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INVESTIGATION OF THE FEASIBILITY OF AN ANALYTICAL METHOD OF ACCOUNTING FOR THE EFFECTS OF ATMOSPHERIC DRAG ON SATELLITE MOTION

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ABSTRACT

An analytic technique for accounting for the joint effects of earth oblateness and atmospheric drag on close-earth satellites is investigated. The technique is analytic in the sense that explicit solutions to the Lagrange planetary equations are given; consequently, no numerical integrations are required in the solution process. The atmospheric density in the technique described in this report is represented by a rotating spherical exponential model with superposed effects of the oblate atmosphere and the diurnal variations. A computer program implementing the process is discussed and sample output is compared with output from program NSEP (Numerical Satellite Ephemeris Program). NSEP uses a numerical integration technique to account for atmospheric drag effects.

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INTRODUCTION

In order to accurately predict orbital lifetimes and obtain trajectory data on close-earth satellites, it is generally accepted that joint effects due to pertubations such as atmospheric drag and the oblateness of the earth must be included in the mathematical model. However, inclusion of these joint pertubative effects usually leads to coupled systems of nonlinear differential equations that are difficult to solve in closed form - i.e.,obtain an exact solution. Consequently, most computational procedures that attempt to account for joint effects make use of numerical integration techniques to generate solutions to the coupled equations. Although these procedures can produce accurate results, they are often found to be time-consuming in their execution because of the numerical integrations involved. Because of this, persons working in real-time satellite tracking activities are continually seeking better ways of accelerating the solution process.

This report is an attempt to evaluate and test an analytic model that includes the joint effects of oblateness and drag and requires no numerical integration in its execution. The theory for this model was developed by Sean C. H. Chen ([4],[5]). Chen's development commences with the Lagrange Planetary equations for the classical orbital elements. From the outset the development considers jointly the coupled effects of oblateness and drag and is not the superposition of two separate developments. Consequently, it is considered to be a mathematically rigorous theory.

The solutions to the variational equations of motion are derived using a two-variable asymptotic expansion. Pertubations due to the second harmonic are considered as first order effects and pertubations due to the third and fourth harmonics and the drag force are considered as second order effects for the series expansion.The atmospheric density model used in the development of the solutions is a rotating spherical exponential model with superposed effects of the oblate atmosphere and the diurnal variation.

A computer program based on the drag theory of Chen is described and sample output is discussed.

GENERAL FORM OF SOLUTIONS

The general form of the solutions to the equations of motion for the drag problem will be briefly described. The solutions themselves are quite lengthy and will not be included in this document. The interested reader can consult references [4] and [5] for complete details.

The equations of motion of a particle of mass m, under the attraction of a mass m ,but acted on by perturbing forces F = (R,S,W), can be derived in terms of the classical orbital elements - $a, e, i, \Omega, \omega, M$. Here, a is the semi-major axis; e is the eccentricity; i is the inclination of the orbit; Ω is the right ascension of the ascending node; ω is the argument of perigee; and M is the mean anomaly. The Lagrange planetary equations for the orbital elements can be expressed in the general form:

$$
\begin{aligned}
da/dt &= ((2a^2)/\sqrt{\mu p})\,[e(\sin v)R + (p/r)S] \\
de/dt &= \sqrt{p/\mu}\,[(\sin v)R + (\{(r+p)\cos v + er\}/p)S] \\
di/dt &= [(r\cos u)/\sqrt{\mu p}\,]W \qquad\qquad (1) \\
d\Omega/dt &= [(r\sin u)/(\sqrt{\mu p}\,\sin i)]W \\
d\omega/dt &= (1/e)(\sqrt{p/\mu})[-(\cos v)R + (r/p + 1)(\sin v)S - \\
&\quad \{(re(\sin u)(\cot i))/p\}W] \\
dM/dt &= n - (1/\sqrt{\mu a}\,)[(2r-(p/e)\cos v)R + \{((r+p)\sin v)/p\}S
\end{aligned}
$$

In the above expressions, r is the magnitude of the position vector between the two bodies, n is the mean motion, v is the true anomaly, u is the argument of latitude, $p = a(1-e^2)$, and $\mu = k(m+m)$, (k = gravitational constant).

In deriving solutions to the planetary equations, Chen assumes that the perturbing forces F are of the form F = F(C) + F(D) where F(C) are the conservative forces which includes the earth's oblateness and F(D) is the drag force. The drag force per unit mass is represented by

$$F(D) = -0.5CD(A/m)\rho V\bar{V} \tag{2}$$

where CD is a nondimensional drag coefficient, m is the mass of the satellite, $\bar{V}$ is the velocity of the satellite relative to the atmosphere, A is the effective cross sectional area of the satellite and ρ represents the local atmospheric density. The analytic model for the density is taken to be:

$$\begin{aligned} \rho &= \rho_0 \exp(-(h-h_0)/H)[1 + F(A\cos v + B\sin v)] \\ F &= (RHO1-RHO2)/(RHO1+RHO2) \\ \rho_0 &= RHO1/(1+F) \\ H &= -2ae/\ln(RHO3/RHO1) \end{aligned} \qquad (3)$$

where RHO1 is a value of the density computed at the center of the diurnal bulge, RHO1 is a density value computed at perigee hight at a point opposite the bulge and RHO3 is a density value computed at apogee height at a point under the bulge. A,B,h, and h_0 are parameters that depend on the position of the point under consideration.

The Lagrange planetary equations (system (1)) with F(C) replaced by the earth gravitational potential up to and including the fourth harmonic and with F(D) replaced by (2) and (3) are solved analytically for the orbital elements. A two variable asymptotic expansion technique is used to generate the solutions. The solution for each orbital element is obtained and expressed in the form:

$$X = X(OBLATE) + X(DRAG)$$

where X represents any of the orbital elements, X(OBLATE) is the part of the solution generated by the conservative forces and X(DRAG) is the portion generated by the drag effects. The solutions are quite lengthy and will not be reproduced here. The X(OBLATE) components are contained in reference [5] (pages 6-3 -6-7) and the X(DRAG) components are contained in reference [4] (pages 6-1 - 6-7) .

PROGRAM DESIGN AND SAMPLE OUTPUT

The Analytic Satellite Ephemeris Program (ASEP) now in use in the Orbital Mechanics branch of the Mission Analysis Division at Marshall Space Flight Center is designed to accurately simulate the motion of a satellite in earth orbit (see [10]). The solution to the satellite motion is completely analytic and requires no numerical integrations for its execution. The program is based on the theoretical work of Sean C.H. Chen as described in [5] and only includes pertubations due to the oblate earth. Atmospheric drag effects are not included. In designing a test program that included both oblateness and drag pertubations, a subroutine (ORBIT) from the ASEP program was selected for modification. Subroutine ORBIT accepts initial values for the orbital elements as input and generates a detailed trajectory. In designing the program, several tasks were executed:

(1) The mathematical development of the analytic solution derived by Chen and contained in [4] and [5] was reviewed;

(2) All subroutines in ORBIT that were impacted by the drag terms were identified and appropriately modified;

(3) A driver (main calling program) for ORBIT was written.

As a result of the above tasks, a test program resulted. The components of the program and a brief description of each are listed below:

BDRAG - the main driver program.

BORBIT - BORBIT is the ORBIT subroutine from ASEP containing minor changes to reflect the inclusion of the drag terms.

BRATES - BRATES contains modifications to subroutine RATES of ASEP that are used to generate the secular terms in the solution of the orbital element.

BSECTM - BSECTM is used together with BRATES to generate the secular terms.

BLPTRM - BLPTRM contains modifications to subroutine LPTERM of ASEP that are used to generate the long period components of the solution.

FIT - Program FIT evaluates the constants H, F and ρ_c , given in (3). These values are needed for the local density.

The drag terms are primarily accounted for in the subroutines BRATES, BSECTM, and BLPTRM. The modifications are quite extensive and are included in the Appendix. The interested reader can compare the expressions in the three subroutines with the solutions for the orbital elements contained in [4] and [5].

The test program described above has been coded and implemented and the results compared with output from NSEP (Numerical Satellite Ephemeris Program). NSEP handles the drag effects by a numerical integration process. The NSEP program has been used extensively and found to be accurate. However, the implementation is slow because of the numerical integrations involved.

Results from the analytic model (no numerical integrations required) described in this document are compared with NSEP output. A comparison of agreement between the two procedures is made simply by subtracting the corresponding values given by the two techniques for the orbital elements. A difference of zero would indicate that both procedures gave the same value for that element at the given time. Charts #1 and #2 give the deviations in five orbital elements over a ten day period. It can be noted from the charts that there is reasonably good agreement for the eccentricity, argument of perigee, and right ascension of the ascending node. However, both samples indicate less than favorable comparisons in the semi-major axis and the mean anomaly. It is believed that better comparisons will result once a good fit of the parameters H, F, and ρ_c needed for the atmospheric density, is obtained.

COMPARISON WITH NSEP OUTPUT
[X(NSEP)-X(ANALYTIC),X= ORBITAL ELEMENT]

INPUT DATA

ρ_0 = 5.80595D-11   F = -.01599   H = 49797.8(M)

INITIAL CONDITIONS:
a = 6755.734(km)   e = .019193
Ω = 3.77821(deg) ω = 180.0(deg)
i = 57.0(deg)   M = 0.00(deg)

ORBITAL ELEMENT	MISSION ELAPSED TIME (DAYS)			
	4	6	8	10
SEMI-MAJOR AXIS(KM)	-.575	-.868	-1.168	-1.467
ECCENTRICITY	-.00032	-.00046	-.00063	-.00076
RIGHT ASCENSION OF ASCENDING NODE(DEG)	.03	.04	.06	.07
ARGUMENT OF PERIGEE (DEG)	.03	.03	.17	.28
MEAN ANOMALY(DEG)	4.50	10.13	17.89	27.93

CHART #1

COMPARISON WITH NSEP OUTPUT
[X(NSEP)-X(ANALYTIC),X= ORBITAL ELEMENT]

INPUT DATA

ρ_0 = 4.45D-11 F = 0.99 H = 1039000(M)

INITIAL CONDITIONS:

a = 6755.734(km)
e = .019193
Ω = 3.77821(deg)
ω = 180.0(deg)
i = 57.0(deg)
M = 0.00(deg)

ORBITAL ELEMENT	MISSION ELAPSED TIME (DAYS)			
	4	6	8	10
SEMI-MAJOR AXIS(KM)	1.635	2.480	3.326	4.182
ECCENTRICITY	-.00024	-.00034	-.00046	-.00053
RIGHT ASCENSION OF ASCENDING NODE(DEG)	.04	.06	.09	.12
ARGUMENT OF PERIGEE (DEG)	-.27	-.39	-.41	-.43
MEAN ANOMALY(DEG)	4.75	10.47	18.33	28.45

CHART #2

CONCLUSIONS

An analytic model for accounting for joint effects of earth oblateness and atmospheric drag on close-earth satellites has been described. Preliminary tests indicate reasonable results when compared with output generated by the NSEP program (Numerical integration routine). However, more test cases will be required before a final determination can be made on the accuracy of the analytic process. Also, it will be useful to compare output from the analytic method with actual data collected from observations of close-earth satellite motion. Such data was not available during the period of this report.

A number of authors have pointed out the difficulty in describing a realistic model of the atmospheric density because it depends on position and time in a very complex manner. In the model tested in this report, it was necessary to fit three constants F, ρ_0, and H in order to describe the atmospheric density. This fitting was done by first using the Jacchia model to calculate values of the density at specific points in the diurnal bulge. These values are used to compute the numerical values of the three parameters. More research is required on this process so that a determination can be made on when a " good " fit has been obtained. Additionally, in the tests conducted in this study, only one fitting of the constants was done for the entire ten-day mission. It is anticipated that better results will be obtained if the constants were updated at least every twenty-four hours. This updating process needs to be tested.

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```
c
c-------------------------------------------------------------+
c                        BR A T E S                           |
c-------------------------------------------------------------+
c                                                             |
c        This routine computes the following quantities :     |
c                                                             |
c      1. Mean Motion                                         |
c      2. Perturbed Mean Motion                               |
c      3. Nodal regression rate                               |
c      4. Rate of rotation of the Line of Apsides.            |
c      5. Lambda dot   (one of the regularized elements)      |
c      6. Nodal Period                                        |
c      7. Kepplerian Period                                   |
c                                                             |
c-------------------------------------------------------------+
c
      SUBROUTINE BRATES
      DOUBLE PRECISION PI,TPI,PO2,OME,AMU,RE,POLR,AMI,POL,OE,ME,
     *IME,CAR,EMI,APMI,RANMI,MAMI,UMI,TAMI,LAMMI,IMI,PM,AK2,AAK2,
     *AK4,PERK,PERN,MMOT,LAMDOT,PMMOT,APDOT,RANDOT,
     *CD,A,DM,RP,B,H,FE,Q0,DRG,RHONUL,G,DB,DBSQR,DBCUBE,ESQR,
     *DD,EL,Q,B0,B1,YA,ZA,YE,ZE,YI,ZI,PA,PE,DPI,P1A,P1E,YISTAR,
     *ZISTAR,PISTAR,Y1ISTR,Z1ISTR,Y3I,Z3I,Q1ISTR,Q3I,P1I,TDG,
     *TPIOT,ELM,BB2,DADOT,DEDOT,DIDOT,DRADOT,DAPDOT,DMADOT,
     *MBF0,MBF1,F,ALI
      REAL NODPER,KEPPER
      COMMON/BLOK1/PI,TPI,AMU,RE,POLR,AJ2,AJ3,OME,PO2,AJ4
      COMMON/BLOK8/AMI,EMI,IMI,APMI,RANMI,MAMI,UMI,TAMI,LAMMI
      COMMON/BLOK11/MMOT,PMMOT,APDOT,RANDOT,LAMDOT
      COMMON/BLOK12/JCIRC
      COMMON/BLOK17/POL(6),OE(6),ME(6),IME(6),CAR(6)
      COMMON/BLOK57/NODPER,KEPPER
      COMMON/BLOK110/DADOT,DEDOT,DIDOT,DRADOT,DAPDOT,DMADOT
      COMMON/BLOK111/PA,PE,DPI,PISTAR,Q3I
      COMMON/BLOK113/DRG,F,RHONUL,H,ALI
      DATA DPR/57.29577951D+0/RPD/.0174532925D+0/EL/.0000729D+0/
      DATA B/6.356780D+6/
      CALL UNSIME
      OME2=1.-EMI**2
      BB=.5*SIN(IMI)**2
      AA=1./3.-BB
```

```
      PM=AMI*OME2
      MMOT=DSQRT(AMU/AMI**3)                        ! mean motion
      AK2=1.5*AJ2*(RE/PM)**2
      AAK2=AK2*(PM/AMI)**2
      AK4=AJ4/(AJ2**2)
      RANDOT=-AK2*MMOT*COS(IMI)*(1.-AK2*(-1.5+10./3.*BB+EMI**2
     **(-1./6.-5./12.*BB)-3.*AA*SQRT(OME2)+35./18.*AK4*(-6./7.+3.*BB)
     **(1.+1.5*EMI**2)))          ! Nodal Regression rate
      APDOT=AK2*MMOT*((2.-5.*BB)+AK2*(4.-103./6.*BB+215./12.*BB**2+
     *EMI**2*(7./12.-.75*BB-15./8.*BB**2)+(AA+15.*AA**2)*SQRT(OME2)
     *+35./18.*AK4*(12./7.-93./7.*BB+21.*BB**2+EMI**2*(27./14.-189./
     *14.*BB+81./4.*BB**2))))      ! Rot. rate of line of apsides
      PMMOT=MMOT*(1.+AK2*SQRT(OME2)*((1.-3.*BB)+AK2*(11./6.-26./3.*BB
     *+125./12.*BB**2+EMI**2*(2./3.-4./3.*BB-5./3.*BB**2)+21./2.*AA**2
     **SQRT(OME2)+35./18.*AK4*EMI**2*(9./14.-45./7.*BB+45./4.*BB**2)))
     *)           ! perturbed mean motion
      LAMDOT=MMOT*(1.+AAK2*((3.-8.*BB)-AAK2*(35./6.-155./6.*BB+85./3.*
     *BB**2+AA+51./2.*AA**2+35./18.*AK4*(12./7.-93./7.*BB+21.*BB**2)))
     *)           ! Lamda dot  (regularized element)
      PERN=(TPI/(PMMOT+APDOT))/3600.     ! Nodal Period



      NODPER=PERN
      PERK=TPI/MMOT/3600.                 ! Kepplerian (2-body) Period
      KEPPER=PERK
C
D        TYPE *,'RHONUL =  ',RHONUL
D        TYPE *,'F      =  ',F
D        TYPE *,'H      =  ',H
         RP  = AMI*(1.-EMI)
         FE  = (RE -B)/RE
         Q0  = RE*FE*SIN(IMI)**2*SIN(APMI)**2/H
         G   = DRG*RHONUL*EXP(-(AMI-RP)/H +Q0)
         DB  = AMI*EMI/H
D        TYPE *,'DB =  ',DB
         DBSQR  = DB**2
         DBCUBE = DBSQR*DB
         ESQR   = EMI**2
         DD     = EL/MMOT*SQRT(OME2)*COS(IMI)
         Q      = (RE-B)/H*SIN(IMI)**2
         B0     = MBF0(DB)
         B1     = MBF1(DB)
```

```
C
C     DEFINITION OF PK (K = A,E,I)
C
      YA     = (1.-DD)**2 + EMI**2*(1.5+DD)
      ZA     = 2.*EMI*(1.-DD**2)-ESQR/DB*(1.5+DD)
      YE     = EMI*(1.-DD)*(1.+3.*DD)-EMI**2/(2.*DB)
      ZE     = (1.-DD)**2 + EMI**2*(0.5-DD)-EMI/(2*DB)*
     1         (1.-DD)*(2.+5.*DD) +EMI**2/DB**2
      YI     = (1.-DD)+EMI**2/2.*(1.-13.*DD)
      ZI     = -2.*EMI*(1.-2.*DD)-EMI**2/(2.*DB)*(1.-13.*DD)
      PA     = YA*B0 + ZA*B1
      PE     = YE*B0 + ZE*B1
      DPI    = YI*B0 + ZI*B1
C
C     DEFINITION OF P1A,P1E,AND P1I
C
      P1A    = -0.5*(1.-3./8.*Q)*PA
      P1E    = -0.5*(1.-3./8.*Q)*PE
      YISTAR = (1.-DD)+ESQR/2.*(1.-13.*DD)-4.*EMI*DD/DB-
     1         3.*ESQR*(1.+3.*DD)/DBSQR
      ZISTAR = -2.*EMI*(1.-2.*DD)-1./DB*(2.*(1.-DD)-2.5*ESQR*
     1         (1.+3.*DD))+8.*EMI*DD/DBSQR+6.*ESQR*(1.+3.*DD)
     2         /DBCUBE
      PISTAR = YISTAR*B0 + ZISTAR*B1
      Y1ISTR = 1./DB*(2.*EMI*DD-6./DB*((1.-DD)-.75*ESQR*(5.-DD))+
     1        48.*EMI/DBSQR)
      Z1ISTR = 1./DB*(1.-DD)-ESQR/2.*(11.+DD)-4.*EMI*(3.+DD)/DB+
     1          12./DBSQR*((1.-DD)-.75*ESQR*(5.-DD))-96.*EMI/DBCUBE
      Y3I    = 1./DB*(2.*EMI*DD-3./DB*(1.-DD)*(1.+ESQR)+
     1         24.*EMI/DBSQR-60.*ESQR/DBCUBE*(2.5-.5*DD))
      Z3I    = 1./DB*((1.-DD)-ESQR/2.*(1.+3.*DD)-2.*EMI/DB*
     1          (3.+2.*DD)+3./DBSQR*(2.*(1.-DD)+ESQR/2.*(29.-9.*DD))
     2        -48.*EMI/DBCUBE+60.*ESQR*(5.-DD)/DB**4)
      Q1ISTR = (1.-ESQR)*(Y1ISTR*B0+Z1ISTR*B1)
      Q3I    = (1.-ESQR)*(Y3I*B0+Z3I*B1)
      P1I    = -0.5*(1.-.5*Q)*(DPI-.5*PISTAR+Q1ISTR+2.*Q3I)
C
C     COMPUTATION OF ORBITAL ELEMENTS WITH DRAG
C

      TDG    = TPI/MMOT*(1.-AK2*SQRT(OME2)*(1.-3.*BB))
      TPIOT  = TPI/TDG
      ELM    = EL/MMOT
```

```
    BB2    = SIN(IMI)**2
    DADOT  = -TPIOT*(2.*G*AMI**2*(PA+Q*P1A))
    DEDOT  = -TPIOT*(2.*G*AMI*OME2*(PE+Q*P1E))
    DIDOT = -TPIOT*(.5*G*AMI*ELM*SIN(IMI)/SQRT(OME2)*(DPI+Q*P1I))
    DRADOT = -TPIOT*(G*AK2*MMOT*AMI*(COS(IMI)*((7./2.*PA-4.*EMI*
1         PE)+Q*(7./2.*P1A-4.*EMI*P1E))+.25*ELM*BB2/
2         SQRT(OME2)*(DPI+Q*P1I)))
    DAPDOT = TPIOT*(G*AK2*MMOT*AMI*((2.-5./2.*BB2)*((7./2.*PA-4.*
1         EMI*PE)+Q*(7./2.*P1A-4.*EMI*P1E))+5./4.*ELM*
2         BB2*COS(IMI)*(DPI+Q*P1I)))
    DMADOT = TPIOT*(G*AK2*MMOT*AMI*(SQRT(OME2)*(1.-3.*BB)*
1         ((7./2.*PA-3.*EMI*PE)+Q*(7./2.*P1A-3.*EMI*P1E))
2         + .75*ELM*BB2*COS(IMI)*(DPI+Q*P1I)))
  RETURN
  END
```

```
c
c-----------------------------------------------------------------+
c                          B S E C T M                             |
c-----------------------------------------------------------------+
c                                                                  |
c          This routine computes the Secular terms of the solution |
c      to the satellite motion.                                    |
c                                                                  |
c-----------------------------------------------------------------+
c
      SUBROUTINE BSECTM
      DOUBLE PRECISION T,TO,TF,TAPO,TPER,GET,AMI,EMI,IMI,APMI,
     *MAMI,UMI,TAMI,LAMMI,GSTC,JULDAY,APSEC,RANSEC,MASEC,
     *MMOT,PMMOT,APDOT,RANDOT,LAMDOT,PRIN,DADOT,DEDOT,
     *DAPDOT,DMADOT,DRASEC,DAPSEC,DMASEC,DASEC,DESEC,DISEC,
     *RANMI,LAMSEC,DIDOT,DRADO
      COMMON/BLOK2/T,TO,TF,TAPO,TPER,GET,DAYNO,JULDAY,GSTC
      COMMON/BLOK8/AMI,EMI,IMI,APMI,RANMI,MAMI,UMI,TAMI,LAMMI
      COMMON/BLOK11/MMOT,PMMOT,APDOT,RANDOT,LAMDOT
      COMMON/BLOK12/JCIRC
      COMMON/BLOK20/APSEC,RANSEC,MASEC,LAMSEC
      COMMON/BLOK110/DADOT,DEDOT,DIDOT,DRADOT,DAPDOT,DMADOT
      COMMON/BLOK112/DASEC,DESEC,DISEC
      CALL UNSIME
      RANSEC=RANMI+RANDOT*(T-TO)
      DRASEC=DRADOT*(T-TO)**2
      RANSEC=PRIN(RANSEC+DRASEC)
      IF(JCIRC.EQ.2) GO TO 1
      LAMSEC=PRIN(LAMMI+LAMDOT*(T-TO))
      RETURN
    1 APSEC  =APMI+APDOT*(T-TO)
      DAPSEC = DAPDOT*(T-TO)**2
      MASEC  =MAMI+PMMOT*(T-TO)
      DMASEC = DMADOT*(T-TO)**2
      DASEC  = DADOT*(T-TO)
      DESEC  = DEDOT*(T-TO)
      DISEC  = PRIN(DIDOT*(T-TO))
      APSEC  = PRIN(APSEC+DAPSEC)
      MASEC  = PRIN(MASEC+DMASEC)
      RETURN
      END
```

```
c
c--------------------------------------------------------------+
c                        B L P T R M                            |
c--------------------------------------------------------------+
c                                                               |
c        This routine computes the Long Period Terms that are a |
c     part of the satellite ephemeris solution equations.       |
c                                                               |
c--------------------------------------------------------------+
c
      SUBROUTINE BLPTRM
      DOUBLE PRECISION OME,PO2,PI,TPI,AMU,RE,POLR,T,TO,TF,TAPO,
     *GET,POL,OE,ME,IME,AMI,PM,REOA,REOP,EMI,IMI,APMI,RANMI,
     *UMI,TAMI,LAMMI,BB,ALP,BO,ABET,QOB,EOB,APSTAR,MMSTAR,
     *ELP,RANLP,MALP,LAMLP,ETAB,ZMB,AK42,AK2,AK32,MMOT,
     *APDOT,RANDOT,LAMDOT,ETAI,TPER,MAMI,ILP,APLP,PMMOT,
     *CD,A,DM,RP,B,H,FE,Q0,DRG,RHONUL,G,DB,DBSQR,DBCUBE,
     *Q,B0,B1,MBF0,MBF1,PA,PE,DPI,PISTAR,Q3I,TDG,TPIOT,ELM,EP,
     *EB,DB1,EBSQR,Y1A,Z1A,Y1E,Z1E,Y1I,Z1I,Y1ISTR,Z1ISTR,Q1A,Q1E,
     *Q1I,Q1ISTR,P2A,P2E,P2I,P3I,Y3W,Z3W,Y3M,Z3M,Q3W,Q3M,P2W,
     *Y5A,Z5A,Y5E,Z5E,Y5I,Z5I,Y5ISTR,Z5ISTR,P5A,P5E,P5I,P5ISTR,Y6I,
     *V6I,Y6W,U6W,Z6W,V6W,U6M,Z6M,V6M,P6I,P6W,P6M,EPS1,ALDOT,
     *SEPS1,COISQ,SISQ,APDT,RADT,EL1,EL2,EL3,EL4,EL5,EL6,EL7,EL8,
     *SLW,SLW0,DLW,DLW0,SLOW,SLOW0,DLOW,DLOW0,SL3W,SL3W0,
     *SLO3W,SLO3W0,DLO3W,DLO3W0,SNA,SNA0,SNB,SNB0,SNC,
     *SNC0,SND,SND0,SNE,SNE0,SNF,SNF0,SNG,SNG0,SNH,SNH0,CSA,
     *CSB,CSB0,CSC,CSC0,CSD,CSD0,AWBAR,APRIM,BPRIM,ACPRM,
     *ADLPM,AOPRIM,B0PRIM,AC0PRM,BS0PRM,A0DLPM,DAPM,DS2W,
     *DS4WI,DC2W,DC2WI,DSI,DSISQR,GOEP,REOPSQ,ALPD,ELPD,DILP,
     *APLPD,DMALP,U6I,Z6I,F,ALI,ESQR,DD,EL,P2M,DLMDA,DL3W,
     *DL3W0,CSA0,BSPRM,DS2WI,DS4W,RANLPD
      COMMON/BLOK12/JCIRC
      COMMON/BLOK1/PI,TPI,AMU,RE,POLR,AJ2,AJ3,OME,PO2,AJ4
      COMMON/BLOK2/T,TO,TF,TAPO,TPER,GET,DAYNO,JULDAY,GSTC
      COMMON/BLOK17/POL(6),OE(6),ME(6),IME(6)
      COMMON/BLOK14/BB,ALP,BO,ABET,QOB,EOB
      COMMON/BLOK11/MMOT,PMMOT,APDOT,RANDOT,LAMDOT
      COMMON/BLOK21/ELP,ILP,APLP,RANLP,MALP,LAMLP,ALPD
      COMMON/BLOK8/AMI,EMI,IMI,APMI,RANMI,MAMI,UMI,TAMI,LAMMI
      COMMON/BLOK111/PA,PE,DPI,PISTAR,Q3I
      COMMON/BLOK113/DRG,F,RHONUL,H,ALI
      DATA PI/3.141592654D+0/TPI/6.283185307D+0/
```

```
      DATA AJ2/1.0827E-3/AJ3/-2.56E-6/AJ4/-1.58E-6/
      DATA RE/6378160.D+0/AMU/3.986012D+14/EL/.0000729D+0/
      DATA DPR/57.29577951D+0/B/6.356780D+6/RPD/.0174532925D+0/
      CALL UNSIME
      AA=1./3.-.5*SIN(IMI)**2
      BB=1./3.-AA
      OME2=1.-EMI*EMI
      PM=AMI*OME2
C     TYPE 2002,APMI,AJ2,RE,EMI,BB,MMOT,T,TO
C2002 FORMAT(D20.12,E20.7,/,3D20.12,3D20.12)
      APSTAR=APMI+1.5*AJ2*(RE/PM)**2*(2.-5.*BB)*MMOT*(T-TO)
      MMSTAR=MAMI+MMOT*(T-TO)*(1.+1.5*AJ2*(RE/PM)**2*
     *SQRT(OME2)*(1.-3.*BB))
      AK2=1.5*AJ2*(RE/PM)**2
      AK32=.5*AJ3/AJ2
      REOA=RE/AMI
      REOP=RE/PM
      AK42=AJ4/(AJ2*AJ2)
      S2I=SIN(2.*IMI)
      C2WS=COS(2.*APSTAR)

      C2AP=COS(2.*APMI)
      SI=SIN(IMI)
      CI=COS(IMI)
      C2I=COS(2.*IMI)
      CTI=CI/SI
      S2WS=SIN(2.*APSTAR)
      S2AP=SIN(2.*APMI)
      SWS=SIN(APSTAR)
      SAP=SIN(APMI)
      CWS=COS(APSTAR)
      CAP=COS(APMI)
      IF(JCIRC.EQ.1) GO TO 1
      ILP=AK2*S2I/(2.-5.*BB)*EMI*EMI*(-7./48.+5./16.*BB+35./18.*AK42*
     *(9./56.-3./8.*BB))*(C2WS-C2AP)-AK32*REOP*EMI*CI*(SWS-SAP)
      ELP=-AK2*2.*OME2*SI*SI/(2.-5.*BB)*EMI*(-7./48.+5./16.*BB+35./18.*
     *AK42*(9./56.-3./8.*BB))*(C2WS-C2AP)+AK32*REOP*(SWS-SAP)*SI
      RANLP=AK2*.5*CI/(2.-5.*BB)*(EMI*EMI*(-7./12.+5./2.*BB)+EMI*EMI*
     *BB/(2.-5.*BB)*(-35./12.+25./4.*BB)+35./18.*AK42*(EMI*EMI*(9./14.-
     *3.*BB)+EMI*EMI*BB/(2.-5.*BB)*(45./14.-15./2.*BB)))*(S2WS-S2AP)+
     *AK32*REOP*EMI*CI/SI*(CWS-CAP)
```

```
      APLP=.5*AK2/(2.-5.*BB)*(BB*(-7./6.+5./2.*BB)+EMI*EMI*(7./12.-79./
     *12.*BB+45./4.*BB*BB)+4.*BB*(13.-30.*BB)/(2.-5.*BB)*EMI*EMI*(7./48.
     *-5./16.*BB)+35./18.*AK42*(BB*(9./7.-3.*BB)+EMI*EMI*(-9./14.+105./
     *14.*BB-27./2.*BB*BB)+4.*BB*(13.-30.*BB)/(2.-5.*BB)*EMI*EMI*(-9./56
     *.+3./8.*BB)))*(S2WS-S2AP)+AK32*REOP*(SI/EMI-EMI*CI*CI/SI)*
     *(CWS-CAP)
      MALP=.5*AK2*SQRT(OME2)/(2.-5.*BB)*(BB*(7./6.-5./2.*BB)*(1.-5./2.*E
     *MI*EMI)-35./18.*AK42*(BB*(9./7.-3.*BB)*OME2))*(S2WS-S2AP)-AK32*REO
     *P*SQRT(OME2)*SI*(1./EMI-EMI)*(CWS-CAP)
        RP  = AMI*(1.-EMI)
        FE  = (RE -B)/RE
        Q0  = RE*FE*SIN(IMI)**2*SIN(APMI)**2/H
        G   = DRG*RHONUL*EXP(-(AMI-RP)/H  +Q0)
        DB   = AMI*EMI/H
        DBSQR = DB**2
        DBCUBE = DBSQR*DB
        ESQR   = EMI**2
        DD      = EL/MMOT*SQRT(OME2)*COS(IMI)
        Q       = (RE-B)/H*SIN(IMI)**2
        B0      = MBF0(DB)
        B1      = MBF1(DB)
        BB      = .5*SIN(IMI)**2
C
C       COMPUTATION OF LONG PERIOD ORBITAL ELEMENTS WITH DRAG
C

        TDG     = TPI/MMOT*(1.-AK2*SQRT(OME2)*(1.-3.*BB))
        TPIOT   = TPI/(TDG)
        ELM     = EL/MMOT
        EP      = 1.5*AJ2
        EB      = EMI/DB
       DB1      = 1./DB
        EBSQR   = EB**2
C
C       DEFINITION OF P2K(K = A,E,I,W,M) AND P3I
C
        Y1A     = EB*(4.*(1.-DD)-3.*EB*(17./2.-5.*DD))
        Z1A         =  DB1*((1.-DD)**2+ESQR*(17./2.-5*DD)-8.*EB*(1.-DD)
     1          + 6.*EBSQR*(17./2.-5.*DD))
        Y1E     = DB1*((1.-DD)**2+ESQR*(11./2.-3.*DD)-3.*EB*(1.-DD)
     1          *(3.+.5*DD)+12.*EBSQR*(11./2.-3.*DD))
        Z1E  = DB1*(EMI*(1.-DD)*(3.+DD)-DB1*(2.*(1.-DD)**2+5.*ESQR*
     1          (11./2.-3.*DD))+3.*(EMI/DBSQR)*(1.-DD)*(6.+DD)-
```

```
     2        24.*ESQR/DBCUBE*(11./2.-3.*DD))
      Y1I    = EB*(2.*DD+3.*EB*(.5+1.5*DD))
      Z1I      =  DB1*((1.-DD)-ESQR*(.5+1.5*DD)-4.*DD*EB-3.*EBSQR*
     1        (1.+3.*DD))
      Y1ISTR = DB1*(2.*EMI*DD-6./DB*((1.-DD)-.75*ESQR*(5.-DD))+
     1        48.*EMI/DBSQR)

      Z1ISTR = DB1*((1.-DD)-.5*ESQR*(11.+DD)-4.*EB*(3.+DD)+
     1          12./DBSQR*((1.-DD)-.75*ESQR*(5.-DD))-96.*EMI/DBCUBE)
      Q1A     = OME2*(Y1A*B0+Z1A*B1)
      Q1E     = OME2*(Y1E*B0+Z1E*B1)
      Q1I    = OME2*(Y1I*B0+Z1I*B1)
      Q1ISTR = OME2*(Y1ISTR*B0+Z1ISTR*B1)
      P2A     = .5*PA-Q1A
      P2E     = .5*PE-Q1E
      P2I    = .5*(DPI-PISTAR-2.*Q1I)
      P3I    = .5*(.5*PISTAR-Q1ISTR+2.*Q3I)
      Y3W    = DB1*((1.-DD)**2/EMI+EMI*(5./2.-DD+1.5*DD**2)-3./DB*
     1               (1.-DD)*(3.+.5*DD)+12.*EMI/DBSQR*(11./2.-3.*DD+
     2        .5*DD**2))
      Z3W   = DB1*((1.-DD)*(2.+1.5*DD)-DB1*(2.*(1.-DD)**2/EMI+EMI*
     1        (43./2.-11.*DD+9./2.*DD**2))+6./DBSQR*(1.-DD)*
     2            (3.+.5*DD)-24.*EMI/DBCUBE*(11./2.-3.*DD+.5*DD**2))
      Y3M   = DB1*((1.-DD)**2/EMI+EMI*(5./2.+.5*DD**2)-3./DB*(1.-DD)
     1           *(3.+.5*DD)+12.*EMI/DBSQR*(11./2.-3.*DD+.5*DD**2))
      Z3M   = DB1*((1.-DD)*(2.+1.5*DD)-DB1*(2.*(1.-DD)**2/EMI+EMI*
     1        (43./2.-9.*DD+5./2.*DD**2))+6./DBSQR*(1.-DD)*
     2            (3.+.5*DD)-24.*EMI/DBCUBE*(11./2.-3.*DD+.5*DD**2))
      Q3W     = SQRT(OME2)*(Y3W*B0+Z3W*B1)
      Q3M     = SQRT(OME2)*(Y3M*B0+Z3M*B1)
      P2W     = -Q3W
      P2M     = -Q3M
C
C     DEFINITION OF P5K ( K = A,E,I,I*)
C
      Y5A    = 2.0*EMI*(1.-DD**2)-ESQR/DB*(9./2.-DD)
      Z5A    = (1.-DD)**2  +ESQR*(1.5+DD)-EB*(1.-DD)*(3.+DD)+
     1        2.*ESQR/DBSQR*(9./2.-DD)
      Y5E    = (1.-DD)**2+ESQR*(.5-DD)-EB*(1.-DD)*(2.+1.5*DD)+
     1        3.*ESQR/(2.*DBSQR)*(5.-DD-3.*DD**2)
```

```
      Z5E      =  EMI*(1.-DD)*(1.+3.*DD)-DB1*(1.-DD)*(1.-DD+ESQR*
     1         (3.+2.*DD))+EMI/DBSQR*(1.-DD)*(4.+3.*DD)-
     2         3.*ESQR/DBCUBE*(5.-DD-3.*DD**2)
      Y5I    = -2.*EMI*(1.-2.*DD)+ESQR/(2.*DB)*(1.+7.*DD)
      Z5I    = (1.-DD)+ESQR/2.*(1.-13.*DD)+EB*(1.-3.*DD)-
     1         ESQR/DBSQR*(1.+7.*DD)
      Y5ISTR  =  -2.*EMI*(1.-2.*DD)-DB1*(2.*(1.-DD)-ESQR/2.*(7.+DD))
     1         +6.*EMI/DBSQR*(1.+DD)
      Z5ISTR  =  (1.-DD)+ESQR/2.*(3.-7.*DD)+EB*(1.-7*DD)+2./DBSQR
     1         *(2.*(1.-DD)-ESQR/2.*(7.+DD))-12.*EMI/DBCUBE
     2         *(1.+DD)
      P5A     = Y5A*B0+Z5A*B1
      P5E     = Y5E*B0+Z5E*B1
      P5I    = Y5I*B0+Z5I*B1
      P5ISTR = Y5ISTR*B0+Z5ISTR*B1
C
C     DEFINITION OF P6K (K = I,W,M)
C
      Y6I    = DB1*((1.-DD)-ESQR/2.*(1.+3.*DD)-3.*EB*(1.+DD)+
     1         6.*ESQR/DBSQR*(1.-DD))
      U6I    = OME2*Y6I
      Z6I    = DB1*(2.*EMI*DD-DB1*(2.*(1.-DD)+ESQR/2.*(1.-9.*DD))+
     1              6.*EMI/DBSQR*(1.+DD)-12.*ESQR/DBCUBE*(1.-DD))



      V6I    = OME2*Z6I
      Y6W      = DB1*((1.-DD)*(2.+1.5*DD)-3.*EB*(5./2.-.5*DD))
      U6W      = SQRT(OME2)*Y6W
      Z6W      = DB1*((1.-DD)**2/EMI+EMI*(5./2.-DD+ 0.5*DD**2)-
     1           2./DB*(1.-DD)*(2.+1.5*DD)+6.*EMI/DBSQR*(5./2.-.5*DD))
      V6W      = SQRT(OME2)*Z6W
      U6M      = U6W
      Z6M    = DB1*((1.-DD)**2/EMI+EMI*(5./2.-.5*DD**2)-2./DB*(1.-DD)
     1         * (2.+1.5*DD)+6.*E/DBSQR*(5./2.-.5*DD))
      V6M      = SQRT(OME2)*Z6M
      P6I    = U6I*B0+V6I*B1
      P6W      = U6W*B0+V6W*B1
      P6M      = U6M*B0+V6M*B1
C
C     DEFINITION  OF  AWBAR,A',A0',B',B0',A",A0",AC',AC0',BS',BS0'
C
C           ALDOT = MEAN RATE OF THE SUN RELATIVE TO THE EARTH
```

```
C        ALI   = INITIAL MEAN LONGITUDE OF THE SUN AT TIME TO
C                     EPS1   = THE OBLIQUITY OF THE ECLIPTIC
C          DLMDA  = ANGLE IN RIGHT ASCENSION THAT CENTER
C                 OF BULGE LAGS THE SUN
      EPS1   = 23.44D0*RPD
      ALDOT  = (0.9856D0*RPD)/86400.D0
      DLMDA  = 30.D0*RPD
      SEPS1  = SIN(EPS1)
      COISQ  = COS(IMI/2.)**2
     SISQ   = SIN(IMI/2.)**2
      APDT   = AK2*MMOT*(2.-5./2.*SIN(IMI)**2)
      RADT   = -AK2*MMOT*COS(IMI)
      EL1    = ALDOT + APDT
      EL2    = ALDOT - APDT
      EL3    = ALDOT-RADT+APDT
      EL4    = ALDOT-RADT-APDT
      EL5    = ALDOT + 3.*APDT
      EL6    = ALDOT - 3.*APDT
      EL7    = ALDOT-RADT+3.*APDT
      EL8    = ALDOT-RADT-3.*APDT
      SLW    = ALI+APMI+EL1*(T-TO)
      SLW0   = ALI+APMI
      DLW    = ALI-APMI+EL2*(T-TO)
      DLW0   = ALI-APMI
      SLOW   = ALI-RANMI+DLMDA+APMI+EL3*(T-TO)
      SLOW0  = ALI-RANMI+DLMDA+APMI
      DLOW   = ALI-RANMI+DLMDA-APMI+EL4*(T-TO)
      DLOW0  = ALI-RANMI+DLMDA-APMI
      SL3W   = ALI+3.*APMI+EL5*(T-TO)
      SL3W0  = ALI+3.*APMI
      DL3W   = ALI-3.*APMI+EL6*(T-TO)
      DL3W0  = ALI-3.*APMI
      SLO3W  = ALI-RANMI+DLMDA+3.*APMI+EL7*(T-TO)
      SLO3W0 = ALI-RANMI+DLMDA+3.*APMI
      DLO3W  = ALI-RANMI+DLMDA-3.*APMI+EL8*(T-TO)
      DLO3W0 = ALI-RANMI+DLMDA-3.*APMI
      SNA    = SIN(SLW)
      SNA0   = SIN(SLW0)
      SNB    = SIN(DLW)
      SNB0   = SIN(DLW0)
      SNC    = SIN(SLOW)
      SNC0   = SIN(SLOW0)
      SND    = SIN(DLOW)
```

```
      SND0  = SIN(DLOW0)
      SNE   = SIN(DL3W)
      SNE0  = SIN(DL3W0)



      SNF   = SIN(SL3W)
      SNF0  = SIN(SL3W0)
      SNG   = SIN(DLO3W)
      SNG0  = SIN(DLO3W0)
      SNH   = SIN(SLO3W)
      SNH0  = SIN(SLO3W0)
      CSA   = COS(SLW)
      CSA0  = COS(SLW0)
      CSB   = COS(DLW)
      CSB0  = COS(DLW0)
      CSC   = COS(SLOW)
      CSC0  = COS(SLOW0)
      CSD   = COS(DLOW)
      CSD0  = COS(DLOW0)
C
      AWBAR  = (RE/PM)**2*MMOT*(2.-5./2.*SIN(IMI)**2)
     APRIM  = .5*SEPSI*SIN(IMI)*(EP/EL2*SNB-EP/EL1*SNA)
     1         +SISQ*EP/EL3*SNC+COISQ*EP/EL4*SND
     BPRIM  = -.5*SEPSI*SIN(IMI)*(EP/EL2*CSB-EP/EL1*CSA)
     1         +SISQ*EP/EL3*CSC-COISQ*EP/EL4*CSD
      ACPRM = EP/4.*SEPSI*SIN(IMI)*(1./EL6*SNE-1./EL5
     1        *SNF-1./EL2*SNB+1./EL1*SNA)+
     2        EP/2.*COISQ*(1./EL8*SNG+1./EL3*SNC)
     3        +EP/2.*SISQ*(1./EL7*SNH+1./EL4*SND)
      BSPRM = EP/4.*SEPSI*SIN(IMI)*(1./EL6*SNE-1./EL5
     1        *SNF+1./EL2*SNB-1./EL1*SNA)+
     2        EP/2.*COISQ*(1./EL8*SNG-1./EL3*SNC)
     3        +EP/2.*SISQ*(1./EL7*SNH-1./EL4*SND)
      ADLPM = -EP**2/2.*SEPSI*SIN(IMI)*(1./EL2**2*CSB-1./EL1
     1        **2*CSA)-SISQ*EP**2/EL3**2*CSC-
     2        COISQ*EP**2/EL4**2*CSD
     A0PRIM  = .5*SEPSI*SIN(IMI)*(EP/EL2*SNB0-EP/EL1*SNA0)
     1         +SISQ*EP/EL3*SNC0+COISQ*EP/EL4*SND0
     B0PRIM  = -.5*SEPSI*SIN(IMI)*(EP/EL2*CSB0-EP/EL1*CSA0)
     1         +SISQ*EP/EL3*CSC0-COISQ*EP/EL4*CSD0
      AC0PRM = EP/4.*SEPSI*SIN(IMI)*(1./EL6*SNE0-1./EL5
     1        *SNF0-1./EL2*SNB0+1./EL1*SNA0)+
```

```
     2         EP/2.*COISQ*(1./EL8*SNG0+1./EL3*SNC0)
     3          +EP/2.*SISQ*(1./EL7*SNH0+1./EL4*SND0)
      BS0PRM = EP/4.*SEPSI*SIN(IMI)*(1./EL6*SNE0-1./EL5
     1         *SNF0+1./EL2*SNB0-1./EL1*SNA0)+
     2         EP/2.*COISQ*(1./EL8*SNG0-1./EL3*SNC0)
     3          +EP/2.*SISQ*(1./EL7*SNH0-1./EL4*SND0)
      A0DLPM = -EP**2/2.*SEPSI*SIN(IMI)*(1./EL2**2*CSB0-1./EL1
     1         **2*CSA0)-SISQ*EP**2/EL3**2*CSC0-
     2         COISQ*EP**2/EL4**2*CSD0
C
      DAPM    = APMI+APDT*(T-TO)
      DS2W    = SIN(2.*DAPM)
      DS2WI   = SIN(2.*APMI)
      DS4W    = SIN(4.*DAPM)
      DS4WI   = SIN(4.*APMI)
      DC2W    = COS(2.*DAPM)
      DC2WI   = COS(2.*APMI)
      DSI     = SIN(IMI)
      DSISQ   = DSI*DSI
      GOEP    = G/EP
      REOPSQ  = (RE/PM)**2
C
      ALPD    = -TPIOT*(2.*GOEP*AMI**2*(Q*P2A/(2.*AWBAR)*
     1         (DS2W-DS2WI)+ F*P5A*(APRIM-A0PRIM)))
      ELPD    = -TPIOT*(2.*GOEP*AMI*OME2*(Q*P2E/(2.*AWBAR)*
     1         (DS2W-DS2WI)+F*P5E*(APRIM-A0PRIM)))



      DILP  = -TPIOT*(.5*GOEP*AMI*ELM*DSI/SQRT(OME2)*((PISTAR+Q*
     1                 P2I)/(2.*AWBAR)*(DS2W-DS2WI)+Q*P3I/(4.*AWBAR)*
     2                    (DS4W-DS4WI)+F*(P5I*(APRIM-A0PRIM)+P5ISTR*
     3         (ACPRM-AC0PRM)-2.*P6I*(BSPRM-BS0PRM))))
      RANLPD = TPIOT*(GOEP*REOPSQ*MMOT*AMI*COS(IMI)*(Q/(4.
     1                   *AWBAR**2)*(7.*P2A-8.*EMI*P2E)*(DC2W-DC2WI)-
     2         F*(ADLPM-A0DLPM)*(7.*P5A-8.*EMI*P5E)))
      APLPD  = TPIOT*(GOEP*AMI*SQRT(OME2)*(Q*P2W/AWBAR*
     1                  (DC2W-DC2WI)-2.*F*P6W*(BPRIM-B0PRIM)))-TPIOT*
     2                 (GOEP*REOPSQ*MMOT*AMI*(2.-5./2.*DSISQ)*(Q/(4.*
     3         AWBAR**2)*(7.*P2A-8.*EMI*P2E)*(DC2W-DC2WI)-
     4         F*(ADLPM-A0DLPM)*(7.*P5A-8.*EMI*P5E)))
      DMALP  = -TPIOT*(GOEP*AMI*(Q*P2M/AWBAR*(DC2W-DC2WI)-
     1                 2.*F*P6M*(BPRIM-B0PRIM)))-TPIOT*(GOEP*REOPSQ*
```

```
2           MMOT*AMI*SQRT(OME2)*(1.-1.5*DSISQ)*(Q/(4.*
3                  AWBAR**2)*(7.*P2A-6.*EMI*P2E)*(DC2W-DC2WI)-F*
4           (ADLPM-A0DLPM)*(7.*P5A-6.*EMI*P5E)))
   ILP    = ILP + DILP
   ELP     = ELP + ELPD
   APLP    = APLP + APLPD
   RANLP  = RANLP + RANLPD
   MALP    = MALP + DMALP
  RETURN
1 ETAB=QOB*ALP*SIN(BO*(T-TO)+ABET)+EOB
  ETAI=QOB*ALP*SIN(ABET)+EOB
  ZMB=ALP*COS(BO*(T-TO)+ABET)
  RANLP=.5*(AJ3/AJ2)*REOA*CTI*(ZMB-ALP*COS(ABET))
  LAMLP=-.5*(AJ3/AJ2)*REOA*C2I/SI*(ZMB-ALP*COS(ABET))
  ILP=-.5*(AJ3/AJ2)*REOA*CI*(ETAB-ETAI)
  RETURN
  END
```