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PROPULSION ESTIMATION TECHNIQUES

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ABSTRACT

This paper gives a brief overview of statistical tools that are needed to perform post flight/test reconstruction of state variables. Linear regression, recursive linear regression, and the exact connection between the Kalman filter and linear regression are discussed. The regression connection is expected to serve as an aid in the application of a recently developed analytical method of flight reconstruction to single engine test firing data.

1. INTRODUCTION

This paper considers the issue of post flight/test state variable reconstruction through the application of observations made on the output of the Space Shuttle propulsion system. The incentive for the concepts discussed here is rooted in a NASA contract report by Rogers (1987). Rogers used the Kalman filtering procedure to reconstruct the state variables of the Space Shuttle propulsion system.

Since its development by Kalman (1960), the Kalman filtering methodology has been a popular procedure with control engineers. Within recent years, it has seen increased applications in other engineering areas as well as in nonengineering areas. Although the applications have been diverse and research on some of its defects has been quite extensive, applied statisticians have not taken full advantage of the tool. This lack of use by some applications areas may be because many publications on the topic have obscured statistical simplicity. Often times the basic statistics concepts are embedded in a discussion of numerical solutions to differential equations.

An objective of this paper is to give the general setup of the Kalman filter and its connection to linear regression. A second objective is to examine the Rogers (1987) reconstruction methodology for application to the reconstruction of the state vector of a single Space Shuttle Main Engine (SSME) by using static test firing data.

Throughout this paper, underlined capital letters are used to denote vectors, capital letters denote matrices, and the identity matrix is denoted by the letter I. All vectors are of the column type and the transpose of any matrix or vector is denoted by using the letter T at the superscript position. The letter E denotes the expectation operator, $N(\underline{U},\Sigma)$ denotes the multivariate normal probability distribution with mean vector $\underline{U}$ and covariance matrix Σ. The caret symbol "^" written directly above a scalar or vector denotes a statistical estimator of that vector or scalar.

2. ESTABLISHMENT OF NOTATION

For any Space Shuttle flight, let $\underline{Z}_t$ denote the observed values of vector $\underline{Z}$ at time t. Each component of $\underline{Z}_t$ represents a relevant output of the Space Shuttle propulsion system that can be measured. For example, the components of $\underline{Z}_t$ may be chamber pressure, oxygen flow rate, hydrogen flow rate, etc., for each of the three SSME's. Rogers (1987) lists 71 components for vector $\underline{Z}_t$ and 35 components for vector $\underline{X}_t$ where $\underline{X}_t$ is defined below.

Vector $\underline{X}_t$ is a state vector of parameters to be estimated at time t. It is assumed that the observation vector $\underline{Z}_t$ is a function of the state vector $\underline{X}_t$. That is,

$$\underline{Z}_t = \underline{h}\ (\underline{X}_t) + \underline{V}_t \qquad (2.1)$$

where $\underline{h}$ is some function and $\underline{V}_t \sim N(\underline{O}, R_t)$. State vector $\underline{X}_t$ is known to change with respect to time according to the equation

$$\dot{X}_t = \underline{f}\ (\underline{X}_t, t) + \underline{W}_t \qquad (2.2)$$

where $\underline{f}$ is some function, $\underline{W}_t \sim N(\underline{O}, Q_t)$ and $E(\underline{W}_t\ \underline{V}_t^T) = 0$.

If it is assumed that $\underline{f}$ and $\underline{h}$ in equations (2.1) and (2.2) are linear, then numerical procedures allow us to transform these equations to the form

$$\underline{X}_K = F_K\ \underline{X}_{K-1} + \underline{W}_K \qquad (2.3)$$

$$\underline{Z}_K = H_K \underline{X}_K + \underline{V}_K \tag{2.4}$$

where K represents discrete values of t, $\underline{W}_K \sim N(\underline{O}, Q_K)$ and $\underline{V}_K \sim N(O, R_K)$.

After a Space Shuttle flight has taken place, the post flight reconstruction procedure seeks to use the observed values of $\underline{Z}_K$ and the equations in (2.3) and (2.4) to reconstruct (estimate) the parameters $\underline{X}_K$ so that the estimation error is minimized. The estimated value of $\underline{X}_K$ is denoted by $\hat{\underline{X}}_K$.

3. A RECURSIVE ESTIMATOR OF $\underline{X}$

In order to complete the development of the reconstruction process mentioned in section 2, we consider a system of equations

$$\underline{Z} = H \underline{X} + \underline{\varepsilon} \tag{3.1}$$

where (3.1) is a system of linear equations that has been generated by making η observations on the single equation

$$\underline{Z}^* = H^* \underline{X} + \underline{\varepsilon}^* \tag{3.2}$$

where $\underline{Z}^*$ and $\underline{\varepsilon}^*$ are vectors or scalars. H^* is a jxp matrix where j = number of components in vector $\underline{Z}^*$ and $\underline{X}$ is fixed. Vector $\underline{X}$ being fixed means that $\underline{X}$ does not change with time or does not change as the number of observations on $\underline{Z}$ increase. If $\text{Cov}(\underline{\varepsilon}) = \Sigma$ and $\det(\Sigma) \neq 0$, then the weighted least squares estimate of vector $\underline{X}$ is given by

$$\hat{\underline{X}} = (H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1} \underline{Z} \tag{3.3}$$

and the covariance of the estimate is given by

$$\mathrm{Cov}(\hat{X}) = E\,[(\hat{\underline{X}} - \underline{X})\,(\hat{\underline{X}} - \underline{X})^T] = (H^T\,\Sigma^{-1}\,H)^{-1} = P\ . \tag{3.4}$$

The reader should be reminded that with the appropriate assumptions, the maximum likelihood, the minimum variance unbiased linear as well as the minimum mean-squared error estimator of $\underline{X}$ are identical to the estimator given by equation (3.3). We also remark that $\mathrm{Cov}(\hat{\underline{X}})$ is a measure of the quality of the estimates in vector $\hat{\underline{X}}$. A detailed discussion of these estimators may be found in Elbert (1984).

In the sequel that follows, the estimator in equation (3.3) will be rearranged so that $\underline{X}$ can be estimated by using a recursive process. The development of the recursive process is started by assuming that there are K observations on equation (3.2) which form the system

$$\underline{Z}_K = H_K\,\underline{X} + \underline{\varepsilon}_K \tag{3.5}$$

where $\mathrm{Cov}\ \underline{\varepsilon}_K = \Sigma_K$ and $\det\,(\Sigma_K) \neq 0$. By equation (3.3)

$$\hat{\underline{X}}_K = (H_K^T\,\Sigma_K^{-1}\,H_K)^{-1}\,H_K\,\Sigma_K^{-1}\,\underline{Z}_K \quad , \tag{3.6}$$

is the estimate of $\underline{X}$ where observations up to and including observation K are used. The covariance of the estimator is denoted by P_K and

$$P_K = E[(\hat{X}_K - \underline{X})\,(\hat{\underline{X}}_K - \underline{X})^T] = (H_K^T\,\Sigma_K^{-1}\,H_K)^{-1}\ . \tag{3.7}$$

Assume that an additional measurement $\underline{Z}_{K+1}$ has been made. Let

$$\underline{Z}_{K+1} = H_{K+1}\,\underline{X} + \underline{\varepsilon}_{K+1} \tag{3.8}$$

where $\mathrm{Cov}\,(\underline{\varepsilon}_{K+1}) = \Sigma_{K+1}$ and $E\,(\underline{\varepsilon}_K\,\underline{\varepsilon}_{K+1}^T) = 0$. Note that $E\,(\underline{\varepsilon}_K\,\underline{\varepsilon}_{K+1}^T) = 0$ means that the (K+1)th observation is independent of the 1st K observations. Putting the K observations with the (K+1)th observation yields the system

$$\begin{pmatrix} \underline{Z}_K \\ \text{----} \\ \underline{Z}_{K+1} \end{pmatrix} = \begin{pmatrix} H_K \\ \text{----} \\ H_{K+1} \end{pmatrix} \underline{X} + \begin{pmatrix} \underline{\varepsilon}_K \\ \text{----} \\ \underline{\varepsilon}_{K+1} \end{pmatrix} \tag{3.9}$$

where

$$\text{Cov} \begin{pmatrix} \underline{\varepsilon}_K \\ \text{----} \\ \underline{\varepsilon}_{K+1} \end{pmatrix} = \left(\begin{array}{c|c} \Sigma_K & 0 \\ \hline 0 & \Sigma_{K+1} \end{array} \right) .$$

By substituting into equation (3.3) we get

$$\hat{\underline{X}}_{K+1} = \left[(H_K^T \mid H_{K+1}^T) \left(\begin{array}{c|c} \Sigma_K^{-1} & 0 \\ \hline 0 & \Sigma_{K+1}^{-1} \end{array} \right) \begin{pmatrix} H_K \\ \text{-----} \\ H_{K+1} \end{pmatrix} \right]^{-1} (H_K^T \mid H_{K+1}^T) \left(\begin{array}{c|c} \Sigma_K^{-1} & 0 \\ \hline 0 & \Sigma_{K+1}^{-1} \end{array} \right) \begin{pmatrix} \underline{Z}_K \\ \text{----} \\ \underline{Z}_{K+1} \end{pmatrix} . \tag{3.10}$$

Note that we can write

$$\hat{\underline{X}}_{K+1} = \hat{\underline{X}}_K + (\hat{\underline{X}}_{K+1} - \hat{\underline{X}}_K) . \tag{3.11}$$

Substituting results from (3.6) and (3.10) into (3.11) and using matrix algebra, we get

$$\hat{\underline{X}}_{K+1} = \hat{\underline{X}}_K + (P_K^{-1} + H_{K+1}^T \Sigma_K^{-1} H_{K+1})^{-1} H_{K+1}^T \Sigma_{K+1}^{-1} (\underline{Z}_{K+1} - H_{K+1} \hat{\underline{X}}_K) . \tag{3.12}$$

By substituting into equation (3.7) and using matrix algebra, we get

$$P_{K+1} = \left[(H_K^T \mid H_{K+1}^T) \left(\begin{array}{c|c} \Sigma_K^{-1} & 0 \\ \hline 0 & \Sigma_{K+1}^{-1} \end{array} \right) \begin{pmatrix} H_K \\ \text{-----} \\ H_{K+1} \end{pmatrix} \right]^{-1} = (P_K^{-1} + H_{K+1} \Sigma_{K+1}^{-1} H_{K+1})^{-1} .$$

Substituting P_{K+1} into equation (3.12) yields

$$\hat{\underline{X}}_K = \hat{\underline{X}}_K + P_{K+1} H_{K+1}^T \Sigma_{K+1}^{-1} (\underline{Z}_{K+1} - H_{K+1} \hat{\underline{X}}_K) \tag{3.13}$$

and

$$P_{K+1} = (P_K^{-1} + H_{K+1}^T \Sigma_{K+1}^{-1} H_{K+1})^{-1} . \tag{3.14}$$

Equations (3.13) and (3.14) provide recursive equations for estimating $\underline{X}$ and its corresponding covariance matrix P.

Recall that the estimator given in equation (3.3) may be identified by using the symbol $\hat{\underline{X}}_n$. Therefore, if $\underline{X}$ is fixed as it is in equation (3.1), the recursive equations of (3.13) and (3.14) have no statistical advantage over the estimator given in equation (3.3). However, if $\underline{X}$ changes with time where there is one observation per time interval, then the recursive equations are quite useful. In fact, equations (3.13) and (3.14) when combined with equations (2.3) and (2.4) form the Kalman filtering process.

4. THE KALMAN FILTER

It is worthwhile to mention that Brown (1983) and Gelb (1974) used matrix theory to express equations (3.13) and (3.14) as

$$P_{K+1} = (I - K_{K+1} H_{K+1}) P_K \tag{4.1}$$

$$\hat{\underline{X}}_{K+1} = \hat{\underline{X}}_K + K_{K+1} (\underline{Z}_{K+1} - H_{K+1} \hat{\underline{X}}_K) \tag{4.2}$$

where

$$K_{K+1} = P_K H_{K+1} (\Sigma_{K+1} + H_{K+1} P_K H_{K+1}^T)^{-1} \tag{4.3}$$

and K_{K+1} is called the gain matrix.

For the convenience of the reader, equations (2.3) and (2.4) are reprinted here. That is,

$$\underline{X}_{K+1} = F_K \underline{X}_K + \underline{W}_K \tag{4.4}$$

and

$$\underline{Z}_{K+1} = H_{K+1} \underline{X}_{K+1} + \underline{V}_{K+1} . \tag{4.5}$$

When equations (4.1) and (4.2) are combined with equations (4.4) and (4.5), two types of estimates of $\underline{X}_{K+1}$ are possible for each K. A notation by Gelb (1974) allows for the two estimators to be distinguished. That notation is

$\hat{\underline{X}}_{K+1}^{(-)}$ = the estimate of $\underline{X}_{K+1}$ using all observations up to and including observation K.

$P_{K+1}^{(-)}$ = the estimate of P using all observations up to and including observation K.

$\hat{X}_{K+1}^{(+)}$ = the estimate of $\underline{X}_{K+1}$ using all observations up to and including observation (K+1).

The values of $\hat{\underline{X}}_{K+1}^{(-)}$ and $P_{K+1}^{(-)}$ are obtained by using equations (4.4) and (4.5). The $\hat{\underline{X}}_{K+1}$ vector is often called the extrapolated or predicted value of $\underline{X}_{K+1}$ and $P_{K+1}^{(-)}$ is called the extrapolated variance of $\hat{\underline{X}}_{K+1}^{(-)}$.

The values of $\hat{\underline{X}}_{K+1}^{(+)}$ and $P_{K+1}^{(+)}$ are computed by substituting $\hat{\underline{X}}_{K+1}^{(-)}$ for $\underline{X}_K$ in equation (4.2) and $P_{K+1}^{(-)}$ for P_K in equation (4.1).

The computation summary is

$$\hat{\underline{X}}_{K+1}^{(-)} = F_K \hat{\underline{X}}_K^{(+)} \tag{4.6}$$

$$P_{K+1}^{(-)} = F_K \, P_K^{(+)} \, F_K^T + Q_K \qquad (4.7)$$

$$\hat{\underline{X}}_{K+1}^{(+)} = \hat{\underline{X}}_{K+1}^{(-)} + K_{K+1} \, (\underline{Z}_{K+1} - H_{K+1} \, \hat{\underline{X}}_{K+1}^{(-)}) \qquad (4.8)$$

$$P_{K+1}^{(+)} = (I - K_{K+1} \, H_{K+1}) \, P_{K+1}^{(-)} \qquad (4.9)$$

$$K_{K+1} = P_{K+1}^{(-)} \, H_{K+1}^T \, (H_{K+1} \, P_{K+1}^{(-)} \, H_{K+1}^T + R_{K+1})^{-1} \; . \qquad (4.10)$$

Equations (4.6) to (4.10) completely describe the Kalman filtering process when the original functions $\underline{f}$ and $\underline{h}$ of equations (2.1) and (2.2) are linear.

However, when functions $\underline{f}$ and $\underline{h}$ are nonlinear, the state vectors $\underline{X}_K$ ($K = t_0, t_1, \ldots t_T$) are estimated through the application of an extended Kalman filtering procedure. This procedure requires a linearization of functions $\underline{f}$ and $\underline{h}$ about some known state value $\underline{X}_K^*$. The next paragraph provides a brief overview of the extended Kalman filter concept.

Let $\underline{X}_K^*$ be some known value of $\underline{X}$ and assume that $\Delta\underline{X}$ is small. A first degree Taylor series approximation of functions (2.1) and (2.2) may be

$$\dot{\underline{X}}_K^* + \Delta\dot{\underline{X}} \approx f(\underline{X}_K^*, K) + \left[\frac{\partial \underline{f}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_K^*} \cdot \Delta\underline{X} + \underline{W}_t \qquad (4.11)$$

and

$$Z_t \approx h(\underline{X}_K^*) + \left[\frac{\partial \underline{h}}{\partial \underline{X}}\right]_{\underline{X}=X_K^*} \cdot \Delta\underline{X} + V_t \; . \qquad (4.12)$$

If it is assumed that $\underline{X}_K^*$ is selected so that $\dot{\underline{X}}_K^* = f(\underline{X}_K^*, K)$, then equations (4.11) and (4.12) become

$$\Delta \dot{\underline{X}} = \left[\frac{\partial \underline{f}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_K^*} \Delta \underline{X} + \underline{W}_K \tag{4.13}$$

and

$$\underline{Z}_K - h(\underline{X}_K^*) = \left[\frac{\partial \underline{h}}{\partial \underline{X}}\right]_{\underline{X}=\underline{X}_K^*} \Delta \underline{X} + \underline{V}_K \quad . \tag{4.14}$$

Equation (4.13) is called the linearized dynamics equation and (4.14) is the linearized measurement equation. Note that equations (4.13) and (4.14) may be transformed to equations that are equivalent to equations (2.3) and (2.4). Hence for each discrete time K, $\Delta \underline{X}_K$ can be estimated and error covariance matrices can be determined. The state vector estimate at time K is then given by

$$\hat{\underline{X}}_K^{(+)} = \underline{X}_K^* + \Delta \hat{\underline{X}}_K^{(+)}$$

where $\Delta \hat{\underline{X}}_K^{(+)}$ is computed by substituting from equations (4.13) and (4.14) into equation (4.8). Vector $\hat{\underline{X}}_{K+1}$ is computed by letting $\underline{X}_{K+1}^* \doteq \hat{\underline{X}}_K$ and repeating the above procedure.

The extended Kalman filter has performed well in a large class of applications. However, there are occasions when divergence occurs in the state vector estimates. Divergence occurs when the computed entries of the error covariance matrix P_K become small as compared to the actual error in the estimate of the state vector. The cause of this divergence is not due to a defect in the filtering procedure, but may be caused by the linear approximation procedure, numerical rounding error, or an inadequate model of the system being studied. Additional possible causes of divergence are mentioned by Gelb (1974).

A publication by Varhaegen and Van Dooren (1986) is representative of the theoretical and experimental analyses that are currently being done on the divergence problem. The Rogers (1987) computer code has implemented the U-D factorized algorithm as a means of controlling numerical roundoff error that may lead to divergence in the state vector. Checks for other sources that may generate divergence are conducted through computer evaluations.

5. CONCLUSION

At this point we have reviewed the general setup for a regression problem where the parameters to be estimated are fixed. This lead to recursive equations (4.1), (4.2), and (4.3) which allowed for the development of an estimation procedure for a time varying parameter. When functions $\underline{f}$ and $\underline{h}$ are linear, it has been clearly stated that equations (2.3) and (2.4) are the essential ingredients for state variable reconstruction. If $\underline{f}$ and $\underline{h}$ are nonlinear, $\underline{X}_K$ can be estimated by linearing $\underline{h}$ and $\underline{f}$ about some known vector and treating the linearized equations as if they are equations (2.3) and (2.4).

The problem of applying the extended Kalman filtering procedure to the static test firing data remains. The basic approach for the application is identical to the presentation given in Section 4. Therefore, the computer codes that have been prepared by Rogers (1987), which are currently operational on the MSFC computer system, may be modified so that static test data may be analyzed. The static test setup differs from the actual flight data in that many flight associated modules will become inactive. Therefore, after adjustments are made for the analysis of static test data, it will be necessary to evaluate the performance of the computer code for the divergence of parameter estimates.

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