ON METEOR STREAM SPATIAL STRUCTURE THEORY

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The classical spatial representation of meteor streams is an elliptical torus with variable cross-section. The position of this torus in space is determined by the mean orbit elements that may be obtained directly from observations of individual meteor stream particles when crossed by the earth. Since the orbits of individual particles of a stream differ from each other, the distance between them on a plane normal to the mean orbit of elliptical torus forms some area, i.e., a cross-section. The size and form of these cross-sections change with the change of the direction among the mean orbit and are completely defined by the dispersion values of the orbit elements in a stream.

This paper deals with an attempt to create an analytical method that would permit description of the spatial and time parameters of meteor streams, i.e., the form and size of their cross-sections, density of incident flux and their variations along the mean orbit and in time. In this case, the stream is considered as a continuous flux rather than a set of individual particles.

The solution to this problem lies in creation of a model of a stream structure that depends on the type of parental matter decay and the stage of a stream evolution. We suggest the following three models giving analytical descriptions of the spatial structure of a meteor stream (ANDREEV 1984a, 1984b).

1. The stream is comparatively young and perturbations have not yet changed its structure. It is clear that in such a model all the orbits intersect at the ejection point.

2. The stream is comparatively old and the perturbations now show up as changes of the meteoroid orbit elements with their orbits intersecting, not at the ejection point but along some arc.

3. The stream is so old that it is impossible to distinguish the common area of crossing orbits but a distinct meteor stream is still spatially defined.

Since the analytical developments of the above models are about the same and the final formulas in some cases are rather bulky, we shall confine our exposition of the elements of our spatial structure theory using as an example only the first model.

Suppose the stream is formed by means of the parental body decay within an infinitesimal volume of its original orbit. Let a, e, i, Ω, ω be Kepler's orbit elements of parental matter having the velocity V at the decay point. Analysis shows that convenient formulas for estimating the velocity increment of particle orbital elements as compared with the comet orbit may be obtained, if the comet orbit plane is taken as the main plane

and the apex of comet motion is taken as the main direction. Let u be the velocity vector magnitude of matter ejection directed at an angle β to the main plane with a projection on the plane of the comet orbit making an azimuthal angle α with the direction to the apex. Vector addition of velocities gives the following variation of the velocity magnitude and its direction

$$\frac{\Delta V}{V} = (1 + \frac{U^2}{V^2} + 2\frac{U}{V} \cos \alpha \cdot \cos \beta)^{1/2} - 1, \qquad (1)$$

$$\sin \Delta\psi = U(V^2 + U^2 + 2UV \cos \alpha \cdot \cos \beta)^{-1/2} \cdot \sin \alpha \cdot \cos \beta \qquad (2)$$

In the formula (2), $\Delta\psi$ is the change of angle of ψ between the radius-vector of r and the velocity vector of V.

Using the expressions (1, 2) we obtained the following correlations for estimating the variation of orbit elements:

a) increments connected with reorientation of the velocity vector in the orbit plane:

$$\Delta a = 2 \frac{a^2}{\mu} V \cdot \Delta V,$$

$$\Delta e = 2 \frac{\Delta V}{V} (e + \cos v) + \frac{r}{a} \Delta\psi \cdot \sin V, \qquad (3)$$

$$\Delta\omega = 2 \frac{\Delta V}{eV} \sin V - (2e + \cos V + e^2 \cos V)(1 + e \cos V)^{-1} \cdot \frac{\Delta\psi}{e},$$

b) increments connected with the reorientation of the orbit plane:

$$\Delta i = \frac{r}{\sqrt{\mu p}} U \cdot \cos (V + \omega) \cdot \sin \beta,$$

$$\Delta\Omega = \frac{r}{\sqrt{\mu p}} U \cdot \sin (V + \omega) \cdot \csc i - \sin \beta = \tan (V + \omega) \cdot \csc i \cdot \Delta i, \qquad (4)$$

$$\Delta\omega = - \frac{r}{\sqrt{\mu p}} U \cdot C \tan i \cdot \sin (V + \omega) \cdot \sin \beta = - \Delta\Omega \cdot \cos i$$

The formulas given in the systems (3 and 4) enable one to analyze easily the orbit elements changes at ejection.

The formulas (1, 2, 3, 4) give the change of an individual particle orbit. For formation of a stream of particles, it is necessary to vary these relations using different values of u, α, β (depending on the required peculiarities of stream formation) and to find the stream's mean orbit and the dispersion of orbit elements. Suppose particles are ejected so that the relative velocities of particles and their directions are within the intervals of $U \pm \sigma u$, $\alpha \pm \sigma\alpha$, $\beta \pm \sigma\beta$. Then the elements of the mean orbit of a stream are ($U \ll V$)

$$\bar{a} = a + \frac{2a^2}{\mu} UV \cdot \cos \alpha \cdot \cos \beta,$$

$$\bar{e} = e + 2 \frac{U}{V} (e + \cos V) \cos \alpha \cdot \cos \beta + \frac{rU}{aV} \sin V \cdot \sin \alpha \cdot \cos \beta,$$

$$\bar{i} = i + \frac{rU}{\sqrt{\mu p}} \cos (V + \omega) \cdot \sin \beta,$$

(5)

$$\bar{\omega} = \omega + \frac{2U}{eV} \sin V \cdot \cos \alpha \cdot \cos \beta - \frac{U}{V} (1 + \frac{\cos E}{e}) \sin \alpha \cdot \cos \beta - \frac{rU}{\sqrt{\mu p}} \cdot$$

$$\frac{\sin (V + \omega)}{\tan i} \cdot \sin \beta,$$

$$\Omega = \Omega + \frac{rU}{\sqrt{\mu p}} \sin(V + \omega) \cdot \mathrm{cosec}\, i \cdot \sin \beta, \quad \cos E = (e + \cos V)(1 + e \cos V)^{-1},$$

and the orbit element dispersions in the stream are defined by means of differentiation of (5) with respect to u, α, β.

If the ejection is considered isotropic rather than directed then the formulas for determining the root-mean-square dispersions of the orbit elements will look like:

$$\sigma a = \frac{a^2}{\mu} V(U^2 + \sigma^2 U)^{1/2}, \quad \sigma e = \frac{1}{2V} (V^2 + \sigma^2 U)^{1/2} [4(e + \cos V)^2 + r^2 a^{-2} \cdot$$

$$\sin^2 V]^{1/2},$$

$$\sigma i = \frac{r}{\sqrt{\mu p}} \cdot \frac{(\sigma^2 U + U^2)^{1/2}}{2} \cdot \cos (V + \omega), \quad \sigma \Omega = \tan (V + \omega) \; \mathrm{cosec}\, i \cdot \sigma i,$$

$$\sigma \omega = \frac{1}{2} (U^2 + \sigma^2 U)^{1/2} \left[(2 \frac{\sin V}{Ve})^2 + (\frac{e + \cos E}{Ve})^2 + \frac{r^2}{\mu p} \cdot \sin^2 (V + \omega) \cdot \right.$$

$$\left. \cotan^2 i \right]^{1/2},$$

and the mean orbit will coincide with that of the parental body.

The formulas (1-5) make it possible to model analytically any matter ejection out of a parental body, and to estimate the mean orbit and the orbit element dispersions about it. However, to study the meteor stream evolution as a whole, it is necessary to have the appropriate analytical formulas to calculate the mean orbit evolution and orbit elements dispersions. For the mean orbit, this calculation is not difficult. In studying the evolution of meteor streams during long time intervals, only secular perturbations that generate systematic change of stream

characteristics are of interest. For numerical investigation of the secular evolution of the mean orbit, one can apply the well-known Halphen-Goryachev method. But it is very difficult to calculate the secular evolution of the orbit element dispersions. However, as our investigations have shown, this problem may be solved by means of intermediate calculations using the Halphen-Goryachev algorithm for each step in time. Indeed, if the initial values of orbit elements and their dispersions are denoted by the zero subscript, new values of the Keplerian elements at the time $t = t_o + \Delta t$ will equal $x(t) = x_o + \Delta x$, where x is any of the elements. It is not difficult then to conclude that the value of the dispersions of element x by the moment of t can be found from the expression:

$$\sigma x(t) = \left| \frac{\partial \partial(t)}{\partial x_o} \right| \sigma x = \left| 1 + \frac{\partial(\Delta x)}{\partial x_o} \right| \sigma x_o \qquad (6)$$

where particular derivatives related to orbit elements can be easily found by the Eulerian equations.

The next step is to obtain the analytical dependences of the form and size of cross-sections and their changes in space and time. The derivation of these formulas depends entirely on the model of stream origin and the evolutionary stage at which it occurs (ANDREEV, 1984). In particular, for a stream arising as a result of a "point" decay, the formula for determining the cross-sectional area may be obtained on the basis of independence of relative distance between orbits in the mean orbit plane and the plane orthogonal to it, i.e.,

$$S = \pi \cdot A \cdot B = \pi \cdot \Delta r \cdot \sin \psi \cdot r \cdot \Delta \phi \cdot \sin (V + \omega - c), \qquad (7)$$

where $A = \Delta r \cdot \sin \psi$ is the size ("width") of the cross-section in the orbit plane, $B = r \cdot \Delta \psi \cdot \sin (V + w - c)$ is the stream's thickness", c an angular distance of decay point from the ascending node and $\Delta \phi$ is a dihedral angle between extreme orbits at the ejection point. The radius-vector variation in an arbitrary direction may be obtained by means of the expression:

$$\Delta r = \left[\left(\frac{\partial r}{\partial a} \sigma a \right)^2 + \left(\frac{\partial r}{\partial e} \sigma e \right)^2 + \left(\frac{\partial r}{\partial w} \sigma \omega \right)^2 \right]^{1/2}, \qquad (8)$$

where radius-vector partial derivatives related to elements occur provided that $\nu(a, e, \omega) = $ const. Representative values $\Delta \phi$ and c may be obtained both analytically and by analyzing orbit catalogues. In particular, applying Delambert's formulas we have

$$\tan^2 \frac{\Delta \phi}{2} = \frac{\tan^2 \frac{\sigma \Omega}{2} \cdot \sin^2 i + \sin^2 \frac{\sigma i}{2}}{\tan^2 \frac{\sigma \Omega}{2} \cos^2 i + \cos^2 \frac{\sigma i}{2}} , \quad \tan c = \tan \frac{\sigma \Omega}{2} \cdot \sin i \cdot \text{cosec} \frac{\sigma i}{2} \cdot$$

Upon substitution of (7) and (8) we shall have

$$(v) = \pi r^2 \Delta \phi \mid \sin \psi \cdot \sin(v + \omega + c) \mid \left[\left(\frac{\sigma a}{a} \right)^2 + (e + \cos E)^2 \left(\frac{\sigma e}{1 - e^2} \right)^2 \right.$$

$$\left. + \text{cotan } \psi \cdot \sigma \omega)^2 \right]^{1/2}$$

The formula (9) makes it possible to determine the cross-section. The mean orbit elements and their dispersions are calculated in the expression (9) according to expression (5) or are determined from existing orbit catalogues. Application of the above relations permits investigation of the stream evolution as a whole. If it is supposed that the number of particles in a steam does not change over time and denote the incident flux density by Q, we have the following explicit expression:

$$\frac{Q(v_1 t_1)}{Q(v_1 t_2)} = \frac{S(v_1 t_2)}{S(v_1 t_1)} \cdot \frac{a^{3/2}(t_2)}{a^{3/2}(t_1)} \tag{10}$$

which makes it possible to estimate the flux density change not only along the orbit (when it is constant) but in time as well. That is the main factor for estimating the meteor stream dynamics as a whole rather than just its individual orbits. It is clear that the space structure and dynamics of comparatively young meteor streams or streams of a common origin but which are only slightly susceptible to perturbations can be investigated by means of the above expressions. It can be expected that among modern meteor streams there are those to which the above method can be applied. To this end, only those streams have been considered which have sufficient orbit statistics. It turns out that the majority of modern active meteor streams are characterized by signs of parent body decay within a relatively small vicinity of their mean orbits.

Table 2 presents the ecliptic coordinates (λ_c, B_c) of the common "point" of orbit crossing, the nodal distance value C, radius-vector of this point r_c, and its true anomaly v_c.

Table 2.

Coordinates of common range of crossing orbits
for some meteor streams

		Perseids	Geminids	Orioids	S-Taurids
λ_c	, degree	80 ± 5	190 ± 11	307 ± 3	130 ± 10
B_c	, degree	74 ± 2	-22 ± 1	15 ± 7	5 ± 1
C	, degree	77 ± 2	-72 ± 10	82 ± 8	79 ± 6
V_c	, degree	-74 ± 4	-40 ± 10	0 ± 10	-40 ± 12
r_c	, a.u.	1.5 ± 0.1	0.16 ± 0.01	0.6 ± 0.3	0.4 ± 0.1

It should be noted that in spite of insufficient accuracy of the present orbit catalogues, the errors in determining the point C are insignificant and the arc length, where the supposed decay of a parent body occured, does not exceed 0.1 per cent of the orbit length for the showers under

consideration. Hence, knowing from observation the structure peculiarities of the above meteor streams, a space model for these streams can be constructed for a given moment by means of the relations (1-10). The main direction and ejection rate of matter from their parent bodies can be determined and the evolution of these streams as affected by gravitational and nongravitational forces can be traced.

References

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