J2

N88-14928

S2-02
117226
19P.

FINITE-VOLUME SCHEME FOR TRANSONIC POTENTIAL FLOW ABOUT AIRFOILS AND BODIES IN AN ARBITRARILY SHAPED CHANNEL

Jerry C. South, Jr.
NASA Langley Research Center
Hampton, Virginia

Michael L. Doria
Valparaiso University
Valparaiso, Indiana

Lawrence L. Green
NASA Langley Research Center
Hampton, Virginia

[redacted]

ABSTRACT

A conservative finite-volume difference scheme is developed for the potential equation to solve transonic flow about airfoils and bodies in an arbitrarily shaped channel. The scheme employs a mesh which is a nearly conformal "O" mesh about the airfoil and nearly orthogonal at the channel walls. The mesh extends to infinity upstream and downstream, where the mapping is singular. Special procedures are required to treat the singularities at infinity, including computation of the metrics near those points. Channels with exit areas different from inlet areas are solved; a body with a sting mount is an example of such a case.

INTRODUCTION

This presentation describes a "Finite-Volume Scheme for Transonic Potential Flow About Airfoils and Bodies in an Arbitrarily Shaped Channel" by Jerry C. South, Jr.; Michael L. Doria; and Lawrence L. Green (ref. 1). The work was done primarily while Dr. Doria was working as an ASEE Research Fellow in the Theoretical Aerodynamics Branch. A 1982 AIAA paper by Doria and South (ref. 2) explains the basic formulation which is summarized here. This work focuses on several improvements which have made the scheme more useful and accurate.

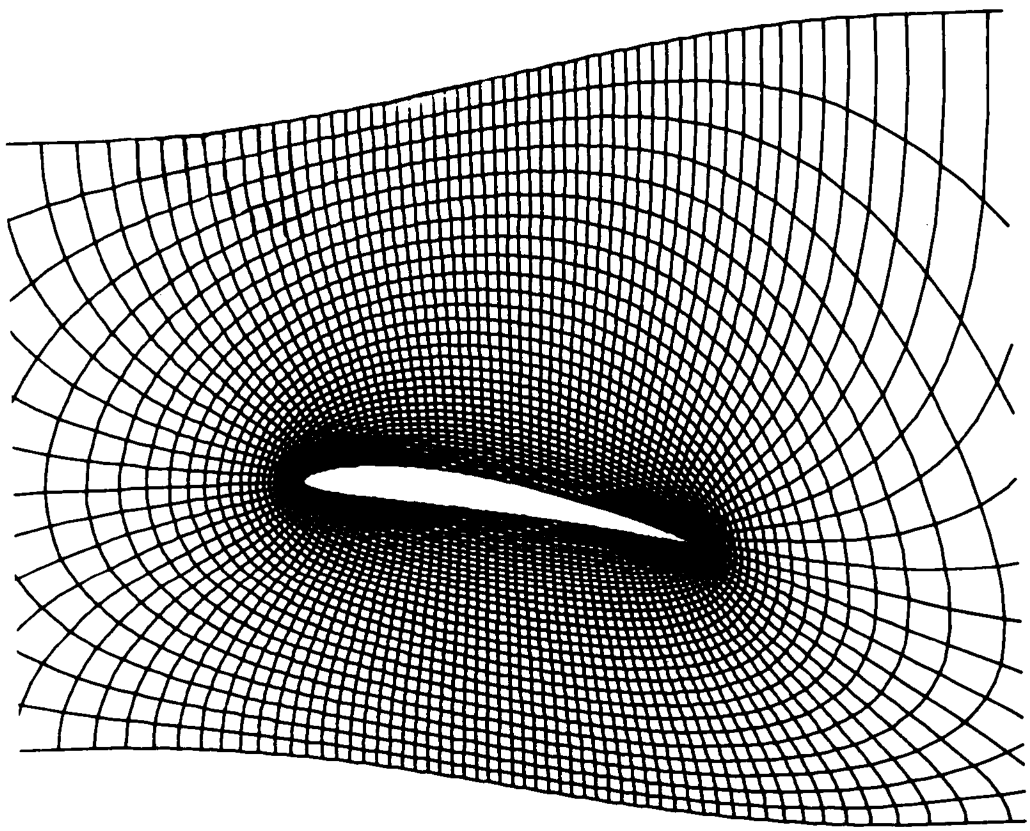
GRID GENERATION

The grid generation procedure uses a sequence of Schwarz-Christoffel and shearing transformations, first proposed by Caughey (ref. 3), to map the physical airfoil-in-channel problem to a uniform rectangular computational domain. The mapping results in a nearly orthogonal "O"-type mesh extending from the airfoil surface to the tunnel wall. The mapping provides for grid point clustering near the airfoil and particularly at the leading and trailing edges. The grid generation procedure now accepts very general body and channel shapes (2-D or axisymmetric) which can be described either analytically or by input coordinates which are spline-fitted.

- Sequence of Schwarz-Christoffel and shearing transformations proposed by Caughey (ref. 3)
- Nearly orthogonal "O" - type mesh
- Clustering of grid points at L. E. and T. E.
- Accepts very general body and channel shapes

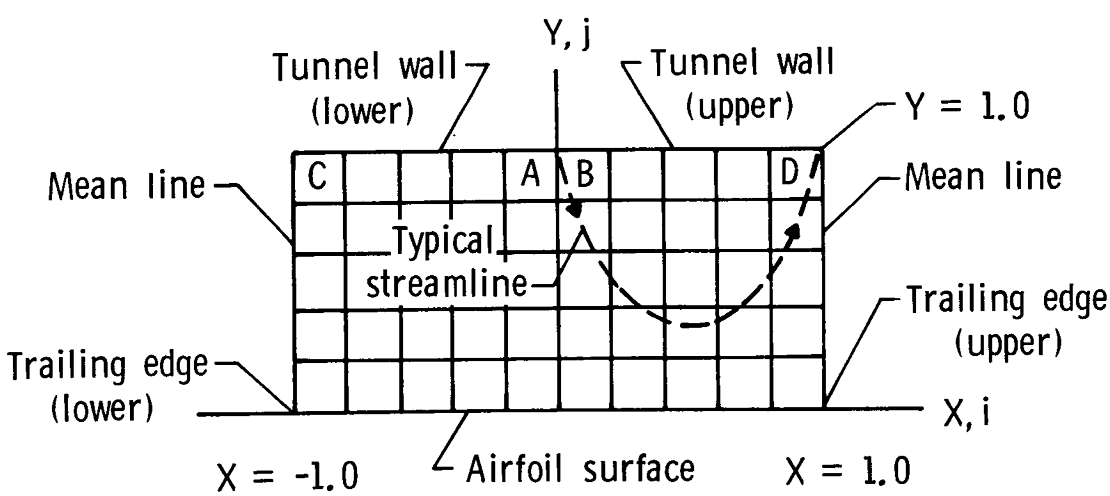
GRID--NACA 4409 IN DIVERGING TUNNEL

This "O"-type grid for an NACA 4409 airfoil in a diverging tunnel demonstrates many of the grid generation procedure capabilities. The airfoil is cambered, at angle of attack, and offset from the tunnel centerline. Coordinates for the airfoil were input and spline-fitted. Also, the tunnel wall is rather arbitrarily shaped. One set of coordinate lines forms rings between the body surface and the tunnel wall. Another set of coordinate lines emanates from the body and terminates at the tunnel wall. One grid line emanates from the body but extends to upstream or downstream infinity, where the mapping is singular and special procedures are required. Grid points are clustered near the leading and trailing edges. The grid produced is twice as fine as that on which the flow problem is solved to allow for accurate metric calculation by central differences.



COMPUTATIONAL DOMAIN

This is a sketch of the computational domain for the airfoil-in-tunnel problem. The airfoil surface is mapped onto the X-axis. The tunnel wall is mapped into the line Y = 1, with the lower wall being on the left and upper wall on the right. Coordinate lines which form rings in the physical plane are mapped into lines of constant Y, and those coordinate lines radiating from the body in the physical plane are mapped into lines of constant X. The line in the physical plane extending to upstream infinity is mapped into the Y-axis and the line extending to downstream infinity is split between X = 1 and X = -1. A typical streamline starting at upstream infinity, passing over the airfoil and terminating at downstream infinity is shown. Cells A, B, C, and D each have as one corner a point in the physical plane at infinity where the mapping is singular and special procedures, described subsequently, are required.



Cells A, B, C and D have singular points at corners

GOVERNING EQUATIONS IN CARTESIAN COORDINATES

The continuity equation is written in Cartesian coordinates for either planar 2-D flow if $\sigma = 0$ or axisymmetric flow if $\sigma = 1$. Assuming isentropic flow, the density is given as a function of the Mach number, M_∞; the total velocity, q; and the ratio of specific heats, γ. Assuming irrotationality, the streamwise and normal components of velocity are related to a disturbance potential, ϕ, by the expressions shown.

$$(y^\sigma \rho u)_x + (y^\sigma \rho v)_y = 0$$

$$\rho = \left[1 + .5(\gamma - 1) M_\infty^2 (1 - q^2)\right]^{1/(\gamma - 1)}$$

$$u = 1 + \phi_x$$

$$v = \phi_y$$

$\sigma =$ 0 for planar 2D

1 for axisymmetric

COORDINATE TRANSFORMATION

For a generalized transformation between physical coordinates (x,y) and computational coordinates (X,Y), the metrics g_{11}, g_{12}, and g_{22} and the Jacobian are shown. In a perfectly orthogonal mapping, g_{12} would be zero. For the mapping considered here, g_{12} is several orders of magnitude smaller than g_{11} and g_{22}. The partial derivative x_Y in g_{22} will require special treatment near the singular points at infinity.

$$x = x(X,Y) \qquad y = y(X,Y)$$

$$g_{11} = (x_X)^2 + (y_X)^2$$

$$g_{12} = x_X x_Y + y_X y_Y$$

$$g_{22} = (x_Y)^2 + (y_Y)^2$$

$$J = x_X y_Y - x_Y y_X$$

GOVERNING EQUATIONS IN CURVILINEAR COORDINATES

The transformed governing equation and expressions for the contravariant velocity components in terms of the disturbance potential are shown. The component U is in the direction around the airfoil in the physical plane and V is in the direction normal to the airfoil surface and tunnel wall. Notice that U depends on g_{22} which requires special treatment near the singular points at infinity. Also, since the mapping is not orthogonal, g_{12} is not zero and U and V both depend on ϕ_X and ϕ_Y.

$$(y^{\sigma}\rho J U)_X + (y^{\sigma}\rho J V)_Y = 0$$

$$JU = y_Y + (g_{22}\Phi_X - g_{12}\Phi_Y)/J$$

$$JV = -y_X + (-g_{12}\Phi_X + g_{11}\Phi_Y)/J$$

FINITE DIFFERENCE EQUATIONS

The mass balance for a typical four-sided cell is shown, using compass-point notation (N, S, E, and W) to designate the cell faces. The retarded-density formulation is used to provide numerical stability in supersonic regions and to allow for shock capturing. In this formulation, the isentropic density, ρ, is replaced by a retarded density, $\bar{\rho}$, which is shifted upwind in the streamwise direction, ξ, if the local Mach number is greater than unity. The contravariant components of velocity are expressed in terms of the metrics and the disturbance potential; the resulting simultaneous equations for ϕ are solved iteratively by AF2, ZEBRA I, or VLOR.

Retarded density method

$$(y^{\sigma}\bar{\rho}JU)_E - (y^{\sigma}\bar{\rho}JU)_W + \alpha(y^{\sigma}\bar{\rho}JV)_N - \alpha(y^{\sigma}\bar{\rho}JV)_S = 0$$

$$\bar{\rho} = \rho - \mu\rho_{\xi}\Delta\xi$$

$\mu = f(M) = 0$ at subsonic points

$0 < \mu < 1$ at supersonic points

$$\alpha = \Delta X/\Delta Y$$

Express JU and JV in terms of Φ, g_{11}, g_{12} and g_{22}

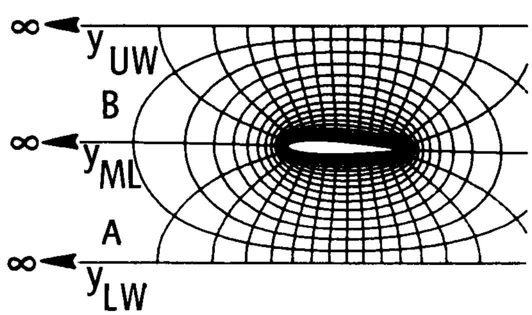
solve for Φ using AF2, ZEBRA I or VLOR

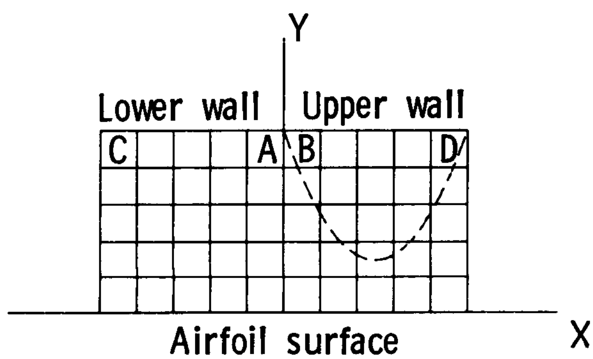
ADDED MASS SOURCES/SINKS

Cells A, B, C, and D contain singularities at one corner and are actually five-sided cells in the physical domain since they extend to upstream or downstream infinity. The mass flowing across the fifth face of these cells has not been accounted for in writing the finite-difference equations, and it is necessary to include for these cells a source or sink term as shown in the mass balance equation. The form of these source or sink terms can be rigorously derived from the form of the singularity of the mapping at these points and must be included to calculate flows in channels where the inlet area is different from the exit area, such as diverging or converging tunnels.

$$S_A = -(\rho_{IN} u_{IN}) \left[(y_{ML})_{IN} - (y_{LW})_{IN} \right]$$

$$S_B = -(\rho_{IN} u_{IN}) \left[(y_{UW})_{IN} - (y_{ML})_{IN} \right]$$





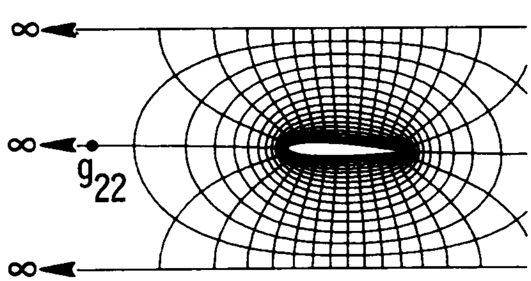
LOGARITHMIC DIFFERENCING OF METRICS

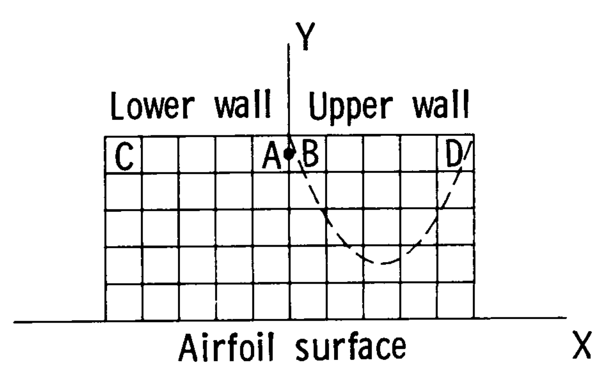
The mass flux across the face adjoining cells A and B (or equivalently, cells C and D) is not, in general, zero and should be calculated as part of the solution. This requires calculating the U contravariant velocity component through this face which implies evaluating g_{22} in the cell face. However, as the computational coordinate $Y \to 1$ along $X = 0$, the physical coordinate x behaves logarithmically in Y. Since the physical x becomes negatively infinite when $Y = 1$, that is, at the upstream singular point, central differencing to obtain x_Y along this face is impossible. Instead, the derivative is evaluated using the form shown, derived from the logarithmic behavior of x in Y along the coordinate line leading to the singularity. Without this modification, the solution could not be converged fully since the maximum residual would "hang up" at fairly large values in cells A, B, C, and D.

As $Y \longrightarrow 1$ along $X = 0$, $\quad x = A \ln(1 - Y)$

$x(1) = -\infty$, central differencing impossible

instead use $(\delta x/\delta Y)_W = 2(x_W - x_{SW})/(\Delta Y \ \ln 2)$



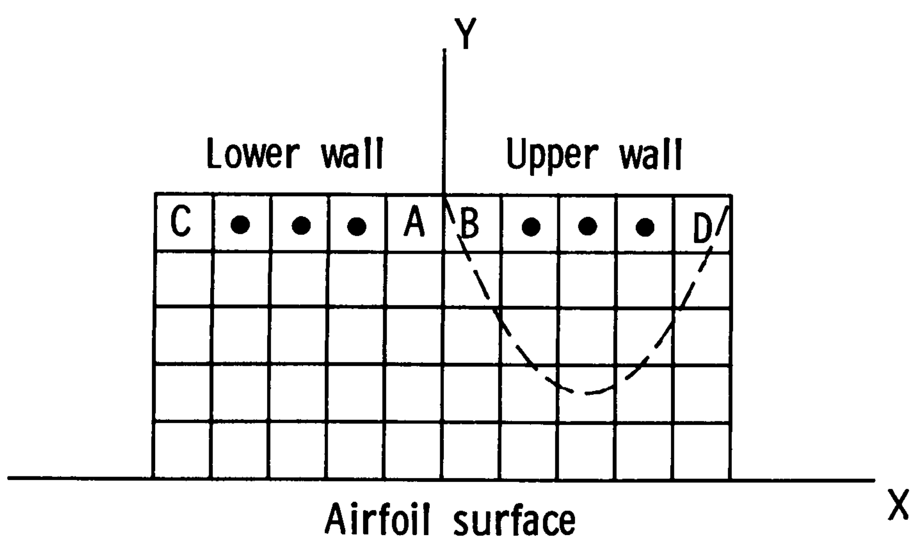


FREE-AIR BOUNDARY CONDITION

For cell faces along solid boundaries at the airfoil and tunnel walls, the mass flux is set to zero to enforce the boundary condition. This is equivalent to a Neumann or derivative boundary condition on ϕ. The far-field behavior in free air, however, is different from that found in a tunnel. Thus, an alternative, Dirichlet condition for ϕ in the outer ring has been included as an option; it properly simulates the far-field behavior in free air if applied sufficiently far away from the airfoil. The expression is derived from the disturbance potential for a compressible vortex. Use of this free-air boundary condition instead of the solid-wall boundary condition has the added benefit that the angle of attack can be changed without remapping the problem, as is necessary for in-tunnel cases.

$$\Phi_{I,JMAX-1} = \frac{\Gamma}{2\pi}\left[\pi - \tan^{-1}(\beta \tan\theta)\right]$$

$$\beta = \left[1 - (M_\infty)^2\right]^{1/2}, \qquad \theta = \left[0, 2\pi\right]$$

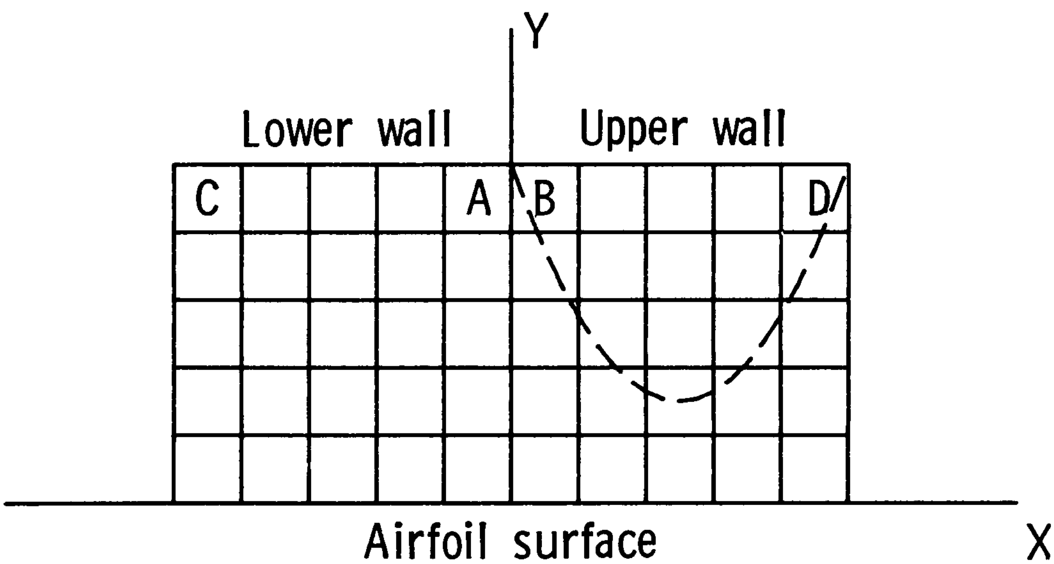


EXIT CONDITION FOR SUPERSONIC FLOW

The final improvement involves specifying the value for the disturbance potential in cells C and D, which is consistent with supersonic flow at the exit. In supersonic flow, all signals propagate downstream only. No influence can be felt upstream from a point farther downstream. To simulate this behavior at a supersonic exit, it is assumed that $\phi_{XX} = 0$. The potential in cells C and D is expressed in terms of those upwind on the same ring. This is substituted into the tridiagonal system and solved using the horizontal scheme ZEBRA I. Thus, the exit mass flux is allowed to adjust to conditions upstream of the exit. Anomalous Mach numbers appeared near the exit if this boundary condition was not used, although convergence was achieved.

$$\left(\Phi_{XX}\right)_{ij} = \left(\Phi_{i-1,j} - 2\Phi_{ij} + \Phi_{i+1,j}\right) / \Delta X^2$$

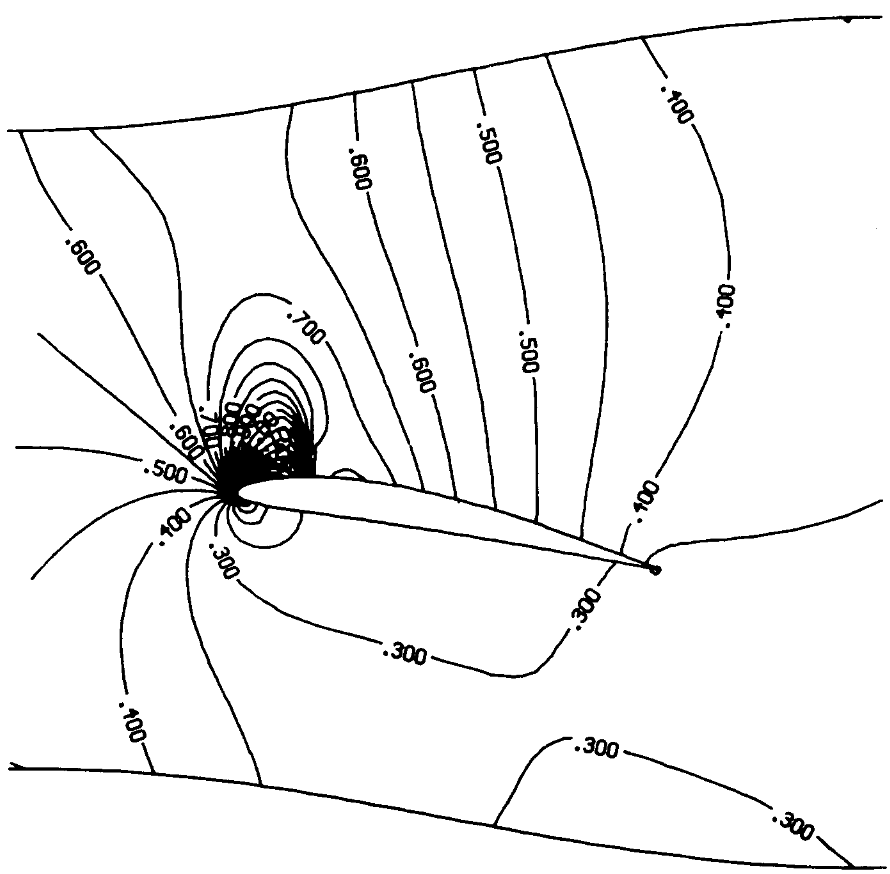
- Solve for Φ in cell D from $\Phi_{XX} = 0$
- Substitute into horizontal tridiagonal system



NACA 4409 IN DIVERGING TUNNEL

Computed constant Mach number contours around an NACA 4409 airfoil in a diverging tunnel for an upstream Mach number of 0.5 are shown. The airfoil is at 8 degrees angle of attack, has camber, and is offset from the tunnel centerline. There is a supersonic region around the upper surface leading edge and a strong shock at about 10 percent of the airfoil chord. Downstream of the divergence the flow has a Mach number lower than the inlet Mach number, as would be expected from continuity.

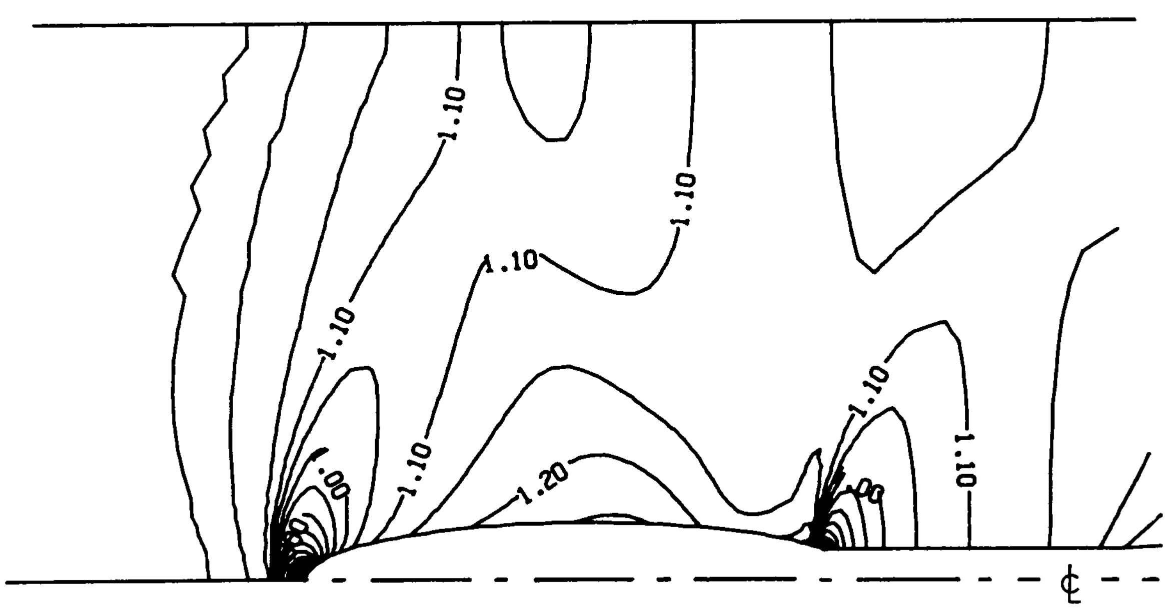
$M_\infty = 0.5, \ \alpha = 8°$



ELLIPSOID ON STING

Computed constant Mach number contours are shown for an axisymmetric case of an ellipsoid with a sting in a straight wall tunnel with supersonic flow at the inlet and exit. For the upstream Mach number of 1.15, a bow shock can be seen ahead of the ellipsoid with an embedded subsonic region between the bow shock and the body. Another subsonic region appears near the ellipsoid/sting junction. If either one of these subsonic regions, surrounded by supersonic flow, was large enough to intersect the wall, the problem would be ill-posed with the current boundary conditions and would eventually diverge. Typical of axisymmetric cases, only the upper half of the physical domain is computed.

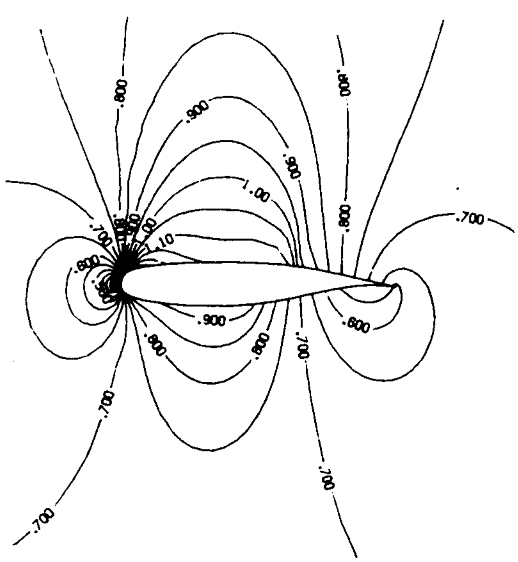
$M_\infty = 1.15,\ H = 1$

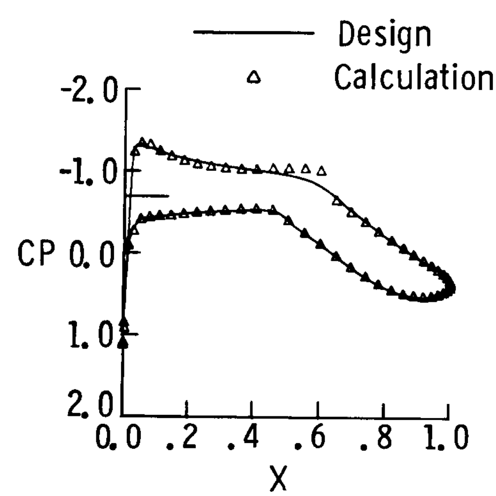


NLR 7301 IN FREE AIR

The computed constant Mach number contours and surface pressure coefficient for the NLR 7301 airfoil are shown. The upstream Mach number is 0.721 and the angle of attack is -0.194 degrees. The airfoil was designed in the hodograph plane to be shock free at these conditions; it is a quite sensitive flow to compute. The present solution shows a weak shock and a slightly underdeveloped supersonic region relative to the design. The calculated lift coefficient is 0.610 compared to the design of 0.595. It is interesting to note that the design point for this airfoil lies in the range of nonuniqueness for the conservative full-potential equation, as shown by Salas and Gumbert (ref. 4).

$M_\infty = 0.721, \ \alpha = -0.194°$





SUMMARY

A conservative finite-volume scheme for transonic potential flow around bodies in an arbitrarily shaped channel, including converging or diverging walls and stings, has been presented. The concept of logarithmic differencing to obtain metrics near singularities has been applied to correct a previous convergence problem. A free-air Dirichlet-type boundary condition has been added as an option. An extrapolation-type boundary condition for supersonic flow at the exit has been included. More details of these and other aspects of this procedure can be found in a 1982 AIAA paper (ref. 2) and in a 1985 paper from the 3rd Symposium on Numerical and Physical Aspects of Aerodynamic Flows (ref. 1).

- Conservative, finite-volume scheme for transonic potential flow
- Arbitrary body and channel shapes (converging/diverging tunnel, flared-sting)
- Logarithmic differencing (convergence)
- Free-air boundary condition
- Exit condition for supersonic flow

REFERENCES

1. South, Jerry C., Jr.; Doria, Michael L.; and Green, Lawrence L.: Finite-Volume Scheme for Transonic Potential Flow About Airfoils and Bodies in an Arbitrarily-Shaped Channel. Conf. Proc. 3rd Symposium on Numerical and Physical Aspects of Aerodynamic Flows, Long Beach, CA, Jan. 1985.

2. Doria, Michael L.; and South, Jerry C., Jr.: Transonic Potential Flow and Coordinate Generation for Bodies in a Wind Tunnel. AIAA Paper 82-0223, Jan. 1982.

3. Caughey, D. A.: A Systematic Procedure for Generating Useful Conformal Mappings - Application to Transonic Aerodynamics. Int. J. Num. Meth. in Engr., vol. 12, no. 11, 1978, pp. 1641-1657.

4. Salas, Manuel D.; and Gumbert, Clyde R.: Breakdown of the Conservative Potential Equation. AIAA Paper 85-0367, Jan. 1985.