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A THEORY FOR THE CORE FLOW OF LEADING-EDGE VORTICES

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ABSTRACT

Separation-induced leading-edge vortices can dominate the flow about slender wings at moderate to high angles of attack, often with favorable aerodynamic effects. However, at the high angles of attack which are desirable for take-off and landing as well as subsonic-transonic maneuver the vortices can break down or "burst" in the vicinity of the aircraft causing many adverse effects; these include lift loss, pitchup, and buffet. The flow in the core of leading-edge vortices is generally affiliated with the vortex breakdown phenomenon.

A theory is presented for the flow in the core of separation-induced, leading-edge vortices at practical Reynolds numbers. The theory is based on matching inner and outer representaions of the vortex. The inner representation models continuously distributed vorticity and includes an asymptotic viscous subcore. The outer representation models concentrated spiral sheets of vorticity and is fully three dimensional. A parameter is identified which closely tracks the vortex breakdown stability boundary for delta, arrow, and diamond wings.

A THEORY FOR THE CORE FLOW OF LEADING-EDGE VORTICES

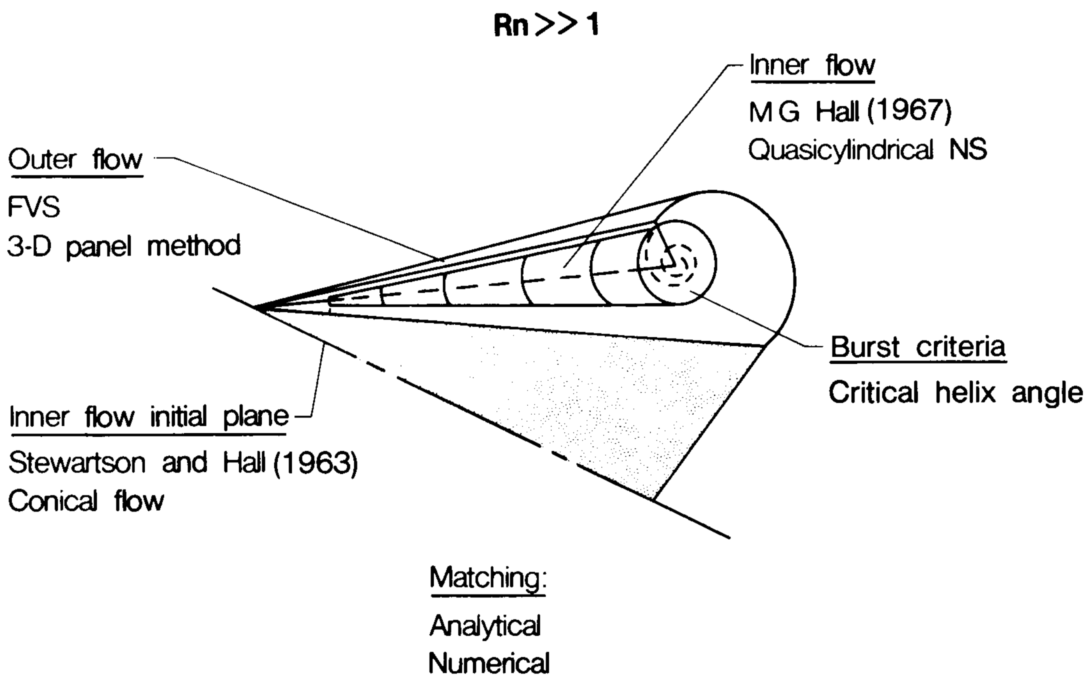
Shown in this figure is an outline of the presentation. The chief concepts of the theoretical formulation will first be reviewed, followed by some computed results which highlight the general character of the solutions. An analysis for incipient vortex breakdown is also discussed. Additional details of the theory have recently been given by Luckring (1985).

- Theoretical formulation
    - Inner/outer representation
    - Initial conditions
    - Boundary conditions
- Computed results
    - Experimental correlation
    - Vortex breakdown analysis
- Concluding remarks

THEORETICAL FLOW MODEL

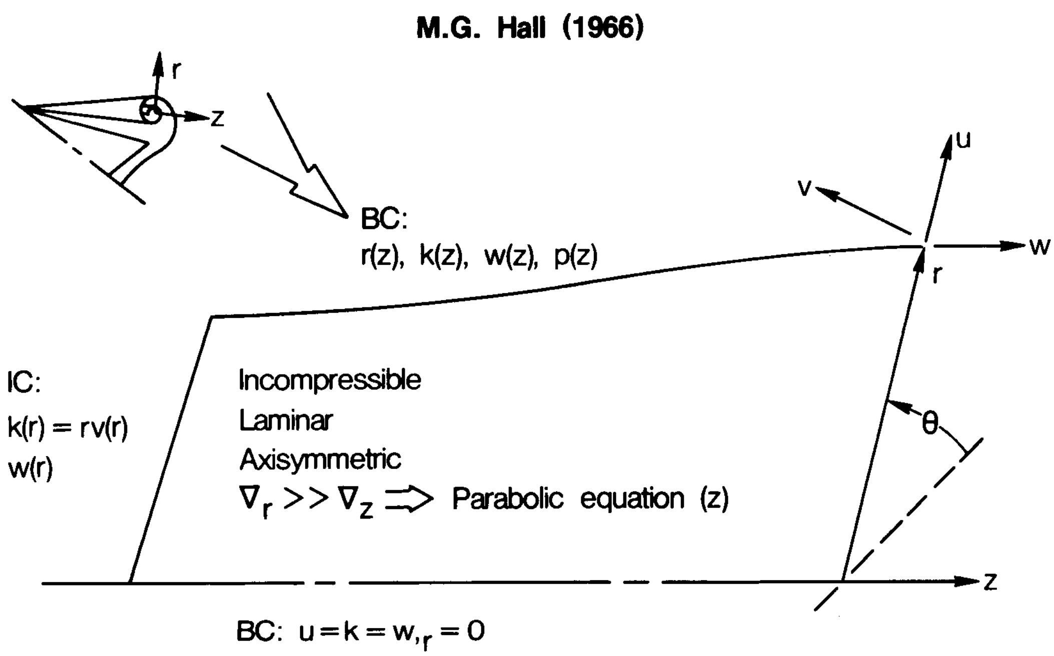
Previous theoretical studies have been focused primarily on either (1) modeling the global vortex flow field with a simplified vortex core representation, or (2) modeling detailed vortex core flow for simplified external conditions. At practical Reynolds numbers, the composite leading-edge vortex flow can be subdivided into overlapping regions which can be modeled with appropriate subclasses of the full governing equations; it is this feature of the flow which is exploited by the present approach. With this approach, considerable advantage can be taken of previous modeling studies of the isolated core and the leading-edge vortex so long as appropriate matching conditions can be established between the two models.

The outer flow is modeled by the free-vortex-sheet theory of Johnson, et al. (1980), a higher-order panel method which solves the Prandtl-Glauert equation including nonlinear boundary conditions pertinent to the concentrated vorticity representation of the leading-edge vortex. This method generally provides good estimates of inviscid wing pressure distributions as well as force/moment properties. The inner flow is modeled by the quasicylindrical Navier-Stokes equations and is initiated with an asymptotic solution valid for conical external conditions. Additional details of the inner flow model as well as matching the inner flow to the outer flow are described subsequently.



QUASICYLINDRICAL VORTEX CORE

The purpose of the inner flow fomulation is to provide a physically realistic representation of the flow in the core of a three-dimensional, leading-edge vortex, chiefly by accounting for the effects of distributed vorticity as well as viscosity. The unburst cores tend to be slender and, as a consequence, exhibit large gradients in the radial direction as compared to the axial direction. Therefore, the quasicylindrical Navier-Stokes equations of Hall (1966) were chosen as the pilot model of the core flow. The steady flow is assumed to be laminar, incompressible, and axially symmetric; in addition, the slenderness condition renders the equations parabolic in the axial direction. The solution is advanced in space by standard finite difference techniques from the initial plane solution of Stewartson and Hall (1963) with centerline boundary conditions appropriate to the axisymmetric assumption and with edge boundary conditions obtained from the free-vortex-sheet theory.



INNER/OUTER MATCHING--NONAXISYMMETRIC EFFECTS

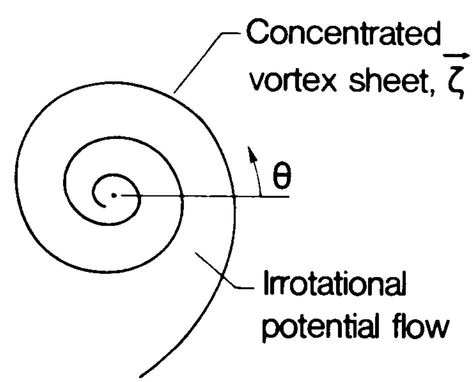
The inner-flow representation of the vortex core requires "edge" values of axial flow, circulation, and pressure as well as the region of the edge itself. The matching must be accomplished in the vicinity of the vortex (away from the wing) and must account for the differences between the inner and the outer representations of the vortex.

In the overlap region, both theories model inviscid vortices, the former modeling axisymmetric, continuously distributed vorticity and the latter modeling nonaxisymmetric concentrated vorticity. These differences can be reconciled with the theoretical solution of Mangler and Weber (1966) for a spiral sheet of concentrated vorticity embedded in an otherwise irrotational potential flow. The most noteworthy aspect of their asymptotic solution is that "the leading terms of the velocity components for a potential flow with vorticity concentrated along a sheet are the same as for an axisymmetric flow with continuously distributed vorticity," as given by Hall (1961) and used herein. To lowest order, the (axisymmetric) velocity and vorticity fields are aligned, and the pressure may therefore be derived from a Bernoulli relationship.

The Mangler and Weber (1966) solution provides a guide for the extraction of axisymmetric boundary condition quantities from the nonaxisymmetric outer formulation. Nonconical effects for the flow in the vicinity of the wing apex must also be addressed as discussed by Luckring (1985). Viscous-inviscid interaction effects are not presently accounted for.

- Mangler & Weber (1966)

  Slender core
  Asymptotic
  Incompressible
  Inviscid
  Conical



- Comparison to axisymmetric, distribution ζ sol'n

  $\vec{V}(r/z, \theta; \vec{\zeta}\text{ concentrated}) = \vec{V}(r/z; \vec{\zeta}\text{ distributed}) + \text{HOT}$

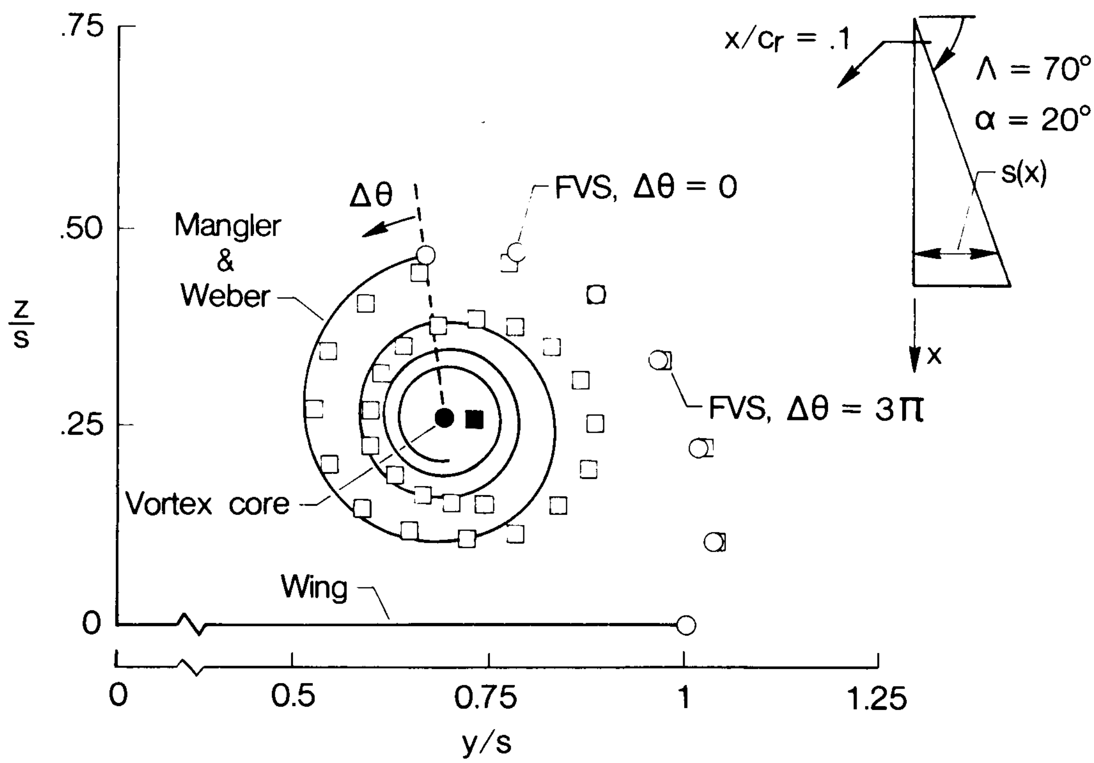
  $\vec{\zeta} \times \vec{V} = 0$

  p - Bernoulli Field

COMPARISON OF VORTEX SHEET TRAJECTORIES

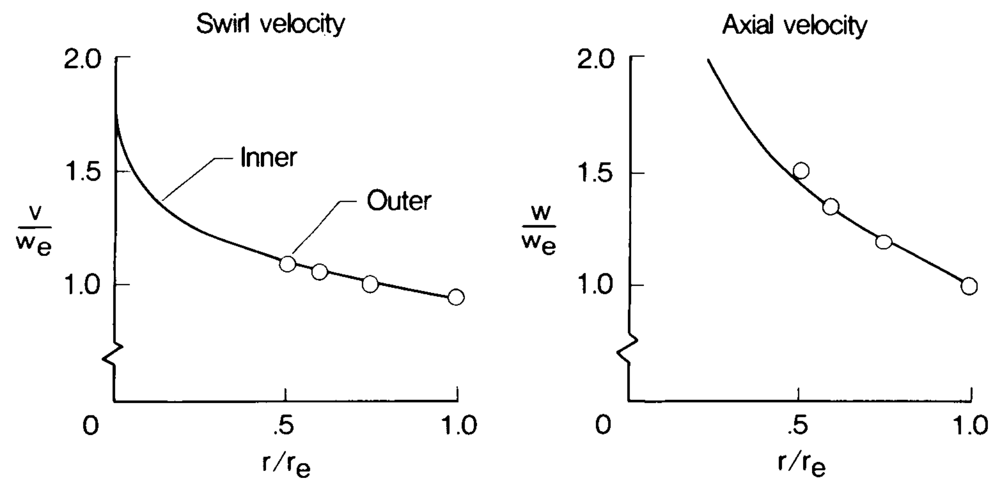
One approach to quantifying the matching of the inner and the outer models of the vortex is to compare the vortex sheet trajectories. This correlation stresses all three velocity components, and the outer region of the inner model (vortex core) should agree with the inner region of the outer model (vortex sheet) if the solutions are reasonably matched.

The Mangler and Weber (1966) solution, based on boundary condition data from the datum free-vortex-sheet solution ($\Delta\theta = 0$), shows reasonable correlation with the extended rollup free-vortex-sheet solution ($\Delta\theta = 3\pi$), except in the region given by $\pi < \Delta\theta < 2\pi$. The free-vortex-sheet solution is seen to be somewhat oblate, and nonaxisymmetric effects are, therefore, one cause for differences between the two solutions. Even so, the correlation is encouraging, and improvements based on advanced matching concepts can be expected.



INNER TO OUTER FLOW MATCHING

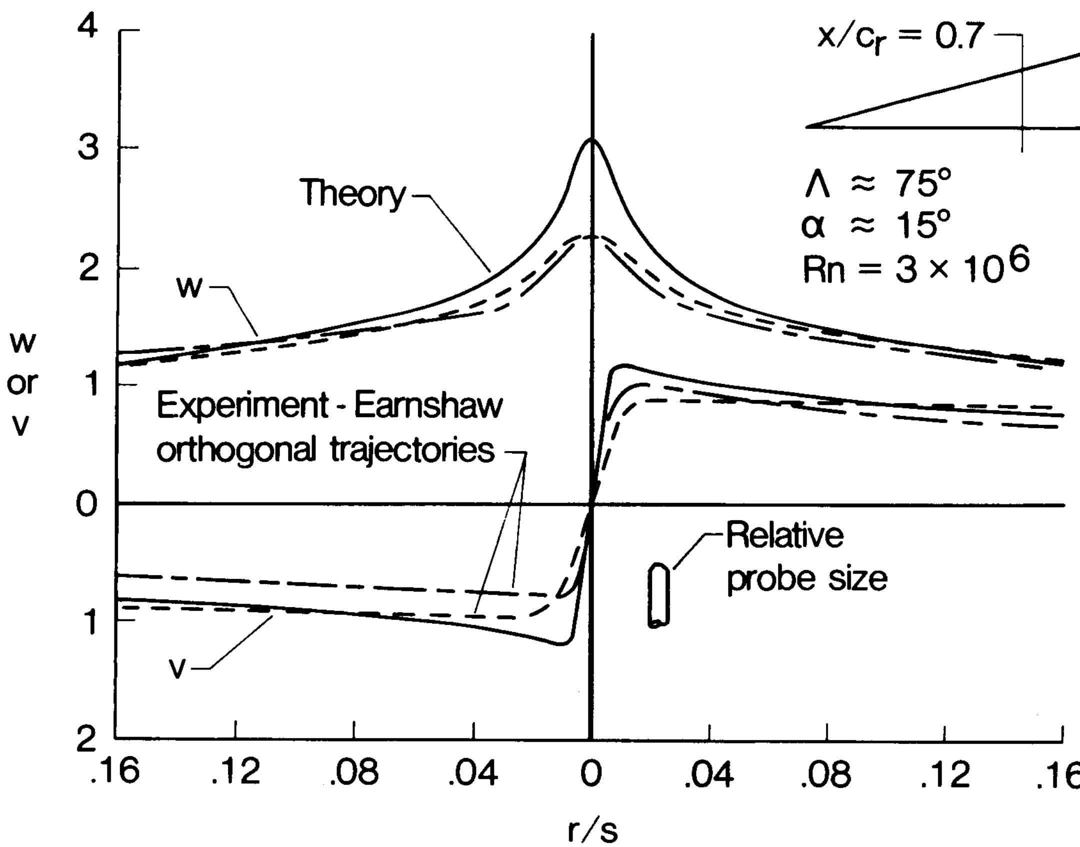
Another approach to quantifying the matching between the inner and the outer models of the vortex is to directly compare the individual velocity components. The radial distributions of swirl and axial velocity were computed in the datum plane ($\Delta\theta = 0$) and show good correlation between the two representations of the vortex. The correlation for velocities in the plane oriented $\pi/2$ radians from the datum plane was not as good, consistent to the correlation of vortex sheet trajectories shown on the previous figure. Additional studies indicated that the shown correlation was independent of the amount of modeled rollup in the free-vortex-sheet theory.



EXPERIMENTAL CORRELATION--VELOCITIES

Calculations have been performed for a wide range of conditions including isolated vortex core flows for generic external conditions as well as composite leading-edge vortex flows for delta, arrow, and diamond wings over a broad range of leading-edge-sweep angles and angles of attack. Typical velocity profiles through the core of the vortex are shown in this figure for a 75-degree delta wing at an angle of attack of 15 degrees. The experimental results of Earnshaw (1962) were obtained for a 76-degree (unit aspect ratio) delta wing at an angle of attack of 14.9 degrees. The freestream reference Mach number was approximately 0.09 and the Reynolds number, based on the wing root chord, was approximately three million.

Comparisons between the theoretical and experimental velocity profiles show reasonable correlations for the outer region of the vortex core and for the radial extent of the viscous subcore. The major discrepancy of this correlation is the centerline axial flow, and both theory and experiment are probable contributors to this discrepancy. Although the five-hole Conrad probe was small as compared to wing dimensions, its diameter is still appreciable as compared to the scale of the viscous subcore. Apart from the probe perturbing the flow itself, gradients across the probe head will also affect the measurements. Theoretically, the major factors affecting the lack of correlation are the incompressible and laminar flow assumptions. Because these flows have a local maximum in velocity at or near the vortex axis, they can be locally compressible at incompressible reference conditions. The inclusion of compressibility effects or turbulence effects would lessen the centerline axial flow.

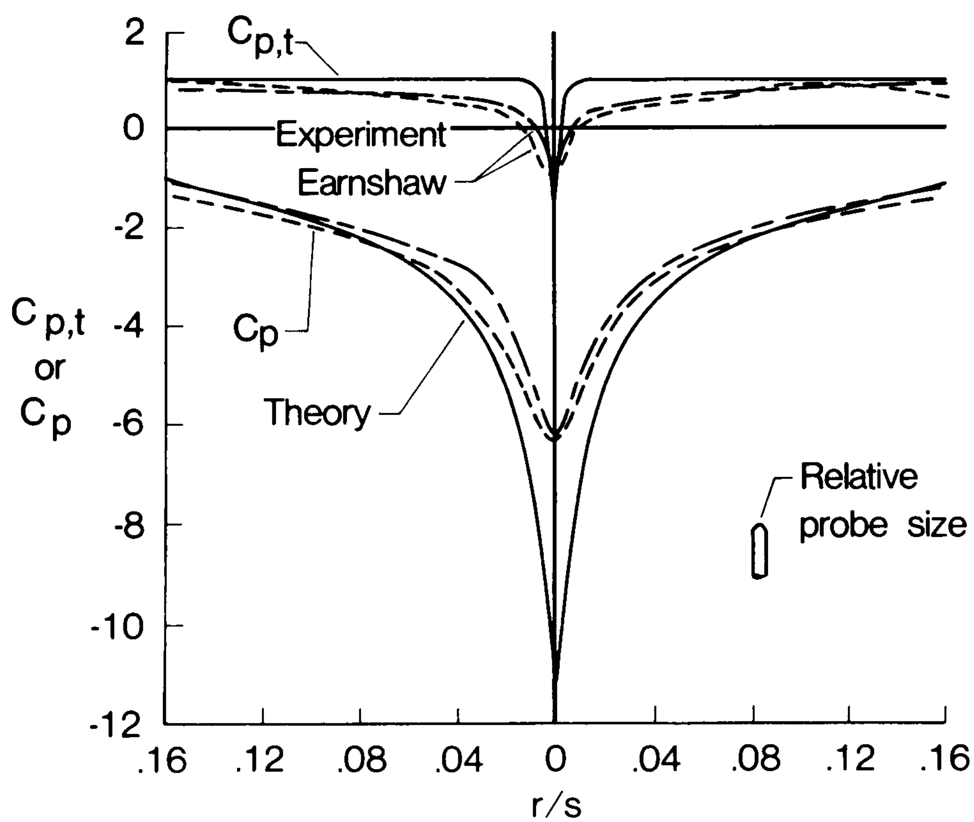


EXPERIMENTAL CORRELATION--PRESSURES

The correlation between theoretical and experimental pressure coefficients is consistent with the velocity correlations. The centerline static pressure coefficient is more negative than the experimental value, chiefly because of the increased centerline axial flow. However, static pressures of this magnitude are not unusual for vortex core flow. At higher angles of attack Earnshaw (1962) recorded C_p values of approximately -24; for a 65-degree swept wing at 15 degrees incidence Lambourne and Bryer (1962) recorded C_p values in the vicinity of -13.

The theoretical total pressure losses are confined to the viscous subcore whereas, experimentally, they are evidenced over the majority of the region shown. However, the theory provides a reasonable estimation of the maximum total losses at the centerline of the vortex.

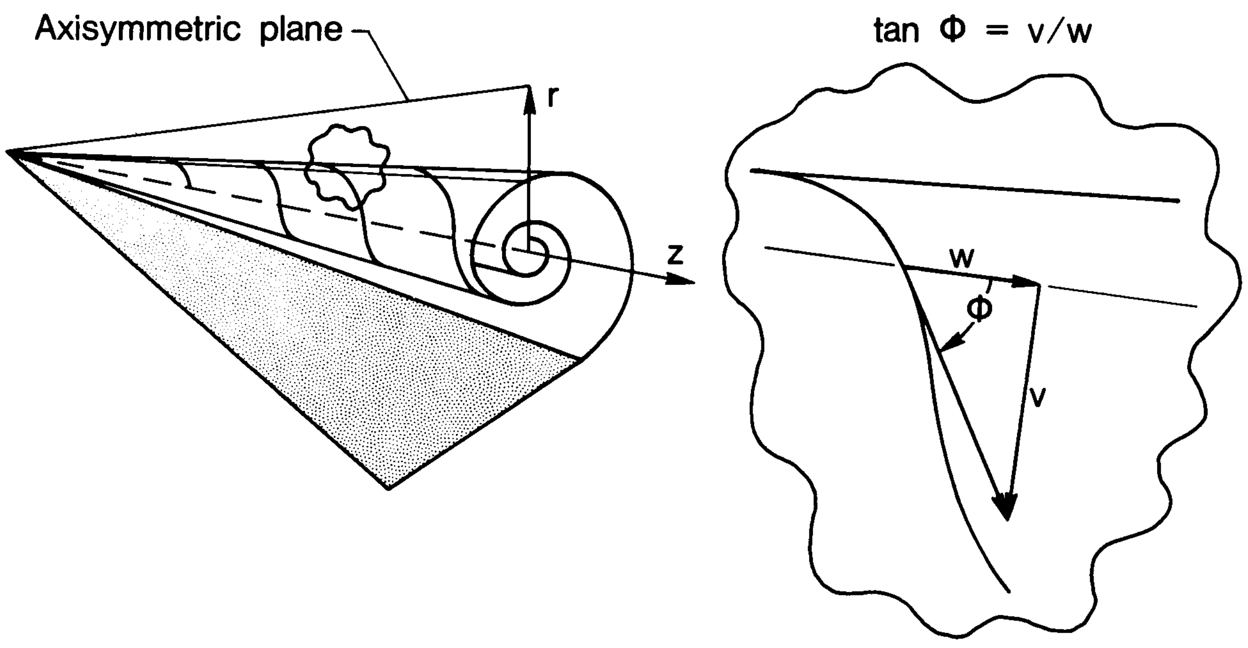
The computed flow exhibits many of the general features of leading-edge vortex core flow; the flow is weakly singular in the radial direction, has a local maximum axial velocity at the vortex axis several times the freestream value, and has total pressure losses in this same region.



DEFINITION OF HELIX ANGLE

The vortex core flow was analyzed a posteriori by several established criteria for evidence of incipient vortex breakdown. Included in this analysis were the boundary-layer analogy of Hall (1967), the hydrodynamic stability criterion of Ludwieg (1962), and a critical helix angle criterion. Of these criteria, the critical helix angle was found to offer the best correlation with experimental trends.

The tangent of the helix angle is defined as the ratio of the swirl to the axial velocity. It provides a local measure of the flow going through the axisymmetric plane to the flow going down the plane. Previous research has shown that vortex breakdown can occur for values of the helix angle in excess of some critical value, generally in the vicinity of 45 degrees, when accompanied by an adverse longitudinal pressure gradient.

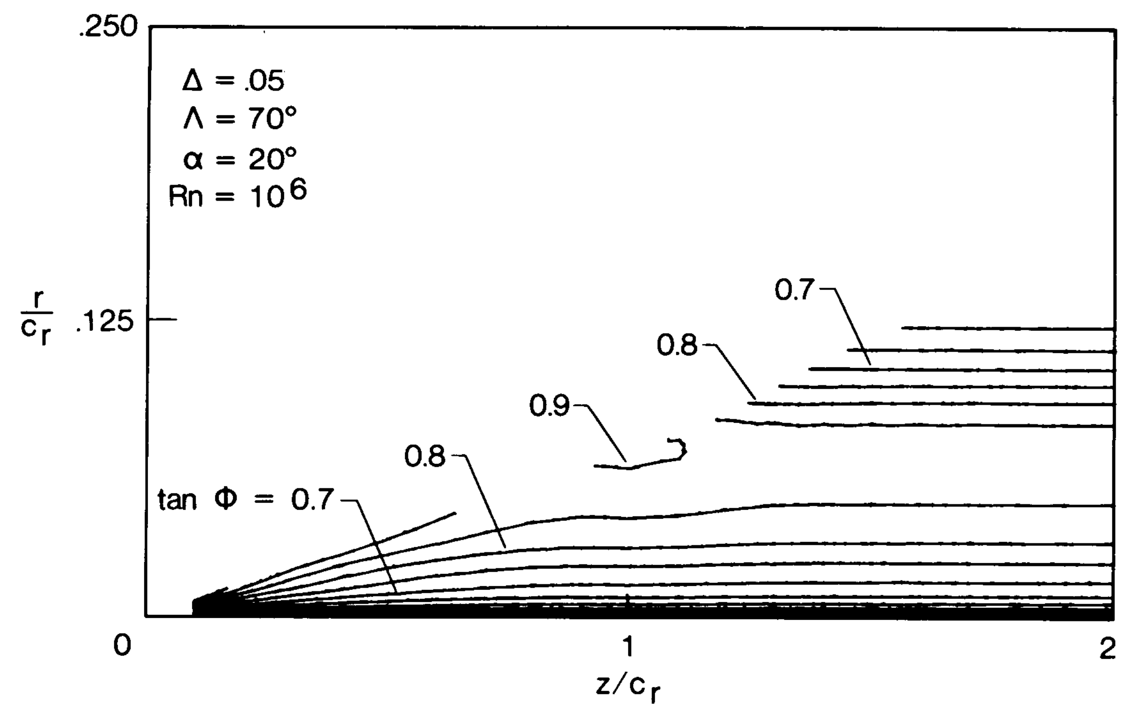


Previous research has shown that tan $\Phi > 1 \Longrightarrow$ breakdown

HELIX ANGLE CONTOURS--ALPHA = 20 DEGREES

Analysis of the vortex core flow for local helix-angle effects is presented in the form of contour distributions. The roughly diagonal edge where the contours terminate corresponds to the edge of the inner computational space as given by the free-vortex-sheet theory. This contour plot is for a 70-degree delta wing at an angle of attack of 20 degrees; the data of Wentz and Kohlman (1968) indicate that breakdown will first occur at the trailing edge of this wing at approximately 29 degrees angle of attack.

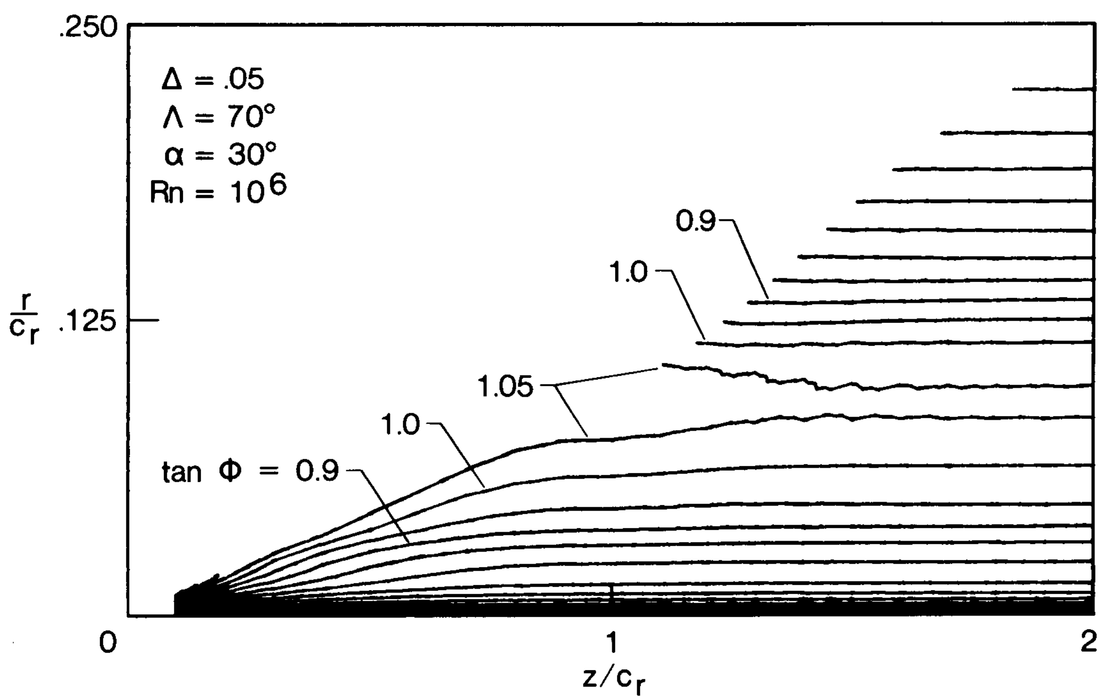
Several aspects of the helix angle distribution are noteworthy. There are two regions of maximum helix angle, both of which occur at the edge of the vortex. One is near the apex and the other is in the vicinity of the trailing edge. In addition, the structure of the vortex in terms of this parameter changes from the wing to the wake; over the wing the maximum helix angle occurs radially at the edge of the vortex, whereas in the wake this maximum occurs well within the vortex. Finally, a region of maximum helix angle persists well downstream from the trailing edge.



HELIX ANGLE CONTOURS--ALPHA = 30 DEGREES

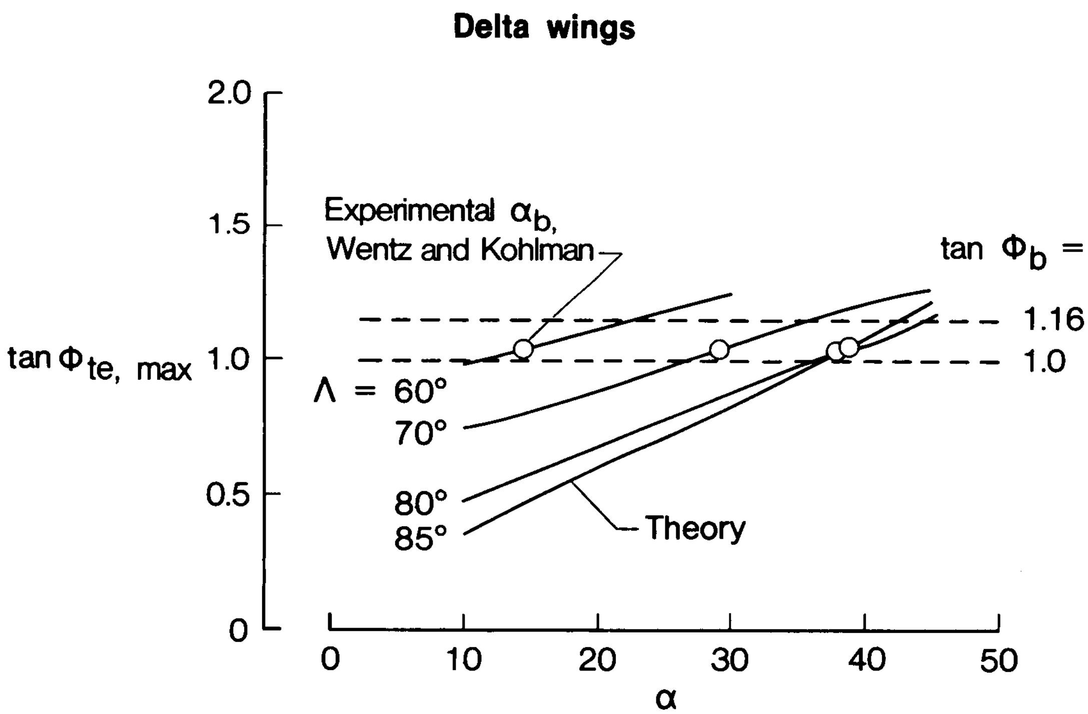
A similar contour analysis of the flow in the core of the vortex is shown for an angle of attack slightly in excess of the critical value for which breakdown occurs at the trailing edge. The general features of this solution are similar to the 20-degree case of the previous figure. For this case the maximal value of the helix angle tangent exceeds unity.

Additional analysis showed that, in general, near the apex the centerline vortex core flow exhibited a proverse longitudinal static pressure gradient whereas near the trailing edge this gradient was adverse.



TRAILING-EDGE MAXIMUM HELIX ANGLE

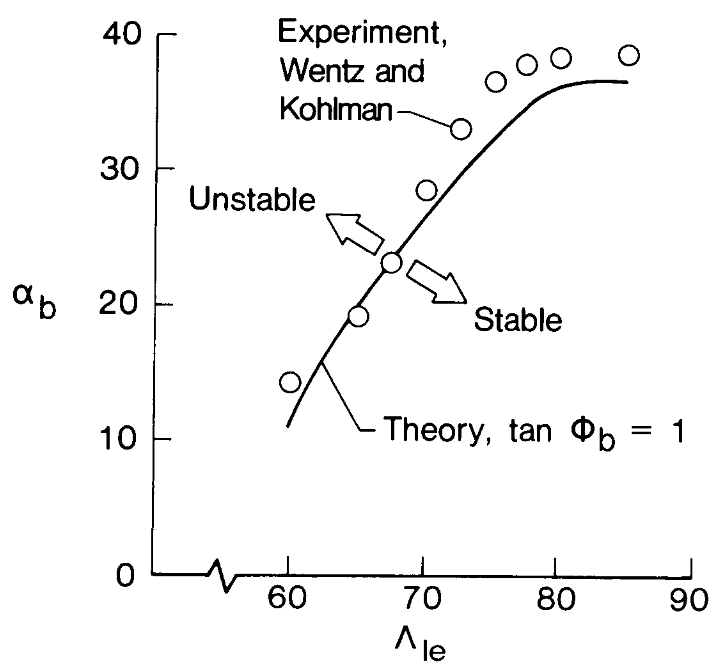
The various solutions were analyzed for the conditions of a maximum helix angle occurring in conjunction with an adverse longitudinal pressure gradient. These conditions occurred in the vortex in the vicinity of the trailing edge. The resultant values are shown for several of the delta wings analyzed along with the experimentally determined values of α_b, the angle of attack for which vortex breakdown first occurs at the trailing edge, from the Wentz and Kohlman (1968) data. The experimental condition of vortex breakdown at the trailing edge of the delta wings correlates roughly with a constant theoretical value of the maximum helix angle at the trailing edge. With the present formulation, this value is slightly greater than one; for reference purposes the conical value of 1.16 is also shown.

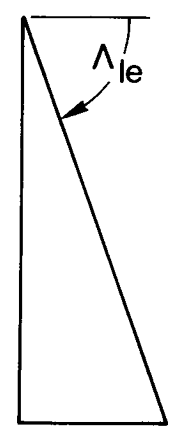


VORTEX BREAKDOWN STABILITY BOUNDARY

This correlation is also shown in a more familiar parameter space; the theoretical results are based on a critical helix angle tangent value of unity. This numerical correlation indicates a good estimation of the strong leading-edge sweep effects on the vortex breakdown stability boundary for delta wings. Additional studies have shown that the same criterion with the same critical value also provided a good estimation of the weak trailing-edge effects on this boundary.

Numerical correlation





CONCLUDING REMARKS

The flow in the core of a three-dimensional, separation-induced, leading-edge vortex can be calculated by appropriately matching inner and outer representations of the vortex. This approach is not strictly limited to the theories or applications of the present formulation. Other vortical flows (e.g., forebody vortices) could be addressed in a similar fashion with models appropriate to the particular flow.

The computed results of the present formulation exhibit many of the prominent features of the subject flow. These include weak radial singularities in the inviscid, rotational flow, axial velocity excesses at the vortex core axis which are several times the freestream reference value, and total pressure losses in the viscous subcore which arise due to modeled viscous effects. The solutions are, in general, highly three-dimensional, and showed reasonable correlation with experiment.

The experimental condition of incipient vortex breakdown at the trailing edge of delta, arrow, and diamond wings was found to closely correlate with the theoretical condition of a critical helix angle in conjunction with an adverse longitudinal pressure gradient.

The method can readily be extended to account for a number of additional effects. These include compressibility, turbulence, elliptic effects, and viscous/inviscid interaction consequences. Systematic extension of the present formulation should provide additional insights to the vortex breakdown phenomena for three-dimensional flows at practical Reynolds numbers.

- Method demonstrated
  - Matched inner/outer representations
  - Nonconical effects
  - Nonaxisymmetric effects
- Computed results
  - Prominent flow features exhibited
  - Reasonable correlation with experiment
  - Vortex breakdown stability boundary
- Method is readily extendable
  - Compressibility
  - Eddy viscosity
  - Elliptic core
  - Viscous/inviscid interactions

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