

NONLINEAR, RELATIVISTIC LANGMUIR WAVES  
IN ASTROPHYSICAL MAGNETOSPHERES

by

Abraham C.-L. Chian\*

Institute of Geophysics and Planetary Physics  
University of California, Los Angeles  
Los Angeles, CA 90024, USA

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\* Permanent address: Institute for Space Research (INPE/CNPq), C.P. 515,  
12200-São José dos Campos, SP, Brazil

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# NONLINEAR, RELATIVISTIC, LANGMUIR WAVES IN ASTROPHYSICAL MAGNETOSPHERES\*

Abraham C.-L. Chian

Institute for Space Research (INPE/CNPq), C.P. 515, 12200-São José dos Campos, SP, Brazil

## ABSTRACT

Large amplitude, electrostatic plasma waves are relevant to physical processes occurring in astrophysical magnetospheres wherein charged particles are accelerated to relativistic energies by strong waves emitted by pulsars, quasars or radio-galaxies. In this paper the non-linear, relativistic theory of traveling Langmuir waves in a cold plasma is reviewed. The cases of streaming electron plasma, electron-ion plasma and two-streams are discussed.

## 1. Introduction

Strong waves capable of driving plasma particles to relativistic energies are of current interest in several fields of research. A wave is strong enough to drive electrons or positrons relativistic if its dimensionless strength parameter  $\nu = eE_{\max}/m_e \omega c$  is comparable to unity.

In astrophysical context, it was first proposed by Pacini<sup>1)</sup> that a rotating neutron star with misaligned magnetic dipole and rotational axes can release its magnetic and rotational energy by emitting low-frequency magnetic dipole electromagnetic waves at the stellar rotation frequency. Gunn and Ostriker<sup>2)</sup> showed that charged particles could be accelerated to ultrarelativistic energies by low-frequency pulsar magnetic dipole radiations with  $\nu \geq 10^{11}$ , thus suggesting that pulsar electromagnetic fields may be a cosmic ray source. Kennel et al<sup>3)</sup> found that when ions as well as electrons are driven relativistic by a plane plasma wave, there is an upper limit to the cosmic ray number flux above which the wave encounters cut-off. Rees<sup>4)</sup> suggested that the rotational energy of a large number of pul-

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sars or collapsing stars could supply the power for quasars or radiogalaxies through the emission of low-frequency strong electromagnetic waves.

Recently, Bodo et al<sup>5)</sup> used the dispersion relation of strong electromagnetic waves to study the propagation conditions of high-frequency coherent radio emission in pulsar magnetospheres. Chian and Kennel<sup>6)</sup> estimated that the strength parameter of coherent pulsar radiowaves in the source region lies in the range  $10^{-2} \leq v \leq 10^3$ ; in the field of those waves the charged particles can acquire from weakly to moderately relativistic energies.

It is worth mentioning that in addition to astrophysical applications, strong waves can be relevant to laser-plasma interaction<sup>7),8)</sup> and particle acceleration by laser beatwave<sup>9)</sup>. At laser intensities of  $10^{16} \text{Wcm}^{-2}$  for CO<sub>2</sub> gas laser and  $10^{18} \text{Wcm}^{-2}$  for Nd glass laser the wave strength parameter  $v \approx 1$  and relativistic effects become important. Lasers of such intensities are already available in laboratories.

Astrophysical applications of strong electromagnetic waves have been reviewed elsewhere<sup>10),11)</sup>. In this paper only the subject of nonlinear, relativistic electrostatic Langmuir waves is reviewed. The Langmuir mode may play an important role in both emission and propagation processes of pulsar radiations.

Ruderman and Sutherland<sup>12)</sup> suggested that the two-stream instability resulting from the relative streaming between electrons and positrons in a pulsar magnetosphere can excite unstable electrostatic plasma waves which then bunch the charged particles, producing coherent microwave radiation. Khakimova and Tsytovich<sup>13)</sup> and Buti<sup>14)</sup> considered the generation of coherent pulsar radio emission by Langmuir solitons arising from the modulational instability that develops from electrostatic plasma waves if their energy exceeds a certain threshold. Mamradze et al<sup>15)</sup> showed that the coalescence of two Langmuir waves can lead to infrared pulsar emission. Mikhailovskii<sup>16)</sup> presented a theory of pulsar radio emission using the decay process of a Langmuir wave into two electromagnetic waves.

As a strong electromagnetic wave propagates in a pulsar magnetosphere, its absorption and scattering is often dominated by collective processes involving electrostatic plasma waves. Tsintsadze and

Tsikarishvili<sup>17)</sup> investigated the parametric instability of a strong electromagnetic wave decaying into Langmuir and ion-acoustic waves. Drake et al.<sup>18)</sup> suggested that such instability may break up strong electromagnetic waves before energy transfer to magnetospheric particles<sup>2),3)</sup> can occur. Arons et al.<sup>19)</sup> considered the parametric Raman scattering of strong pulsar radiations by subluminal Langmuir waves.

The subject of Langmuir waves has been studied extensively. To the linear approximation, Tonks and Langmuir<sup>20)</sup> demonstrated that a small displacement of a uniform plasma results in an electrostatic force which gives rise to harmonic oscillations at the plasma frequency. As the amplitude of oscillations increases and the relativistic variation of particle mass appears, many nonlinear effects can alter significantly the behavior of Langmuir waves. As an initial step to understand the complex role that the Langmuir mode plays in various strong wave problems, we examine the traveling wave solutions of nonlinear, relativistic Langmuir waves. Three cases are studied: (i) streaming electron plasma, (ii) electron-ion plasma and (iii) two-streams.

## 2. Streaming Electron Plasma

The traveling wave theory of nonlinear, relativistic, Langmuir waves was first studied by Akhiezer and Polovin<sup>21)</sup>. The case they treated was a stationary electron plasma. In many astrophysical problems one encounters electron (or positron) streams moving at relativistic stream velocities. In this section the treatment of Akhiezer and Polovin<sup>21)</sup> is generalized to the case of streaming electron plasma<sup>22),23)</sup>.

Consider a large-amplitude Langmuir wave propagating with a constant speed  $V$  in a cold, unmagnetized plasma. The average electron charge and current densities are neutralized by positive ions, but the ion dynamics is ignored. Since we are seeking traveling wave solutions we can write the basic equations in terms of a combined space-time variable  $\theta = t - nx/c$ , where the index of refraction  $n$  ( $-\infty \leq n \leq \infty$ ) is a measure of the wave speed  $V = c/n$ . The basic equations are the relativistic equation of motion, the continuity equation and Poisson's equation

$$(1 - nv/c) \frac{d(\gamma v)}{d\theta} = - \frac{e}{m} E \quad , \quad (1)$$

$$\frac{dN}{d\theta} - \frac{n}{c} \frac{d(Nv)}{d\theta} = 0 \quad , \quad (2)$$

$$- \frac{n}{c} \frac{dE}{d\theta} = 4\pi e(N_0 - N) \quad , \quad (3)$$

supplemented by Maxwell's equation

$$\frac{dE}{d\theta} - 4\pi eNv + 4\pi J_0 = 0 \quad , \quad (4)$$

where  $\gamma = (1 - v^2/c^2)^{-1/2}$ ,  $N_0$  and  $J_0$  are neutralizing ion charge and current densities. In nonlinear theory, the plasma stream velocity can be defined as

$$V_s = \langle Nv \rangle / \langle N \rangle \quad , \quad (5)$$

which can be obtained by taking the phase-average of (3) and (4), with  $J_0 = eN_0 V_s$ .

The basic equations (1) - (5) can be combined to yield a single equation

$$\frac{1}{2} \left( \frac{du}{d\tau} \right)^2 = \frac{W - \gamma + V_s u/c}{(1 - nu/\gamma)^2} \quad , \quad (6)$$

where  $u = \gamma v/c =$  reduced electron velocity,  $\tau = \omega_{pe} \theta$ ,  $\omega_{pe}^2 = 4\pi N_0 e^2/m$  and  $W$  is a constant that characterizes the wave amplitude. Evidently (6) indicates that periodic wave solutions exist. The nonlinear dispersion relation is obtained by a further integration of (6)

$$P = \frac{\sqrt{2}}{\omega_{pe}} \int_{u_1}^{u_2} \frac{1 - nu/\gamma}{(W - \gamma + V_s u/c)^{1/2}} du \quad , \quad (7)$$

where  $P$  is the wave period. (7) describes Langmuir wave oscillations for arbitrary wave amplitudes. It recovers the result of a stationary plasma<sup>21)</sup> if  $V_s$  is set to zero, and can be obtained from the dispersion relation for a stationary plasma by a Lorentz transformation<sup>22)</sup>. Fig. 1 shows the variation of the cut-off frequency ( $n = 0$ )

of nonlinear, relativistic Langmuir waves with the amplitude of electron oscillations (where  $v_\alpha = |v_1 - v_2|$ ,  $v_{1,2}$  being the turning points of  $v$ ) for different stream velocities. It demonstrates that, due to relativistic variation of particle mass ( $m \rightarrow \gamma m$ ), the Langmuir wave frequency decreases as the wave amplitude increases. In Fig. 2, the product of  $c^2/\omega_{pe}^2$  with  $k^2$  (where  $k = n\omega/c$ ) is plotted against the wave speed for different wave amplitudes. It indicates that, for a given wave velocity, relativistic effects cause the wavenumber of Langmuir waves to decrease as the wave amplitude increases. It also shows that there are regions in the dispersion curves where periodic wave oscillations are not possible due to the phenomenon of wave-breaking.

The wave speed of nonlinear, relativistic Langmuir waves can be either superluminal ( $V > c$ ) or subluminal ( $V < c$ ). Superluminal waves can attain arbitrary amplitudes, but the amplitude of subluminal waves is limited. For nonlinear subluminal waves moving at a given speed, there is a critical wave amplitude above which wave breaks whereby ordered wave energy is transformed to random plasma thermal energy. In laser fusion research it has been found that wavebreaking of electrostatic plasma waves can produce highly energetic electrons<sup>24)</sup>; similar processes are expected to take place in astrophysical plasmas. The problem of wavebreaking can be clarified by analyzing the first integral corresponding to (6) in the time-independent frame<sup>22),23)</sup>

$$\frac{1}{2} \left( \frac{d\gamma}{d\tau} \right)^2 = W - f(\gamma) \tag{8}$$

with

$$f(\gamma) = |1/n - V_s/c| (\gamma^2 - 1)^{1/2} + (1 - V_s/nc)\gamma \tag{9}$$

and  $\tau = (1 - n^{-2})^{-1/4} \omega_{pe} z/c$ . The mathematical form of (8) suggests that the nonlinear wave problem is analogous to the motion of a particle of unit mass at position  $\gamma$ , at time  $\tau$ , in Newtonian dynamics. Thus, (8) corresponds to the conservation of energy with  $(d\gamma/d\tau)^2/2 =$  kinetic energy,  $f(\gamma) =$  potential energy and  $W =$  total energy. A typical plot of  $f$  as a function of  $\gamma$  is displayed in Fig. 3, which

shows that there is a maximum value of  $W$  for a periodic solution to exist

$$W_{\max} = f(\gamma = 1) = 1 - V_s/nc \quad (10)$$

above which wavebreaking then occurs. Transforming the wavebreaking condition back to the laboratory frame, it shows that traveling Langmuir wave solutions exist only if the electron velocity lies in the range

$$\begin{cases} v_c < v < c/n & \text{if } V_s < V \\ c/n < v < v_c & \text{if } V_s > V \end{cases} \quad (11)$$

where

$$v_c/c = \frac{1 + V_s^2/c^2 - 2nV_s/c}{2V_s/c - n(1 + V_s^2/c^2)} \quad (12)$$

Alternatively, the wavebreaking condition can be expressed in terms of  $R^2 = (eE_{\max}/mc\omega_{pe})^2$  as

$$R^2 \geq R_c^2 = 2[(1 - n^{-2})^{-1/2} (1 - V_s/nc) - (1 - V_s^2/c^2)^{1/2}] \quad (13)$$

Graphical displays of (11) and (13) are given by Fig. 4, where the shaded areas represent the regions where wavebreaking occurs. We see that subluminal Langmuir waves can attain large amplitudes if their speeds are close to the speed of light; as the wave speed gets away from  $c$  the critical wave amplitude decreases and becomes zero if  $V = V_s$ .

Typical waveforms are shown in Fig. 5 for the  $V_s = 0$  case. It is seen that, as  $v_{\max}$  approaches the critical value  $c/n$  the waveforms steepen, indicating the possibility of wavebreaking. For low wave amplitudes ( $v < 1$ ) waveforms are essentially sinusoidal, whereas for large wave amplitudes ( $v \gg 1$ ) waveforms become triangular.

### 3. Electron-Ion Plasma

In the previous section, only the electron motion was considered

whereas the motion of the positive ions was ignored. The results obtained thus apply to comparatively small wave amplitudes for which the ion dynamics are insignificant because of its large rest mass. In the presence of very large-amplitude waves, ions can attain considerable directed velocities, becoming relativistic if  $\mu v \gg 1$ , where  $\mu = m_e/m_i$ . Extremely intense sources of electromagnetic radiation do exist in astrophysical plasmas. For instance, in a Crab pulsar magnetosphere, the strength parameter  $v$  associated with the low-frequency magnetic dipole radiation can be  $\geq 10^{11}$  and the ions can acquire ultrarelativistic velocities<sup>2)</sup>.

Consider now a two-fluid electron-ion plasma in the absence of external fields<sup>25)-29)</sup>. In this case the basic equations for Langmuir waves are the relativistic equations of motion and of continuity for electrons and ions, respectively, and Maxwell's equations

$$-\frac{n}{c} \frac{dE}{d\theta} = 4\pi e(N_i - N_e) \quad , \quad (14)$$

$$\frac{dE}{d\theta} + 4\pi e(N_i v_i - N_e v_e) = 0 \quad . \quad (15)$$

Taking the phase-average of (14) and (15), we see that in the absence of external fields the stream velocities of electrons and ions are equal

$$V_s = \langle N_e v_e \rangle / \langle N_e \rangle = \langle N_i v_i \rangle / \langle N_i \rangle \quad . \quad (16)$$

Stationary electron-ion plasma corresponds to the particular case in which the average particle fluxes are zero.

The basic equations can be combined to give two conservation relations, one expresses conservation of energy<sup>28)</sup>

$$E^2/8\pi + Nm_e \gamma_e c^2 + Nm_i \gamma_i c^2 = Nm_e c^2 W \quad , \quad (17)$$

and another expresses conservation of particle momentum

$$(u_i + \mu u_e) + \mu(u_e - n\gamma_e) = D \quad , \quad (18)$$

where  $N = \langle N \rangle (1 - nV_s/c)$ ,  $u = \gamma v/c$ ,  $W$  and  $D$  are constants. The dis-



persion relation for nonlinear, relativistic, Langmuir waves in an electron-ion plasma can be obtained from (17) and (18)

$$P = \frac{\sqrt{2}}{\omega_{pe}} \int_{u_1}^{u_2} \frac{1 - nu_e/\gamma_e}{(W - \gamma_e - \gamma_i/\mu)^{1/2}} du_e \quad (19)$$

When evaluating a particular solution for an electron-ion plasma it is necessary to decide what values should be assigned to the two conservation constants  $W$  and  $D$ . The constant  $W$ , as shown in the previous section, can be considered as a wave amplitude parameter. However, the role of the constant  $D$  is more subtle. Most papers treat  $D$  either as an arbitrary constant or as a plasma drift velocity. This leads to mathematically correct, nonetheless unphysical solutions. In the absence of a wave,  $v_e = v_i = V_s$ , (18) gives

$$(1+\mu)(1 - V_s^2/c^2)^{-1/2} V_s^{-1} - n(1+\mu)(1 - V_s^2/c^2)^{-1/2} = D \quad (20)$$

which shows that  $D$  is a function of  $V_s$ ,  $n$  and  $\mu$ . For large wave amplitudes,  $D$  also becomes a function of the wave intensity. In fact, it was established that  $D$  is uniquely related to the amplitude parameter  $W$ <sup>30)</sup>. Hence, in order to obtain a specific solution with given plasma stream velocity, wave velocity, electron-ion mass ratio and wave intensity, the dynamic parameter  $D$  must be adjusted accordingly so as to render self-consistency to the solution. In Fig. 6, the variation of the self-consistent dynamic parameter  $D$  with  $u_\alpha$  (where  $u_\alpha = |u_1 - u_2|$ ) for a stationary electron-ion plasma is displayed for different values of  $n$ .

The general behavior of nonlinear, relativistic Langmuir waves in an electron-ion plasma is similar to the case of an electron plasma. The major effect of ion dynamics is to decrease the period of oscillation; this effect gets more pronounced as the wave amplitude increases. In fact, Kennel and Pellat<sup>26)</sup> showed that for extremely large wave amplitudes the contribution of ions toward wave properties is equal to electrons; when the wave energy density greatly exceeds the rest mass energy, the inertia of plasma particles is not determined by their rest mass but by the kinetic energy they acquire from the waves. Hence, in the limit of extremely large wave intensities, an electron-ion plasma behaves like an electron-positron plasma.

#### 4. Two-Streams

Two-stream instability can be of fundamental importance to the generation of coherent pulsar radio emission<sup>12)</sup>. In this section, we briefly discuss the relativistic effects in the two-stream instability by analyzing the traveling wave solution of nonlinear, relativistic Langmuir waves. As an illustration, we treat the case of two interpenetrating electron and ion streams. In order to permit relative streaming between electrons and ions, (14) and (15) are modified to

$$-\frac{n}{c} \frac{dE}{d\theta} = 4\pi(eN_i - eN_e + \rho_o) \quad , \quad (21)$$

$$\frac{dE}{d\theta} + 4\pi(eN_i v_i - eN_e v_e + J_o) = 0 \quad , \quad (22)$$

where  $\rho_o$  and  $J_o$  are respectively the constant charge and current densities required to neutralize the average charge and current densities of the streams. Evidently, (21) and (22) show that electron stream velocity  $V_{se}$  is different from ion stream velocity  $V_{si}$ , namely

$$\langle N_e v_e \rangle / \langle N_e \rangle \neq \langle N_i v_i \rangle / \langle N_i \rangle \quad , \quad (23)$$

with

$$\rho_o = e(\langle N_e \rangle - \langle N_i \rangle) \quad , \quad J_o = e(\langle N_e v_e \rangle - \langle N_i v_i \rangle) \quad . \quad (24)$$

The dispersion relation for this case can be obtained by making use of the Lorentz transformation technique<sup>22),25)</sup>. The detailed results will be presented in a forthcoming paper<sup>31)</sup>. Fig. 7 shows some examples of dispersion curves for electron-ion streams, with various wave amplitudes. We see that the two-stream instability occurs for wavenumbers below a certain critical value. Similar to the case of a single stream (see Fig. 2), for a given wave velocity, relativistic effects cause the wavenumber to decrease as the wave amplitude increases. Hence, for nonlinear, relativistic Langmuir waves in electron-ion streams, there is a range of wavenumbers that are unstable according to the linear theory but becomes stable as the wave amplitude gets sufficiently large. Moreover, Fig. 7 indicates that wavebreaking plays an important role in determining the dispersion

characteristics of electron-ion streams. For, as the wave amplitude increases, the range of wave velocities for which wavebreaking occurs also increases. If the wave amplitude becomes very large, wavebreaking may occur for all subluminal wave velocities that lie between the electron and ion stream velocities.

## 5. Conclusion

The traveling wave solutions of nonlinear, relativistic, Langmuir waves studied in this paper serve to understand some basic properties of large-amplitude electrostatic plasma waves when wave intensity induced relativistic and nonlinear effects are important. Astrophysical implications of these wave properties for strong wave problems remain to be explored. In addition, many effects on nonlinear, relativistic, Langmuir waves such as plasma temperature<sup>32)-34)</sup> and wave stability<sup>35),36)</sup>, left out in this paper, await further investigations.

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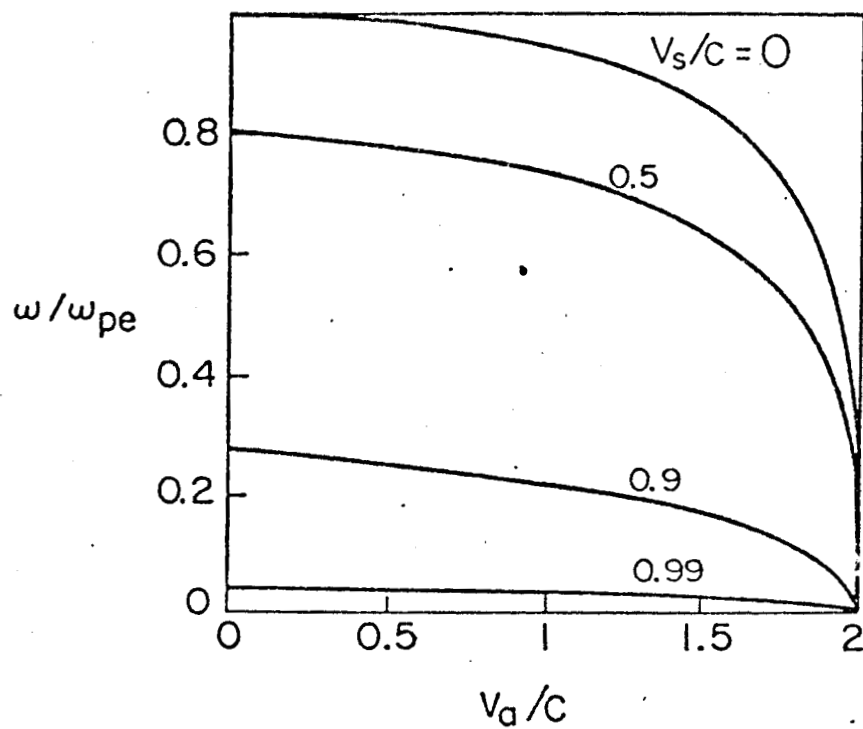


Figure 1. Variation of  $\omega/\omega_{pe}$  with  $v_a/c$  for indicated values of  $v_s/c$ ;  $n = 0$ .

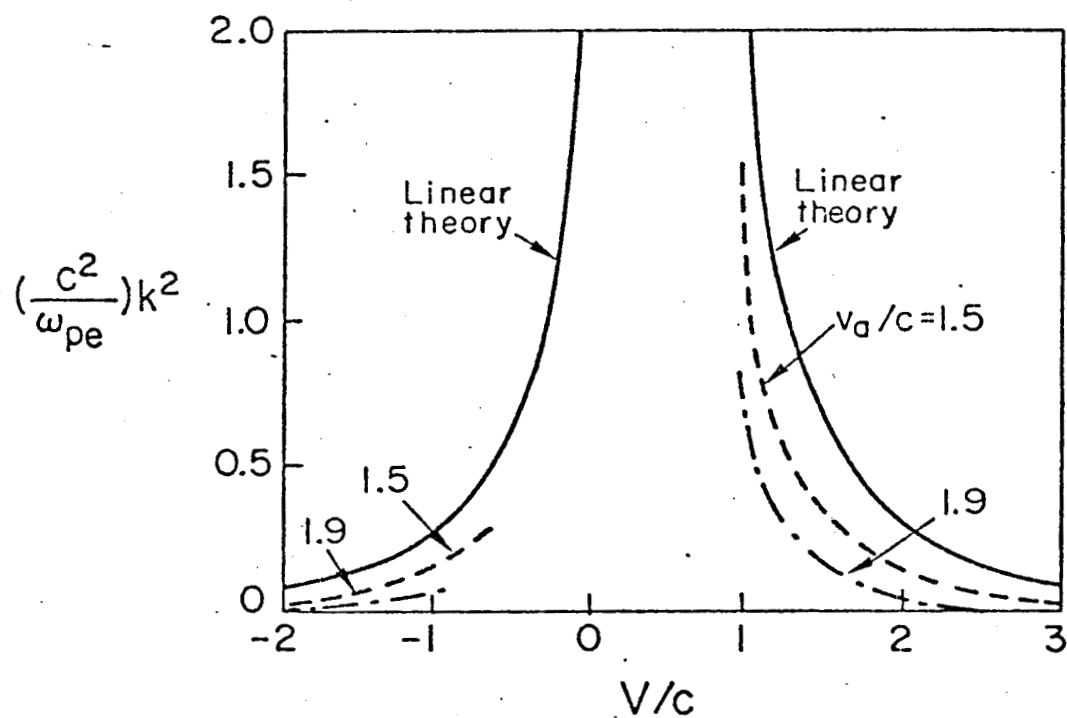


Figure 2. Variation of  $(c^2/\omega_{pe}^2)k^2$  with  $V/c$  for indicated values of  $v_a/c$ ;  $v_s = 0.5c$ .

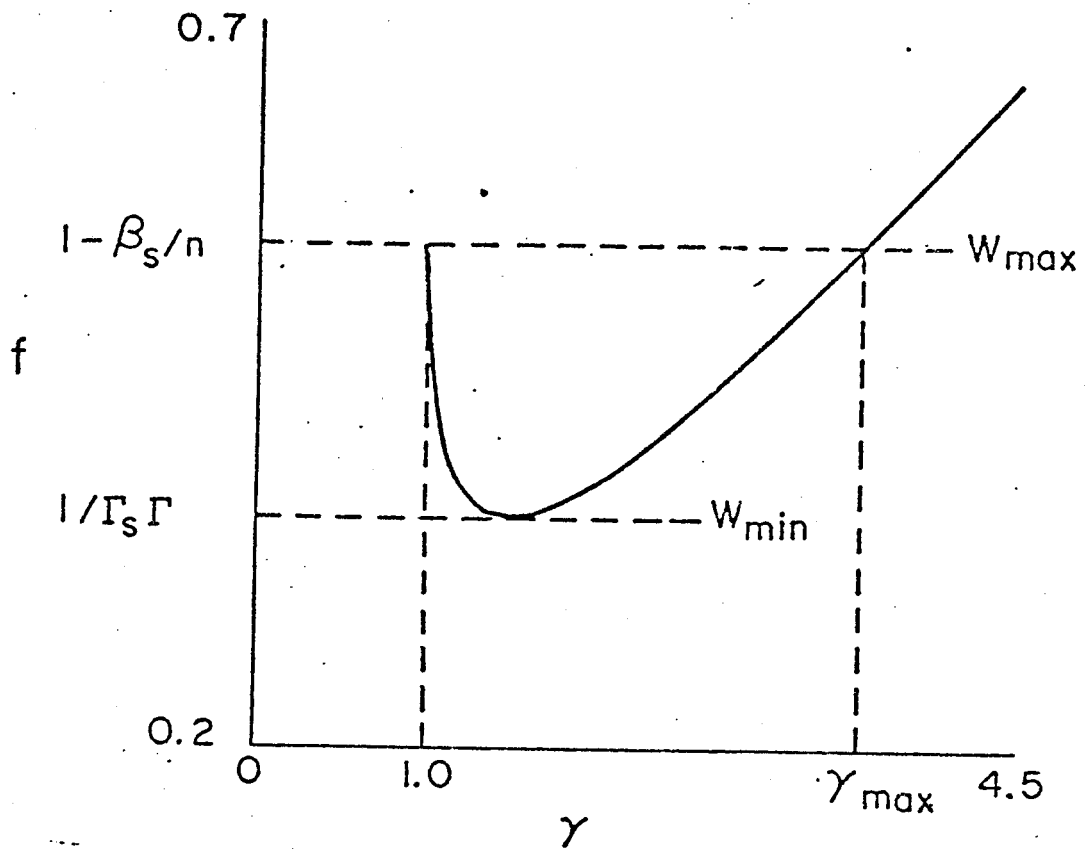


Figure 3. Variation of  $f$  with  $\gamma$  for  $V_s = 0.5c$  and  $n = 1.1$ .

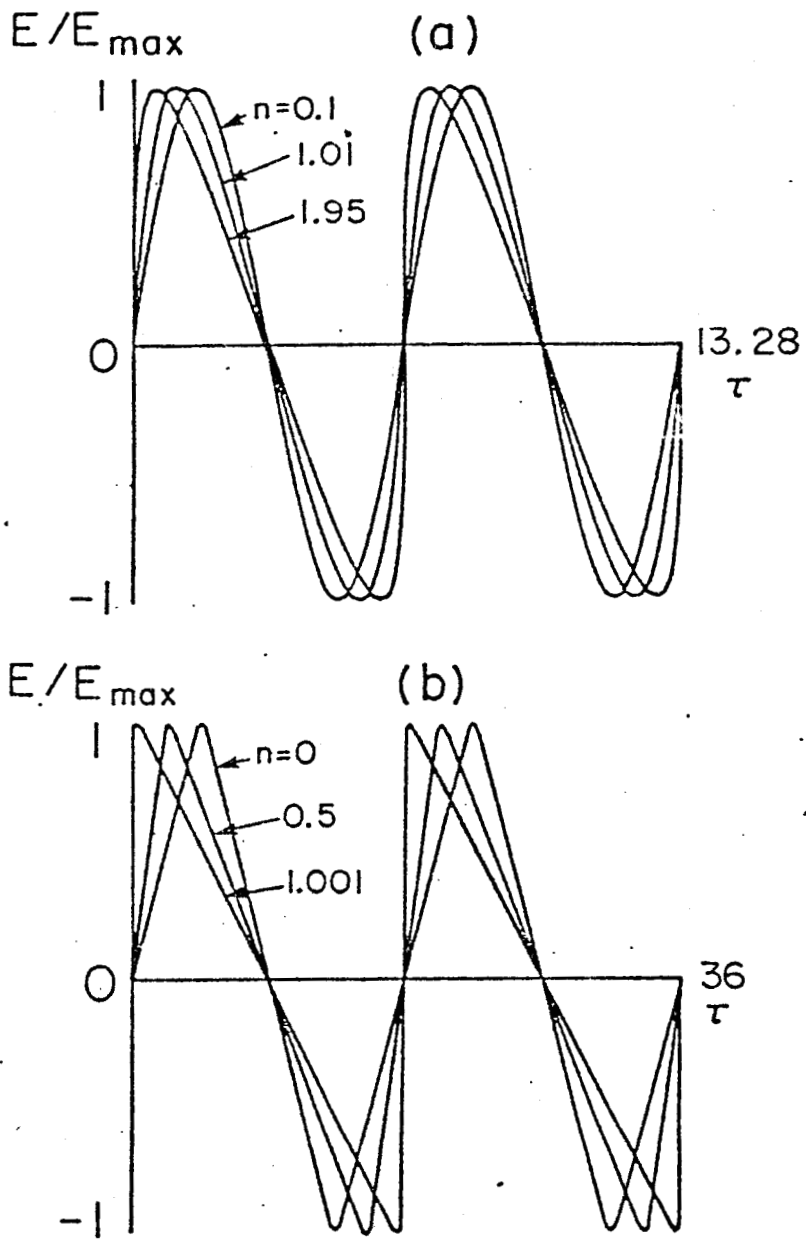


Figure 5. Variation of  $E/E_{max}$  with  $\tau$  for (a)  $v_0 = 0.5c$  ( $u_0 = 0.58$ )  
(b)  $v_0 = 0.995c$  ( $u_0 = 10$ ).

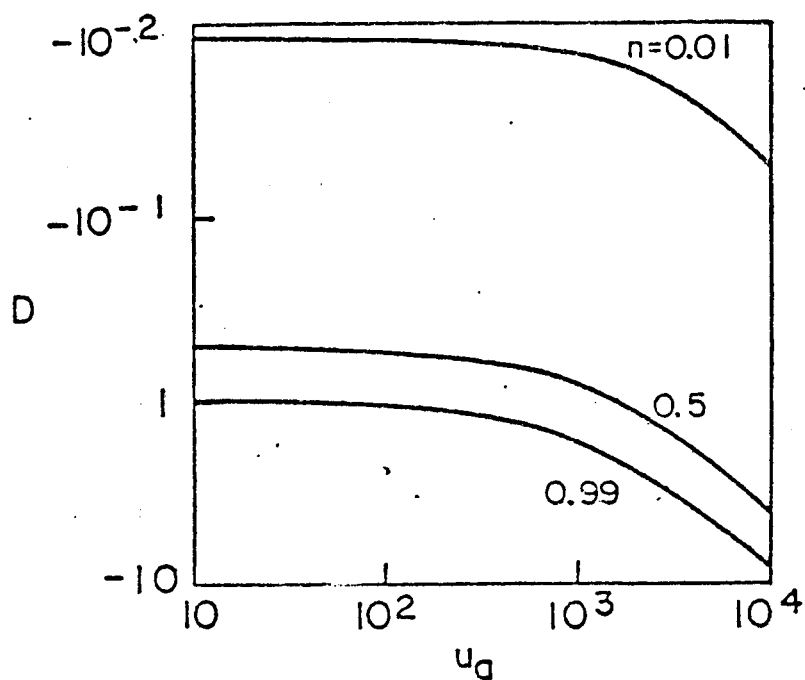


Figure 6. Variation of  $D$  with  $u_d$  for indicated values of  $n$ ;  $V_s = 0$ .

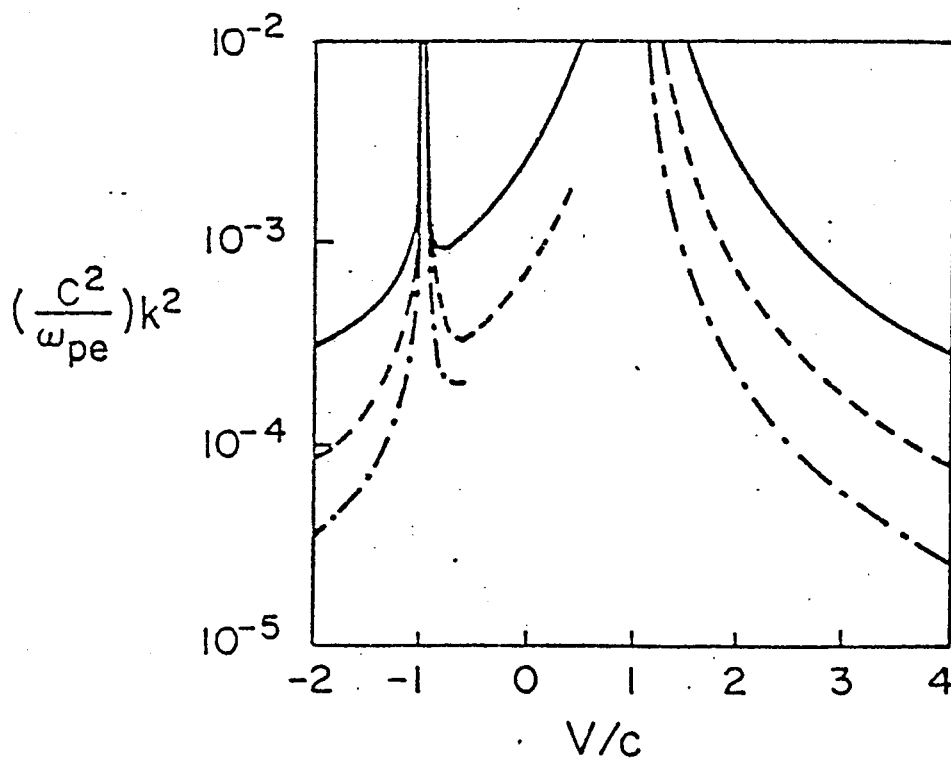


Figure 7. Variation of  $(c^2/\omega_{pe}^2)k^2$  with  $V/c$  for  $V_{se} = 0.99c$ ,  $V_{si} = -0.99c$ ,  $\langle N_e \rangle = \langle N_i \rangle$ ,  $\mu = 1/1837$ : — linear theory; — — —  $|v_e|/c = 0.6$ ; - · - ·  $|v_e|/c = 1.6$ .



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