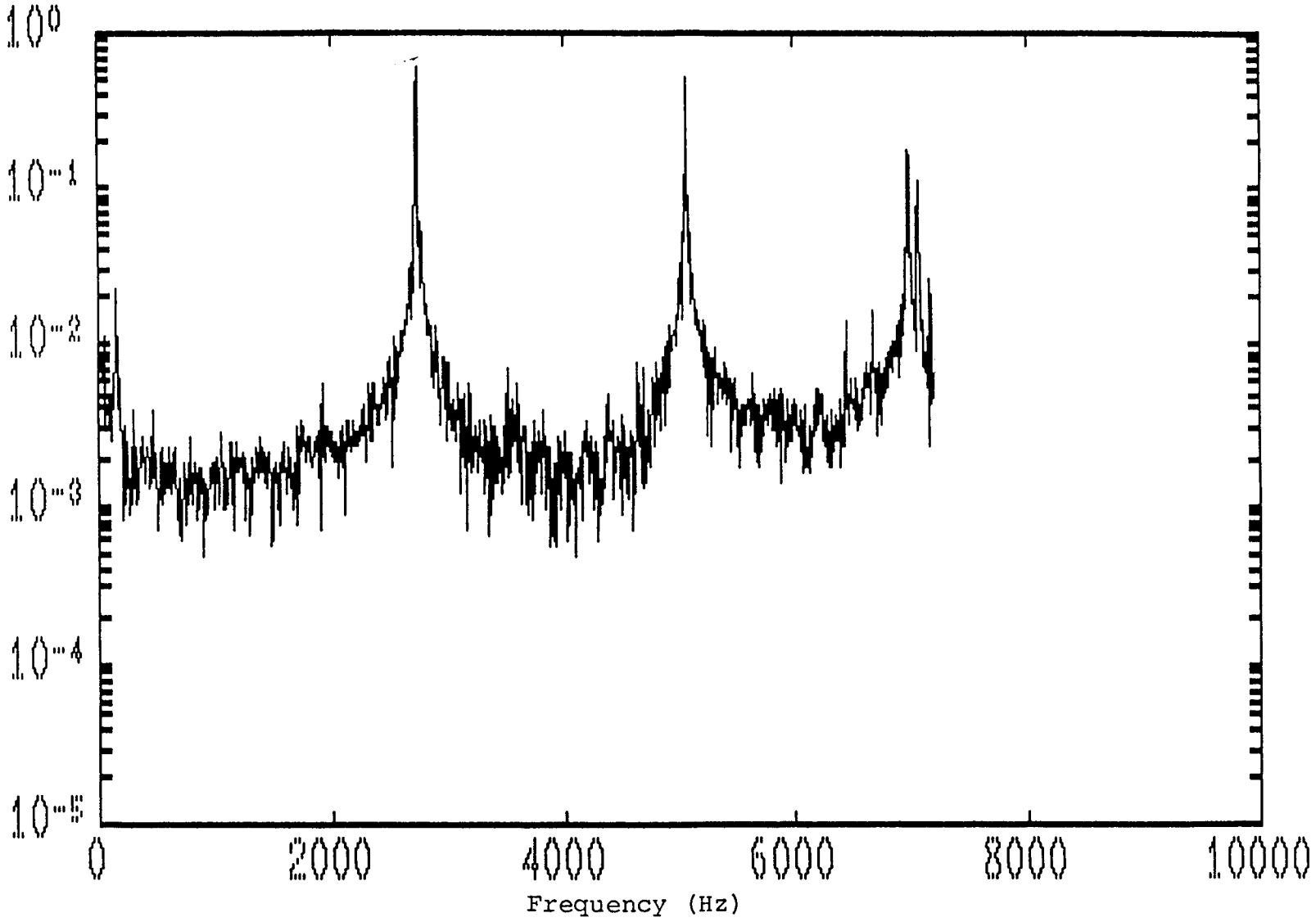

Figure 8: Structural Frequency Response Function for Penetrator Case with a 100 klb, 100 μs, Point Forcing Function at Measurement Point 24 in. from Rear, Exterior (6Hz Frequency Resolution).

A HIGH-LEVEL, DISTRIBUTED FORCING FUNCTION TEST

An impulsive, distributed forcing function was generated by three different explosive configurations. The three impulsive, distributed loads were designed to have approximately the same axial impulse of 20 lb-sec and a pulse duration of 20 μs or less. The pulse duration was chosen so that the frequency content in the forcing function would be essentially constant over the bandwidth of interest, 10 kHz. This unique forcing function simulates an ideal mechanical impulse at force levels which approximate the field conditions. The characteristics of the three distributed forcing functions are summarized in Table 1.

TABLE 1: CHARACTERISTICS OF THREE DISTRIBUTED, IMPULSIVE FORCING FUNCTIONS.

Deta Sheet Layers	Distance along Penetrator Axis (in)*	Ogival Area Covered	Spectrum Decrease-10kHz
2	4.39	48.4 %	7 %
3	3.09	30.5 %	4 %
4	2.40	23.1 %	3 %

* The penetrator nose is solid for 3.20 in along its longitudinal axis.

Since the area of the explosive was limited by the restriction that the pulse duration be less than 20 μs, only one of the three distributed forces covered enough area to include a hollow portion of the penetrator case. Figure 9 is a typical time history of the strain gage response to a distributed forcing function. The 32768 sample points have been decimated to 1024 points for Figure 9 because the plotting device is restricted to 1024 points. However, the time history does show the general envelope of the structural response typically measured from the explosive forcing function. The large number of sample points (32768) was necessary in order to obtain a reasonable frequency resolution (15 Hz) with the sample period of 2 μs for the structural response.

In order to calculate a FRF for the explosive forcing function, three spectra were calculated for the theoretical prediction of the explosive time histories in Mr. Cawlfield's paper. These spectra have essentially constant magnitude for frequencies up to 10 kHz as shown by the values for percent magnitude decrease in Table 1; on the basis of the spectra, the distributed forcing functions may be considered mechanical impulses. The constant magnitude of the spectra means that all the structural frequencies were equally excited in the 10 kHz bandwidth which

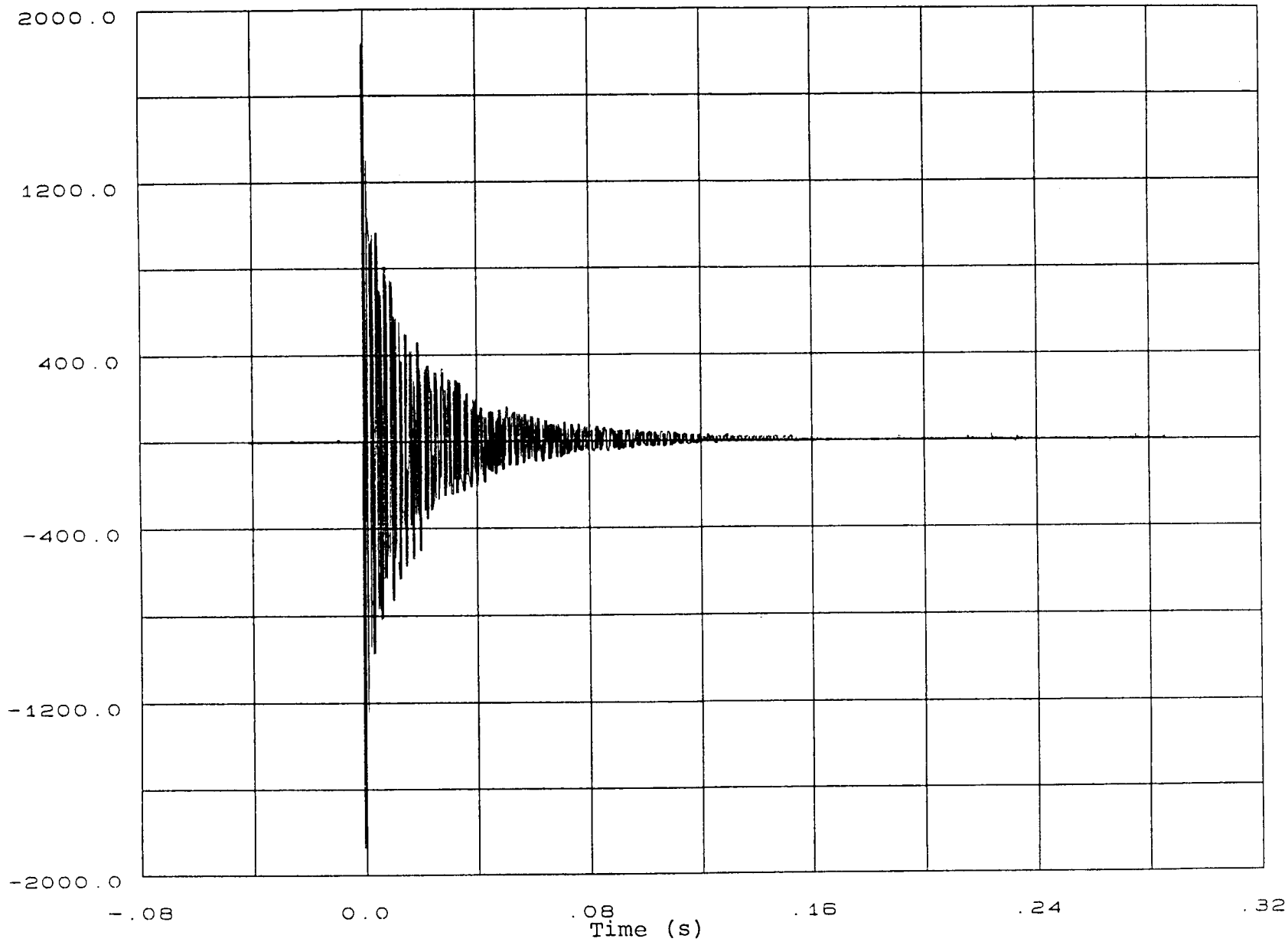

Figure 9: Strain Gage Response for Penetrator Case with a Distributed (30.5% of Ogival Area) Forcing Function (32768 Sample Points Decimated to 1024 Points).

was the motivation for restricting the explosive pulse duration to 20 μs. The spectra for the explosive forcing functions were interpolated so that the FRF's could be calculated.

The results of the FRF calculations for the distributed forcing function are shown in Figures 10-21. A FRF for each distributed force is presented for the four strain gage locations. The figures are grouped according to strain gage location so that the first three figures are for the location 7 in from the rear, interior. The FRFs for the other locations follow. All the FRFs were calculated for 10 kHz and show the same order of magnitude. The three axial modes present in the high-level, point force test are evident in all the FRFs; their frequencies agree with those of the other tests to within the frequency resolution of 15 Hz. The magnitude of the mode peaks varies according to location as expected. Additionally, there are some higher order modes between 8 and 10 kHz which have not been uniquely identified. The frequencies in the 8-10 kHz range are not noise because their amplitude is a decade above the noise floor which is less than 0.001 $\mu\epsilon$/lb and are consistently present in all the FRFs. These modes represent case motion in which all four strain gages at a location move together. This indicates that the modes may be either higher order axial modes or "breathing" modes of the penetrator case.

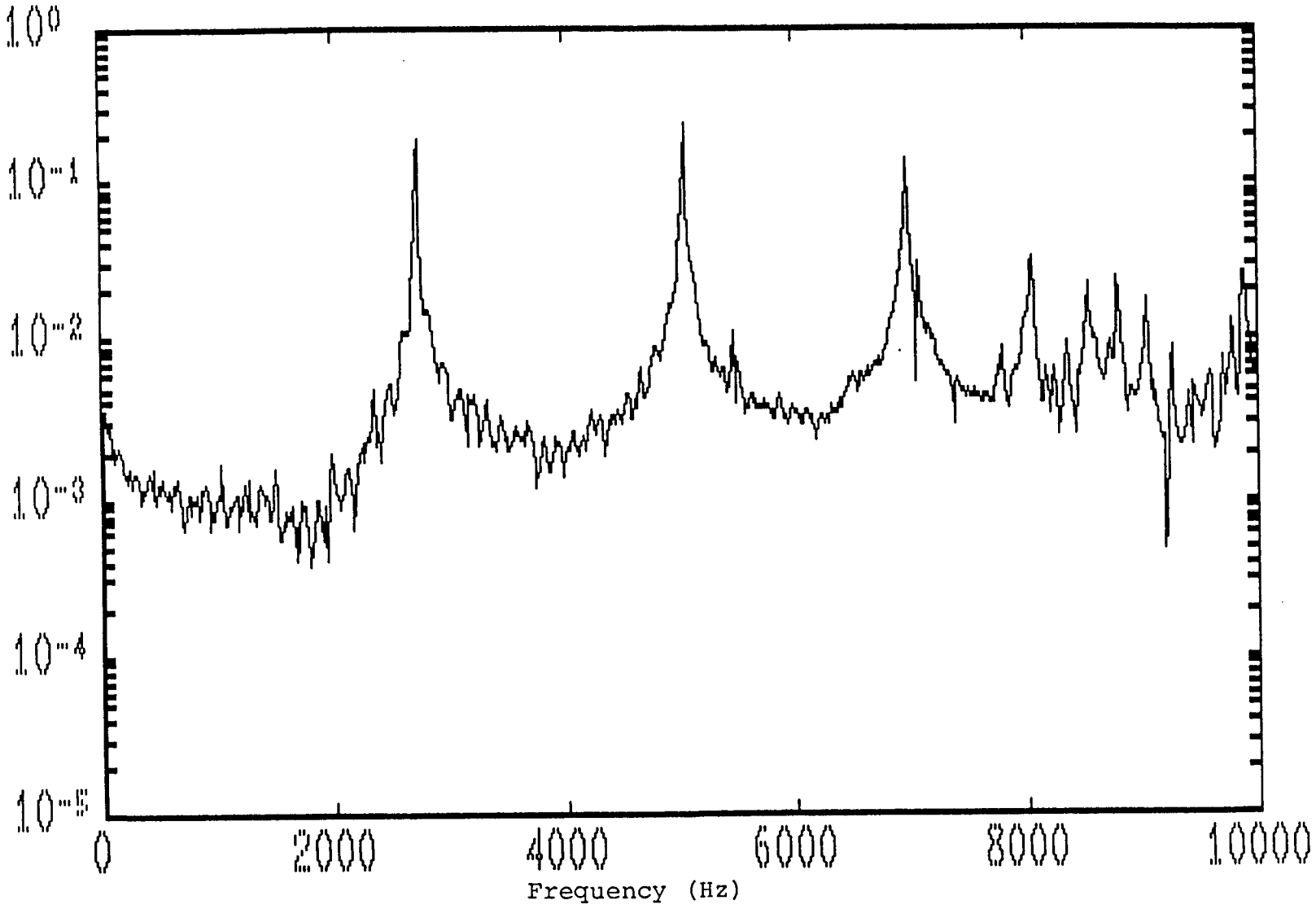

Figure 10: Structural Frequency Response Function for Penetrator Case with a Distributed (23.1% of Ogival Area) Forcing Function at Measurement Point 7 in. from Rear, Interior (15 Hz Frequency Resolution).

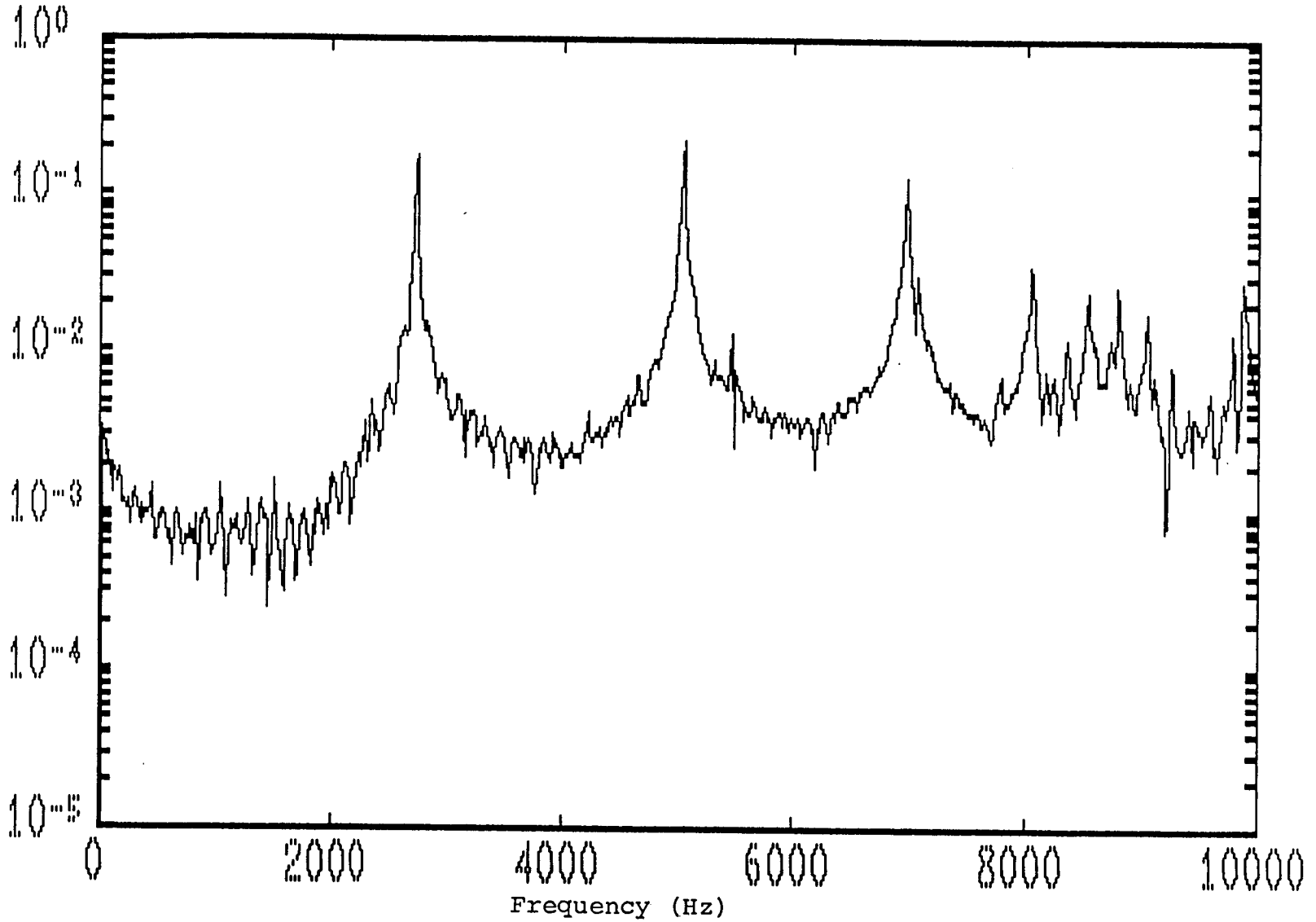

Figure 11: Structural Frequency Response Function for Penetrator Case with a Distributed (30.5% of Ogival Area) Forcing Function at Measurement Point 7 in. from Rear, Interior (15 Hz Frequency Resolution).

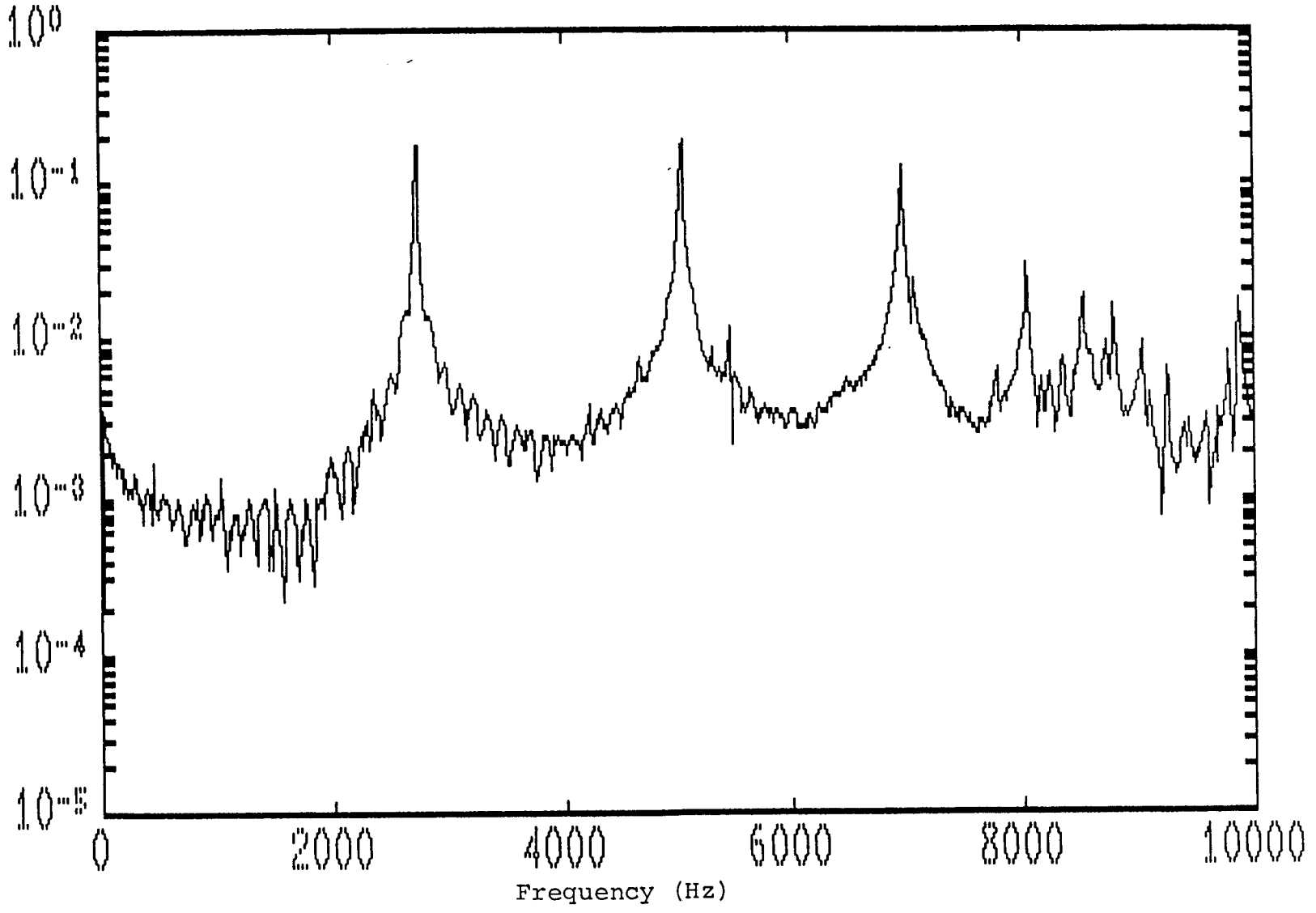

Figure 12: Structural Frequency Response Function for Penetrator Case with a Distributed (48.4% of Ogival Area) Forcing Function at Measurement Point 7 in. from Rear, Interior (15 Hz Frequency Resolution).

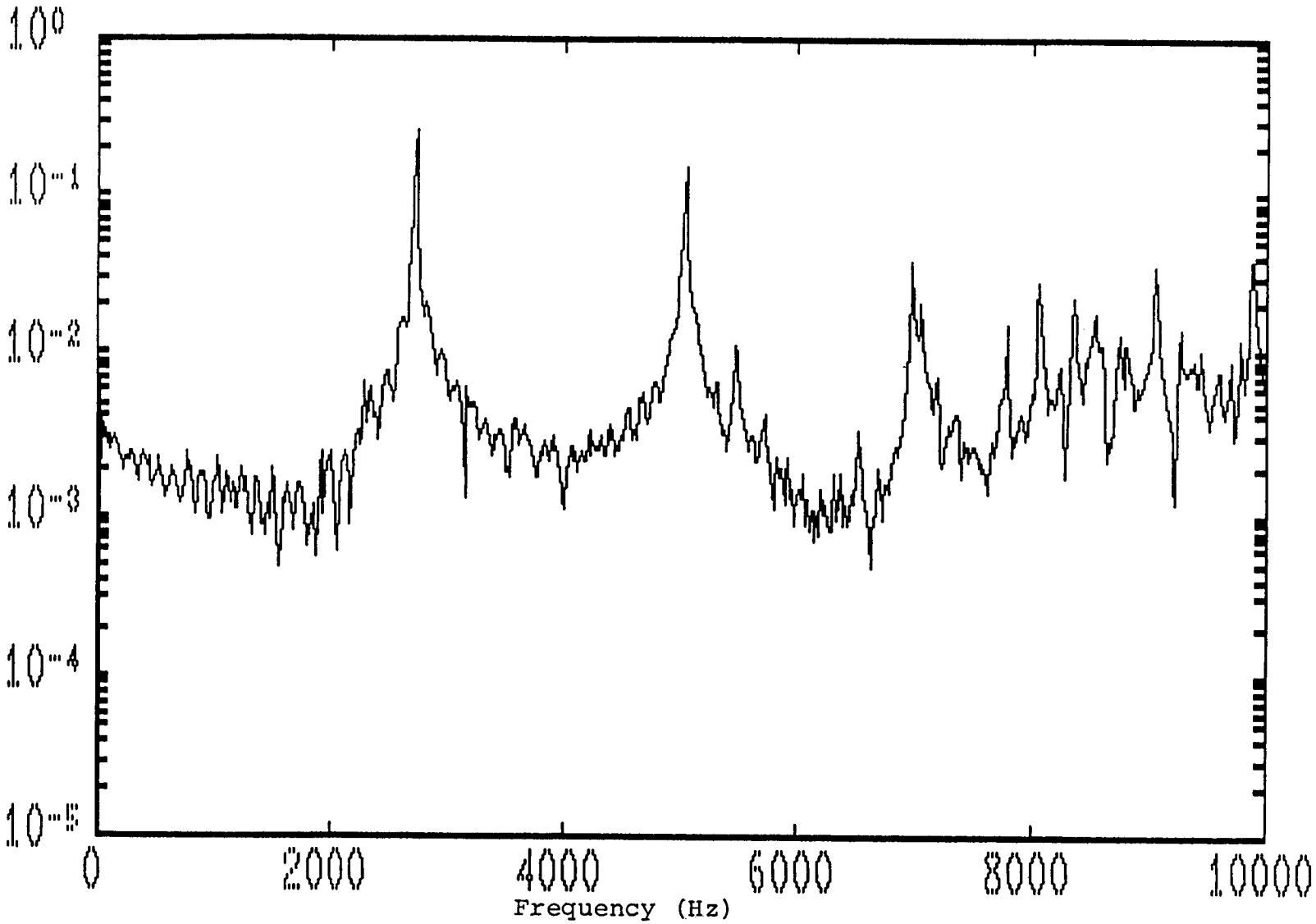

Figure 13: Structural Frequency Response Function for Penetrator Case with a Distributed (23.1% of Ogival Area) Forcing Function at Measurement Point 14 in. from Rear, Interior (15 Hz Frequency Resolution).

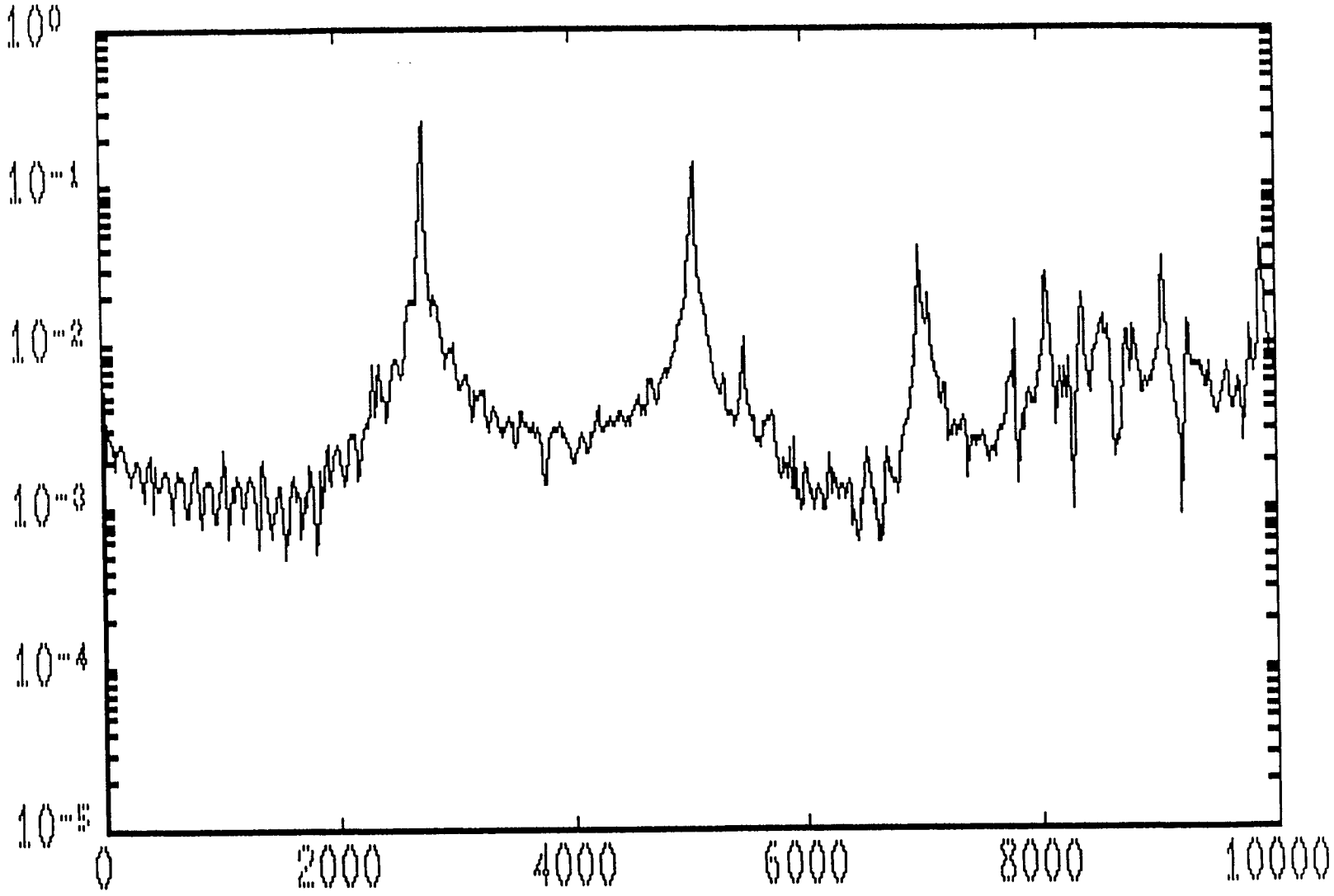

Figure 14: Structural Frequency Response Function for Penetrator Case with a Distributed (30.5% of Ogival Area) Forcing Function at Measurement Point 14 in. from Rear, Interior (15 Hz Frequency Resolution).

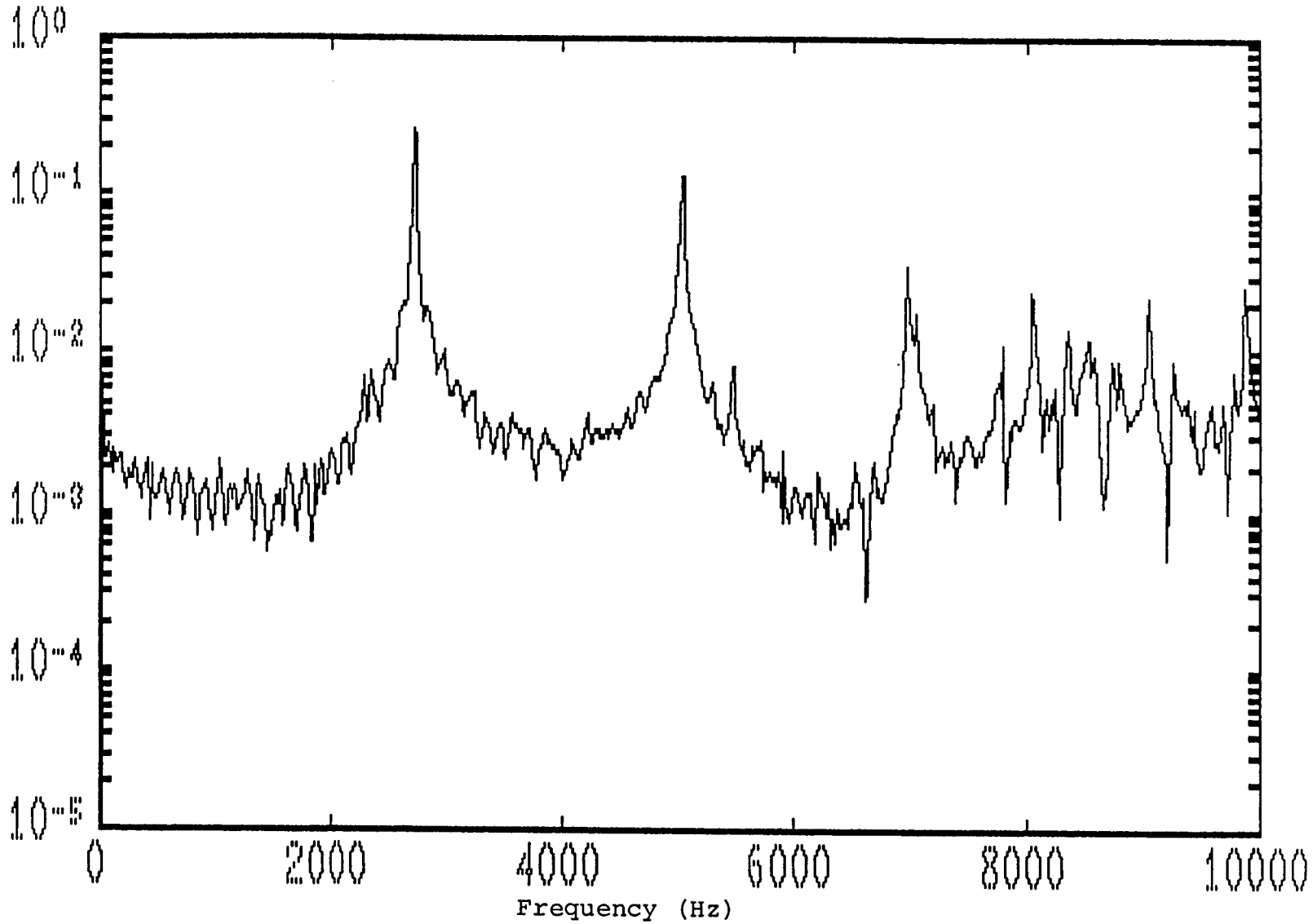

Figure 15: Structural Frequency Response Function for Penetrator Case with a Distributed (48.4% of Ogival Area) Forcing Function at Measurement Point 14 in. from Rear, Interior (15 Hz Frequency Resolution).

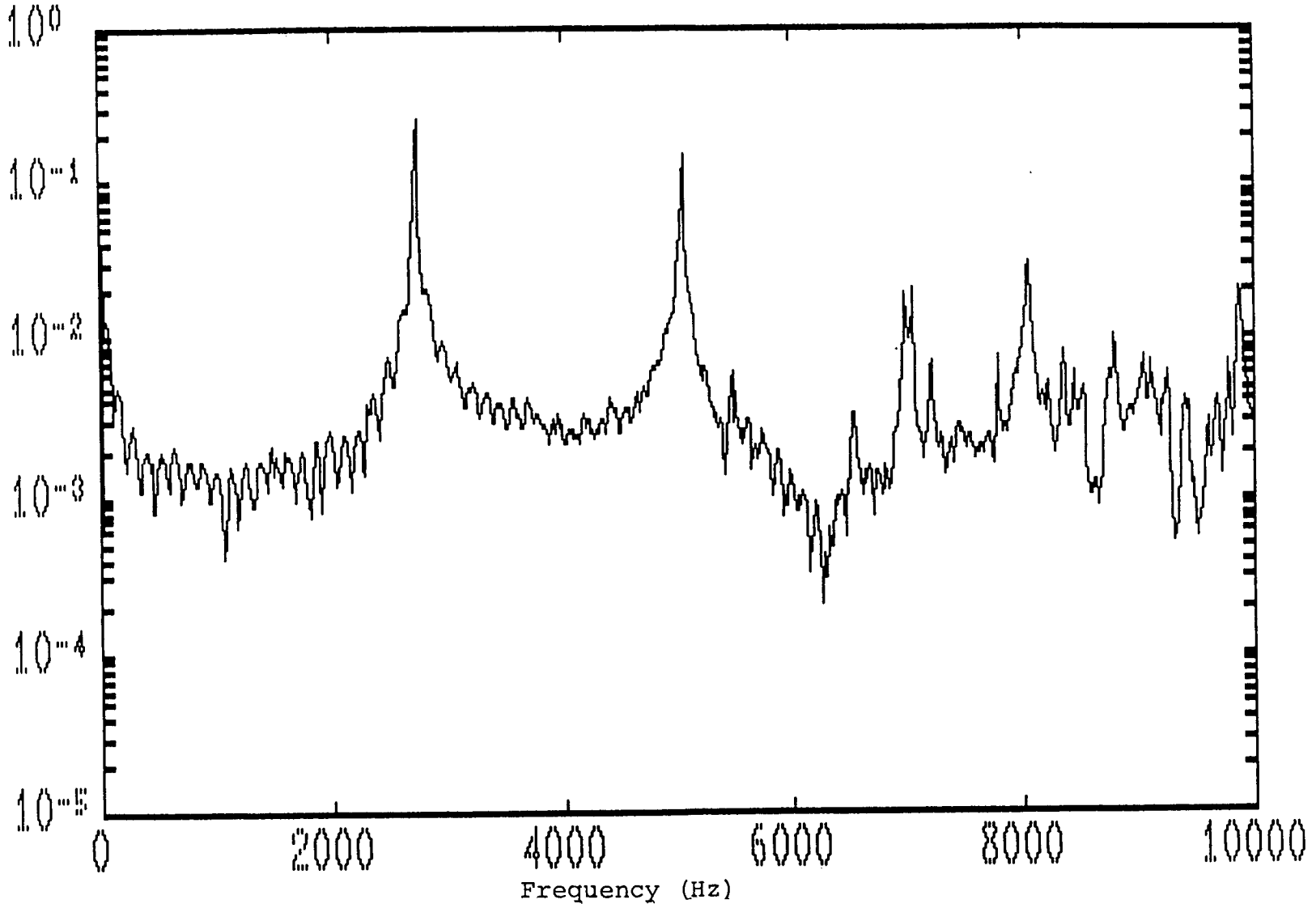

Figure 16: Structural Frequency Response Function for Penetrator Case with a Distributed (23.1% of Ogival Area) Forcing Function at Measurement Point 14 in. from Rear, Exterior (15 Hz Frequency Resolution).

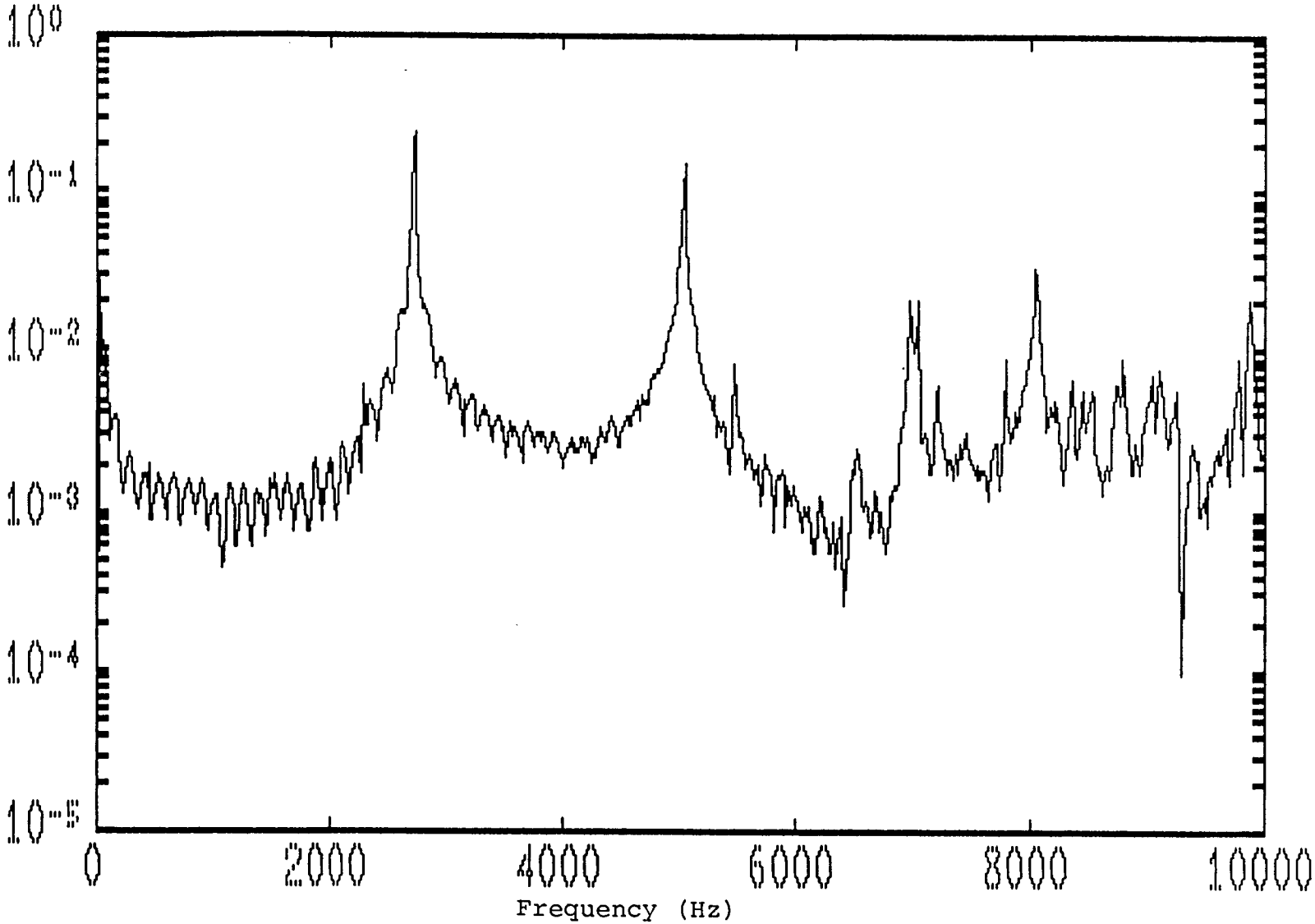

Figure 17: Structural Frequency Response Function for Penetrator Case with a Distributed (30.5% of Ogival Area) Forcing Function at Measurement Point 14 in. from Rear, Exterior (15 Hz Frequency Resolution).

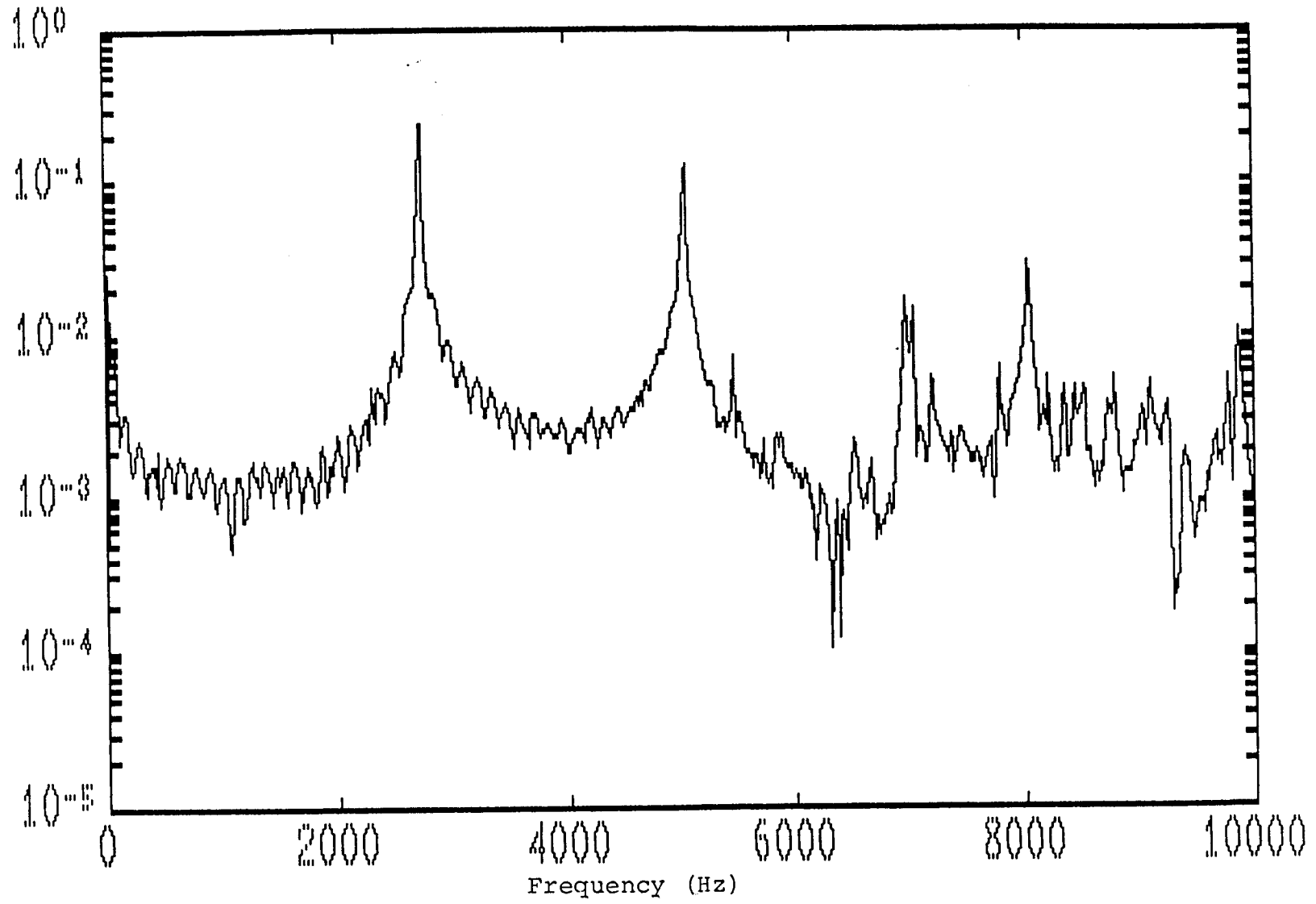

Figure 18: Structural Frequency Response Function for Penetrator Case with a Distributed (48.4% of Ogival Area) Forcing Function at Measurement Point 14 in. from Rear, Exterior (15 Hz Frequency Resolution).

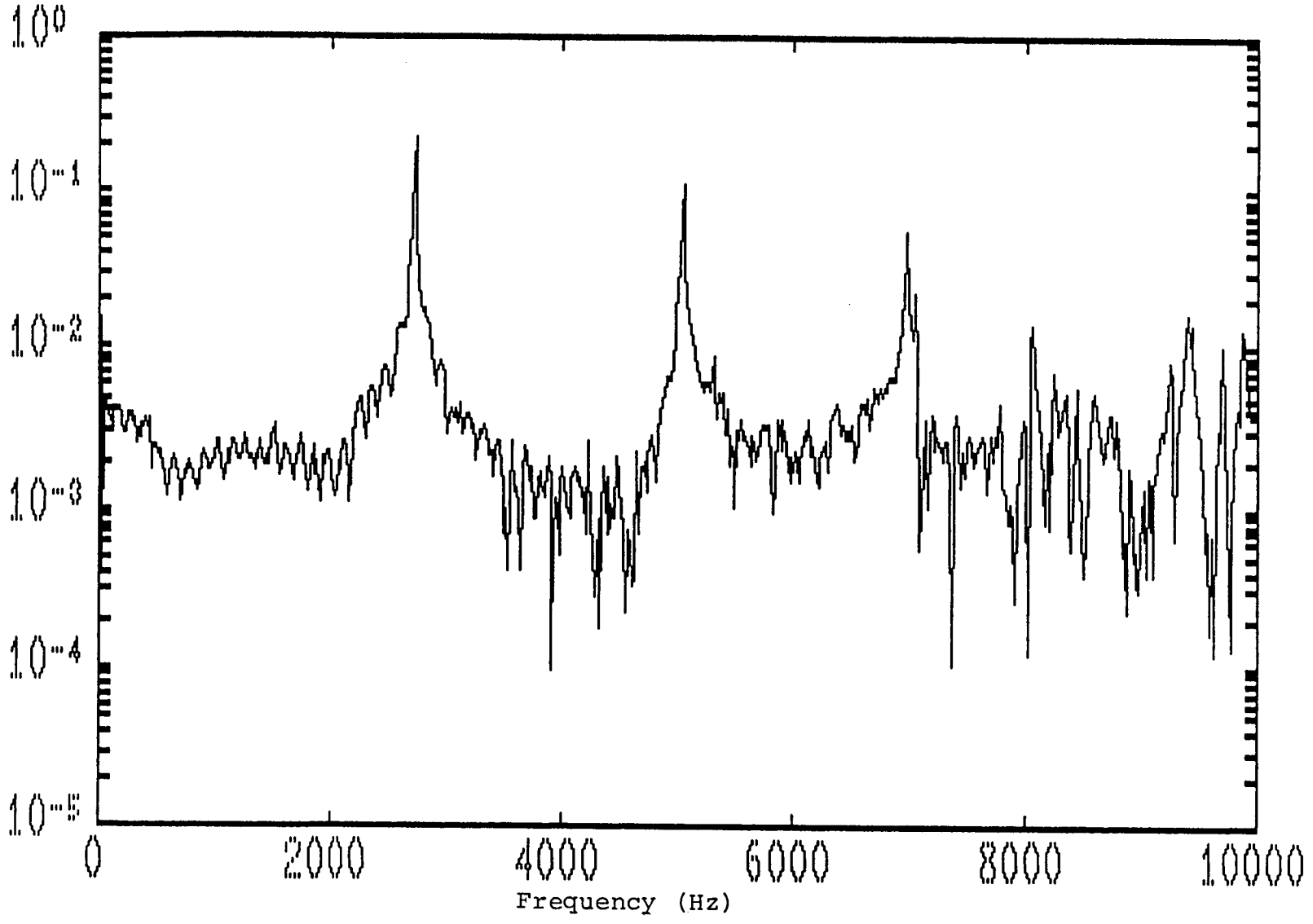

Figure 19: Structural Frequency Response Function for Penetrator Case with a Distributed (23.1% of Ogival Area) Forcing Function at Measurement Point 24 in. from Rear, Exterior (15 Hz Frequency Resolution).

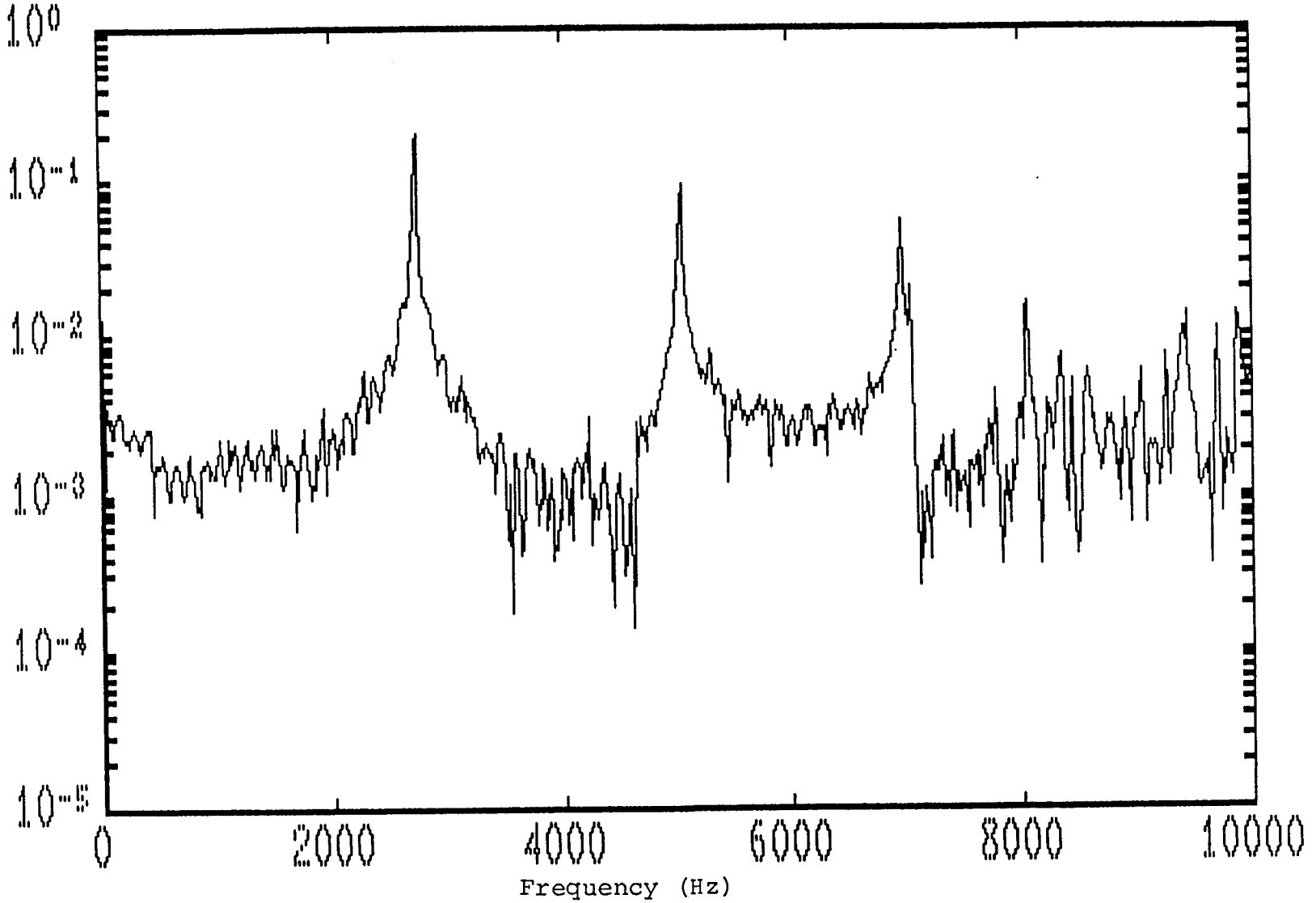

Figure 20: Structural Frequency Response Function for Penetrator Case with a Distributed (30.5% of Ogival Area) Forcing Function at Measurement Point 24 in. from Rear, Exterior (15 Hz Frequency Resolution).

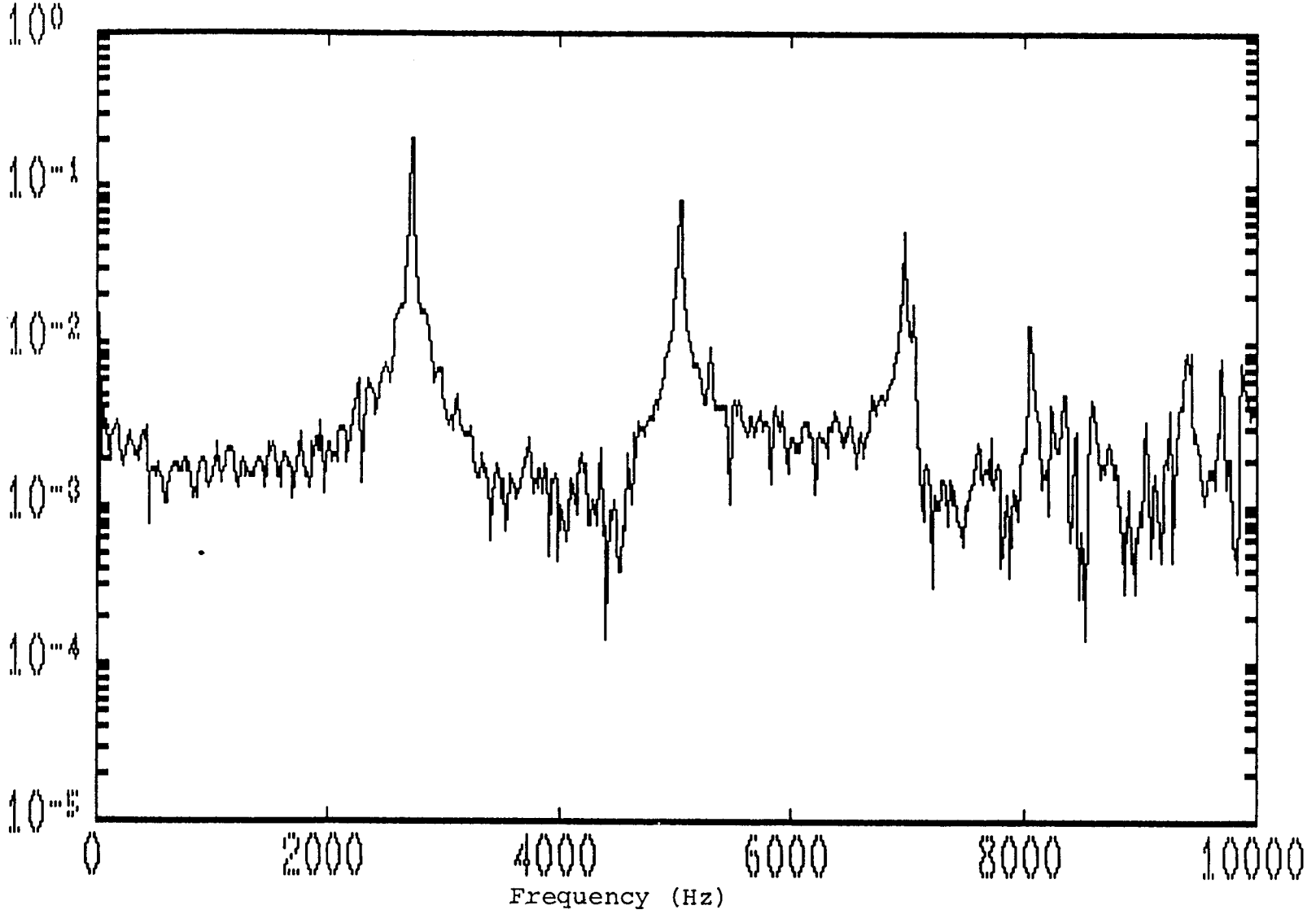

Figure 21: Structural Frequency Response Function for Penetrator Case with a Distributed (48.4% of Ogival Area) Forcing Function at Measurement Point 24 in. from Rear, Exterior (15 Hz Frequency Resolution).

CONCLUSIONS

A series of tests have been conducted to characterize the response of a penetrator case to axial forces. All three goals established prior to the tests have been accomplished. The FRFs for the penetrator case were calculated from responses at the four axial locations for three axial forcing functions. Good structural response measurements were obtained as demonstrated by the consistent structural characteristics in the FRFs. A spatially distributed load was successfully implemented with Deta Sheet in three configurations.

The parameters of the three forcing functions used to characterize the structural response of the penetrator case are summarized in Table 2. The frequency

TABLE 2: THREE AXIAL FORCING FUNCTIONS APPLIED TO A PENETRATOR CASE.

Type	Force Level (lb)	Duration (μs)	Frequency Resolution	Impulse (lb-sec)
Instrumented Hammer	1.9 k	300	4 Hz	0.6
Reverse Hopkinson Bar	100 k	100	6 Hz	10
Distributed Impulsive	1-2 M	20	15 Hz	20

response functions for these forcing functions have been presented and show repeatable characteristics of magnitude and frequency content. The penetrator case exhibited linear behavior over the wide range of force magnitudes applied in this test series. The forcing functions included a mechanical impulse whose magnitude of 1-2 Mlb is the same order of magnitude force that the penetrator experiences in the field.

The structural response depicted in the FRFs indicates that the strain gages respond similarly to a point axial load and to a distributed axial load. There was no detectable difference in the shape or frequency of the first three axial modes of the penetrator case when they were excited by a 100 klb, point forcing function and by a 1 Mlb, distributed forcing function. This result suggests that the distributed axial force generated in a penetration event may be reconstructed as a point axial force when the penetrator behaves in a linear manner.

FUTURE WORK

The tests described in this paper will be used to characterize the lateral response of the penetrator case and the combined axial and lateral response of the penetrator unit with the internal components. The FRFs which result from these tests will indicate the linearity of the structure to lateral loads and to combined axial and lateral loads. The FRFs will also be used with field test response measurements to resolve the applied forces into point or distributed loads by various force reconstruction techniques.

ACKNOWLEDGEMENTS

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