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OPTIMAL CONTROL OF LARGE SPACE STRUCTURES VIA GENERALIZED INVERSE MATRIX

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ABSTRACT: *Independent Modal Space Control (IMSC) is a control scheme that decouples the space structure into n independent second-order subsystems according to n controlled modes and controls each mode independently. It is well-known that the IMSC eliminates control and observation spillover caused when the conventional coupled modal control scheme is employed. The independent control of each mode requires that the number of actuators be equal to the number of modelled modes, which is very high for a faithful modelling of large space structures. In this paper, we propose a control scheme that allows one to use a reduced number of actuators to control all modelled modes suboptimally. In particular, the method of generalized inverse matrices is employed to implement the actuators such that the eigenvalues of the closed-loop system are as closed as possible to those specified by the optimal IMSC. Computer simulation of the proposed control scheme on a simply supported beam is given.*

1. INTRODUCTION

The development of the space shuttle has opened the possibility of constructing very large structures in space for space explorations. Two control problems for LSS are attitude control and shape control. Complex missions impose many stringent requirements on shape and attitude of the LSS, which lead the control researchers to the concept of distributed active control where several actuators and sensors are placed on the structure to in order to optimize its performance and behavior. There has been a considerable interest in the area of active control of large space structures (LSS) [1]-[13]. A number of control schemes were studied, but they represent one form or another of modal control [6]. Two main modal control schemes are the coupled modal control and the Independent Modal Space Control (IMSC). The former uses an active controller that consists of a state estimator and a state feedback; the latter decouples the LSS into n independent subsystems according to n controlled modes and controls each mode independently by means of a modal filter [5] and an optimal controller. Coupled modal control causes control and observation spillover that together can destabilize the LSS [10]. IMSC does not have the spillover problem since each mode is controlled independently. However in order to implement the IMSC the number of actuators is required to be equal to the number of controlled modes which is usually very huge for a faithful modelling of the LSS. This fact presents a fundamental limitation of IMSC since the required number of actuators is unrealizable. The main objective of this paper is to implement the IMSC with a milder requirement of the actuator number. In other words, we will develop a control scheme that uses a reduced number of actuators to control all modelled modes in such a way that the modes of the closed-loop system are as closed as possible to the optimal modes specified by the IMSC scheme. In particular, the method of generalized inverse matrices is employed for the implementation of IMSC.

Matrix notations used in this paper is given below:

(1) $$\text{Block diag}(M_1, M_2, \ldots, M_n) = \begin{bmatrix} M_1 & 0 & \ldots & 0 \\ 0 & M_2 & \ldots & 0 \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & M_n \end{bmatrix}$$

(2) 0_{mm} = mm null matrix

(3) I_n = mm identity matrix

2. SUMMARY OF INDEPENDENT MODAL SPACE CONTROL

The description of a large flexible space structure is given by the following partial differental differential equations [3]:

$$M(P)\,\partial^2 u(P,t)/\partial t^2 + Lu(P,t) = f(P,t) \qquad (1)$$

that must be satisfied at every point P of the domain D, where u(P,t) is the displacement of Point P, L a linear differential self-adjoint operator of order 2p, expressing the system stiffness, M(P) the distributed mass, and F(P,t) the distributed control force. The displacement u(P,t) is subject to the boundary conditions:

$$T_i u(P,t) = 0;\ i = 1,2,\ldots,p \qquad (2)$$

where T_i, i=1,2,....,p are linear differential operator of order ranging from 0 to (2p-1).

The associated eigenvalue problem is formulated by:

$$L\phi_r(P) = \lambda_r M(P)\,\phi_r(P);\quad r=1,2,\ldots \qquad (3)$$

with the boundary conditions:

$$T_i\phi_r(P) = 0\ ;\ i=1,2,\ldots,p;\ r=1,2,\ldots \qquad (4)$$

where λ_r is the rth eigenvalue and $\phi_r(P)$ is the eigenfunction (sometimes also known as Mode Shape) associated with λ_r. Suppose the operator L is self-adjoint and positive definite, and all eigenvalues are positive and are ordered so that $\lambda_1 < \lambda_2 < \ldots$. Since L is self-adjoint, the eigenfunctions are orthogonal and therefore can be normalized such that:

$$\int_D M\phi_r\phi_s\,dD = \delta_{rs} \qquad (5)$$

$$\text{and} \int_D \phi_s L\phi_r\,dD = \lambda_r\delta_{rs}\ ;r,s=1,2,\ldots \qquad (6)$$

where δ_{rs} is the Kronecker Delta.

Using the expansion theorem [3], the solution of u(P,t) can be obtained as:

$$u(P,t) = \sum_{r=1}^{\infty}\phi_r(P)u_r(t) \qquad (7)$$

where $u_r(t)$ is the modal coordinate. Substituting (7) into (1), multiplying both sides of the resulting expression by ϕ_s integrating over D and employing (5) and (6), we obtain

$$\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t);\ r=1,2,\ldots \qquad (8)$$

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In (8), the mode (or natural) frequency ω_r is defined as

$$\omega_r = (\lambda_r)^{1/2} \;;r=1,2,\ldots \quad (9)$$

and the modal control force $f_r(t)$ is computed by:

$$f_r(t) = \int_D \phi_r(P)\; f(P,t)\; dD \quad (10)$$

In practice, the infinite series in (7) is truncated as

$$u(P,t) = \sum_{r=1}^{n} \phi_r(P)\; u_r(t) \quad (11)$$

where n is chosen to be sufficiently large so that $u(P,t)$ can be represented with good fidelity. In this case we are dealing only with the first n modes.

Eq. (8) can be transformed into state equation form as follows:

$$\dot{x}(t) = A\, x(t) + W(t) \quad (12)$$

where $$x(t) = [x_1^T(t)\; x_2^T(t) \ldots x_n^T(t)]^T \quad (13)$$

$$W(t) = [W_1^T(t)\; W_2^T(t) \ldots W_n^T(t)]^T \quad (14)$$

$$A = \text{Block diag}(A_1, A_2, \ldots, A_r) \quad (15)$$

$$x_r(t) = [u_r(t) \quad \dot{u}_r(t)/\omega_r]^T \quad (16)$$

$$W(t) = [0 \quad f_r(t)/\omega_r]^T \quad (17)$$

and $$A_r = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix} \quad (18)$$

for $r=1,2,\ldots,n$.

The system described by (12) consists of n subsystems given by

$$\dot{x}_r(t) = A_r\, x_r t + W_r(t);\; r = 1,2,\ldots, n. \quad (19)$$

The essence of IMSC is to choose $W_r(t)$ such that it depends on $x_r(t)$ alone. Thus

$$W_r(t) = G_r\, x_r;\; r=1,2,\ldots,n \quad (20)$$

where $$G_r = \begin{bmatrix} g_{r11} & g_{r12} \\ g_{r21} & g_{r22} \end{bmatrix}; \quad r=1,2,\ldots,n \quad (21)$$

are (2x2) gain matrices.

Substituting (16) and (17) into (20), we find that G_r must assume the following form:

$$G_r = \begin{bmatrix} 0 & 0 \\ g_{r21} & g_{r22} \end{bmatrix} \;;\; r=1,2,\ldots,n \quad (22)$$

For optimal control, g_{r21} and g_{r22} should be determined such that the following quadratic cost function is minimized (linear regulator problem):

$$J = \sum_{r=1}^{n} J_r \quad (23)$$

where $$J_r = (x_r^T Q_r x_r + W_r^T R_r W_r)\,dt \quad (24)$$

Q_r and R_r are positive semidefinite and positive definite weighting matrix, respectively, associated with the rth mode.

The form of G_r given by (22) requires that R_r assume the form given below:

$$R_r = \begin{bmatrix} \infty & 0 \\ 0 & r_r \end{bmatrix} \;;\; r=1,2,\ldots,n \quad (25)$$

Since W_r depends on x_r alone as seen in (20), J can be minimized by minimizing each J_r independently. From optimal control theory [13], the optimal solution for G_r is given by

$$G_r(t) = R_r^{-1}\, K_r(t) \;;r=1,2,\ldots,n \quad (26)$$

where $K_r(t)$ is the solution of the Riccati equation:

$$\dot{K}_r(t) = -K_r A_r - A_r^T K_r + K_r R_r^{-1} - Q_r \;;r=1,2,\ldots,n \quad (27)$$

with boundary condition $K_r(T) = 0$.

From [4] the solution for $G_r(t)$ was obtained by

$$G_r = \begin{bmatrix} 0 & 0 \\ \omega_r - b & -[2\omega_r(-\omega_r + b) + r_r^{*-1}]^{1/2} \end{bmatrix} \quad (28)$$

where
$$\left.\begin{aligned} r_r^* &= r_r/\omega_r^2 \\ a_r &= (\omega_r r_r^*)^2 + r_r^* \\ b &= (\omega_r^2 + r_r^{*-1})^{1/2} \;;\; r=1,2,\ldots,n. \end{aligned}\right\} \quad (29)$$

We note that IMSC requires that the number of actuators be equal to that of modelled modes.

3. ACTUATOR IMPLEMENTATION OF INDEPENDENT MODAL SPACE CONTROL

Equations (19) and (20) represent the concept of IMSC. In order to implement (20), the modal state vectors x_r for $r=1,2,\ldots,n$ must be available for measurement. In [5] a modal filter was developed to provide an estimate of the modal state vectors. Since this paper focuses on the problem of the implementation of actuators, we assume that the modal state vectors $x_r(t)$ are available for the state feedback in order to avoid the complexity of getting involved in the state estimation problem that can well be the subject of a subsequent paper.

Since it is impossible to control force at every point in the domain D, the distributed control force is realized by m ($m<n$) discrete point force actuators applied at m points $P_1, P_2, \ldots, P_m$ in the domain D as given below:

$$f(P,t) = \sum_{i=1}^{m} \delta(P-P_i)\; F_i(t) \quad (30)$$

where $\delta(P-P_i)$ is a spatial Dirac Delta function and $F_i(t)$ is the force applied by the ith actuator on the point P_i.

Now substituting (30) into (10) yields

$$f_r(t) = \int_D \phi_r(P) \sum_{i=1}^{m} \delta(P-P_i) F_i(t)\; dD \quad (31)$$

From the property of the Dirac Delta function, (31) can be reduced to

$$f_r(t) = \sum_{i=1}^{m} (P_i)\; F_i(t). \quad (32)$$

If we define a force vector $F(t)$ such that

$$F(t) = [\, F_1(t)\; F_2(t) \ldots F_m(t)]^T, \quad (33)$$

then using (32) the relation between $F(t)$ and $W(t)$ can be expressed by

$$W(t) = B\, F(t) \quad (34)$$

where $B = [B_1^T \; B_2^T \; \ldots \; B_{2n}^T \,]^T$ (35)

$$B_{(2i-1)} = 0_{1xm};\; r=1,2,\ldots,n \qquad (36)$$

$$B_{2i} = [\phi_i(P_1)/\omega_i \;\; \phi_i(P_2)/\omega_i \;\; \ldots \;\; \phi_i(P_m)/\omega_i] \qquad (37)$$

for r=1,2,...,n.

4. PROBLEM STATEMENT

The implementation of the IMSC scheme is illustrated in Figure 1 where the optimal state feedback law is defined by

$$H(t) = G\,x(t) \qquad (38)$$

where $H(t) = [H_1^T(t) \; H_2^T(t) \; \ldots \; H_n^T(t) \;\;]^T$ (39)

and $G = \text{Block diag}(G_1,\, G_2,\, \ldots,\, G_n).$ (40)

The optimal solution for the IMSC scheme was obtained in Section 3 as:

$$W(t) = G\,x(t). \qquad (41)$$

The optimal solution is achieved if (41) is satisfied. In order to make W(t) equal to Gx(t), the matrix D in Fig. 1 is designed such that W(t) = H(t). From Fig. 1 we also have

$$F(t) = D\,H(t). \qquad (42)$$

Now substituting (42) into (34), we obtain

$$W(t) = B\,D\,H(t). \qquad (43)$$

In (43) to make W(t) = H(t), it is obvious that D is designed such that

$$B\,D = I_{2n} \qquad (44)$$

From the structure of B as given by (35)-(37), each (2i-1)th row (odd row) of BD, for i=1,2,...,n is a row of zeros. We realize that (44) can never be satisfied. However, noting that each odd row of W(t) is also a row of zeros, if we define

$$\bar{B}(t) = [B_2^T \; B_4^T \ldots \; B_{2n}^T]^T \qquad (45)$$

and $\bar{D}(t) = [D_2 \;\; D_4 \ldots \;\; D_{2n}]$ (46)

where B_i is the ith row of B and D_i the ith column of D, then choosing a matrix D such that

$$\bar{B}\,\bar{D} = I_n \qquad (47)$$

will ensure that W(t) = H(t). It is noted that if (47) holds, then BD is a modified identity matrix of order (2nx2n) whose main diagonal elements are 0 at the (2i-1,2i-1) position and 1 at the (2i,2i) position for i=1,2,...,n.

One obvious solution for (47) is to choose D such that

$$\bar{D} = \bar{B}^{-1}. \qquad (48)$$

However for the inverse of $\bar{B}$ to exist, $\bar{B}$ must be a nonsingular square matrix, requiring that the number of actuators be equal to the number of modelled modes (m=n). Since the number of modelled modes is usually very huge for a faithful modelling of LSS, the required number of actuators is practically unrealizable.

The problem considered in this paper can be formulated as to design the matrix $\bar{D}$ for a nonsquare matrix $\bar{B}$ (m<n) such that (47) is satisfied as well as possible. In other words, if this can be done, then the LSS can be controlled by a reduced number of actuators such that the closed-loop system is as optimal as possible.

5. MAIN RESULT

Lemma1: Consider the following equation:

$$\bar{W}(t) = \bar{B}\bar{F}(t) \qquad (49)$$

where $\bar{W}(t)$ and $\bar{F}(t)$ are matrices consisting of even rows of W(t) and F(t), respectively. If the (nxm) matrix B has rank m, then the solution for (49) that minimizes the weighted norm of error

$$\|e(t)\|_S^2 = \|\bar{W}-\bar{B}\bar{F}\|_S^2 = (\bar{W}-\bar{B}\bar{F})^T S(\bar{W}-\bar{B}\bar{F}) \qquad (50)$$

is given by

$$\bar{F}(t) = (\bar{B}^T S\bar{B})^{-1}\bar{B}^T S\bar{W}(t). \qquad (51)$$

The matrix B^* given by

$$B^* = (\bar{B}^T S\bar{B})^{-1}\bar{B}^T S \qquad (52)$$

is called the generalized inverse of $\bar{B}$.

Proof: A proof of the above Lemma can be found in [12].

The main result of this paper is given in the following theorem:

Theorem 1 : Consider a large flexible space structure whose description and solution are given by (1) and (11), respectively. If the operator L is self-adjoint, then there exists a control scheme with m (m<n) actuators that is suboptimal with respect to (24) in the sense that the closed-loop eigenvalues are assigned as closed as possible to those optimal eigenvalues specified by IMSC.

Proof: A control scheme with a recuced number of actuators would be optimal if $\bar{D}$ could be selected to be a right inverse of $\bar{B}$ in (47). However based on the form of $\bar{B}$ given in (45) and (37) we can assume that $\bar{B}$ has rank m since the discrete actuators apply point forces at m distinct points $P_1, P_2, \ldots, P_m$. From [12] it is well-known that an (nxm) matrix (m<n) having rank m does not possess any right inverse. Consequently B does not have any right inverse. According to Lemma 1, because $\bar{F}(t)$ as given in (51) minimizes (50), selecting a matrix $\bar{D} = B^*$ will minimize the difference $\bar{B}\bar{D}-I_n$, making (47) be satisfied as well as possible. Selecting $\bar{D} = B^*$ also make the closed-loop eigenvalues as identical as possible to those specified by IMSC. Thus there exists a control scheme with a reduced number of actuators that is suboptimal with respect to (24). Q.E.D.

6. EXAMPLE

To illustrate the proposed control scheme we consider the control of a simply supported beam whose dynamic is given by the Euler-Bernoulli partial differential equation:

$$EI(\partial^4/\partial x^4)u(x,t) + m(\partial^2/\partial t^2)u(x,t) = f(x,t) \qquad (53)$$

where for simplicity we set the mass m, the moment of inertia I, thae modulus of elasticity E and the length of the beam to unity. The boundary conditions fo this simply supported beam are:

$$u(0,t) = u(1,t) = 0 \qquad (54)$$

$$\partial^2/\partial x^2 u(0,t) = \partial^2/\partial x^2 u(1,t) = 0 \qquad (55)$$

The solutions for the eigenvalue problem are given by:

$$\lambda_k = (k\pi)^2 \qquad (56)$$

and $\phi_k = \sqrt{2}\,\sin(k\pi x).$ (57)

Computer simulation was done to test the performance of the proposed control scheme. Results show that the dynamics of the closed-loop system is affected considerably by the number and placement of actuators. Suppose we consider 20 modelled modes and divide the whole length of the beam into 20 sections specified by x(k)=k/21 for k=1,2,...,21. As expected, simulation results show that in every case the number of stable closed-loop eigenvalues increases with an increase in actuator number. The maximum numbers of stable closed loop eigenvalues when using 1, 2, 3, 4 actuators are 5, 18, 19 and 20, respectively. The remaining eigenvalues are pure imaginary complex conjugate pairs, thus causing no instability but unwanted oscillations. It is also found that the actuator locations maximizing the number of closed loop eigenvalues are centered around both ends of the beam, namely at x=1/21 and x=20/21. Fig. 2 presents the case for one actuator. As we see, the number of stable eigenvalues falls off as the actuator moves toward the center of the beam. Similar results were obtained for the cases of 2, 3 and 4 actuators. Table 1 provides the closed loop eigenvalues for IMSC with 20 actuators and the proposed control scheme with 4 actuators. As the table shows even with a reduced number of actuators the proposed control scheme assigns eigenvalues that are very close to those specified by the optimal IMSC. Figures 3 and 4 show the beam movement for the case of IMSC with 20 actuators and the proposed control scheme with 4 actuators, respectively, when the beam is excited by an impulse. As the figures show, the proposed control scheme with a reduced number of actuators performs as well as the IMSC with 20 actuators, bringing the beam movement down to zero after 5 seconds. The time responses of mode 1 and 3 are given in Figures 5 and 6, respectively. We note that the time response of mode 1 of the control scheme is almost the same as that of IMSC. There are some insignificant differences in the time response of mode 2. The maximum difference is about $3x10^{-3}$.

7. CONCLUSION:

In this paper, we first summarized the theory of IMSC in the context of optimal control. Then the principal limitation of IMSC was pointed out in terms of the required number of actuators, being equal to that of modelled modes. After that, we proposed a control scheme which employs a reduced number of actuators to control a large space structure that is self-adjoint. Computer simulation showed that the number and placement of actuators play an important role in the stability of the closed-loop system. The research of this paper can be extended into studying the optimization problem of actuator placement and designing a weighting matrix S that maximizes the number of stable closed-loop eigenvalues. The problem of arbitrary assignment of eigenvalues [8] can also be addressed for the control of LSS.

8. ACKNOWLEDGMENT

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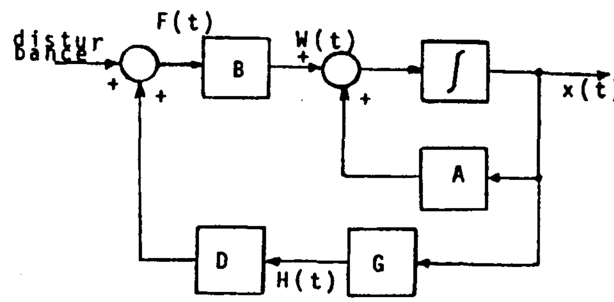

Fig. 1: Implementation of IMSC

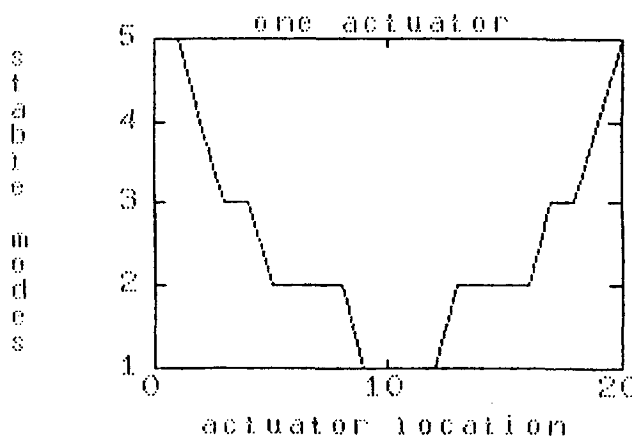

Fig. 2: Relationship between the number of stable closed-loop modes and and the number of actuators

1.0e+003 *

IMSC(20 Actuators)		Proposed control scheme(4 Actuators)	
-0.0014 + 0.0099i	-0.0047 + 1.1942i	-0.0001 + 3.9478i	-0.0009 + 0.9870i
-0.0014 - 0.0099i	-0.0047 - 1.1942i	-0.0001 - 3.9478i	-0.0009 - 0.9870i
-0.0020 + 0.0395i	-0.0049 + 1.4212i	-0.0004 + 3.5629i	-0.0008 + 0.7994i
-0.0020 - 0.0395i	-0.0049 - 1.4212i	-0.0004 - 3.5629i	-0.0008 - 0.7994i
-0.0024 + 0.0888i	-0.0051 + 1.6680i	-0.0007 + 3.1977i	-0.0007 + 0.6317i
-0.0024 - 0.0888i	-0.0051 - 1.6680i	-0.0007 - 3.1977i	-0.0007 - 0.6317i
-0.0028 + 0.1579i	-0.0053 + 1.9344i	-0.0010 + 2.8523i	-0.0006 + 0.4836i
-0.0028 - 0.1579i	-0.0053 - 1.9344i	-0.0010 - 2.8523i	-0.0006 - 0.4836i
-0.0032 + 0.2467i	-0.0055 + 2.2207i	-0.0011 + 2.5266i	-0.0006 + 0.3553i
-0.0032 - 0.2467i	-0.0055 - 2.2207i	-0.0011 - 2.5266i	-0.0006 - 0.3553i
-0.0035 + 0.3553i	-0.0057 + 2.5266i	-0.0010 + 2.2207i	-0.0007 + 0.2467i
-0.0035 - 0.3553i	-0.0057 - 2.5266i	-0.0010 - 2.2207i	-0.0007 - 0.2467i
-0.0037 + 0.4836i	-0.0058 + 2.8523i	-0.0008 + 1.9344i	-0.0007 + 0.1579i
-0.0037 - 0.4836i	-0.0058 - 2.8523i	-0.0008 - 1.9344i	-0.0007 - 0.1579i
-0.0040 + 0.6317i	-0.0060 + 3.1978i	-0.0007 + 1.6680i	-0.0012 + 0.0099i
-0.0040 - 0.6317i	-0.0060 - 3.1978i	-0.0007 - 1.6680i	-0.0012 - 0.0099i
-0.0042 + 0.7994i	-0.0062 + 3.5629i	-0.0008 + 1.4212i	-0.0004 + 0.0395i
-0.0042 - 0.7994i	-0.0062 - 3.5629i	-0.0008 - 1.4212i	-0.0004 - 0.0395i
-0.0045 + 0.9870i	-0.0063 + 3.9478i	-0.0009 + 1.1942i	-0.0005 + 0.0888i
-0.0045 - 0.9870i	-0.0063 - 3.9478i	-0.0009 - 1.1942i	-0.0005 - 0.0888i

Table 1: Closed-loop eigenvalues comparision

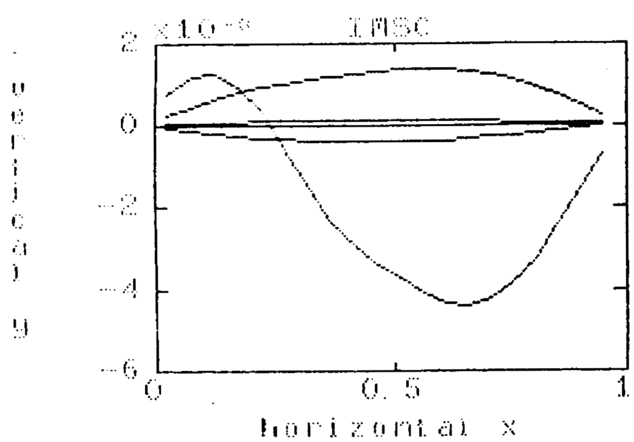

Fig. 3: Beam movement when the optimal IMSC is applied with 20 actuators.

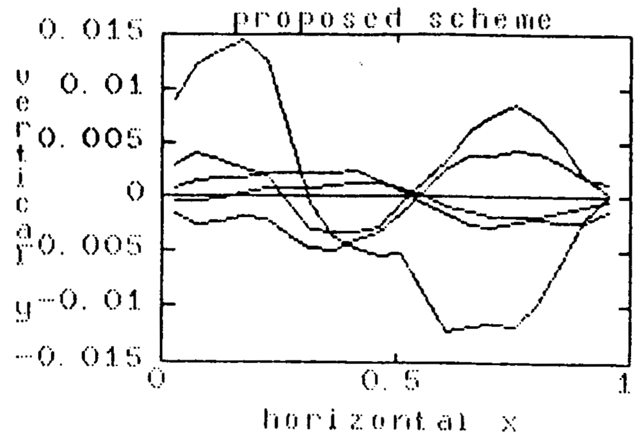

Fig. 4: Beam movement when the proposed control scheme is applied with 4 actuators.

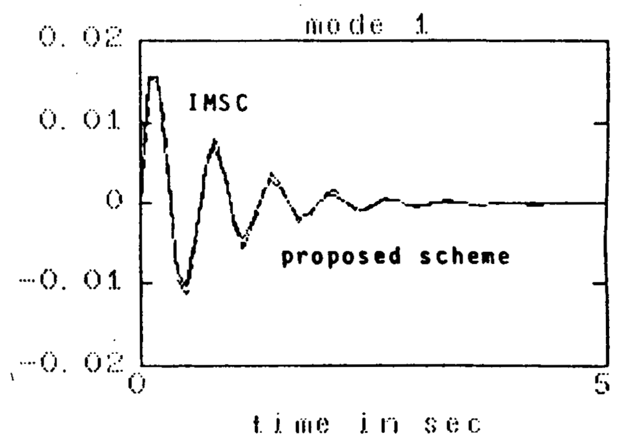

Fig. 5: Time response of the first mode

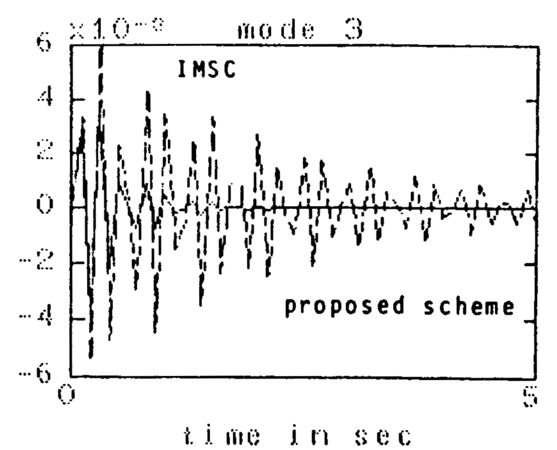

Fig. 6: Time response of the third mode