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NUMERICAL SIMULATION OF A CONTROLLED BOUNDARY LAYER

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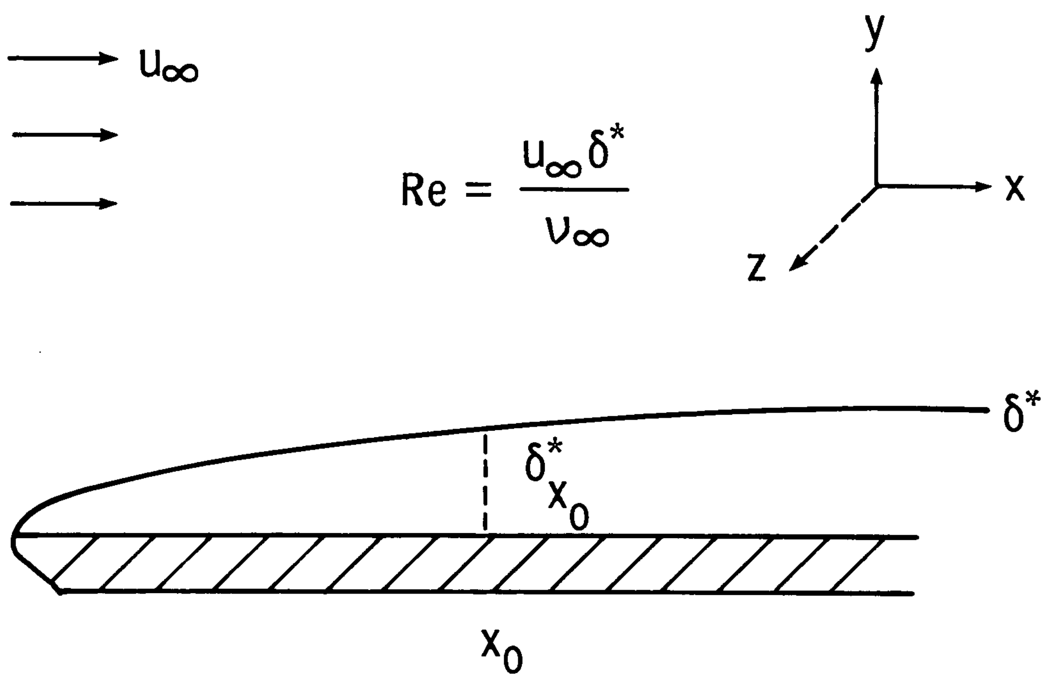
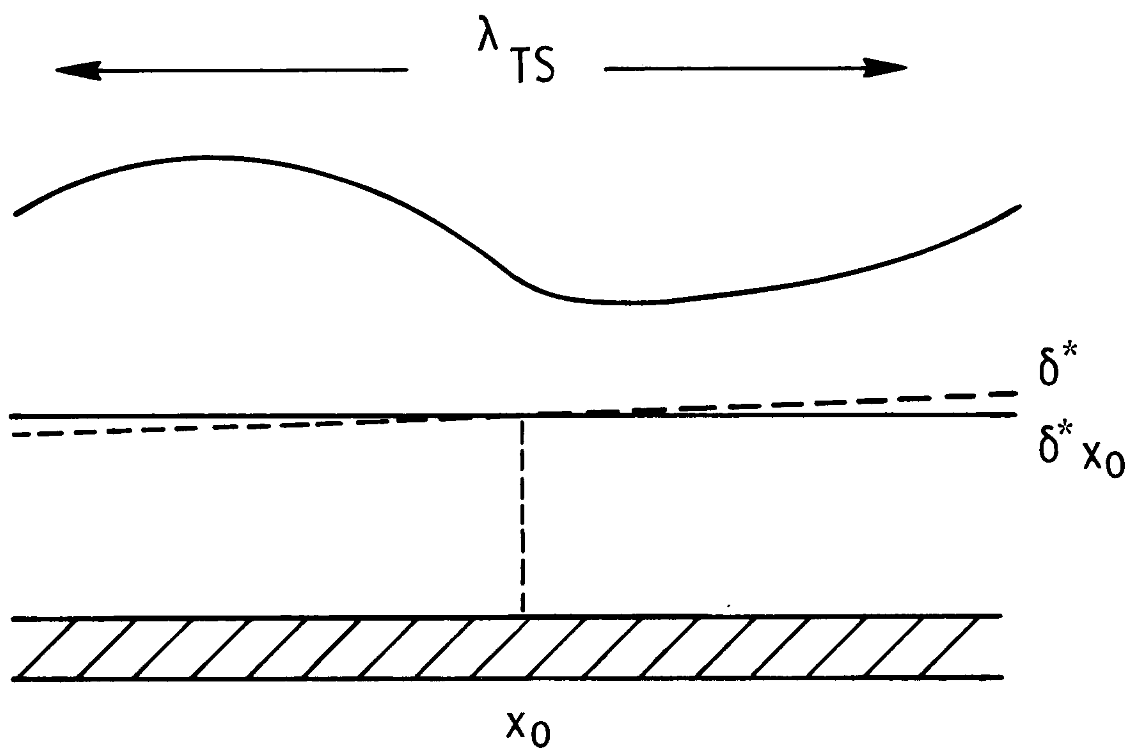
[redacted]WING BOUNDARY LAYER

The problem of interest is the boundary layer over a flat plate. The boundary layer grows in space and the transition process is one which occurs in space (in the streamwise direction along the plate). The figure illustrates the growth of the displacement thickness δ^*. The Reynolds number Re depends on x and is based on the free-stream velocity u_∞, the displacement thickness, and the free-stream kinematic viscosity ν_∞. With present computers, 3-D nonlinear simulations of the growing boundary layer can cover only a very small portion of the transition process due to the extreme demands on resolution in the streamwise direction.



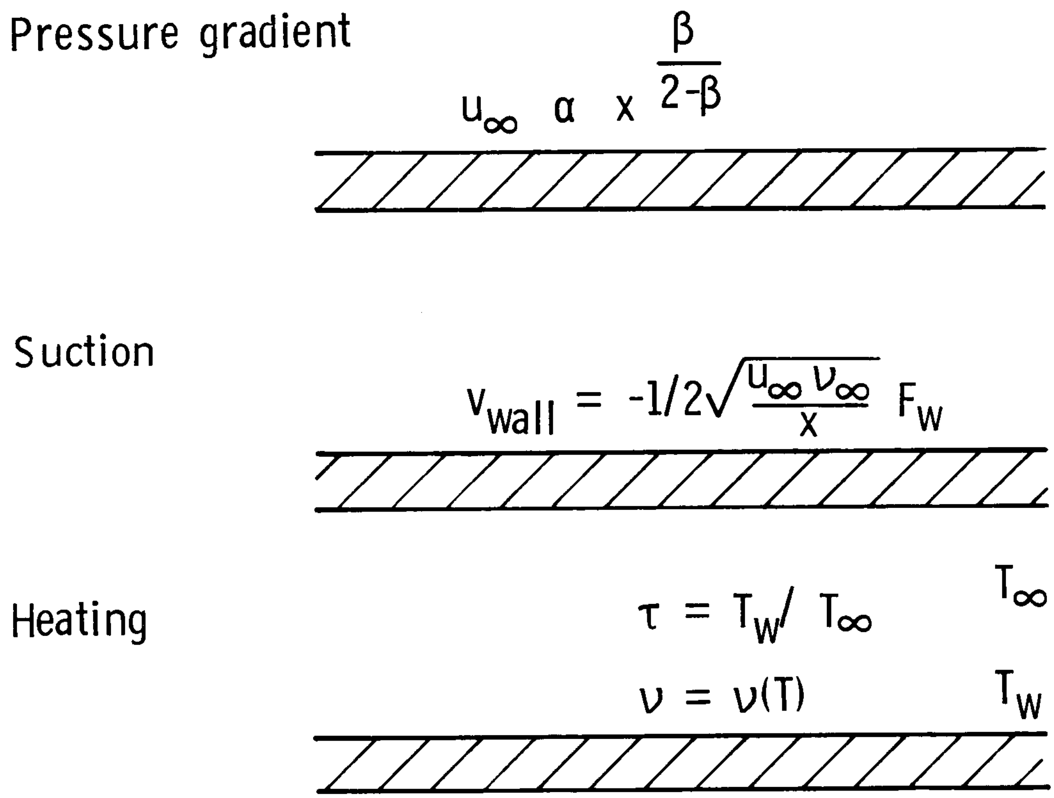
PARALLEL BOUNDARY LAYER

The parallel flow assumption ignores the growth of the boundary layer in the streamwise direction. It reduces the resolution demands by making feasible the use of periodic boundary conditions in the streamwise direction. Some typical point x_o is chosen as the reference location, and the computational domain extends over 1 or 2 Tollmein-Schlichting (TS) wavelengths λ_{TS}.



BOUNDARY LAYER CONTROLS

The three standard laminar flow control (LFC) techniques are pressure gradient, suction, and heating. The figure illustrates how each technique affects the mean flow. It also introduces the parameters used to describe the amount of control in the context of the boundary layer equations: β for the pressure gradient, F_w for suction, and τ for heating. The latter influences the flow through the dependence of the viscosity upon temperature.



CONTROLLED, PARALLEL BOUNDARY LAYER

The parallel flow assumption permits the use of Fourier series in the streamwise (x) and spanwise (z) directions. The wavenumbers α and β are real. The basic equations describing the mean flow are discussed on the figure.

- Parallel flow assumption

$$U(X, Y, Z, T) = \hat{U}(Y)\, e^{i(\alpha X + \beta Z - \omega T)}$$

- Mean flow described by Faulkner-Skan equation with pressure gradient, suction and/or heating controls
- Viscosity and conductivity based on empirical formulas for water
- Reynolds numbers based on displacement thickness and free-stream conditions

NUMERICAL METHODS

Numerical methods are required to find the mean flow, the linear eigenvalues of the Orr-Sommerfeld equation, and the full, nonlinear, 3-D solution of the Navier-Stokes equations. The specific numerical methods used in this work are outlined on the figure.

- Nonlinear mean flow

  4th-order compact finite difference scheme

- Linear modes

  Chebyshev Tau method (ref. 1)

- Navier-Stokes solution

  - Fourier-Chebyshev collocation in space
  - 3rd-order Adams-Bashforth on explicit terms
  - Crank-Nicholson on implicit terms (vertical diffusion, pressure and continuity)

NAVIER-STOKES ALGORITHM

The full, nonlinear 3-D, incompressible codes have been implemented on the NASA Langley VPS 32 in both 32-bit and 64-bit arithmetic. The characteristics of the machine and the performance of the code are listed on the chart for 64-bit arithmetic. The storage and speed figures are twice as large for 32-bit arithmetic. Calculations have been performed on 128^3 grids with the Fourier-Chebyshev code in 32-bit arithmetic. They take 25 sec/step and run at a sustained speed of 220 MFLOPS. About 25% of the CPU time is devoted to transposing the data. Hence, the actual computations are being performed at nearly 300 MFLOPS.

- Implemented on VPS 32
  - Cyber 205 architecture (2 pipes)
  - 16 million words (64-bit)
  - 200 Mflops peak speed (64-bit)
  - 80-120 Mflops sustained (64-bit)
- Performance of the Fourier-Chebyshev code
  - 32^3 grid: 2.5 sec/step
  - 64^3 grid: 10 sec/step
- Performance of the Fourier-FD code
  - 32^3 grid: 0.5 sec/step
  - 64^3 grid: 3.0 sec/step

INITIAL CONDITIONS

The initial conditions for the nonlinear simulations consist of the mean flow (U_O) plus a combination of 2-D and 3-D eigenvalues. The eigenfunctions U_{2D} and U_{3D} are normalized so that their maximum value is 1.

$$
\begin{aligned}
U(X, Y, Z, 0) = RE \Big\{ & U_O(Y) \\
& + \varepsilon_{2D} U_{2D}(Y) e^{i\alpha X} \\
& + \varepsilon_{3D} U_{3D}(Y) e^{i(\alpha X + \beta Z)} \Big\} \\
& + \varepsilon_{3D} U_{3D}(Y) e^{i(\alpha X - \beta Z)}
\end{aligned}
$$

$$
\max_{Y} |U_{2D}(Y)| = \max_{Y} |U_{3D}(Y)| = 1
$$

FOURIER DECOMPOSITION

The importance of individual components of the flow field can be ascertained by examining the amplitudes of the horizontal Fourier harmonics of the solution. The individual harmonics are labelled by the rational numbers k_x and k_z which measure the horizontal wavenumbers relative to those present in the initial condition.

$$U(X, Y, Z, T) = \sum_{k_X} \sum_{k_Z} U_{k_X, k_Z}(Y, T) \, e^{i(k_X \alpha X + k_Z \beta Z)}$$

$$E_{k_X, k_Z}(T) = \int \left| U_{k_X, k_Z}(Y, T) \right|^2 DY$$

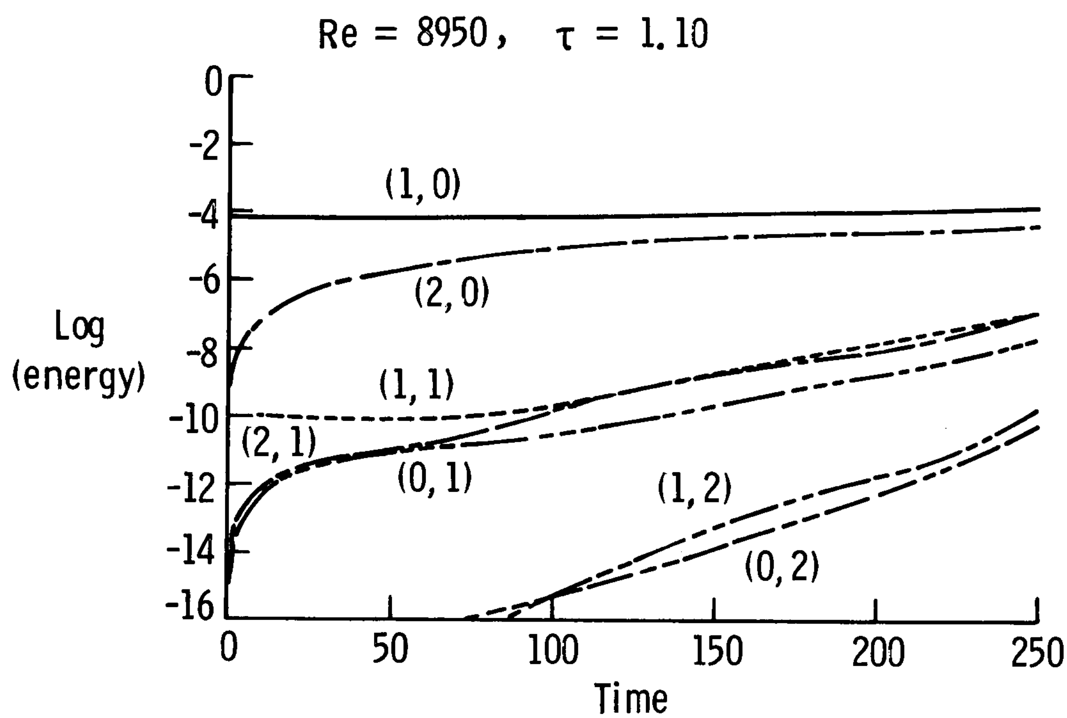
BOUNDARY LAYER MODES FOR Re = 8950

This table lists the linear TS waves which are used to form the initial conditions for the numerical simulation. The control parameters are $\beta = 0.55$, $F_w = 0.895$, and $\tau = 1.10$. The numerical code does an excellent job of reproducing the linear results.

Control	Mode	α	β	Linear growth rate	Computed growth rate
Pressure	TS 2-D	0.168	0.000	0.000095	0.000096
	TS 3-D	0.168	0.168	-0.001012	-0.001028
Suction	TS 2-D	0.162	0.000	0.000093	0.000093
	TS 3-D	0.162	0.162	-0.000968	-0.000993
Heating	TS 2-D	0.150	0.000	0.000093	0.000097
	TS 3-D	0.150	0.150	-0.000798	-0.000793

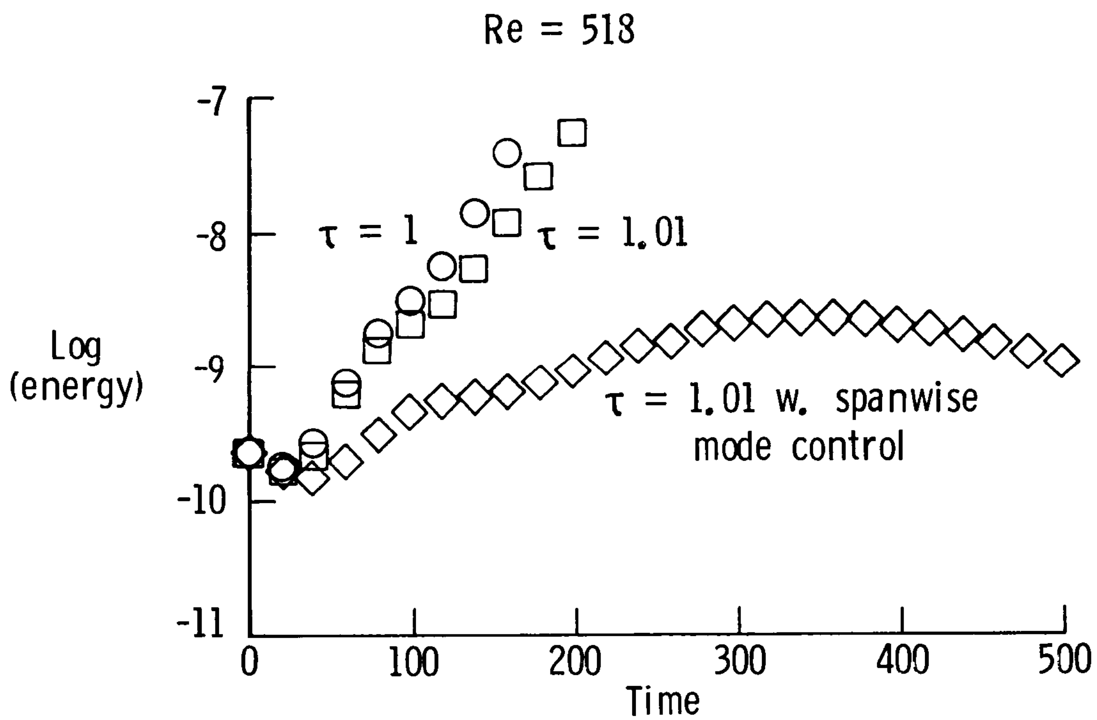
SECONDARY INSTABILITY OF HEATED BOUNDARY LAYER

This figure lists the energy in several Fourier harmonics for a simulation of a boundary layer at Re = 8950 with τ = 1.10 and a wall temperature of 293K. The initial conditions were a 2-D and a 3-D TS wave, with ε_{2D} = 0.05 and ε_{3D} = 0.0001. The harmonics are labelled by (k_x, k_z). Both TS waves are linearly stable. Nonlinear effects induce an instability which ultimately leads to turbulence.



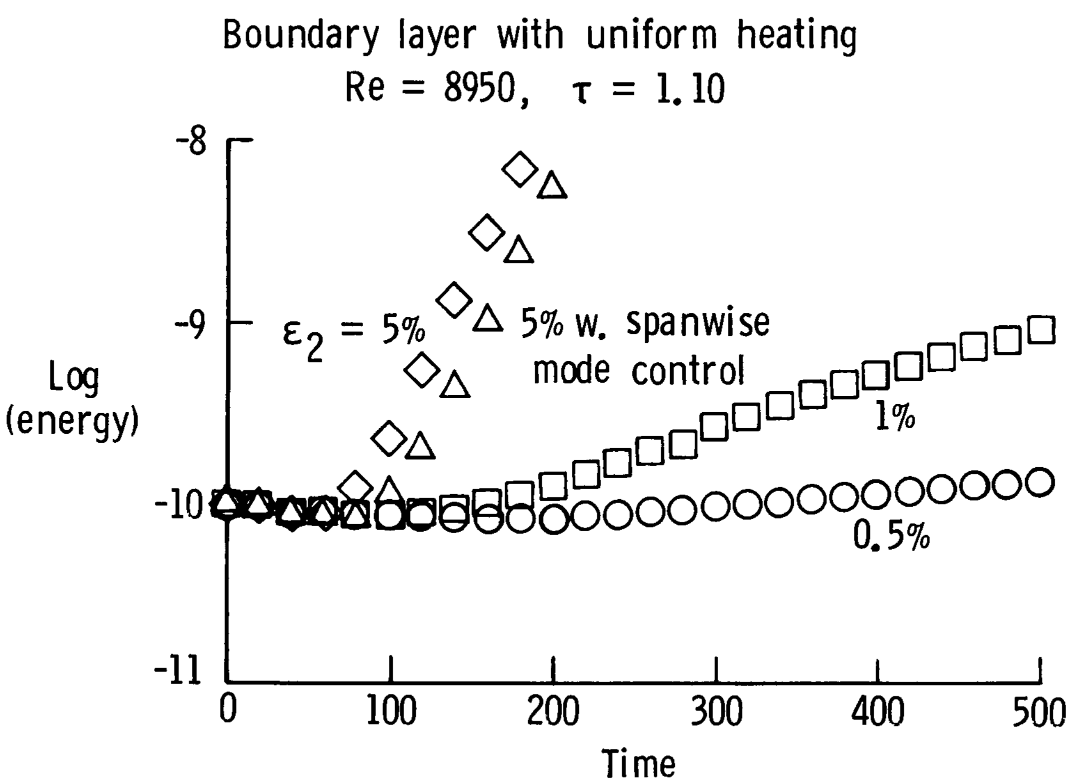
3-D ENERGY VERSUS WALL HEATING

The secondary instability is sensitive to the amount of heating. In these calculations $\varepsilon_{2D} = 0.05$ and $\varepsilon_{3D} = 0.0001$, and the energy for the 3-D mode is shown. It is the mode labelled (1,1) in the earlier figure. A 1% wall heating has a slight stabilizing effect. If this is combined with a selective control of the (0,1) spanwise mode, then the secondary instability is eliminated entirely.



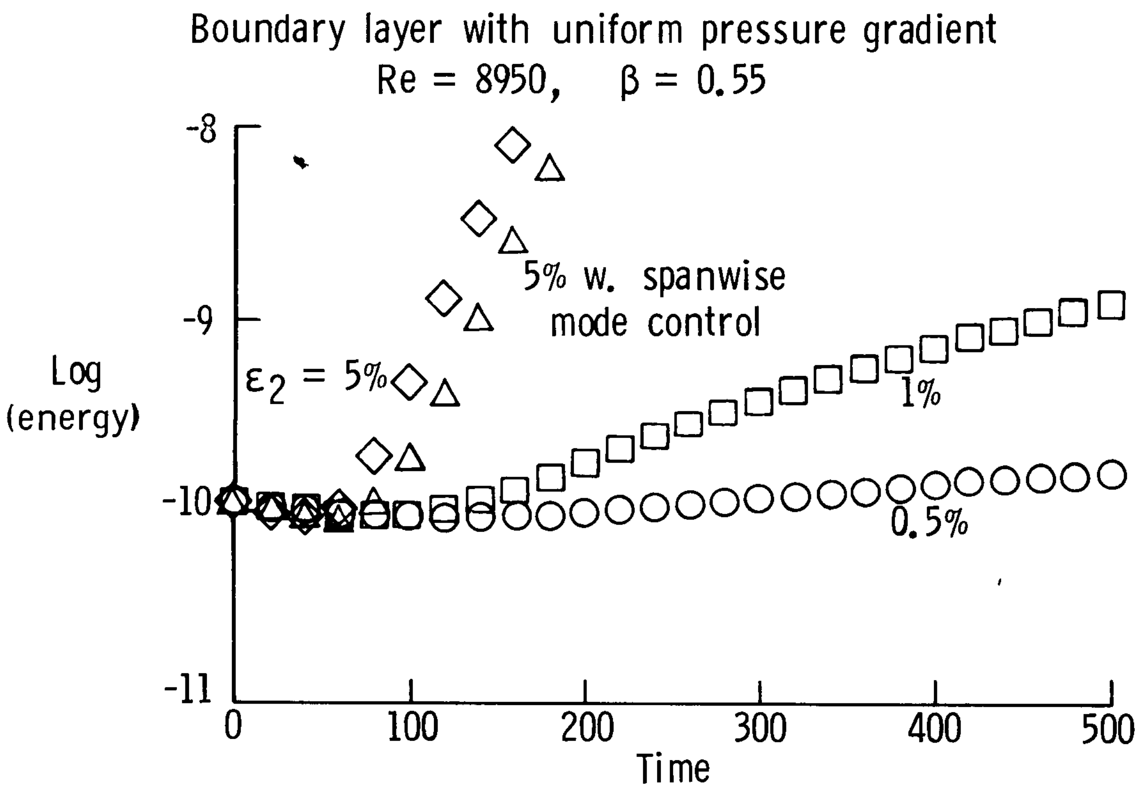
3-D ENERGY VERSUS 2-D AMPLITUDE (UNIFORM HEATING)

Even a 1% 2-D TS wave is sufficient to induce the secondary instability in the boundary layer with wall heating. The instability at this Reynolds number is so severe that selective control of the spanwise mode cannot remove it.



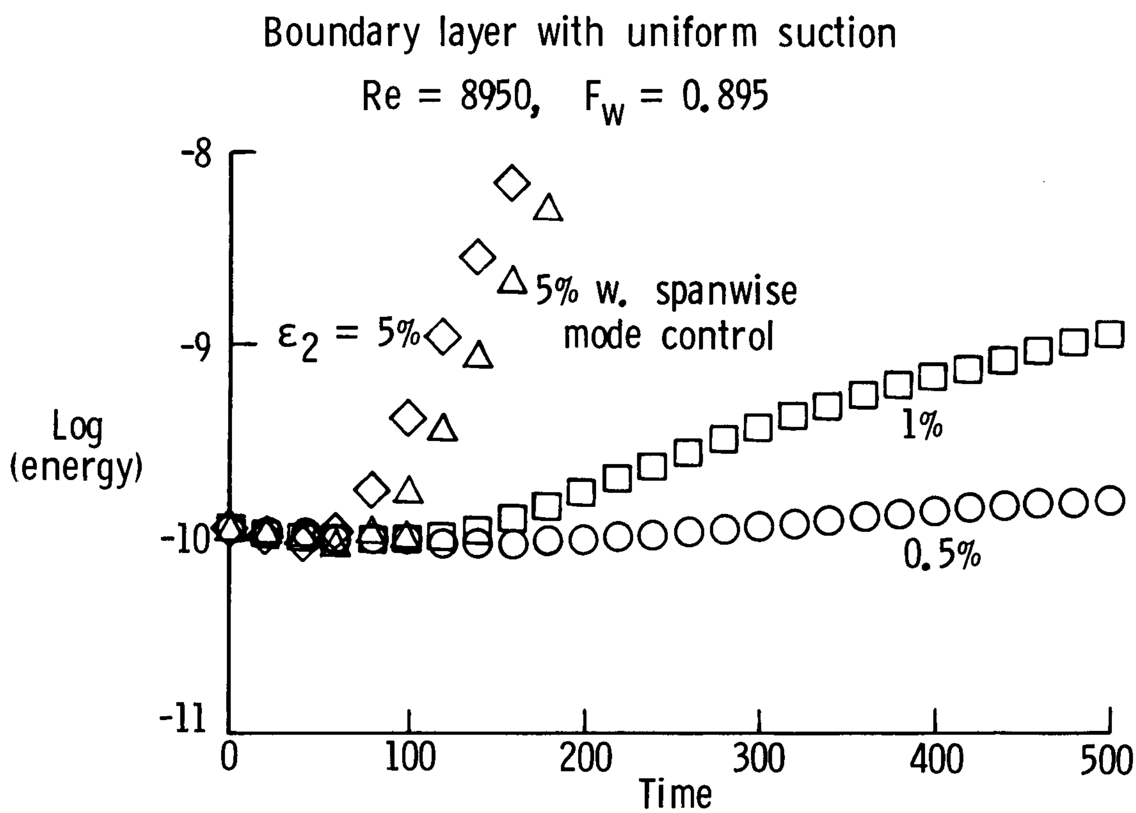
3-D ENERGY VERSUS 2-D AMPLITUDE (UNIFORM PRESSURE GRADIENT)

Even a 1% 2-D TS wave is sufficient to induce the secondary instability in the boundary layer with pressure gradient. The instability at this Reynolds number is so severe that selective control of the spanwise mode cannot remove it.



3-D ENERGY VERSUS 2-D AMPLITUDE (UNIFORM SUCTION)

Even a 1% 2-D TS wave is sufficient to induce the secondary instability in the boundary layer with suction. The instability at this Reynolds number is so severe that selective control of the spanwise mode cannot remove it.



SUMMARY

- A secondary instability exists for the parallel boundary subject to uniform pressure gradient, suction or heating

- Selective control of the spanwise mode reduces the secondary instability in the parallel boundary layer at low Reynolds number

REFERENCE

1. Gottlieb, D. and Orszag, S.: Numerical Analysis of Spectral Methods. SIAM, Philadelphia, PA, 1977.