A STUDY OF THE MECHANISM OF INTERNAL GRAVITY WAVE GENERATION BY QUASIGEOSTROPHIC METEOROLOGICAL MOTIONS

A. S. Medvedev

Leningrad State University
Leningrad-Petrodvorets, USSR

Numerous experiments on the detection of atmospheric waves in the frequency range from acoustic to planetary at meteor heights have revealed that important wave sources are meteorological processes in the troposphere (cyclones, atmospheric fronts, jet streams, etc.). A dynamical theory based on the works of OBUKHOV (1949) and MONIN (1958) include describing the adaptation of meteorological fields to the geostropic equilibrium state. According to this theory, wave motions appear as a result of constant competition between the maladjustment of the wind and pressure fields due to non-linear effects and the tendency of the atmosphere to establish a quasi-geostrophic equilibrium of these fields.

Barotropic Atmospheric Model

To demonstrate the above, we consider an approximation of the atmosphere to be a two-dimensional liquid film located in the Coriolis force field. The equations for this model are obtained by averaging over height the equations of motion and continuity for a three-dimensional atmosphere (OBUKHOV, 1949). Then, passing over to dimensionless variables, we choose values corresponding to large-scale synoptic motions (OBUKHOV, 1949) by way of the scales of length L and velocity W. The natural scale $(2 \omega_u)^{-1}$ corresponding to OBUKHOV's (1949) characteristic adaptation time will serve as the temporal scale. In the dimensionless variables we have equations:

$$\frac{\partial U}{\partial \tau} + R_o \left(U \frac{\partial U}{\partial \zeta} + V \frac{\partial U}{\partial \eta} \right) - V = - \frac{\partial \Pi}{\partial \zeta} \; ; \; \frac{\partial V}{\partial \tau} + R_o \left(U \frac{\partial V}{\partial \zeta} + V \frac{\partial V}{\partial \eta} \right) + U = - \frac{\partial \Pi}{\partial \eta} \; ;$$

$$\frac{\partial U}{\partial \tau} + R_o \left(U \frac{\partial \Pi}{\partial \zeta} + V \frac{\partial \Pi}{\partial \eta} \right) + \frac{D}{\beta^2} = 0 \; ; \quad D = \frac{\partial V}{\partial \zeta} + \frac{\partial V}{\partial \eta} \; , \quad (1)$$

where $2\omega_z$ is a Coriolis parameter; U, V are the wind velocity components; $\zeta = x/L$; $\eta = y/L$; $U = u/W$; $V = v/W$; $\Pi = c^2 x/2\omega_z WL$;

$$\tau = 2\omega_z t \quad ; \quad x = \ln (p/p_o);$$

p is pressure, p_o is a standard pressure near the earth. The dimensionless parameters of Rossby-Kibel $R_o = W/2\omega_z L$ and $\beta = 2\omega_z L/c$ have been introduced into (1). To analyze the system (1), we use a small Rossby number for motions of the synoptic scale and apply the method of asymptotic series and multiple time scale successive approximations (BLUMEN, 1972; JEFFREY and KAWAHARA, 1982), according to which we introduce, apart from the "fast" dimensionless time $\tau = 2\omega_z t$, the "slow" dimensionless time $T = R_o \tau = tw/L$ and change

$$\partial/\partial\tau \rightarrow \partial/\partial\tau \; + Ro \; \partial/\partial T$$

The zero-order system in Ro

$$\frac{\partial U_o}{\partial\tau} - V_o = \frac{\partial\Pi_o}{\partial\zeta}; \quad \frac{\partial V_o}{\partial\tau} + U_o = -\frac{\partial\Pi_o}{\partial\eta}; \quad \frac{\partial\Pi_o}{\partial\tau} + \frac{D_o}{2} = 0; \quad D_o = \frac{\partial U_o}{\partial\zeta} + \frac{\partial V_o}{\partial\eta} \quad (2)$$

was investigated by OBUKHOV (1949). OBUKHOV showed that a complete solution of the nonlinear system (2) is the superposition of the non stationary waves and the stationary geostrophic components.

In the first-order approximation in Ro the system (1) takes the form:

$$\frac{\partial U_1}{\partial\tau} + \frac{\partial\Pi_1}{\partial u} - V_1 = -\frac{\partial V_o}{\partial\tau} - U_o\frac{\partial V_o}{\partial u} - V_o\frac{\partial V_o}{\partial\eta}; \frac{\partial V_1}{\partial\tau} + \frac{\partial\Pi_1}{\partial\eta} + U_1 = -\frac{\partial V_o}{\partial T} - U_o\frac{\partial V_o}{\partial u} - V_o\frac{\partial V_o}{\partial u};$$

$$\text{(3)}$$

$$\frac{\partial\Pi_1}{\partial\tau} + \frac{D_1}{\beta^2} - V_1 = -\frac{\partial\Pi_o}{\partial T} - U_o\frac{\partial\Pi_o}{\partial u} - V_o\frac{\partial\Pi_o}{\partial\eta}; \quad D_1 = \frac{\partial U_1}{\partial u} + \frac{\partial V_1}{\partial\eta} \quad .$$

In the analysis of wave generation within the flux under the influence of nonlinear terms we consider the atmosphere in a quasi-adjusted state when the process of adaptation either has been completed and the wave component has already been scattered in space or the wave motion has not occurred at all due to the initial adjustment of the meteorological fields. Therefore, in what follows, we shall consider the zero-order terms in (3) to be purely geostrophic and independent of the "fast" time τ. Proceeding in the usual way we obtain from (3) equations for the vorticity

$$\Omega_1 = \partial V_1/\partial\zeta - \partial U_1/\partial\eta$$

and divergence D_1

$$\partial D_1/\partial + \Delta\Pi_1 - \Omega_1 = -q_o; \quad \partial\Omega_1/\partial + D_1 = -\partial\Omega_1/\partial T - S_o; \quad \Delta = \partial^2/\partial\zeta^2 + \partial^2/\partial^2\eta;$$

$$q_o = \frac{\partial U_o}{\partial\zeta}^2 + 2\frac{\partial U_o}{\partial\eta}\frac{\partial V_o}{\partial\eta} + \frac{\partial V_o}{\partial\eta}^2; \quad S_o = (\vec{v}_o \; \nabla\omega_o) = U_o\frac{\partial\Omega_o}{\partial\zeta} + V_o\frac{\partial\Omega_o}{\partial\eta} \quad (4)$$

By excluding D_1 from the third equation of (3) and the second equation of (4), we obtain

$$\frac{\partial}{\partial\tau}(\Omega_1 - \beta^2\Pi_1) = -\frac{\partial\Pi_o}{\partial T} + \frac{1}{\beta^2}\frac{\partial\Omega_o}{\partial T} + \frac{1}{\beta^2}(\vec{v}_o \; \nabla\Omega_o) \quad (5)$$

In the right-hand side of (5) there is a function T, in the left-hand one there are functions τ and T. Therefore, to eliminate divergence in (5), according to the requirements of the method of multiple time scales, we assume that the right- and left-hand sides of (5) separately equal zero. By equating to zero the right-hand side of (5) we obtain the equation of slow quasi-geostrophic evolution of a potential vortex (BLUMEN, 1972). Then, from the condition of the left-hand side of (5) being zero, we obtain $\Omega_1 - \beta^2 \Pi_1 = 0$. Thus, the first order terms in Ro in the expansion (5) are purely wave terms. Making use of the condition $\Omega_1 - \beta^2 \Pi_1 = 0$, it is possible to derive wave equations with a nonlinear forcing term on the right-hand side. For instance, the equation for Ω_1 has the form:

$$\frac{\partial^2 \Omega_1}{\partial \tau^2} - \frac{\Delta \Omega_1}{\beta^2} + \Omega_1 = q_o \tag{6}$$

The expression on the right-hand side of (6) describes the rate of wave generation by the background quasi-geostrophic motions. This process exists owing to the nonlinear transfer of energy between motions of different types: slow quasi-geostrophic and fast waves.

Influence Functions of Tropospheric Meteorological Sources

We can obtain an equation for the spectral density of the vertical velocity W (GAVRILOV, 1985) for an isothermal three-dimensional atmosphere:

$$\frac{d^2 W}{d\zeta^2} + n^2 W = F_w(\zeta, \omega, k_x, k_y); \quad \zeta^2 = \zeta/H; \quad k^2 = kx^2 + k_y^2 ;$$

$$n^2 = (N^2 - \omega^2)/(\omega^2 - \ell^2) \; k^2 H^2 + \omega^2 H^2/C_s^2 - 1/4;$$

$$F_w = \frac{iH^2}{\omega^2 - \ell^2} \left(\frac{1}{H} \frac{d}{d\zeta} - \frac{N^2}{g} \right) (\omega/Q - i\ell S) \exp(-\zeta/2); \tag{7}$$

where N is the Brunt frequency; W, Q, and S are the Fourier transforms of the vertical components of wave velocity W and the source terms (4) q and s, respectively; ω, k_x, k_y are the frequency and horizontal projections of the wave vector, respectively. Analysis of (7) shows that the contribution of the source Q increases in the region of high frequencies. For the acoustic frequencies $\omega \gg 2\omega_z$ the main contribution to the generation of acoustic waves is made by Q, the expression for which coincides with that for the source of the sound generation by turbulence obtained by LIGHTHILL (1952). For the low frequency IGW the contribution of the addendum containing s is essential whereas the values of Q and S are smaller and of the same order.

Using the condition of radiation $W \sim \exp(i\eta u)$ at $u \to \infty$ and being interested in a solution for large heights only, one can find the solution of (7) by the method proposed in KAMKE, (1976) and GAVRILOV (1985):

$$W = - e^{i\eta\zeta} I; \quad I = \int_{0}^{\zeta} \frac{\sin n\nu}{n} F_w(\nu) \, d\nu \qquad (8)$$

Here the wave field of the vertical velocity W is the Fourier integral:

$$w = e^{\frac{\zeta}{2}} \iiint_{-\infty}^{\infty} \exp[i(\omega t - k_x x - k_y y + n\zeta)] \int_{0}^{\zeta} \frac{\sin n\nu}{n} F_w(\nu) \, d\nu \, d\omega \, dk_x \, dk_y \qquad (9)$$

In (9) we proceed from integration over spectral variables Ω, k_x, k_y to integration over the natural variables t, x, y. With this in view, we use the theorem of the Fourier-transform of the convolution of functions and find the influence functions of the sources q and S:

$$G_i(\nu, \zeta, x, y, t) = \iiint_{-\infty}^{\infty} \frac{\sin n\nu}{n} \exp[i(\omega t - k_x x - k_y y + n\zeta)].$$

$$\frac{iH^2}{\omega^2 - 4\omega_u^2} \phi_i(\omega, \ell) \, d\omega \, dk_x \, dk_y; \quad \phi_1 = \omega; \quad \phi_2 = - i\ell \qquad (10)$$

The expressions for W through G_1 and G_2 have the form:

$$-W(x, y, t, \zeta)e^{-\zeta/2} = \int_{0}^{u} G_1 \, \theta \left(\frac{1}{H} \frac{d}{d\nu} \frac{N^2}{g} \right) q(\nu)e^{-\nu/2} \, d\nu +$$

$$\qquad (11)$$

$$+ \int_{0}^{\zeta} G_1 \, \theta \left(\frac{1}{H} \frac{d}{d\nu} \frac{N^2}{g} \right) s(\nu)e^{-\nu/2} \, d\nu$$

where the symbol θ denotes the convolution of functions in the variables x, y, t. It can be seen from (11) that the wave amplitudes increase exponentially with height. Therefore, of interest is the behavior of G_1 and G_2 at $u \to \infty$. To calculate the integral

$$= \iint_{-\infty}^{\infty} \frac{\sin n\nu}{n} \exp[i(n\zeta - k_x x - k_y y)] \, dk_x \, dk_y, \qquad (12)$$

contained in (10) we use an asymptotic expansion by the stationary phase method (FELSEN and MARCUVITZ, 1973) at $\zeta \to \infty$. We obtain, for example, for G_1

$$G_1 = - \frac{(2\pi)^{3/2}H}{\sqrt{3Nt}\ R}(1 - \mu)^{2/3}{}^{-1/4} \sin(\nu\ \frac{\zeta H}{2R}\ \overline{\frac{\mu^{2/3}}{1-\mu^{2/3}}}\ )\cos[Nt(1-\mu^{2/3})^{3/2} - \frac{\pi}{4}\ ]$$

$$- \frac{(2\pi)^{3/2}H\ c^*/R}{NR\ \sqrt{t}}(1-\mu^2)^{-3/4} \sin(\nu\ \frac{\zeta H}{2R}\ \frac{N(1-\mu^2)^{1/2}\ R}{c^*\mu})\cos[c^*t/R\cdot(1-\mu^2)^{1/2} + \frac{\pi}{4}]$$

$$(13)$$

where $\mu = R/(2HN)$; $R = \sqrt{x^2 + y^2}$; $c^* = 4R^2\omega_z^2 + N_\zeta^2 H^2$.

The expression for G_2 is derived in a similar way but is omitted here because of its bulkiness. The first addendum in (13) describes the high frequency mode of IGW with the frequencies $\omega \sim N$, and the second addendum is the lower frequency waves with periods of the order of hours.

The described procedure of obtaining the Green's function was used in the calculation of the wave field of the vertical velocity W for a typical meteorological formation, such as the cyclonic vortex. The mathematical model of the latter is given by the expression

$$p = p_o - \delta p\cdot\exp(- t^2/\tau_o^2 - x^2/a^2 - y^2/b^2); \qquad (14)$$

where the cyclonic pit depth p changes with height

$$p(u) = p_n \exp[-(z - z_S)^2/h^2]$$

In (14) p_o is the standard pressure at a given height; a, b are the scales of the vortex length along the corresponding axes, τ_o is the vortex characteristic lifetime; z_s is the vortex maximum height; k is the vertical halfwidth of the source. In the calculations the values which correspond to mesoscale synoptical vortices of the moderate type were used: a = 50 km, b = 100 km, h = 2 km, δp_m = 20 m bar, z_s = 8 km, τ_o = 3 hrs. Fig. 1 shows the directional pattern of radiation in the plane x, y for the vertical velocity W. It can be seen that the maximum radiation intensity takes place along the semimajor axis of the vortex. The ratio of the radiation intensities along the main semiaxes increases in proportion to the square of the a/b ratio. Fig. 2 shows a horizontal cross-section along the x axis of the wave field of the vertical velocity W. It can be seen that the vertical velocity values reach tens of cm s^{-1}, and the decrease of the amplitude is due to the source attenuation with time. The temporal variation of W at the point x = 1,500 km, y = 0.3, H = 100 km is presented in Fig. 3. It can be seen that oscillations at a given point occur with a period T ~ 80 min and are modulated by a wave with the period T ~ 16 hrs.

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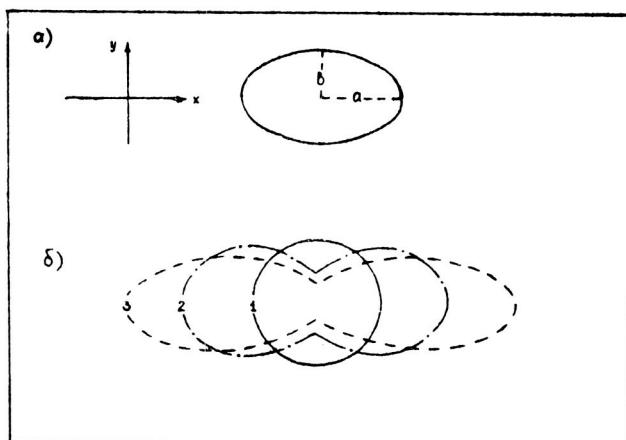
Fig. 1 Radiation orientation direction diagram (δ) by
a vortex (α): 1 - for a/b = 1; 2 - for a/b = 2;
3 - for a/b = 3.
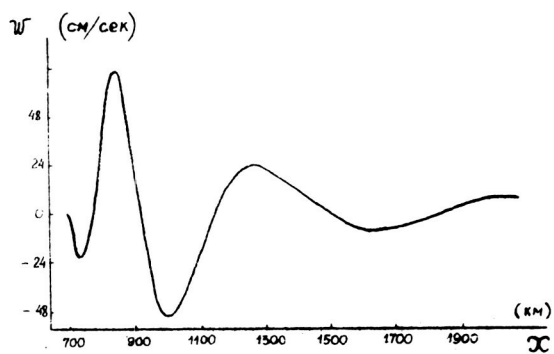

Fig. 2 A horizontal cross-section of vertical velocity
wind field ω caused by a mesoscale vortex.

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