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# A SUMMARY OF RECENT NASA/ARMY CONTRIBUTIONS TO ROTORCRAFT VIBRATIONS AND STRUCTURAL DYNAMICS TECHNOLOGY

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#### ABSTRACT

The requirement for low vibrations has achieved the status of a critical design consideration in modern helicopters. There is now a recognized need to account for vibrations during both the analytical and experimental phases of design. Research activities in this area have been both broad and varied and notable advances have been made in recent years in the critical elements of the technology base needed to achieve the goal of a "jet smooth" ride. The purpose of this paper is to present an overview of accomplishments and current activities of government and government-sponsored research in the area of rotorcraft vibrations and structural dynamics, focusing on NASA and Army contributions over the last decade or so. Specific topics addressed include: airframe finite-element modeling for static and dynamic analyses, analysis of coupled rotor-airframe vibrations, optimization of airframes subject to vibration constraints, active and passive control of vibrations in both the rotating and fixed systems, and integration of testing and analysis in such guises as modal analysis, system identification, structural modification, and vibratory loads measurement.

#### INTRODUCTION

Since the first U.S. helicopter went into production over four decades ago (fig. 1), excessive vibrations have plagued virtually all new rotorcraft developments. The problem transcends national boundaries and is not unique to the U.S. helicopter community. An account of the vibration problems encountered in the development of an early Soviet helicopter (fig. 2) is given by Alexander Yakovlev in refer-Yakovlev's account was popularized when excerpts from his ence 1. book appeared in the magazine Aviation Week (December 28, 1959). The frustration of trying to solve an elusive vibration problem became so intense that, as the designer writes, "It got to the point where, instead of calling greetings when we met in the morning, we shouted at each other: 'How is it going - still shaking?' 'It's shaking; it's shaking!' 'When will this damned shaking stop?'" More recent accounts of the impact of vibrations on Army helicopter developments

are given in references 2 and 3 in which problems experienced during initial flight testing of the UH-60 Black Hawk (fig. 3) and AH-64 Apache (fig. 4) are described. The problems encountered on these helicopters included: higher than expected rotor vibratory loads, unanticipated rotor-airframe interactions, airframe resonances near excitation frequencies, excessive empennage vibrations, and ineffective vibration control devices. As a result, vibration levels on the prototype aircraft were significantly above Army specifications throughout the flight envelope.

Helicopters are susceptible to vibrations due to the inherent cyclic nature of the airloads acting on the rotors. The vibrations normally pervade both the rotor and the airframe and can seriously degrade both service life and ride qualities. Vibrations also frequently limit the maximum speed in forward flight. Considerable progress has been made over the past 40 years in reducing the level of vibration in helicopters as indicated in figure 5. While improvements have been significant, it should be noted that the procurement specifications have consistently been for levels of vibration lower than could usually be achieved on production helicopters. In the case of the Army UTTAS (Utility Tactical Transport Aircraft System) and AAH (Advanced Attack Helicopter) development programs in the mid-1970s, for example, the specifications originally required vibration levels not exceeding 0.05g. Because none of the competitors could meet this specification, it had to be increased to 0.10g. However, even with this relaxed requirement, the vibration levels in the UH-60 and AH-64 (the winning designs in the two competitions) were reduced to 0.10g only after making numerous structural and configuration changes which included raising main rotors, adding aerodynamic fuselage fairings, modifying hub absorbers, installing airframe absorbers, changing local stiffnesses, modifying crew seats, and isolating stabilators. (It should be pointed out that the 0.10g levels achieved are for the delivered aircraft and that structural changes which occur during normal aircraft operations tend to degrade vibration characteristics. Levels of 0.20g are more typical of fielded Army helicopters). The dramatic reduction in the level of vibration noted in figure 5 has, for the most part, been achieved through the use of add-on vibration control devices of one type or another. These devices, while quite effective in reducing vibrations, have tended to cost an increasing percentage of the design gross weight. The weight penalty associated with the addition of absorbers to reduce vibration levels to 0.10g can be as high as 2.5 percent of design gross weight. For a fixed design gross weight, this represents a reduction of from 10 to 15 percent in primary mission pay-Isolation systems have also gained popularity in recent years. load. These mechanisms, which are designed to uncoupled the rotor dynamic system from the fuselage, appear to have somewhat reduced weight penalties with respect to other passive vibration control devices.

Even though excessive vibrations have always been prevalent in new helicopter developments, until recently, helicopter manufacturers have not addressed vibrations as part of the regular structural design process. The UTTAS RFQ in 1971 was the first instance when a procuring agency specified the level of vibration to be addressed in a competitive design. With only a few exceptions, helicopters have been designed to performance requirements while relying on past experience to account for vibrations. Excessive vibrations (which invariably occur) are "tinkered out" during ground and flight testing. The vibration levels to be regarded as acceptable are usually negotiated during this tinkering process (recall the UTTAS and AAH experience). Oftentimes modifications to reduce vibrations to acceptable levels continue well into the operational phase of a helicopter.

The cost required to solve vibration problems during the development cycle is qualitatively illustrated in figure 6 which shows the trend of engineering manpower requirements dedicated to vibration During the design phase, effort increases gradually until reduction. At this point an abrupt increase occurs (the beginning first flight. of the so-called "crisis period") that extends well into the develop-This increase significantly raises development costs and ment cycle. leads to slipped delivery schedules. Operational costs are also increased both due to the attendant weight penalties associated with vibration treatments and due to the increased maintainability requirements for vibration control devices. Clearly, the payoff from minimizing crisis engineering and eliminating overruns is signifi-As previously mentioned, helicopter companies have relied cant. little on analysis during design to limit vibrations. However, because of the vibration problems encountered in the UTTAS and AAH development programs, there has emerged a consensus within the industry on the need to account for vibrations more rigorously during both the analytical and experimental phases of design. This need has resulted in the subject of helicopter vibrations receiving considerably increased attention in recent years (see, for example, refs. 4 The goal (unofficially) set down by the industry is to to 9). achieve the vibration levels associated with fixed-wing aircraft, the so-called "jet smooth" ride. To achieve this goal will require the development of advanced vibration design methodologies (ref. 10).

Vibration design can be broadly classified into three interdependent activities: (1) passive design to select rotor and airframe parameters which yield low inherent vibrations; (2) design of vibration control devices to minimize rotating and fixed-system vibratory loads; and (3) vibration testing to verify design concepts and to compensate for any deficiencies in analytical capabilities. The interactive nature of these activities is depicted in figure 7 which shows one representation of the helicopter vibration design cycle. The diagram indicates that the problem involves analytical and experimental considerations of the rotor, the airframe, and the coupling between the rotor and the airframe. The primary sources of high vibrations are cyclic loads transmitted to the airframe by the main and tail rotors as well as aerodynamic excitation of the tail boom and empennage by the main rotor wake. For the most part, passive vibration design combines past experience with rudimentary analysis. Special and general-purpose aeroelastic analyses are used to design for minimum blade vibratory loads. Large-scale finite-element models are used to verify adequate placement of airframe natural frequencies with respect to operating frequencies. Comprehensive rotor-airframe

coupling analyses which account for flexible hub structural dynamics and interactional aerodynamics have only become available recently and have not yet been validated. Correlation with test and comparative studies of these state-of-the-art helicopter rotor and airframe vibration analyses have confirmed what the Black Hawk and Apache experiences have demonstrated, namely, the inadequacies of existing passive vibration design methods.

Underlying all considerations related to vibrations and serving as a unifying element is structural dynamics. Every consideration of a helicopter system includes dynamic phenomena in some form (fig. 8), and the importance of structural dynamics is well recognized (ref. 11). The key role played by structural dynamics in the broader context of aerospace vehicle design as well as an assessment of structural dynamics needs are given in references 12 and 13. However, while structural dynamics clearly plays a principal role in determining the vibration characteristics of modern rotorcraft, it is not regarded as a sufficiently mature discipline by the helicopter industry on which to base vibration design decisions. (It is interesting to note that such is not the case for stability, with analytical predictions often influencing design decisions). Good structural dynamic characteristics are essential for the success of any rotor-The modern helicopter is more susceptible to high vibrations craft. because of increased operational demands for high-speed and nap-ofthe-earth flight, high maneuverability and agility, improved crew effectiveness, advanced weapons delivery, increased structural integrity, high reliability, and low maintenance. As a result, vibrations has achieved the status of a critical design consideration in modern helicopters. The challenge is now, more than ever, passed on to the Indeed, it may well emerge that the success or failure dynamicist. of future rotorcraft developments will rest on the dynamicist.

Research activities in the U.S. in the area of rotorcraft vibrations and structural dynamics have been both broad and varied. Notable advances have been made in recent years in the critical elements of the technology base needed to achieve the goal of a "jet smooth" The purpose of this paper is to present a management overview ride. in the style of an executive summary of accomplishments and current activities of government and government-sponsored research in the area of rotorcraft vibrations and structural dynamics. The overview focuses on NASA and Army contributions over the last decade or so. Both in-house and contracted research and development efforts pertaining to design analyses for vibrations, vibration control, and vibration testing are described. Emphasis throughout is placed on the airframe. Rotorcraft aeroelastic stability, rotor blade vibratory airloads, rotor dynamics, and associated wind-tunnel testing are not addressed except if needed to provide for continuity. This separation between the rotor and the airframe is primarily a separation between aerodynamics and structural dynamics. In practice, this separation is not possible because of the interaction between the rotor and the airframe in producing vibrations. Specific topics addressed include: airframe finite-element modeling for static and dynamic analyses, analysis of coupled rotor-airframe vibrations, optimization of airframes subject to vibration constraints, active and passive control of vibrations in both the fixed and rotating systems, and integration of testing and analysis in such guises as modal analysis, system identification, structural modification, and vibratory loads measurement. NASA and Army funded efforts with the university community are also included. The information used as a basis for the overview was obtained by reviewing the material identified in a computerized literature search and from the extensive personal libraries of the authors. Of the hundreds of potentially relevant reports and papers reviewed those that were judged to be significant for the purposes of the paper are cited as references.

# PREPARATORY REMARKS

With a view toward providing a better perspective of NASA and Army vibrations research, some material of a background nature is given in this section.

Current NASA rotorcraft research has evolved from the autogyro research begun by NACA in the 1930's. Valuable contributions to rotorcraft development have resulted from NACA/NASA research since While there has always been a close association between that time. NACA/NASA and the military rotorcraft research and development agencies, particularly with the Army, the relationship with the Army was strengthened in 1965 when the Army Aeronautical Research Lab was In 1970 the Army estabestablished at the Ames Research Center. lished research labs at the Langley Research Center and the Lewis Research Center and formed what is currently called the Aviation Research and Technology Activity (ARTA) of the U.S. Army Aviation Systems Command (AVSCOM). These labs represented an important adjunct to the NASA organization and sparked a resurgence in NASA rotorcraft research activities aimed at strengthening and exploiting the joint research which was made possible by the collocated Army labs.

The first major NASA program addressing vibrations was the Civil Helicopter Technology Program (refs. 14 and 15). Although the primary goal of this program was ride quality research aimed at civil acceptance of helicopters for transports, vibrations was of interest because it was a major factor contributing to public acceptance of In March 1978, NASA's Office of Aeronautics and Space helicopters. Technology formed a special Rotorcraft Task Force to review rotorcraft technology needs and to prepare an appropriate rotorcraft research program aimed at advancing technology readiness. The Task Force solicited inputs from the rotorcraft industry, NASA research The National Research Councenters, and other government agencies. cil (NRC) and the Rotorcraft Subcommittee of the NASA Aeronautics Advisory Committee conducted independent reviews of the proposed NASA As a result of counsel received from all quarters, a plan program. was finalized and published in October 1978 (ref. 16). The review conducted by the NRC was published under separate cover (ref. 17). The Task Force proposed a 10-year, \$398 million (FY 78 dollars) program with four major elements: aerodynamics and structures, flight control and avionic systems, propulsion, and vehicle configurations.

Each of the four major elements was divided into two or more specific areas of emphasis. Vibrations was cited as one of three key areas under aerodynamics and structures. As enunciated in the Task Force Report, the focus was to be on providing the technology and design methodology for accurate prediction and substantial reduction of air-The Task Force Report was the catalyst for the frame vibrations. NASA Langley Research Center to begin formulating a rotorcraft structural dynamics program to meet the needs of the helicopter industry with respect to airframe vibrations. The overall objective of the proposed program, which was defined in close cooperation with the industry and coordinated with the Army, was to establish in the U.S. a superior capability to utilize airframe finite-element analysis to support the design of helicopter airframe structures. Viewed as a whole, the program includes efforts by NASA, universities, and the helicopter industry. In the initial phase of the program, teams from the major manufacturers of helicopter airframes would formulate finite-element models of selected airframes of both metal and composite construction and carry out ground vibration tests and correlations to evaluate the analysis models. To maintain the necessary scientific observation and control, emphasis throughout these activities would be on advance planning, documentation of methods and procedures, and thorough discussion of results and experiences, all with industry-wide critique to allow maximum technology transfer between companies. The finite-element models formed in this phase would then serve as the basis for the development, application, and evaluation of both improved modeling techniques and advanced analytical and computational techniques to enhance the technology base which supports design of helicopter airframe structures. Here again, procedures for mutual critique have been established which call for a thorough discussion among the program participants of each method prior to the applications and of the results and experiences after Because of the emphasis on design methodology, the the applications. aforementioned rotorcraft structural dynamics program was given the acronym DAMVIBS (Design Analysis Methods for VIBrationS).

In 1979, primarily because of the problems experienced during the UTTAS and AAH development programs, the Director of what is now the U.S. Army Aviation Research and Technology Activity requested that an assessment of helicopter vibration research be made. Information for this assessment was obtained by surveying the helicopter industry, Army research labs, and appropriate NASA research centers. This review addressed the status of past, present, and planned research efforts within the Army as well as joint Army/NASA programs. The results of this assessment were published in 1982 (ref. 18). The five major disciplines which were critically reviewed included: rotor vibratory loads, airframe structural dynamics, rotor-airframe coupling, vibration control devices, and vibration testing. As a result of this comprehensive review, and with a consensus of the rotorcraft As a result community, significant technology voids were identified and areas for future research were recommended. The technology deficiencies can be summarized into two areas of concern relative to helicopter vibrations. First, the inability of present design methods to accurately predict rotor vibratory loads and coupled rotor-airframe vibrations. Hence, the need to resort to add-on vibration control devices. Second, a lack of definitive procedures which make maximum use of vibration test data, instead of trial-and-error testing, to resolve vibration problems. To address these technical concerns, Army vibration research in recent years has been directed to rotor-airframe coupling analysis, advanced active and passive vibration control demonstration, and improved vibration testing methodology development.

The Army program (ref. 18) was reaffirmed and the proposed NASA DAMVIBS program was formally presented to the helicopter industry at a finite-element modeling workshop focusing on rotorcraft structures which was held at Langley Research Center in February 1983 (refs. 19 and 20). Because of the complementary nature of the two programs, industry consensus was to proceed with both programs. Army funding for the contracted activities envisioned under their program did not materialize so only the in-house work was initiated. NASA funding for the DAMVIBS program was approved and the program was implemented in April 1984 with the awarding of task-type contracts to each of the four primary helicopter airframe manufacturers (Bell Helicopter Textron, Boeing Vertol, McDonnell Douglas Helicopter Company (at that time Hughes Helicopters, Inc.), and Sikorsky Aircraft). Work completed to date under the NASA and Army programs as well as the status of current activities and near-term plans are also discussed in appropriate sections of the paper.

#### DESIGN ANALYSIS FOR VIBRATIONS

As discussed in the Introduction, designing a helicopter for low vibrations may be viewed as consisting of essentially three interdependent activities: (1) design technology, wherein the use of analysis during design (i.e., design analysis) is employed to establish dynamically passive or vibration-benign rotors and airframes; (2) control technology, whereby vibration control devices are designed to further reduce rotating and fixed-system vibratory loads; and (3) test technology, wherein vibration testing is used to verify design concepts and to compensate for any deficiencies in analytical capabilities. This section is concerned with the first of these activities, namely, the use of vibration analysis to support design of airframe structures. Three specific areas are discussed: (1) airframe finite-element modeling; (2) analysis of coupled rotor-airframe vibrations; and (3) airframe structural optimization.

#### Airframe Finite Element Modeling

Structural analysis methods employed in the aerospace industry today are based mostly on the finite-element method. The finiteelement method is a numerical matrix technique for obtaining approximate solutions to a wide variety of engineering problems. Although originally developed about 25 years ago to analyze complex aircraft structures, it has since been extended and applied to a wide variety of problems spanning many fields of engineering. In particular, the finite-element method has assumed a premier role in the design and analysis of aerospace structures both in this country and abroad.

The idea of the finite-element method is to provide a library of structural elements (rods, beams, shear panels, plates, etc.) which can be connected together so as to model any structure of interest. A computer then automatically carries out the computations necessary to determine specified categories of behavior of the structure under Finite-element analysis is the standard method for specified loads. airframe structural analysis in the U.S. and is now routinely used as a design tool to calculate static internal loads on each airframe element to permit sizing and stress analysis. Within the U.S. helicopter industry, finite-element analysis as embodied in the NASTRAN computer code is used exclusively. NASTRAN (ref. 21) is the very widely emplaced, general-purpose computer code for finite-element analysis of structures originally developed under NASA sponsorship in the late 1960s. (Several commercial versions of the code have become available since that time, with the version developed by the MacNeal-Schwendler Corporation (ref. 22) being the most widely used). The remarkable collection of terms and symbols referring to various entities of the code has become a highly effective universal vocabulary. The increased accuracy of finite-element-analysis based methods (such as NASTRAN) over earlier strength-of-materials based methods of analysis for prediction of internal load distributions has contributed significantly to the ability to design more efficient (lighter weight) aircraft structures.

The major fixed-wing aircraft manufacturers developed their own special-purpose finite-element codes soon after the emergence of the finite-element method in the late 1950's and well in advance of the introduction of NASTRAN in 1970. Hence, the use of NASTRAN in this industry, while extensive, has been generally no more than supplemental to their own well-established codes in airframe design work. The U.S. helicopter industry, on the other hand, lagged the fixed-wing industry in the development of their own finite-element analysis codes for design so when NASTRAN became available in 1970 it was promptly adopted by the helicopter industry. NASTRAN is now used exclusively in this industry to support both static and dynamic design.

Some early accounts of the use of NASTRAN in the helicopter industry are contained in references 23-26. The integration of NASTRAN into the airframe design process at Bell Helicopter is described in reference 23. The reference outlines pre-processing procedures for automatic generation of the airframe finite-element model and distribution of non-structural weight to the threedimensional model and a post-processing procedure for reformatting the output so that it is more directly useful to the stress analyst. Initial experiences at Bell with the use of the various options in NASTRAN for static and dynamic analysis are described in reference A brief historical perspective of the adoption and subsequent 24. application of NASTRAN for analysis of helicopter airframes at Sikorsky Aircraft are given in reference 25. With respect to the ability of a finite-element analysis to design a lighter weight aircraft, Sikorsky credits the use of NASTRAN during the design of the UH-60 Black Hawk with reducing the structural weight by about ten percent. Some additional industry accounts of the early use of

NASTRAN in design may be found in reference 26. As NASTRAN became more firmly established in the helicopter industry, analytical and experimental investigations based on the use of finite-element models began to become more common. Some of the more noteworthy of these finite-element modeling applications are summarized in the remainder of this section.

Combined experimental/analytical investigations conducted on Army OH-58A and OH-6A helicopters are reported in references 27 and 28, respectively. Those studies were some of the earliest aimed at determining engine response to airframe vibrations. The objective was to provide the data needed to establish a set of improved engine vibration specifications for engine manufacturers. The finiteelement models developed as part of those studies are shown in figures 9 and 10, respectively. In each case, the finite-element model of the airframe was coupled to a model of the engine based on mobility data supplied by the engine manufacturer. Analytical predictions were reported to have agreed reasonably well with test data in both studies.

Some early modeling and correlation work conducted by Sikorsky The initial on the CH-53A is reported in references 29 to 31. finite-element model, described in reference 29, was based on an inhouse code originally developed for civil engineering structures. The model was rather simple, with the forward and aft portions of the fuselage modeled as beams cantilevered from a detailed threedimensional model of the center fuselage section. A companion simplified NASTRAN model (ref. 30) was later used to develop a complete, three-dimensional finite-element model of the CH-53A used in the NASA Civil Helicopter Program (fig. 11). This program (refs. 14 and 15), which was directed at evaluating helicopters for short-haul transportation, utilized a CH-53A modified to incorporate an airline passen-The modified CH-53A underwent an extensive shake ger compartment. test program and a detailed comparison was made between test results and NASTRAN results (ref. 31). Good agreement was noted for the fundamental airframe bending and transmission pitch frequencies, but poor agreement resulted for the lateral/torsion modes and the higher frequency transmission modes. The predominant vibratory loads imposed on an airframe by the rotor occur at the blade passage frequency which equals N times the rotational frequency, where N is the number It is customary to refer to this frequency as N-per-rev of blades. For the six-bladed CH-53A this frequency is 18.5 Hertz. or N/rev. Since the higher frequency transmission modes control the 6/rev vibratory response in the CH-53A airframe, the analysis was judged to be an unreliable design tool for predicting even the primary vibration levels. It was thus concluded that further development of finite-element modeling techniques was required before such analyses could reliably predict N/rev response at critical stations on an airframe.

The role of NASTRAN in the design of the Rotor Systems Research Aircraft (RSRA) is discussed briefly in reference 32. The RSRA (fig. 12) was intended to serve as a flying test bed for a variety of advanced rotors for helicopters. The requirement to mount different rotors posed several unique vibration design problems for the airframe. NASTRAN was used extensively to provide the structural dynamics representations for the usual analytical checks on vibrations. An upgraded version of the original finite-element model of the RSRA in a compound configuration is shown in figure 12.

In 1973 the Army initiated a program to evaluate NASTRAN as a tool for vibration analysis of helicopter airframes. The first part of the program was to develop a NASTRAN model of the AH-1G helicopter that would represent the low-frequency (below 30 Hertz) vibration characteristics of the airframe. The documentation of the model was required to be clear and complete so that government personnel could independently make changes to the model and use it for in-house analyses. Following development of the NASTRAN model, the validity of the model was assessed by comparing the model with static and dynamic tests. References 33 to 37 describe the results obtained under this The NASTRAN finite-element model, which was developed under program. the technical direction of a NASA/Army team, is shown in figure 13 and described in detail in reference 33. Figure 14 illustrates the type of documentation which was provided for the stiffness modeling The figure shows a drawing of the actual strucunder the contract. ture (with skins removed) of the fuselage portion of the airframe. An exploded view of the finite-element model corresponding to the aft (shaded) part of the fuselage is depicted in the middle of the fig-This sketch is the familiar "wire-frame" diagram that is custoure. marily shown when graphically illustrating a finite-element model. The sketch at the bottom of the figure is an exploded view of one of the bulkheads in the model and shows the individual rods and shear panels which represent that particular bulkhead. Detailed sketches of this type appear for every bulkhead, frame, panel, etc. in the Each sketch is also accompanied by a set of tables which airframe. describes the structural elements, constraints which need to be imposed on the model, and an explanation of the basis for omitting degrees of freedom not employed for the dynamic analysis. Reference 34 contains the results of static and dynamic tests and comparisons of results from those tests with results from NASTRAN analysis. Some frequency response comparisons which are typical of those obtained from the ground vibration test are given in figure 15. In general, measured frequency response characteristics were found to be in fair to good agreement with NASTRAN predictions only through about 15-20 Hertz (This corresponds to about 4/rev for the two-bladed AH-1G). Α report (ref.38) recently generated under the DAMVIBS program in support of an industry-wide coupled rotor-airframe vibrations activity (to be described in the next section) summarizes all the modeling and testing which has been conducted on the AH-1G, including some recent testing conducted by Kaman Aerospace Corporation. As a consequence of these well-documented activities on the AH-1G, the AH-1G is probably the best known airframe of any aircraft described in the open literature. This has resulted in the AH-1G finite-element model being used extensively throughout government, industry, and academia.

The vibrations portion of the rotorcraft research program plan laid out in 1978 by the Rotorcraft Task Force (ref. 16) contained an airframe modeling/test assessment activity. This proposed task area

was to involve participation by NASA and industry in a workshop environment to assess and document industry design procedures, difficulties with software, modeling techniques, and shake test procedures. All work was to be conducted on a production aircraft. NASA funding for that activity was approved and, as a result of a competitive procurement, a contract was awarded Boeing Vertol in 1980. The subject vehicle was to be the CH-47D. An unusual requirement of the contract was that each major step of the program be presented to and critiqued by the other three primary helicopter airframe manufacturers. Also unique was the requirement that plans for the modeling, testing and correlation be formulated and submitted to both NASA and industry representatives for review prior to undertaking the actual modeling and testing. Boeing was also required to make a study of current and future uses of finite-element models and to keep meticulous records on the manhours required to form the vibrations model. The latter "time and motion" study was intended to provide a basis on which to schedule finite-element modeling for any new helicopter development program. The contract also called for thorough documentation of the model, but not to the level of detail which had been required for the References 39-43 constitute the formal documentation of all AH-1G. work done under the contract. A concise summary of the program may be found in reference 44. The finite-element model developed under the program is shown in figure 16. An example of the type of modeling guides required as part of the modeling plan is given in figure 17, which shows static and mass modeling guides for a typical frame in the CH-47D. Figure 18 illustrates the types of comparisons which were obtained between measured and computed frequency responses. In general, the agreement between test and analysis was acceptable only through about 15-20 Hertz (3/rev for the 3-bladed CH-47D corresponds to 11.25 Hz). The modeling activity demonstrated that a finiteelement model suitable for internal loads, structural member sizing, and vibrations can be developed, and that there is no need to form separate static and dynamic models as has usually been the practice. The study further showed that the cost of such a combined static and dynamic model is about five percent of the manhours of a typical airframe design effort. Of the five percent, four percent is already typically expended in most companies to form the internal loads model; the vibrations model is another one percent. The "time and motion " study showed that a vibrations model could be formed early enough in a new helicopter development program to influence the airframe design. A number of items were identified during the modeling and correlation effort which have the potential for improving the These include: consideration of nonuniformly distribcorrelation. uted modal damping, the inclusion of secondary effects such as stringer shear area, assumptions on stringer continuity across splice joints, and the inclusion of suspension system effects. An example of the type of improvement which could be achieved by better treatment of damping is indicated in figure 19. Usual practice is to use the same (assumed) value of damping for each mode in forced response The figure shows the results of a preliminary exercise in analyses. which modal damping has been adjusted in some of the more important modes in an effort to improve correlation with test results. In the case shown the damping has been varied to obtain the best match away from the response peaks.

As a consequence of the CH-47D modeling and correlation activities, it became clear that the key to engendering in the industry the needed confidence to use finite-element models for vibration design was more industry hands-on experience along the lines of the CH-47D Also identified as being essential was a workshop environprogram. ment which fostered the discussion of modeling details and the interchange of ideas. Prior to the CH-47D program, finite-element modeling work conducted by the industry was fragmented for the most part with each company going its own way and (sometimes) preparing a report (which wasn't always available to competitors). The transfer of technology related to modeling was minimal at best. The NASA rotorcraft structural dynamics program, known as DAMVIBS, was defined with a view toward providing the necessary focus and environment of shared experiences for the common good of all. As previously mentioned, the DAMVIBS program was implemented in April 1984 with the award of contracts to the four primary helicopter airframe manufac-The industry participants, working under task-type conturers. tracts, have already been issued several tasks for the modeling and testing of both metal and composite airframes. Three NASA/industry meetings have already been held under the DAMVIBS program (September 24-25, 1984; October 1-3, 1985; December 2-4, 1986) at which industry participants have either presented their plans for conducting an activity or the results and experiences of a completed activity. Draft final reports for the completed tasks have been submitted and are in various stages of NASA review. Finite-element modeling and correlation activities have been completed on the McDonnell Douglas AH-64A (fig. 20). Modeling of the Sikorsky UH-60A and Bell D-292 (ACAP) are complete and correlations are under way (figs. 21 and 22). The ground vibration test of the Boeing Model 360 (fig. 23) has been completed; modeling is nearing completion at which time correlation studies will begin. The results of the unfinished studies will be presented at the next DAMVIBS meeting (tentatively scheduled for late 1987). From the modeling and correlation results obtained to date under the DAMVIBS program, metal airframes continue to exhibit acceptable agreement through only about 15-20 Hertz. Preliminary results also show that the dynamics of composite airframes are essentially the same as metal airframes. While correlations are not yet completed, preliminary results indicate that agreement between test and analysis for composite airframes is similar to that obtained for metal airframes (still a problem above about 15-20 Hz). Preliminary results also indicate that damping levels in composite airframes are about the same as in metal airframes (2-4 percent critical).

The CH-47D modeling activities and attendant industry critique demonstrated that all companies are using essentially the same techniques to model metal aircraft. The DAMVIBS program has demonstrated that the same is true for composite airframes. In the basic modeling studies being conducted under the DAMVIBS program only the primary (major load carrying) structure is represented fully (stiffness and mass) when forming the finite-element model. This is consistent with usual modeling practice. There are many components (e.g., transmissions, engines, and stores) and secondary structure (e.g., fairings, doors, and access panels) which are represented in the model only as

This may be a major contributing factor to the dislumped masses. agreement noted between analysis and test at the higher frequencies. In an attempt to answer this question, a DAMVIBS activity called "Finite-Element Modeling of Difficult Components" has been recently The aim of the "difficult components" activity is to isoinitiated. late the effects of modeling assumptions and to develop improved modeling guides for components which require more detailed modeling The first study is being conducted by Bell utilizing representation. an AH-1G helicopter. The airframe will be stripped down to primary structure and sequentially built back up to its full configuration, as suggested by figure 24. At each stage, a ground vibration test and an analysis based on a suitably modified finite-element model will be performed and the results compared. The end results will be the identification of modeling procedures which need to be improved. Current plans are to conduct a similar type activity on a composite airframe.

Effects of support systems and excitation systems on airframe elastic responses measured in a ground vibration test are typically assumed to be negligible. However, if there are differences between test and analysis, the question of possible extraneous effects associated with these systems often arises. It is clear that correlations would be interpreted with more confidence if these effects were included in the analysis. NASA has devised a scheme for including the effects of support systems and excitation systems in the finiteelement dynamic analysis while taking into account the prestiffening effects due to gravity. Boeing Vertol applied this method to the While only minor effects were noted for the CH-47D (refs. 42 CH-47D. and 43) the effects may not be negligible for other configurations. The method appears promising but additional investigation is needed before the method can be routinely applied. The work of fully developing and verifying the method is continuing at Langley using the finite-element model of the CH-47D airframe. In connection with this latter effort, several areas in which the finite-element model could These latter refinements be improved have recently been identified. are to be done by a joint NASA/Boeing team.

Steady-state vibration response analyses are currently being used in evaluating the dynamic response of structures to cyclic excitation forces. An undocumented vibration response analysis based on modal superposition was developed at Langley about 13 years ago in support of RSRA dynamic studies. (This program was used to do the forced response analyses for the CH-47D contained in references 42 and 43). Recently, several enhancements were made to the program making it interactive for rapid evaluation and plotting of responses. The improved version of this computer program is thoroughly documented in reference 45.

There are two in-house Army activities of note relating to finite-element modeling of composite structures. One activity, recently completed, was aimed at examining the modeling and testing complexities of composite structures. A prototype composite tail boom of the type installed on several OH-58A helicopters for environmental evaluation purposes was selected as the test specimen. The

Engineering Analysis Language (EAL) finite-element computer program (ref. 46) was used to model the tail boom (fig. 25). Interest was focused on studying the effect of graphite fiber-volume fraction on static and dynamic behavior because material tests had indicated that the volume varied by as much as ten percent. Results (refs. 47 and 48) indicated that there was improved agreement with test if measured values of material properties were used in the analysis. The other composite modeling activity relates to a blade rather than an airframe but it seems appropriate to include it because the blade is being modeled as a three-dimensional structure. The interest here is to investigate the potential for improving the dynamic and aerodynamic performance characteristics of composite rotor blades through the exploitation of structural coupling associated with ply orientation. Extension-torsion coupling is currently being studied. Α three-dimensional model of a highly twisted blade such as might be employed for a tilt rotor is being formed, both to support the design of a model blade and to support subsequent comparisons with both static and dynamic tests. A preliminary model of the D-spar of an untwisted blade as well as of a more recent twisted blade which includes the trailing edge are shown in figure 26. The model is being refined and work is under way to include the proper rotational effects.

# Analysis of Coupled Rotor-Airframe Vibrations

There are four technical factors that should be recognized when dealing with vibrations of a helicopter: (1) vibratory loads induced by the rotor actions; (2) response of the rotor; (3) coupling of the rotor and airframe; and (4) response of the airframe. The major source of vibrations arises from the cyclic loads acting on the rotor blades due to their interactions with the airstream. The dynamic characteristics of the rotor and the airframe and the coupling of these two systems determine the manner in which the helicopter responds to this excitation. As mentioned in the Introduction, the purpose of this paper is to present an overview of accomplishments and contributions associated only with factors (3) and (4) noted The response of airframe structures regarded as separate sysabove. tems was addressed in the previous section. In this section attention is directed to factor (3), namely, the coupling of the rotor and the airframe to account for their interaction in producing vibrations. The emphasis here, as before, is on the response of the airframe as part of a coupled rotor-airframe system.

The analysis methods now employed by industry applicable to helicopter vibrations generally fall into two categories, namely, (1) methods for analysis of airframe behavior and (2) methods for analysis of rotor behavior. For nonrotating airframe components, the NASTRAN computer code, as discussed in the previous section, has become the standard finite-element analysis tool used throughout the helicopter industry for structural design. For rotating components, there has been extensive work on formulating and solving equations of motion of rotors (see, for example, refs. 49 to 57). These references include a number of existing computer simulations of the helicopter in flight. Such simulations, of course, incorporate representations of both the rotor and the airframe and the connections between the two and thus theoretically could be applied to calculate vibrations. However, there is little note in the literature of their use to calculate airframe vibrations. These simulations have been applied mainly to evaluate flight controls, to analyze rotor stability, and to calculate blade vibratory loads. As a rule, the current simulations incorporate only cursory, if any, treatment of the airframe elasticity, and are cumbersome to use for airframe structural design work.

It has long been recognized that the interaction or coupling of the rotor and the airframe is important in analysis of helicopter vibrations and there has been at least one early attempt at addressing the problem analytically (ref. 58). From a practical point of view, however, the complexity of the problem has been so overwhelming that it has been customary to separately compute rotor vibratory loads and then apply them to an analytical model of the airframe for determining airframe responses. In this method, a (usually) sophisticated aeroelastic rotor airloads program is employed to calculate the rotor vibratory forces and moments acting at the hub assuming the hub can not move (rotor rotation is, of course, permitted). These vibratory loads are then imposed on an airframe finite-element model to analyze vibrations. In an attempt to approximately account for the effect of the rotor, an "equivalent" rotor mass is usually included in the airframe finite-element model. Historically, most predictions of vibrations have been based on the approach which has just been described. It is clear that this approach can not account for interactions between the rotor and the airframe. A simplified view of how the rotor and the airframe interact to produce vibrations is depicted in figure 27. Due to the cyclic nature of the airloads acting on the blades of a turning rotor, the blades respond dynamically and the resulting vibratory loads are transmitted to the airframe causing it to respond. The resulting airframe motions cause the hub to vibrate which alters the aerodynamic loading on the blades and hence the loads transmitted to the airframe. Depending on the type and configuration of the hub, this interaction can substantially alter the loads which are transmitted to the airframe and hence its vibratory response. However, because of the complexity of such an analysis, the simplistic approach described above was adapted by industry as an early expedient to permit a rudimentary consideration In this regard the method has served the industry of vibrations. However, because of increasing demands for further reductions well. in vibrations to achieve the goal of a "jet smooth" ride, it is now recognized that the simplistic approach is no longer sufficient. Analysis methods which accurately account for rotor-airframe coupling must be employed in vibration design analysis.

Two of the earliest descriptions of practical methods for calculating vibrations of a helicopter as a single system may be found in references 59 and 60. The analyses described in these references are impedance coupling techniques which effect a solution in the frequency domain rather than in the time domain. The impedance coupling technique has been widely used for the vibration analysis of mechanical systems which are composed of an assembly of point-In this approach each component is analyzed connected components. separately and then coupled together by requiring equilibrium and compatibility (i.e., matching forces and displacements) at each connection point. In its application to the solution of the coupled rotor-airframe problem (see, for example, ref. 60), the loads transmitted by the rotor to the airframe are given by the hub loads calculated assuming the hub is fixed and a (linear) correction term which accounts for small hub motions. The correction term is the so-called rotor hub impedance matrix and is obtained by prescribing small hub motions at the frequencies of interest and calculating the resulting constraint forces and moments at the hub. It should be pointed out that the gross vibratory forces exerted by the rotor on the airframe are given by the fixed-hub forces and that these forces are not, in general, computed by linear theory. The fixed-hub forces come from the solution of the underlying nonlinear rotor equations with the constraint that the rotor-airframe interface points are fixed. The rotor impedance matrix represents a correction to the gross rotor forces resulting from small displacements of the rotor from equili-It is a tenet of design to avoid resonant conditions, and if brium. such conditions are avoided, the displacements from equilibrium should be small. Thus, a rotor model linearized in the guise of a rotor impedance matrix should be nearly as good for vibration prediction as the underlying nonlinear model. The impedance matrix of the airframe at its interface with the rotor is calculated in a similar manner. Compatibility relations are then written for the interface forces and displacements leading to a set of coupled equations in terms of impedances. The resulting "harmonic balance" equations are a set of simultaneous linear algebraic equations which are solved for the hub motions, from which the airframe vibrations are computed.

Calculations based on the theory developed in reference 59 are compared with flight test data obtained on a Sikorsky H-34 rotor blade for several rotor-related quantities. However, only limited analytical results are shown for airframe vibrations and these are for a different helicopter. Reference 60 reports correlations for a tandem-rotor helicopter with three-bladed rotors. The correlations are reproduced in figure 28. While these results fall outside the period of time surveyed by this paper, they do represent some of the earliest published comparisons of a coupled rotor-airframe analysis with airframe vibrations measured in flight. Reference 61 reports a correlation for a different tandem-rotor helicopter using the analysis of reference 60. The relevant results are reproduced in figure The rotor model was very crude. Specifically, only the fixed-29. hub forces obtained from the equilibrium solution were retained in The rotor impedance was ignored. the linearized rotor equations. Reference 62 reports correlations for a compound helicopter with a four-bladed hingeless rotor. Plots indicative of the correlations are reproduced in figure 30. Nonlinear rotor equations were used in that analysis, but the airframe was represented by impedances calculated using a simple stick model representation of the airframe. The results of an early application of the C-81 flight simulation analysis for computing airframe vibrations on a helicopter with a fourbladed hingeless rotor are reported in reference 63. Computed

results for the 4/rev hub vibrations are compared with measured flight vibrations in figure 31. The airframe was represented by only three modes: pitch and roll of the pylon about its focal point (the test vehicle was equipped with a "focused pylon" vibration isolation system) and a vertical rigid-body mode. A correlation performed for a helicopter with a two-bladed teetering rotor is reported in refer-In this case, the analysis did not incorporate a model of ence 37. the rotor system. The procedure was to measure the flight vibratory accelerations at the rotor hub and then to impose the measured values of acceleration on a NASTRAN finite-element model of the airframe. The calculated response of the airframe was compared with the response measured in flight. Typical results for the major responses are shown in figure 32. Reference 64 describes procedures developed for correlating stresses derived from a NASTRAN finite-element model of the Bell 214A helicopter with stresses measured in flight. Although the flight tests were aimed at static structural qualification of the airframe in design maneuvers and not vibration, it seems appropriate to mention it here because C-81 was used to compute the external forces which were applied to the NASTRAN model. Analytical stresses were calculated by applying the internal loads calculated by NASTRAN to the effective cross-sectional area at each of the straingauge positions in the airframe as outlined in reference 64. Excellent correlation was noted.

In an analysis of helicopter vibrations based on a finiteelement model of the airframe, the number of degrees of freedom in the finite-element model must be reduced. Two approaches are currently recognized for making this reduction and still preserving the essence of the finite-element model: (1) representing the airframe by forced responses (i.e., impedances) calculated at a few frequencies corresponding to the rotor harmonics of interest; and (2) representing the airframe by superposition of a few of the natural modes of Whichever approach is used, data needed to represent the vibration. airframe with a reduced number of degrees of freedom are calculated by using a finite-element model of the airframe alone. Modal representations can be used for reducing the number of degrees of freedom when calculating any of the linear structural responses of interest This includes problems of aeroelastic in practical flight dynamics. stability and transient response as well as the present problem of steady-state vibrations. This broad applicability has caused the modal representation of the airframe to be the choice of developers of computer simulations of the helicopter in flight (e.g., C-81, CAMRAD, REXOR). Modal representations of the airframe are also used in more specialized coupled rotor-airframe formulations (see, for example, refs. 49 and 65). However, for vibration analysis done to support design of airframe structures, there are several attendant advantages to representing the airframe by harmonic forced responses. Hence, developers of new codes specifically for computing coupled rotor-airframe vibrations have tended to represent the airframe in terms of harmonic forced responses.

There have been several research studies using simple math models of coupled rotor-airframe systems to gain physical insight into the helicopter vibrations problem and to identify governing parame-

References 66 to 74 contain solutions of such simplified ters. rotor-airframe systems and relevant subsidiary analysis procedures. These and other studies have all shown that the coupling between the rotor and the airframe has a major effect on all aspects of vibra-In addition to studies using simplified models, there has been tion. some work in developing equations of motion of coupled rotor-airframe systems which devotes particular attention to nonlinearities associated with the rotor contributions to the coupled equations of motion (refs. 75 to 78). Reference 75 addressed the problem of developing a general approach for the dynamic analysis of gyroscopic structures composed of point-connected substructures by a component mode synthesis technique. The resulting formulation was intended to permit the determination of the modal characteristics of a helicopter. The mathematical model underlying the formulation, as well as the simplified model of a helicopter used to illustrate the formulation, are shown in figure 33. A computational procedure for deriving explicit equations of motion for such dynamical systems using symbolic manipulation is described in reference 76. Reference 77 derived the governing equations of motion for a helicopter rotor with blades having freedom in flap, lag, and torsion coupled to an airframe modeled as a rigid body with three translational and three rotational degrees of The resulting differential equations are nonlinear and confreedom. tain periodic coefficients associated with forward flight. Reference 78 derived the governing equations for rotor and airframe subsystems to use in an impedance matching approach to coupling. The reference also described a procedure for solving the resulting nonlinear equations for the coupled vibratory response by an iterative, combined harmonic-balance, impedance-matching method.

In recent years there have been several attempts to formulate a general method of vibration analysis suitable for airframe structural design work. These efforts have specifically addressed practical methods for calculating helicopter vibrations. Some of these endeavors are discussed below.

Dissatisfaction with first generation predictive capability for helicopter performance, loads, and vibrations motivated the Army to begin development of the Second Generation Comprehensive Helicopter Analysis System (2GCHAS). As a consequence of predesign studies related to 2GCHAS, several special-purpose codes have been developed by industry for solution of dynamics problems of coupled rotorairframe systems, including vibrations. Two of these are RDYNE (ref. 79) and DYSCO (refs. 80 and 81). RDYNE (Rotorcraft System Dynamics Analysis) employs a time-history analysis for computing rotorcraft response (stability or vibrations). A substructures approach is employed to model the helicopter. The program has been applied to at least one flight vibrations analysis, which is dis-Another code that had its genesis in the 2GCHAS predecussed later. sign studies is DYSCO (DYnamic System COupler). The DYSCO program has been under development since 1978 with both corporate (ref. 80) and government (ref. 81) funding. The program forms coupled equations of motion using the uncoupled equations of each component. Each component may contain periodic, nonlinear, and nonanalytic effects. Solutions can be effected in either the time or frequency

domain. There is no note in the literature of its use to calculate coupled rotor-airframe vibrations.

The SIMVIB (Simplified Vibration Analysis) code was developed under Army sponsorship to provide a design tool for predicting vibrations and for use in research studies (ref. 82). The analysis is based on a substructures approach and consists of a base program and a set of external programs (fig. 34). While emphasis is placed on obtaining solutions for steady-state vibrations by a harmonic balance method, other types of solutions are available. The results of limited correlations with data obtained from wind-tunnel tests of dynamically scaled models which include higher harmonic control effects are presented in that report. On the basis of these comparisons it was concluded that trends of vibration with airspeed could be pre-A recent "application" of SIMVIB to the SH-60B Sea Hawk is dicted. reported in reference 83. In this case the rotor impedance was not calculated by the program. Instead, 4/rev vibratory hub loads measured on the UH-60 were scaled to the SH-60B and imposed (within SIMVIB) as known exciting forces on a six-mode representation of the airframe. Comparisons of predicted vibration levels with those measured in flight are given in figure 35.

Reference 84 is an outcome of recent efforts at the NASA Langlev Research Center to establish foundations for adequate representation and treatment of the airframe structure in design analysis of helicopter vibrations. The report presents a body of formulations for coupling airframe finite-element analysis models to rotor analysis The rotor is represented models and calculating airframe vibrations. by a general set of linearized differential equations with periodic coefficients, and the connections between the rotor and airframe are specified through general linear equations of constraint. Coupling equations are derived and then applied to combine the rotor and airframe equations into one set of linear differential equations governing vibrations of the rotor-airframe system. These equations are solved by the harmonic balance method to yield the system steadystate vibrations. A key feature of the solution process is to represent the airframe in terms of forced responses calculated at harmonics of the rotor rotational frequency. A method based on matrix partitioning is presented for quick recalculations of vibrations in design studies when only relatively few airframe members are varied. A parallel development is given for the case in which the rotor is represented by impedances. All relations are presented in forms suitable for direct computer implementation. An illustration of this is given in figure 36 in which the coefficient matrix in the general harmonic balance equations retaining all the harmonics has been pulled out to show its structure. The explicit and practical nature of the formulation is illustrated by the example of the formula for the rotor contributions to the harmonic balance equations shown at the bottom of figure 36. Matrices appearing in the formula, such as KRLP, come directly from the linearized rotor equations and parameters, such as ULC, are computed by very simple algorithms which are provided. Such explicit formulas, FORTRAN-like notation, and the blueprint-like representation of matrices are used throughout the report to facilitate computer implementation.

Among the many activities being conducted under the DAMVIBS program is one aimed at evaluating existing analysis methods for calculating coupled rotor-airframe vibrations. In the initial effort in this area Bell, Boeing, McDonnell Douglas, and Sikorsky have applied in-house methods for coupled rotor-airframe analysis to calculate vibrations of the AH-1G helicopter. Comparisons were also made with existing Operational Loads Survey data (refs. 85 and 86). A finiteelement model of the AH-1G airframe was adjusted by Bell to correspond to the aircraft configuration used in the loads survey. The updated model was furnished to the other participating manufacturers as part of the common data utilized for the subject study. Bell was also required to provide to the other companies a summary of all modeling, testing and correlation work conducted on the AH-1G Bell was further required to assemble the flight vibra-(ref. 38). tion data to be used in the correlation and to describe the rotor system both mechanically and aerodynamically to the other participants (ref. 87). The aforementioned exercise on the AH-1G has been completed and the results have been presented at NASA/industry meetings held under the DAMVIBS program. Draft final reports have been submitted and are under NASA review. The comparisons shown in figures 37 and 38 are illustrative of the results obtained. Figure 37 shows a comparison of measured 2/rev and 4/rev vertical vibrations with predictions made by Bell using C-81. A summary of their results may be found in reference 88. Figure 38 shows a comparison of 2/rev vertical and lateral vibrations predicted by each of the four industry participants. These results were also compared with measurements at two locations in the airframe. The analytical results obtained by the four companies for the 2/rev vertical, lateral, and longitudinal vibrations are in fair to poor agreement with measured flight data. It should be noted that 2/rev is the primary main rotor excitation in the airframe. Best agreement was generally obtained for vertical vibrations; the worst for the lateral vibrations.

Boeing Vertol has recently implemented an impedance-based coupled rotor-airframe analysis (developed in-house) based on the concepts in references 60 and 61. The method (which was employed in the aforementioned AH-1G activity) is described in reference 89. Analytical results obtained for a wind-tunnel model and compared to test data showed, as had earlier studies, that results which include coupling differ significantly from results obtained without coupling. More important, however, their analyses also indicated that mechanical impedance effects predominate over aerodynamic effects for the scale model tested. If this result remains true for full-scale configurations, it would mean that a good approximation of rotor impedance for use in coupled rotor-airframe vibrations analyses could be obtained by neglecting (or at least drastically simplifying) the rotor aerodynamics. Because the computational effort required to compute rotor impedances which include aerodynamic effects is usually significant, any substantial reduction in the level of aerodynamic sophistication would greatly reduce these computations. This is an area that needs to be investigated further.

There are two NASA in-house activities of note related to

coupled rotor-airframe vibrations being conducted in support of the DAMVIBS program. The first activity is part of a continuing effort aimed at evaluating existing methods of analysis for coupled rotorairframe vibrations. Work has been initiated toward the application of the SIMVIB analysis to the OH-6A helicopter (fig. 39) used in a recently completed NASA/Army Higher Harmonic Control flight test program (ref. 90). Analyses will be made with and without higher harmonic control and compared with similar results obtained in flight Current plans are to also evaluate the DYSCO analysis with test. respect to its applicability for computing coupled rotor-airframe vibrations. The other activity is aimed at developing new computational procedures for coupled rotor-airframe vibration analyses. The primary effort here will be to encode the computational procedures for coupled rotor-airframe analysis and reanalysis which are outlined in reference 84.

It is clear that further work is needed in analysis of coupled rotor-airframe vibrations. Current plans are to conduct another industry-wide coupled rotor-airframe vibrations analysis under the DAMVIBS program, this time utilizing a helicopter with a four-bladed articulated rotor. Also, in an attempt to identify the importance of aerodynamics in rotor impedance calculations, parametric studies will be conducted in-house by NASA to evaluate the effects of rotor aerodynamic and structural modeling assumptions on predicted airframe vibrations. Current Army plans call for some combined in-house and contractual efforts aimed at validating existing codes for coupled rotor-airframe vibrations analysis using both model and full-scale data.

## Airframe Structural Optimization

The design of aerospace vehicle structures to satisfy static and dynamic specifications is a complex process. This has become especially true for modern helicopters primarily because of increasingly stringent requirements for low vibrations. The structural design process involves the merging of an analysis procedure with a resizing and reanalysis procedure in which changes are made to the structure in an iterative process until a converged design that is best or optimum in some sense is obtained. With regard to the airframe structural design process, the selection of the best airframe that meets all the requirements, in particular the vibration requirements, is a difficult task. It would appear that structural optimization tools, properly brought to bear by the design engineer, could go a long way toward achieving the goal of a design analysis capability for vibrations. Indeed, even the automation of as much of the current design process as possible would clearly serve to reduce design time and hence cost.

The objective of structural optimization is to design a structure that minimizes a specified function while satisfying a set of restrictions imposed on the design. The function with respect to which the design is optimized is called the objective function (alternative names which are sometimes used are performance index and

merit function). For aircraft structures, weight is usually taken to be the objective function. However, the objective function can be any quantity of interest. The restrictions placed on the design that must be satisfied to produce an acceptable design are collectively called constraints. Typically, constraints impose upper or lower limits on quantities such as stresses, displacements, natural frequencies, and structural parameters which are varied. Optimization procedures start with an arbitrary (but usually feasible) initial design and proceed by varying structural parameters in stepwise fashion so that the value of the objective function is reduced. The search is terminated when no further reduction can be made in the objective function without violating some of the constraints. The parameters which are varied during the iterative design process are called design variables. Examples of design variables include member sizes such as thicknesses of panels and cross-sectional areas of stringers, ply thicknesses and fiber orientation angles in composite material laminates, and physical properties of materials. The optimization problem is nonlinear if either the objective function or any of the constraints are nonlinear functions of the design variables. This is the usual case for the class of structural optimization problems which are of interest here.

A design-optimization algorithm consists of an analysis of the structure and modification of the design variables at each iteration. The number of iterations depends on the number of design variables and on the nature and number of constraints. Analyses of most aerospace vehicle structures are based on some type of finite-element model. Modification of design variables can be achieved by employing an optimizer which is based on either a nonlinear mathematical programming method or an optimality criterion method. Optimality criteria methods have the longest history. The basis for this approach is the a priori specification, based either on intuition or rigorous mathematical considerations, of a set of conditions to be satisfied by the optimum design. The premise is that when the structure is sized to satisfy the condition, the objective function automatically attains an optimum value. The algorithm which is formulated to resize the structure is usually recursive in nature. The concept of a fully-stressed design, which has been widely used in static structural design, is perhaps the best example of these methods. Nonlinear programming (NLP) methods have their origins in the field of operations research. These rigorous methods are applicable to a wide range of problems, of which structural optimization represents only one particular application. NLP methods use derivatives to determine move directions in the design variable space. Their main drawback is that the derivatives may be costly to calculate, especially when the number of design variables is large. However, the capability to treat all types of objective and constraint functions makes these methods very versatile. This is the method of choice for most current work related to structural optimization.

Since the beginning of the "modern" field of structural optimization in 1960 (ref. 91), the published literature in the field has literally exploded with new papers. For example, reference 92, which summarizes aeronautical applications of formal optimization methods,

identified over 8000 aeronautically related titles (including 1381 on structural optimization) covering various periods between 1964 and 1980. However, despite its long history and continued widespread interest, as noted in reference 92, there have been few successful genuine applications to aeronautical problems. In so far as the helicopter community is concerned, interest in optimization as it might be employed in helicopter design goes back only a few years. Α preliminary evaluation of optimization techniques as they relate to typical helicopter design problems is reported in reference 93. The paper describes the manner of combining nonlinear programming algorithms with conventional engineering analyses and summarizes the results of applying such algorithms to four different rotor design The results obtained demonstrated that closed-loop designproblems. oriented analyses can significantly reduce design time. The 39th American Helicopter Society Forum the following year featured a panel devoted to the subject (ref. 94) as well as two papers (refs. 95 and 96). The composition of the panel and the topics addressed are indicated in figure 40. References 95 and 96 treated the related topics of designing a rotor blade for minimum hub vibrations and of designing a blade for placement of natural frequencies, respectively. More recently in 1984, a NASA Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization held at Langley Research Center (ref. 97) devoted an entire session to rotorcraft applications (fig. 41). Additional applications are reported in references 98 and Two recent surveys of the application of structural optimization 99. methods to helicopter design problems are given in references 100 and 101. All of the aforementioned references reporting on rotorcraft applications of structural optimization have addressed the rotor sys-There has been very little published work within the rotorcraft tem. community relating to structural optimization of the airframe subject to vibration response constraints. The remainder of this section will address work which has been done that is applicable to the air-The section concludes with a status report of related inframe. house work.

The basic idea of airframe structural optimization under vibration constraints is to design the airframe structure in a way that minimizes the vibratory response in the important areas. It is beyond the scope of current design-optimization codes to treat each element of a structure as a variable in the iterative process. Hence, it is necessary to identify those few elements in a structure that should be treated as variables and modified to effect a reduction in vibrations. This identification process constitutes a task in sensitivity analysis. In its formal implementation sensitivity analysis involves calculating changes in the structural response with respect to (small) changes in the design variables. Such sensitivity derivatives are used by all NLP-based optimization methods. As mentioned earlier, the computation of these derivatives may be costly when the number of design variables is large. Informal implementations of sensitivity analysis are usually based on considerations related to some physical characteristic or behavior of the system, such as the distribution of element strain energies. Hence, they are usually employed in optimality criteria based methods. To date, most applications of optimization to helicopter airframe structures have

employed optimality criteria type methods. Reference 102 considers two strain-energy methods for structural modification (detuning) to achieve vibration reduction. The first method is based on the modal strain energy concept wherein elements having the highest strain energy density in a mode are taken to be the best candidates for modification to obtain a maximum frequency change of that mode for a minimum weight penalty. The second method is an extension of the concept of modal strain energy to the case of damped forced response wherein the strain energy density is determined for all the structural elements under steady-state vibratory loading. The elements with the highest strain energy densities are taken as the best candidates for modification of the structural response condition under The damped forced response (DFR) method is an extension of study. the optimality criterion of uniform strain energy density proposed in reference 103 for modes to the case of forced response. Several applications of the DFR method are described in reference 102, one of which is reproduced in figure 42. The figure shows the results of using modal strain energy to tune the frequency of the fourth elastic Based on the calculated strain energies, the mode of the CH-47A. structure was stiffened (the thickness of ten elements in the forward pylon and main cabin side panels was increased with a weight penalty of 2.5 percent) to move a natural frequency (12.03 Hz) to a higher position (12.74 Hz) with respect to the excitation frequency (11.45 Hz), thereby reducing the dynamic response. As the table shows, only the single frequency of interest was significantly altered. A DFR analysis of the modified airframe confirmed that the vibration levels had been reduced with respect to those in the original structure in the area of interest. Based on the studies conducted in reference 102, it was concluded that the DFR method is more general and thus has a broader range of applicability than the modal strain energy method. However, the modal approach is appropriate if the structure is excited close to a resonance, as in the case of the CH-47A in figure 42. Application of the modal strain energy approach to the CH-47C is reported in reference 104.

As part of an investigation of structural optimization techniques for vibration reduction, reference 105 evaluated two techniques for vibration reduction through local structural modification, the forced response strain energy method of reference 102 and the Vincent Circle method (ref. 106). The latter method is based on a dynamic property of (damped) linear structures, first noticed by Vincent of Westland Helicopters, Ltd. Vincent observed that under sinusoidal excitation the response of a point removed from the point of excitation traces out a circular locus in the complex plane when any single structural element stiffness or mass parameter is continuously varied from minus infinity to plus infinity. The radius of the circle and the location of its center are indicative of the extent to which the parameter change can affect the response. Both methods were applied to an elastic line model of the AH-1G airframe (fig. 43). The objective was to reduce 2/rev vertical vibration at the pilot seat due to 2/rev vertical excitation at the main rotor The results (fig. 43) indicated discrepancies between the two hub. The DFR method points to the tail boom as the area having methods. the most potential for reducing vibrations at the pilot seat, while

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the Vincent Circle method points to the pylon area. Based on the studies conducted in reference 105 it was concluded that the Vincent Circle method was appropriate as an identifier of important elements when considering local effects in relatively simple structures. However, for complex structures involving many elements the DFR method appeared to be preferable for indicating which structural elements are most responsible for the dynamic amplification.

Other approaches to local structural modification aimed at vibration reduction are described in references 107 and 108. Reference 107 describes a sensitivity analysis procedure based on taking derivatives of the stiffness matrix to identify the elements most influential on vibratory response. The method is demonstrated by using a modified version of the elastic line model used in reference 72 and by choosing as design variables Young's modulus of elasticity in each of the beam elements comprising the model. Reference 108 describes an approach for structural modification which utilizes not only the analytical model but also dynamically scaled models, optimization techniques (via optimality criteria) with frequency constraints, and system identification methods. The reference illustrates the approach by applying it to a simple cantilever beam structure.

The papers dealing with structural modification cited above are somewhat misleading. While the term "structural optimization" is used, none of the papers apply structural optimization in the usual way. Rather, the term is used to indicate that any local structural modifications which have been made are the best based on ad hoc considerations such as reduction of dynamic response or reduction of strain energy in a member. It should be recognized, however, that the methods described in those papers can be used as the first step in a procedure for formal optimization because they can identify the best few elements that can be treated as variables for reducing vibrations. Once the sensitive elements are identified a formal optimization procedure can be used to set the precise values of the parameters characterizing those elements.

There has been considerable research on structural optimization subject to dynamic constraints. Most of this work, however, is related to studies in which the only dynamic constraints are those imposed on natural frequencies. There is much less literature dealing with the problem of structural response under dynamic loading in which constraints are imposed on both dynamic responses and frequen-References 109 to 111 are representative of work which is cies. applicable to this more general problem. These papers discuss a phenomenon known as disjoint design space which complicates the structural optimization process for structures under harmonic excitation. The problem is associated with airframe natural frequencies which may move toward coincidence with a (fixed) forcing frequency as design variables are changed during iteration. These resonances form barriers which cause the feasible design space to be disconnected or disjoint.

The success of any optimization procedure rests primarily on the

efficiency of the analysis tool which is used to analyze the structure after every update to the design variables, and to a lesser extent on the efficiency of the optimizer. If the finite-element model is large (which is usually the case), the analysis step contributes significantly to the time for each iteration in the design There has been considerable effort directed toward means process. for reducing the time required for each iteration. Approximate mathematical models obtained from a first-order Taylor series expansion of the full finite-element model have been proposed to lessen the analysis time. Other expedients such as the use of design variable linking, reciprocal variables and constraint deletion have also Such methods are described in reference 112, for been proposed. There have also been attempts to develop algorithms for example. efficient reanalysis of structures which have been locally modified (see, for example, refs. 84, 113, and 114).

Motivated by participation in the initial planning stages of the DAMVIBS program in early 1983, Ames Research Center began building a breadboard structural optimization code for helicopter vibrations in late 1983. The resulting code, called NASOPT, combines MSC/NASTRAN (ref. 22) with the CONMIN optimization program (ref. 115) and is described in reference 116. A recent application of NASOPT to the problem of tuning a helicopter airframe for vibrations is described in reference 117. One case addressed in that paper was to minimize the vertical displacement at the pilot seat under 2/rev vertical forcing at the main rotor hub while subject to a frequency constraint on the first vertical bending mode. The design variables were taken to be the sectional area moments of inertia of each of the 22 beam elements comprising the longitudinal beam in the elastic line model. The resulting iteration history for three of the design variables is shown in figure 44.

The NASA Langley Research Center has a long history of research in structural optimization (see, for example, the summary of ref. 118). Most of this activity has, until quite recently, been centered in the Multidisciplinary Analysis and Optimization Branch (MAOB). In 1984 the Interdisciplinary Research Office (IRO) was formed, with optimization personnel from MAOB as its nucleus, to provide a more focused repository of optimization research. While most of the early Langley work on optimization has been directed to fixedwing aircraft, it has been generic in nature and should be applicable to rotorcraft. Of particular interest in this regard is the method for decomposing large optimization problems into smaller subproblems described in reference 119. Some recent work directed to the dynamics of rotor blades are reported in references 120 and 121.

As part of the NASA/industry rotorcraft structural dynamics program, DAMVIBS, an in-house study was recently initiated at Langley on optimization of rotorcraft airframe structures for vibration reduction. The objective of the research is to evaluate and develop practical computational procedures for structural optimization of airframe structures subject to steady-state vibration constraints. One of the key ingredients to any approach based on a NLP method is design sensitivity analysis. A method for computing the sensitivity coefficients for forced response behavior has recently been formulated and implemented in MSC/NASTRAN as a new solution sequence to complement the already available static and frequency sensitivity The results of an initial application of this design senanalyses. sitivity analysis to a simplified elastic line model of the AH-1G helicopter are presented in reference 122. Some of the results from that study are reproduced in figure 45 which shows computed dynamic response sensitivities for the pilot seat with respect to elements in The forced response strain energies associated with the tail boom. the tail boom elements are also shown. The results show that elements in the tail boom would be likely candidates for modification to effect a favorable change in the response at the pilot seat. It should also be noted that elements with large sensitivities also generally have higher strain energies.

The Langley in-house work on airframe structural optimization described above is continuing. Current near-term plans are to include structural damping in the formulation for calculating forced response sensitivities, to study the implications of computing sensitivities of large finite-element models, and to interface the CONMIN optimizer with the sensitivity analysis. Long-term plans are to merge this airframe optimization activity with IRO activities on rotor blade optimization and establish a joint activity aimed at providing a rudimentary technology base for optimization of coupled rotor-airframe systems. Current plans are to also initiate some type of airframe optimization activities (as yet undefined) with industry under the DAMVIBS program. With respect to the NASOPT code developed at Ames Research Center, current plans are for Ames to maintain the code as a research tool for conducting basic research in structural optimization; long-term plans for the code are unclear at this time.

#### VIBRATION CONTROL

The most significant vibration levels in a helicopter are caused by the cyclic airloads acting on the main rotor as it rotates. The resulting oscillating aerodynamic loads are transmitted to the fuselage as vibratory forces and moments of a frequency equal to the number of rotor blades N times the rotational frequency or N/rev. The character and magnitude of these vibratory loads have resulted in the design of vibration control devices to reduce or minimize these rotor-induced forced vibrations. Vibration reduction concepts may be separated into passive or active methods. Passive devices, as discussed in this paper, are absorbers in the rotating system, absorbers in the fixed system, or rotor isolation systems. Active systems sense vibration levels at one or more locations on the helicopter and attempt to minimize the sensed vibration levels by use of some type of active control feedback system. A variety of passive vibration control systems have been developed and tested over the past 25 The Army and NASA have sponsored considerable research in vears. rotor isolation systems, hub absorbers, and blade absorbers. References 8 and 123 provide an excellent historical and technical prospective of vibration control system development. Since 1975 the Army and NASA have funded major vibration control system demonstration efforts in total main rotor isolation and higher harmonic control. There have also been some contracted research efforts for the analysis and testing of hub-mounted and blade-mounted absorbers. As previously pointed out, only Army and NASA research conducted in the past decade will be specifically discussed.

#### Rotating-System Passive Absorbers

One of the simplest passive mechanisms for reducing vibratory loads in the rotating system is the pendulum absorber. It consists of a simple mass attached at a distance R from the center of rotation by a mechanical linkage of smaller radius r (fig. 46). The spring rate of the pendulum is controlled by centrifugal forces on the mass. The pendulum natural frequency is proportional to the rotational speed and the ratio of radii R/r. Therefore, the pendulum acts as a vibration absorber when the pendulum natural frequency equals the excitation frequency. Both blade-mounted and hub-mounted pendulum absorbers have been used in production helicopter. Reference 124 describes a blade-mounted pendulum absorber system that was designed for the Army AH-64. A general analytical study of pendulum absorber dynamics is reported in reference 125. This analysis was later extended to a frequency response analysis in which the spanwise airload distribution was varied harmonically to excite the rotor (ref. 126). The response of this absorber is shown in figure 47. Another type of rotating-system vibration absorber, the bifilar absorber, is a centrifugally tuned, pendulum-like device mounted to the main rotor hub. A bifilar absorber is shown in figure 48. Components of a bifilar absorber consist of a support arm and sets of bifilar masses each of which is comprised of a dynamic mass, and two cylindrical tuning pins. These pins constrain the mass radially and, together with the circular tracking holes in the support arm and mass, define the pendular radius of the mass (ref. 127). The bifilar rotor hub absorber has been used since the late 1960s. In support of the bifilar development, a coupled rotor-bifilar-airframe analysis was used to study the dynamic characteristics. This analysis was validated by correlation with UH-60 and S-76 helicopter flight test data as shown in figure 49 (ref. 128). In addition to industrysponsored bifilar research, the Army funded research to develop advanced hub absorber concepts. A two degree-of-freedom rotating system absorber, the monofilar (fig. 50), was analyzed and tested in the early 1980's (refs. 129 and 130). The advantages of this concept compared to the bifilar were reduced weight and the ability to provide vibration reduction at two frequencies. Coupled rotormonofilar-airframe analyses were conducted to design a monofilar configuration for a four-bladed rotor under contract to the Army (ref. 131). The system was tuned to reduce 3/rev and 5/rev rotatingsystem forces. Ground test results showed a significant attenuation of 3/rev in-plane rotating-system hub forces. However, attenuation of the 5/rev loads was poor as a result of physical binding of the monofilar components (ref. 131).

# Nonrotating-System Vibration Isolation

Although no Army or NASA in-house research has been conducted to develop specific vibration reduction hardware, in the past ten years efforts have been funded to demonstrate company-developed systems. The most successful passive isolation systems have been based on the anti-resonant (nodalization) principle. A schematic of an antiresonant isolator is shown in figure 51. By proper selection of the tuning weight and arm length, the inertial force can be made equal and opposite to the spring force, and therefore no N/rev vibratory forces Several antiresonant vibration are transmitted to the fuselage. reduction concepts have been investigated. One concept, described in reference 132, is the Dynamic Antiresonant Vibration Isolator or DAVI which was implemented by the Kamam Aerospace Corporation. The Kaman DAVI is a passive isolator that provides a high degree of isolation at low frequencies with low static deflections. Research and development has been conducted on one-dimensional, two-dimensional, and three-dimensional DAVIs (fig. 52). A two-dimensional DAVI system was tested on a modified Army UH-1H helicopter to provide isolation in the vertical, pitch, roll, and fore-and-aft degrees of freedom. This test demonstrated that the DAVI-modified UH-IH had substantially lower vibration levels (over 70 percent) when compared to the unmodified vehicle (fig. 53). The results of this test also demonstrated that the use of the DAVI could, without affecting flying qualities, reduce aircraft weight and lower operating costs due to lower maintenance requirements (ref. 132). In a parallel development, Bell tested a DAVI-type system called the NODAMATIC isolation system (ref. 133). The NODAMATIC system consists of a focused pylon to isolate rotor inplane hub shears and moments and a nodal beam to isolate rotor vertical shears (fig. 54). Boeing Vertol improved the DAVI by replacing the elastomeric springs with metal springs to reduce inherent damping. This new system, called the Improved Rotor Isolation System (IRIS), also provided isolation at twice N/rev (refs. 134-136) (fig. 55). The IRIS was designed and tested on a Boeing-owned BO-105 (fig. 56).

To demonstrate the full potential of passive isolation, the Army in 1979 initiated a program for total (six degree-of-freedom) main The program was conducted in several phases which rotor isolation. included predesign studies, design and bench test, and flight test. Predesign studies were conducted of two different mechanical isolation system concepts (refs. 137 and 138). Both designs were derivatives of the Kaman DAVI. A third concept, which used hydraulic isolator units to achieve antiresonance, was also evaluated (ref. 139). This hydraulic isolator is called the Liquid Inertia Vibration Eliminator (LIVE) and is depicted in figure 57. The LIVE unit consists of an inner cylinder which is bonded to an outer cylinder with a layer of rubber. The inner cylinder cavity is filled with a high-density fluid (mercury). Isolation is achieved when the dynamic pressures create inertial forces which cancel the spring forces associated with deformations of the rubber. Reference 140 describes an application of LIVE. As a result of the predesign effort, the LIVE concept was selected for detail design and bench testing of total main rotor isolation. The success of this phase resulted in an Army-funded contract to install a Total Rotor Isolation System (TRIS) on the Bell 206LM helicopter. The testbed aircraft and the LIVE unit installation are shown in figure 58. The flight test data indicated that over 95 percent reduction of hub 4/rev (26.3 Hz) vibration levels had been achieved. Pilot seat vibrations were reduced to 0.04g throughout the flight envelope, including the transition region which traditionally has high vibration levels (fig. 59). The prototype TRIS installation had a weight penalty of 1.7 percent of the aircraft maximum gross weight. It was projected that the weight penalty could be reduced to less than 1 percent (ref. 141) by manufacturing the LIVE units out of lightweight material, instead of the stainless steel used for the proof-of-concept test.

The Rotor Systems Research Aircraft (RSRA), which is shown in figure 60, incorporated a passive isolation system. The system was designed to provide a satisfactory aircraft vibration environment for "any" rotor system installed on the aircraft. Although labeled the "RSRA Active Isolation/Rotor Balance System" or AIBS, this system is not "active" in the conventional sense. The AIBS (fig. 61) consists of four piston-in-cylinder units which combine the effects of an air spring for 4/rev passive isolation with a low frequency centering action (for active control of transmission alignment). The effective spring rate of the passive isolation system is controlled and set prior to flight by the precharge pressure of the system accumulators. Thus, the AIBS does not sense and react to changing flight vibration levels in the normal sense of "active" control. The hydropneumatic isolation system is described in reference 142. Although the reduced vibration levels measured during the RSRA isolation system shakedown flight test program were encouraging, the isolation system was not optimized for minimum cockpit vibrations and the potential for additional improvement exists (ref. 142).

#### Active Vibration Suppression

Active vibration suppression systems, as discussed in this section, sense vibration levels at one or more locations on the airframe and actively minimize the sensed vibration levels by the use of an automatic feedback system (fig. 62). Because the primary source of helicopter vibrations is the rotor, it is logical to use the feedback system to manipulate the rotor blades to modify the aerodynamic excitation forces, thus reducing the airframe vibrations. The potential of direct rotor control to minimize vibrations has been studied since the 1960's. The early work, however, was limited to analytical studies because adequate hardware did not exist to implement a system (ref. 143). The use of active means for suppressing vibratory loads transmitted to the airframe in flight has become feasible with advances in high-speed, lightweight microcomputers and with advances in hydraulic servo-actuator technology.

One promising method of active vibration control, called Higher Harmonic Control or HHC, superimposes nonrotating swashplate sinusoidal motions at the blade passage frequency upon the basic collective and cyclic flight control inputs. This approach to control vibratory loads has been the subject of several analytical studies by both NASA Ames and the Army (refs. 144 and 145) and wind-tunnel tests by both government and industry (refs. 146-148). These investigations, conducted on significantly different types of rotor systems, all showed that HHC produced substantial reductions in vibration levels transmitted to the airframe. Furthermore, the results indicated that the amplitude of HHC blade pitch inputs required to achieve the desired reductions was small, on the order of one degree.

In 1976, NASA Langley and the Army began some preliminary research into applications of HHC. This work resulted in two major activities which included: (1) wind-tunnel tests; and (2) a flight The initial wind-tunnel tests were conducted test demonstration. open-loop using trial-and-error for setting the amplitudes and phases of the HHC inputs. While these open-loop tests validated the concept, computerized control was needed to achieve optimum control of The open-loop and closed-loop HHC all vibratory forces and moments. test results on a dynamically scaled wind-tunnel model rotor were reported in references 149 and 150. The HHC method for reducing vibrations was demonstrated under contract using an OH-6A helicopter. The preliminary design work, control law development, and flight test results were reported in references 151 to 153. The open-loop and closed-loop flight testing of the OH-6A showed conclusively that HHC can reduce vibration levels in helicopters (fig. 64). Reference 90 constitutes a summary report for that program. Research has continued on HHC with Sikorsky flying the concept, open-loop, on an S-76 (ref. 154) and the Army funding two preliminary design studies for implementation of HHC on current and future generation helicopters.

Although individual blade control has been promoted for vibration reduction using HHC, the complications of moving any control system into the rotating system have slowed down advances in this area. Several concepts of direct rotor control with individual blades have been studied earlier (ref. 155) but to date none have been tested. Higher Harmonic Control shows much promise for reducing helicopter vibrations, especially for the next generation helicopters that may have a fly-by-wire/light control system and a variable speed rotor. The Army plans to extend HHC technology by sponsoring a flight demonstration program using a modern, four-bladed, high-speed helicopter. This program has been given the acronym SOFVIBS (Suppression Of Flight VIBrationS).

The helicopter vibration problem is complex and much time, effort, money, and man-power have been expended to reduce vibrations. Nevertheless, the problem has not been completely solved and a great deal more work remains for the helicopter community before the "jet smooth" ride is achieved. The vibration reduction systems discussed in this section only reduce vibrations that are transmitted mechanically to the fuselage from the rotor. While the helicopter industry has been able to significantly reduce these "mechanicallytransmitted" vibrations in the last 30 years, another source of rotor-induced vibrations still must be addressed, in particular, wake impingement on the airframe. Blade tip vortices create pressure fluctuations on the fin and stabilizer that cause significant fuselage vibrations. These rotor downwash induced vibrations need to be controlled or isolated from the fuselage before vibrations in the helicopter fuselage can ever be totally eliminated (ref. 132).

## VIBRATION TESTING

Vibration testing of helicopters involves experimental investigations to establish and to verify airframe dynamics, flight vibrations, and rotor-induced vibratory loads. Ground and flight vibration testing along with wind-tunnel testing are used to guide helicopter design and to evaluate vibration problems. For the most part, wind-tunnel testing is conducted to verify rotor performance and basic stability and control characteristics for straight and level flight. In recent years, wind-tunnel testing has been conducted to investigate the effects of main rotor wake geometry and aerodynamic interactions on control surface effectiveness and vibration. As mentioned in the Introduction, rotor aeroelastic research and associated wind-tunnel testing will not be specifically addressed in this paper. What will be addressed in some detail is progress in helicopter ground and flight vibration testing methodology. The emphasis of this paper is on the fixed system, i.e., from the rotor hub through the airframe.

Most structural dynamicists would probably agree that helicopter vibration testing requirements are much more critical than corresponding fixed-wing requirements. Vibration testing serves two valuable purposes in helicopter development. First, these tests provide loads and vibrations data to verify design concepts. Second, vibration testing compensates for voids in existing analytical capabili-Helicopter vibration problems have been extremely difficult to ties. quantify and, as a result, have been solved during the development cycle by trial-and-error testing. A major reason for these cut-andtry methods has been a lack of definitive procedures which make maximum use of vibration test data. As conventionally practiced, most helicopter ground and flight vibration tests provide limited information for resolving vibration issues. However, techniques have evolved over the past decade from combined Army and NASA research that provide systematic, as opposed to trial-and-error, procedures for testing, correlating, and evaluating helicopter vibrations. the purposes of this paper, vibration testing is separated into four categories, namely: (1) modal analysis; (2) system identification; (3) structural modification; and (4) vibratory loads measurement. Many scientists and engineers are engaged in rotorcraft vibration research, and vibration testing research in the categories listed above has increased substantially over the past ten years. The majority of references listed in this paper emphasizes in-house and contractual work conducted by the Army and NASA.

#### Modal Analysis

Modal analysis is the name given to techniques which extract from test data the natural frequencies, orthonormal modes, and modal dampings of a structure. These modal parameters are most often used to verify analytical models and to determine which parts of a structure contribute to a given mode of excitation. The theory of modal analysis dates back to the 1940's (ref. 156). There have been some methods which use time domain data (refs. 157 and 158) but structural dynamicists traditionally perform modal analysis using frequency domain data (refs. 159 to 167). The most common frequency domain approach uses complex plane data (the so-called Kennedy and Pancu plots or Nyquist circles). Figure 65 shows an example of these frequency domain circles. The rate of change of arc length around the circle and the diameter of the circle are used to determine the modal parameters. Reference 168 presents a complete derivation and application of this modal analysis methodology. The availability of Fast Fourier Transform signal analyzers in the early 1970's provided the means to apply the Kennedy and Pancu theory (ref. 156) with speed, accuracy, and fidelity. Modal analysis accuracy is typically verified by comparing measured frequency responses with synthesized frequency responses which are calculated using the identified modal parameters. Figure 66 shows a comparison between test and analysis for frequency response measurements on an AH-1G helicopter. The ordinate shift evident in the real part of the response is caused by the rigid-body contribution which was not included in the synthesized The rigid-body part is normally calculated from weights and curve. geometry information.

In the past twenty years helicopter designers have used sophisticated finite-element computer programs for sizing the structure to meet static load requirements and to provide for the normal analytical checks on vibrations. Accurate dynamics models of airframes are necessary not only to assess vibration design against specifications but to evaluate the vibration effects of configuration changes. Numerous researchers have conducted correlation efforts of finiteelement model predictions with vibration test measurements (refs. 29-31, 34, 38, 42, 47 and 48). From a dynamics perspective, natural frequencies and mode shapes have been used as fundamental parameters for verifying the accuracy of analytical models. For example, figure 67 compares calculated and measured mode shapes of an OH-58 composite tail boom. Elaborate correlation efforts of Army CH-47D, UH-60A, AH-64, and ACAP airframes have also been conducted by the helicopter industry under contract to NASA (the DAMVIBS program) to evaluate the state of the art in finite-element modeling. Besides comparing the fundamental modal parameters, frequency response comparisons between shake test and analysis were used to assess the airframe modeling accuracy. The results of these correlation programs have much in common. First, the presence of modes in analysis which are not present during test and vice versa. Second, good accuracy on natural frequencies (less than 5 percent) but correspondingly poor accuracy on frequency response. And finally, the frequency range of acceptable correlation is only from 5 Hertz to about 20 Hertz. Detailed discussions of these correlation efforts were presented in the vibration analysis section of this paper.

Extensive ground vibration testing of an AH-1G helicopter (fig. 68) was conducted about seven years ago to obtain data for ver-

ifying shake test methods and modal analysis techniques. A significant finding from this shake test program was the measurement of complex modes (modes that have real and imaginary components) in the frequency range of interest (fig. 69). Reference 163 provides an excellent description of the cause and effect of complex modes. In short, complex modes can result when damping is not uniformly distributed throughout the structure. As a result, the phase between response and excitation is not constant and the mode shapes change with time. In the case of the AH-1G, this "nonproportional" damping was more than likely caused by the highly damped elastomeric mounts used to attach the transmission to the airframe. For classical modes the real part of the frequency response has two turning points near resonance while the imaginary part has one turning point. The first mode of figure 66 is an example of a classical mode. However, the character of the real and imaginary frequency responses can reverse for complex modes. Figure 69 illustrates this effect for an almost pure imaginary complex mode at 45 Hertz. As a consequence of this research, improved shake test methods have been developed in terms of both frequency response measurement and modal analysis accuracy (refs. 169 to 173). These improved measurement techniques include criteria for determining response linearity, reciprocity, complex modes, local modes, and frequency resolution. The improved modal analysis techniques which are now available provide a more accurate and consistent data base for system identification and finite-element correlation of complex helicopter structures.

# System Identification

Uncertainties inherent with analytical modeling techniques have made experimental modeling a viable approach for augmenting structural dynamics analysis (refs. 168, 174 to 192). The process of obtaining structural dynamics equations of motion or improving existing mathematical models using ground vibration data has been termed system identification. System identification deals with finding impedance-type matrices which are abstract inverses of measurable natural properties of a structure. The objective of system identification is to use these mathematical abstracts for estimating structural response characteristics. The origin of system identification goes back to the 1960's (ref. 174), but most of the theoretical development and validation work was performed in the mid to late The data required to experimentally derive the equations seventies. of motion are the natural frequencies, orthonormal mode shapes, and modal dampings which characterize the frequency spectrum of interest. These parameters are used to determine mass, stiffness, and damping matrices which define the equations of motion. The model which is formulated from this system identification process is called a "truncated model" because there are fewer modes used to determine the model than degrees of freedom in the structure (ref. 175). Multiple regression is used to solve for the constant coefficient matrices which make up the equations of motion. The regression parameter is the difference between the actual frequency response and the approximated frequency response obtained by using a finite number of modes. The primary application of this truncated model is to predict the

effects of mass and stiffness changes on natural frequencies and mode shapes. Computer experiments have verified the accuracy and limitations of the method (refs. 176 and 177). In addition, the truncated model methodology has been applied using AH-1G airframe modal test data. Predicted changes in natural frequencies and mode shapes were compared with test results to assess its usefulness (ref. 168).

Another system identification technique which provides a capability for improving an existing analytical model is the so-called "incomplete model" theory (refs. 178 to 181). This method uses natural frequency and mode shape test data to update or improve mass and The approach which is used to create the incomstiffness matrices. plete model assumes that the measured modal data are correct and forces the analytical mass matrix to be orthogonal with the measured Multiple regression is used to solve for the smallest posmodes. sible changes (in a least-squares sense) that satisfy the specified conditions. In a similar manner, the modal data and improved mass matrix are combined to improve the stiffness matrix. The requirement for small changes is not necessary and is only assumed so that the improved model still represents the physical structure. Engineering judgement is required to determine acceptable values for these small changes. A measure of accuracy of the improved analytical model is obtained by comparing predicted frequency responses with test data. It should be pointed out that current finite-element models do not incorporate nonproportional damping and hence can not account for the effects which lead to complex modes. There has been some research to develop methods for converting complex modes obtained from test into classical (or real) modes for model improvement purposes (ref. 182). The usefulness of these procedures is questionable if the improved model cannot be used to calculate frequency response for the structure being tested. Another criterion for evaluating the usefulness of the incomplete model is its ability to predict the effects of a Figure 70 illustrates how the incomplete model predicts mode change. shape changes due to mass and stiffness configuration changes. The solid curves represent the original mode shapes for a simply supported beam. The new mode shapes, shown by the dashed curves, were calculated using the exact beam equations. The data points in figure 70 were determined using the incomplete model. However, one of the major problems associated with system identification technology is the inability to physically interpret the changes which are identified by the analysis. Model improvement techniques must be developed such that mass and stiffness changes to the original models are physically meaningful as well as mathematically sound. There has been some research outside of the Army and NASA on methods which use test data to identify modeling errors (ref. 183). These techniques provide information to the analyst as to which model parameters are causing discrepancies between test and analysis. New ideas such as these may provide the means for implementing system identification as part of helicopter vibration design.

# Structural Modification

Research into techniques which predict changes in structural

dynamics or flight vibrations due to structural modifications has been under way for about fifteen years. One of the first concepts for evaluating vibration reduction through structural modification was reported in reference 193. The so-called Vincent Circle method (ref. 193) was described earlier in the Airframe Structural Optimiza-This methodology has been applied and extended by tion section. numerous researchers (refs. 105-108 and 194-202) over the past ten years. For the most part the Army requirement for these procedures was motivated by vibration problems which surfaced during helicopter development testing. More recent Army research has concentrated on combining structural modification methodology with ground and flight vibration data to evaluate the effects on vibration. This integration of structural modification with vibration testing has also been referred to as "analytical testing" (ref. 195). Unlike the typical finite-element modeling approach, there is no airframe math model that has to be created or modified such that it correlates with shake test results. The only analytical model required is the structural change as characterized by single-point or multi-dimensional impedance adjustments. These modifications include simple mass, absorber, isolator, and collinear stiffness changes as well as more elaborate skin, stringer, or component changes. The operational equations require only baseline vibration data and the impedance change dynamics. Computer experiments have been conducted to demonstrate the usefulness of this methodology (refs. 195 to 197). The method has also been applied using AH-1G ground and flight vibration test data (refs. 195 and 198). Figure 71 illustrates how the method can be used to predict changes in cockpit vibration due to an absorber located on the vertical fin. In this example, the "remote" absorber was tuned for both frequency and damping to produce zero vibration at the required flight condition. Additional work is under way by Army researchers to validate the analytical testing methodology. The approach taken is to analytically make a change, predict its effect, and then to physically make the change, test the change, and compare the test results with analysis. This methodology has been verified on a generic helicopter model (fig. 72). Further research is being conducted to validate analytical testing using OH-58A ground vibration data and simulated flight test data. Successful implementation of this structural modification methodology will provide a much needed capability to respond to Army field problems and to eliminate costly trial-and-error testing.

#### Vibratory Loads Measurement

Higher than expected vibratory loading is a fundamental cause of high maintenance manhours and low component reliability. In general, the most critical vibratory loads are generated by the main rotor and occur at the blade passage frequency or N/rev. An accurate knowledge of these vibratory loads is needed to improve rotor design, to evaluate vibration control devices, and to establish fatigue characteristics. In particular, the helicopter industry spends substantial resources to reduce vibratory loads in an effort to increase reliability. If the vibratory loads were known, then "ground" flying could be performed on the complete helicopter using these loads to simulate flight. Thus, around-the-clock fatigue testing could be used to evaluate system reliability. Besides reliability testing, ground flying can be used with structural modification testing to evaluate and implement potential fixes to vibration problems before failures occur in the fleet.

The most common approach for measuring vibratory rotor loads (both hub shears and moments) uses strain gages on the main rotor shaft. Slip rings are required to transmit signals from the rotating system to the fixed system. Because this method was costly, slow, and often unreliable, Army research began in the mid 1970's on a method called Force Determination which uses airframe response measurements and shake test calibration data to determine the main rotor hub vibratory loads (refs. 203 to 206). Force Determination is a multiple regression technique (least-squares curve fit) which minimizes the differences between measured responses and calculated responses. All instrumentation is located in the fixed system (no slip rings are needed). Accelerometers and strain gages are distributed throughout the airframe to introduce a high degree of measurement independence and redundancy. The method has been verified on a generic helicopter dynamic model and full-scale aircraft (refs. 203 Figure 73 compares flight test, Force Determination calcuand 205). lated, and ground flying vibration levels at several points along an AH-1G airframe. These results demonstrated that the calculated main rotor hub loads can be used to synthesize actual flight vibrations accurately and with the correct distribution. Force Determination was also applied to a UH-1 helicopter (fig. 74) to evaluate rotor isolation system effectiveness (ref. 203). In this case the calculated loads for the baseline aircraft were combined with forced response measurements obtained from shake testing the aircraft with the isolation system installed. The predicted "new" flight vibrations were consistent with flight measurements and gave credibility to the method. Additional work has been performed by other researchers to improve Force Determination (refs. 207 and 208). Army in-house research is being conducted to evaluate the limitations of the method and to develop a full-scale reliability testing capabil-Several technical issues which are being investigated include itv. shaker cross talk, load versus response linearity, phase shift sensitivity, and shaker attachment (boundary condition) effects on the frequency response calibration data. There are other applications of this technique which are planned through Army in-house research. For example, Force Determination will be used to study the vibration effects of main rotor downwash impingement and main rotor wake interactions on tail surfaces.

There is considerable Army and NASA research in vibration testing planned for the next five years. Most of the work emphasizes verification of current methodologies such as System Identification, Analytical Testing, and Force Determination. Emphasis will be placed on using these new vibration testing methods to develop systematic procedures for solving vibration-related problems. Research will also be performed to demonstrate the applicability of these new methods on composite rotorcraft. Finite-element modeling correlation of composite structures, in particular the Army ACAP airframes, is also planned. Other research issues which will be addressed through combined Army and NASA research include standardization of vibration testing methodology. This standardization will include not only vibration testing procedures but also data acquisition and analysis methodology. The results of these efforts will identify how tests should be performed, what data should be taken to meet vibration testing objectives, and what data analysis procedures give the best results.

#### CONCLUDING REMARKS

Excessive vibrations have plagued virtually all new rotorcraft developments since the first U.S. helicopter went into production over forty years ago. The problem is pervasive and transcends The impact of excessive vibrations on new helinational boundaries. copter development programs is significant, both with respect to increased development costs and slipped delivery schedules. Helicopter companies have relied little on analysis during design to limit vibrations. With few exceptions, helicopters have been designed to performance requirements and excessive vibrations were then "tinkered out" during ground and flight testing. With continued expansion of flight envelopes and more stringent requirements for crew and passenger comfort and component reliability in modern helicopters, the requirement for low vibrations has achieved the status of a critical design consideration. It is clear that vibrations can no longer be addressed in an ad hoc fashion. There is now a recognized need to account for vibrations more rigorously in both the analytical and experimental phases of design. With this as a background, this paper has presented a summary of NASA and Army contributions, both in-house and contractual, to rotorcraft vibrations and structural dynamics technology over the last decade or so. Specific topics that were addressed include: airframe finite-element modeling for dynamic analysis, coupled rotor-airframe vibrations, airframe structural optimization, active and passive control of vibrations, and integration of testing and analysis in such guises as experimental modal analysis, system identification, structural modification, and vibratory loads measurement (force determination). The status of current activities being conducted under major NASA and Army programs, as well as nearterm plans, were also described. Viewed as a whole, it is fair to say that the work described constitutes an important contribution to the critical elements of the technology base needed to achieve the goal of a "jet smooth" ride. However, much work still needs to be done before this goal can be reached. To this end, both NASA and the Army have substantial in-house and contractual research activities planned over the next five to ten years. The ultimate success of these efforts will depend not only on the development of more reliable vibration design tools but also on the practical implementation of these tools into the design process by industry. It is left for a status report ten years hence to judge whether we have been successful.

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Figure 1.- The Sikorsky R-4, the first U.S. production helicopter.



Figure 2.- Soviet Yak-24.



Figure 3.- UH-60 Black Hawk.



Figure 4.- AH-64 Apache.

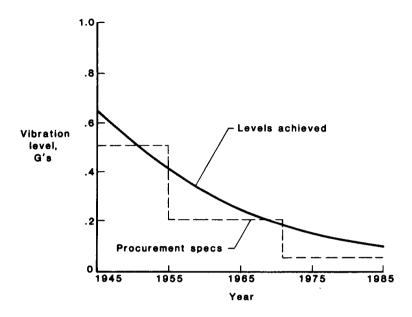


Figure 5.- Trend of helicopter vibration levels.

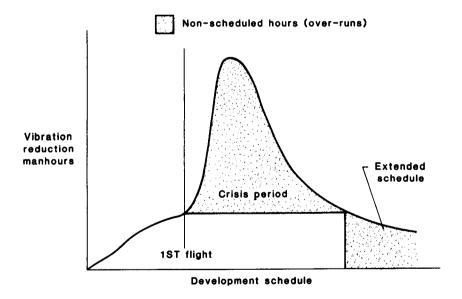


Figure 6.- Impact of vibrations on helicopter development.

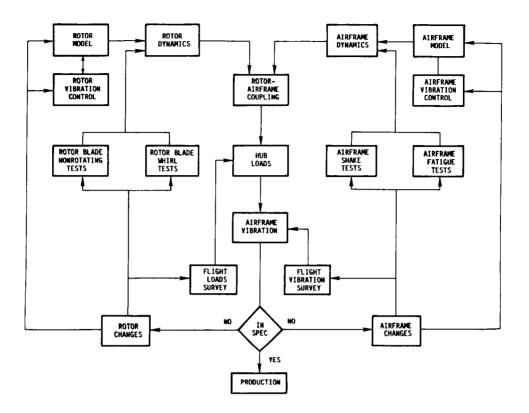


Figure 7.- Helicopter vibration design cycle.

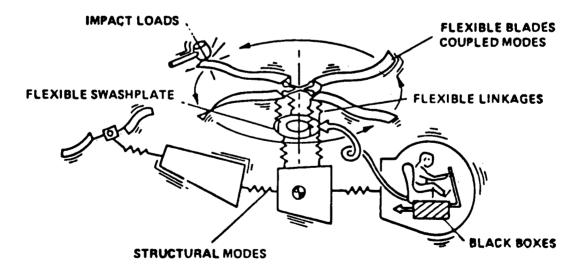


Figure 8.- The helicopter as might be viewed by a dynamicist.

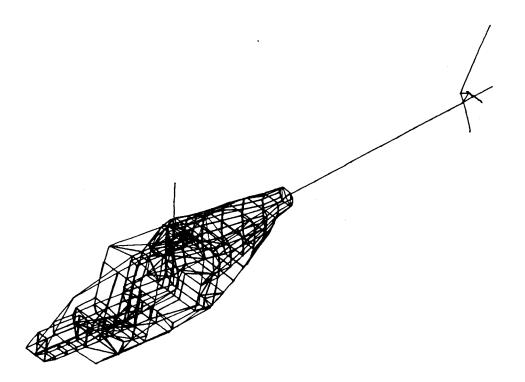
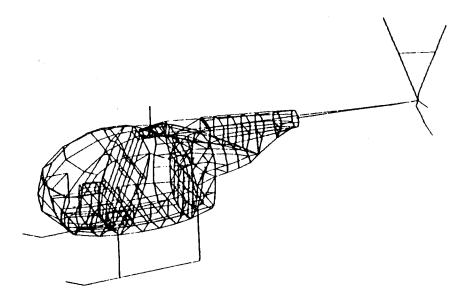
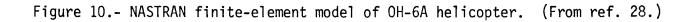


Figure 9.- NASTRAN finite-element model of OH-58A helicopter. (From ref. 27.)







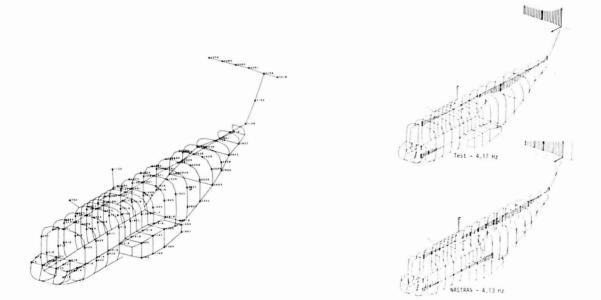


Figure 11.- NASA CH-53 Civil Helicopter Research Aircraft showing NASTRAN model with dynamic degrees of freedom indicated and correlation for the first vertical bending mode. (From ref. 31.)



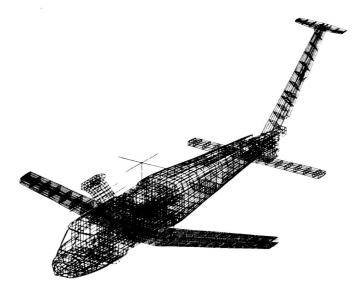


Figure 12.- Rotor Systems Research Aircraft (RSRA) in compound configuration and associated NASTRAN model. (Courtesy Sikorsky Aircraft.)



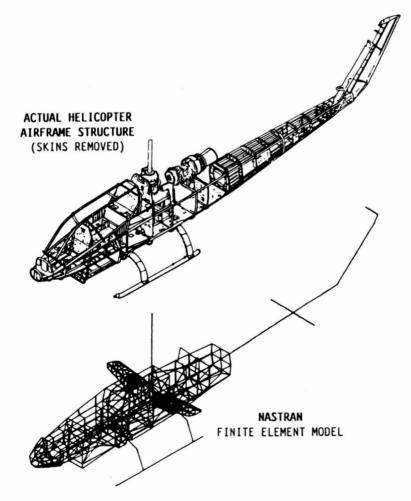
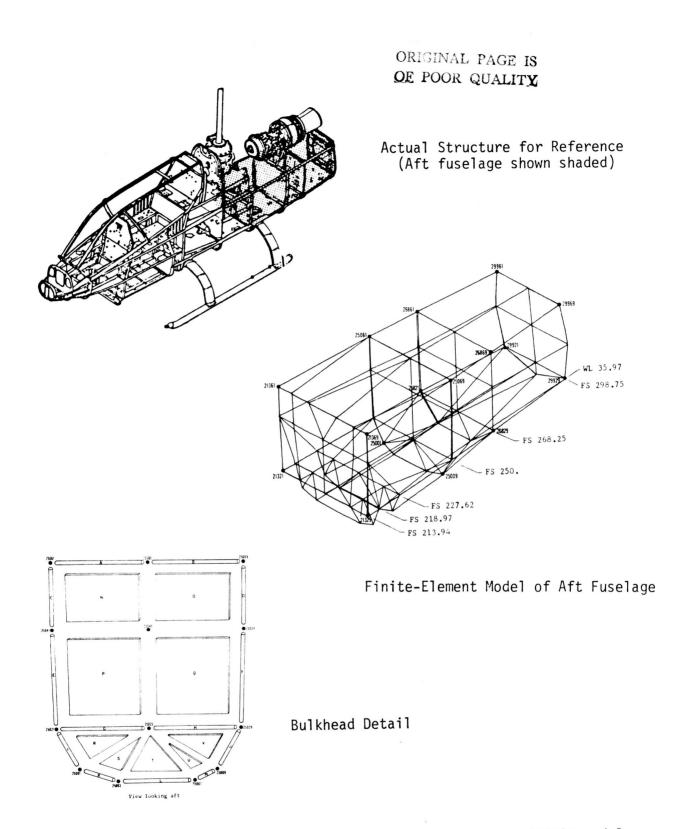
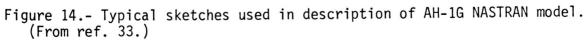
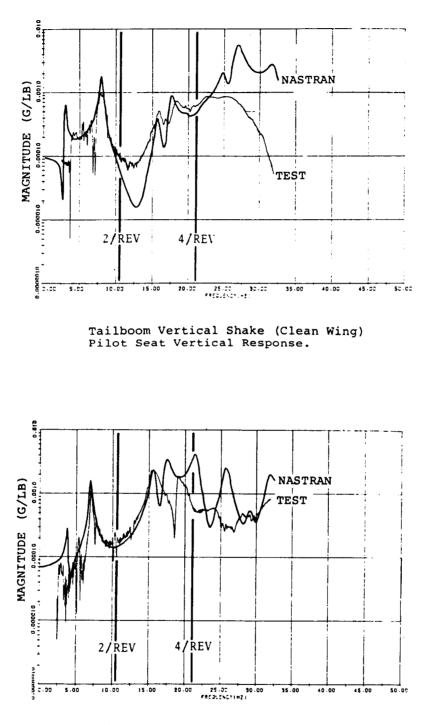


Figure 13.- AH-1G helicopter showing airframe structure with skins removed and NASTRAN finite-element model. (From ref. 33.)

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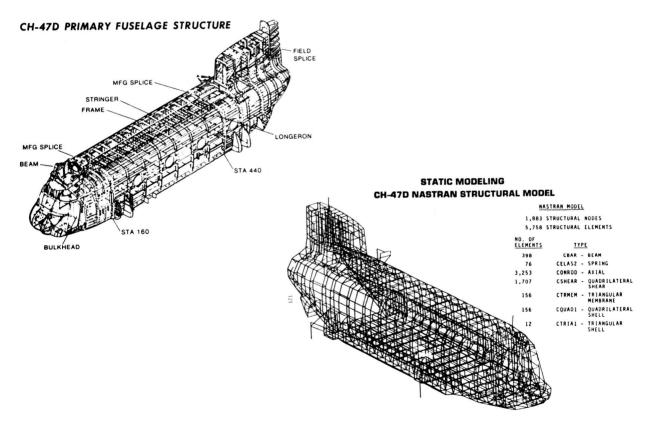


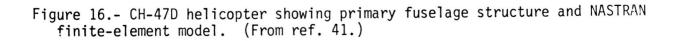


Tailboom Lateral Shake (Clean Wing) Pilot Seat Lateral Response.

Figure 15.- Comparisons of frequency response results for AH-1G. (From ref. 34.)

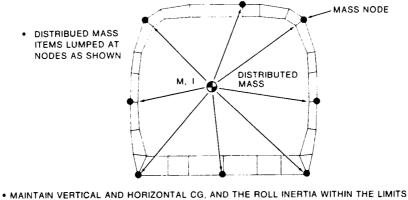






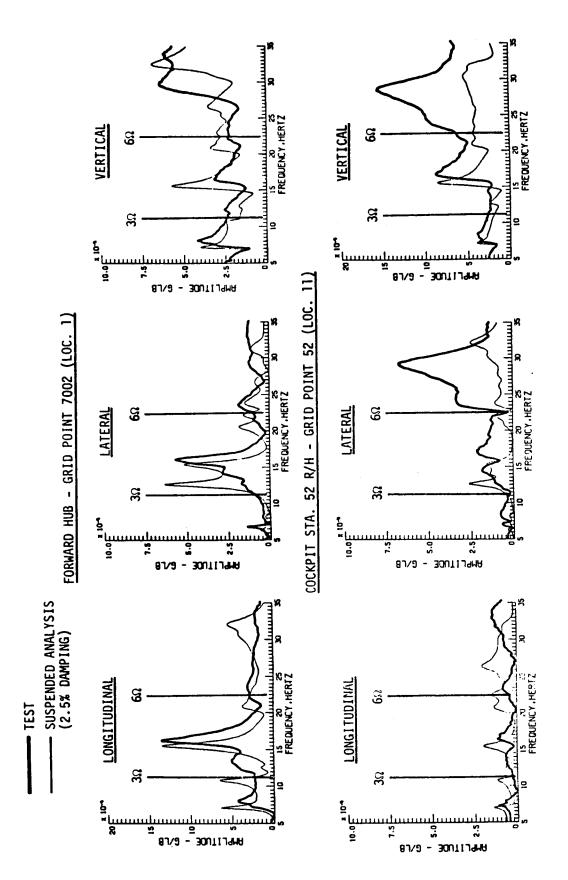
#### **STATIC MODELING GUIDES — FRAMES** FRAME CAPS FRAME WEB CSHEAR CONROD TYPICAL FWD HOLE CONROD TYPE OF STRUCTURAL ELEMENT TYPE SECTION A-A CAP/STIFFENER AXIAL CONROD WEBS SHEAR CSHEAR

# MASS MODELING GUIDES — FRAME STATION MASS TREATMENT



OF THE NASTRAN MODEL FOR DISTRIBUTED MASS ITEMS

Figure 17.- CH-47D static and mass modeling guides for a typical frame. (From ref. 41.)





### SUGGESTED IMPROVEMENTS EFFECT OF VARIATION IN MODAL DAMPING

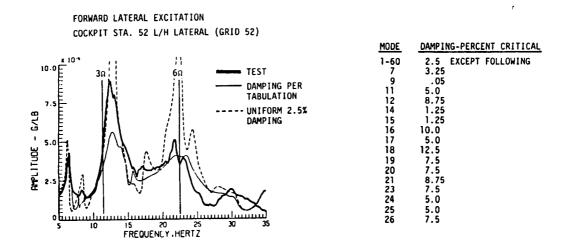


Figure 19.- Effect of variation in modal damping used in analysis on correlation for CH-47D. (From ref. 42.)



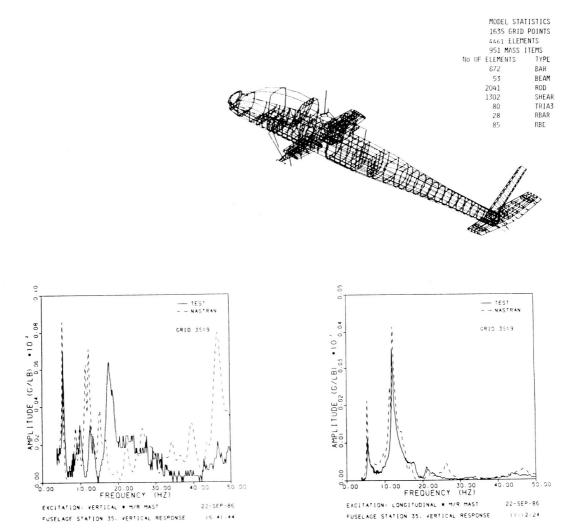


Figure 20.- AH-64A helicopter, NASTRAN finite-element model and typical frequency response comparisons. (From draft final report submitted under DAMVIBS program.)



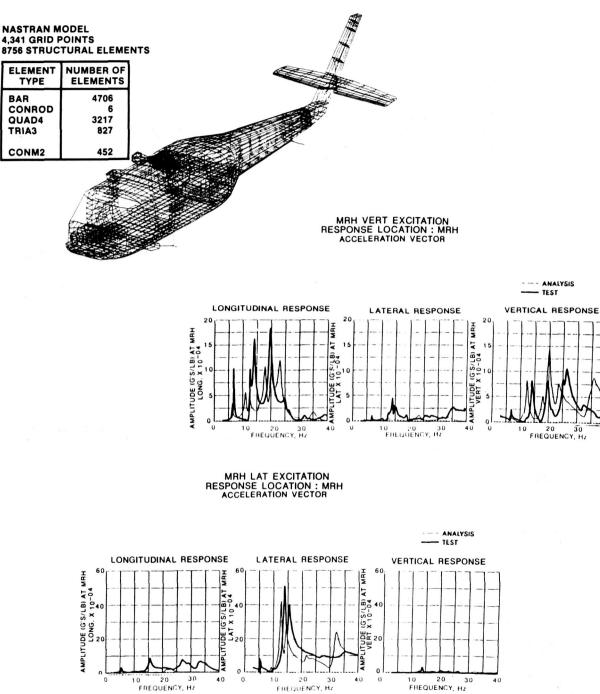


Figure 21.- UH-60A helicopter, NASTRAN finite-element model and typical frequency response comparisons. (From draft final report submitted under DAMVIBS program.)

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FREQUENCY RESPONSE COMPARISONS EXAMPLE

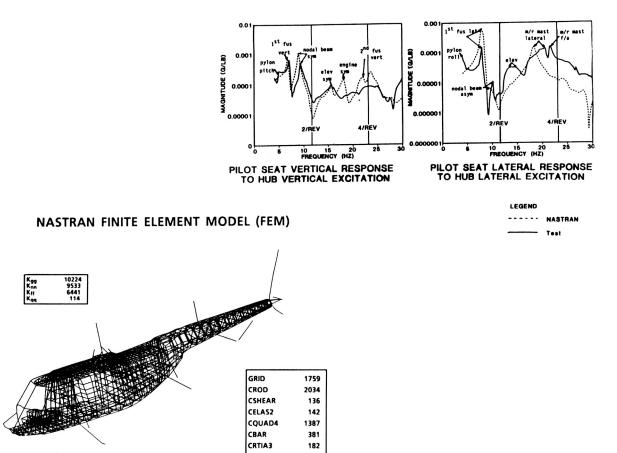
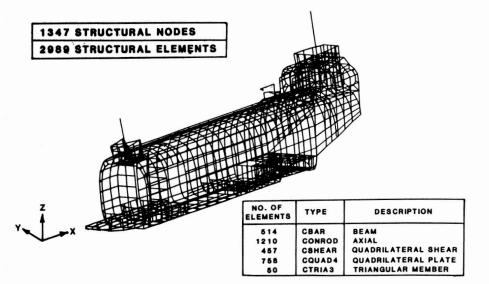


Figure 22.- D292 (ACAP) helicopter, NASTRAN finite-element model and typical frequency response comparisons. (From draft final report submitted under DAMVIBS program.)



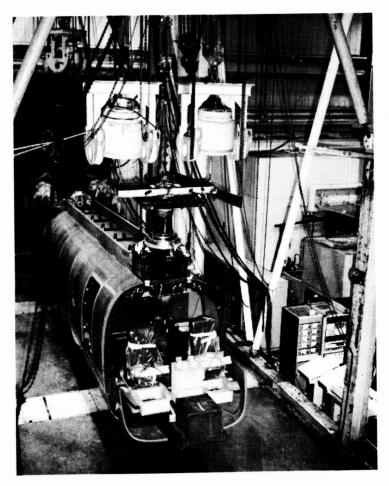
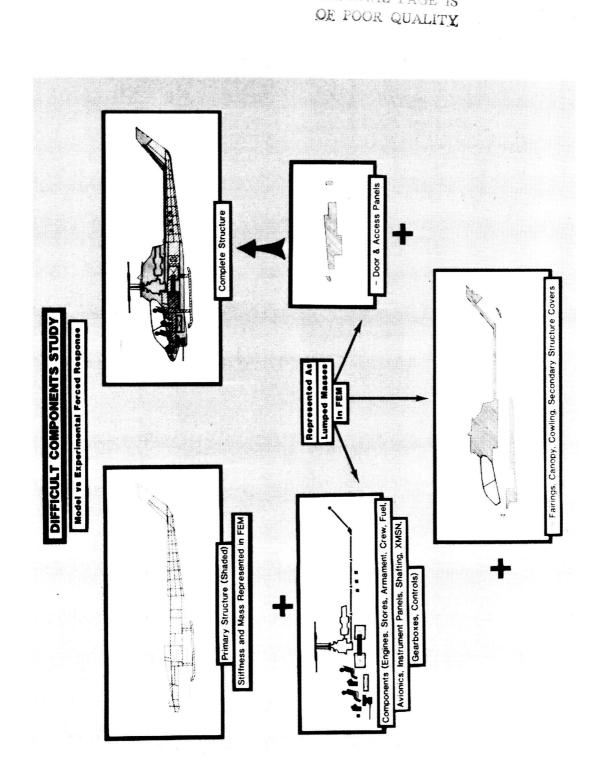


Figure 23.- NASTRAN finite-element model of Model 360 composite airframe and airframe during ground vibration test. (From draft final report submitted under DAMVIBS program.)



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Figure 24.- Difficult components study being conducted on the AH-1G under DAMVIBS program.

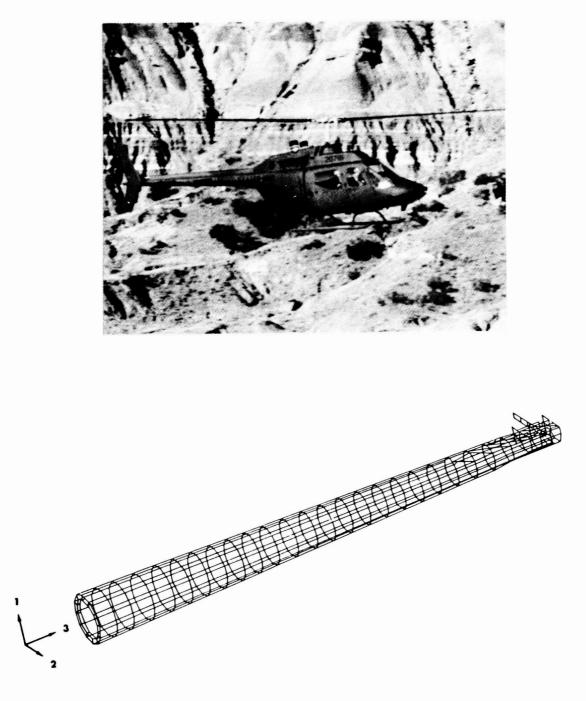
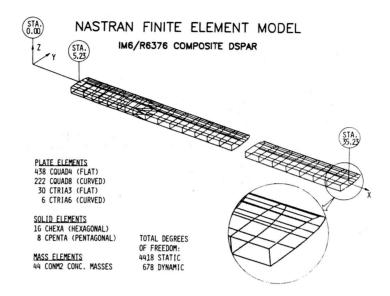


Figure 25.- OH-58A helicopter and EAL finite-element model of composite tail boom. (From ref. 47)



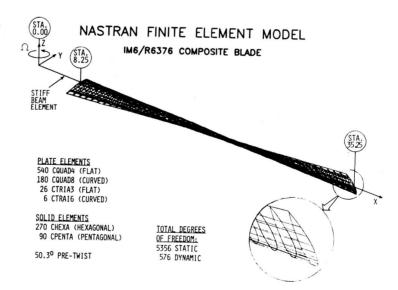
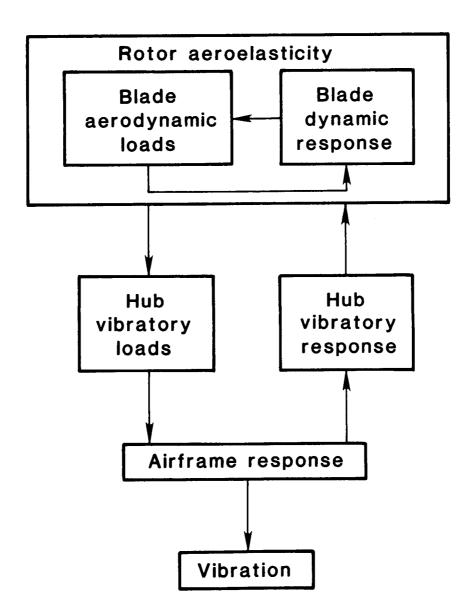
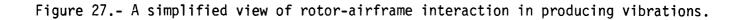


Figure 26.- NASTRAN finite-element models of composite model rotor blades.





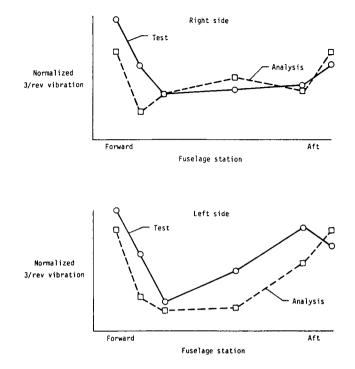


Figure 28.- Correlation of flight test data with a coupled rotor-airframe analysis for 3/rev vibration of a three-bladed tandem-rotor helicopter in high-speed level flight. (From ref. 60.)

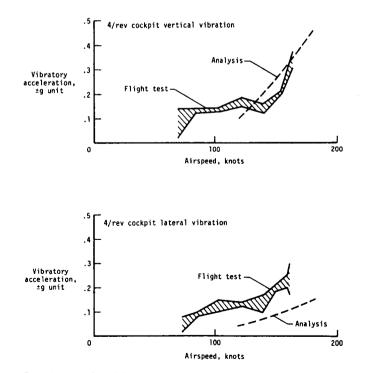
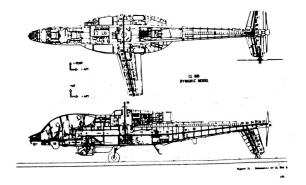


Figure 29.- Correlation of flight test data with an uncoupled rotor-airframe analysis for 4/rev vibrations of a four-bladed tandem-rotor helicopter. (From ref. 61.)





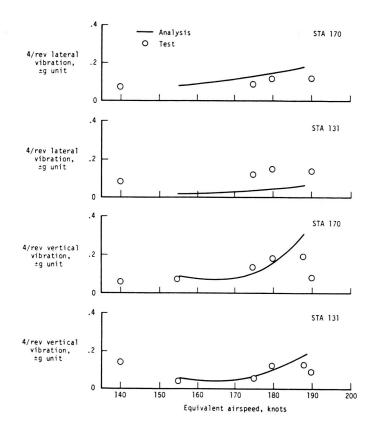


Figure 30.- Comparison of computed coupled rotor-airframe vibrations with measured data for a compound helicopter with four-bladed hingeless rotor. (From ref. 62.)

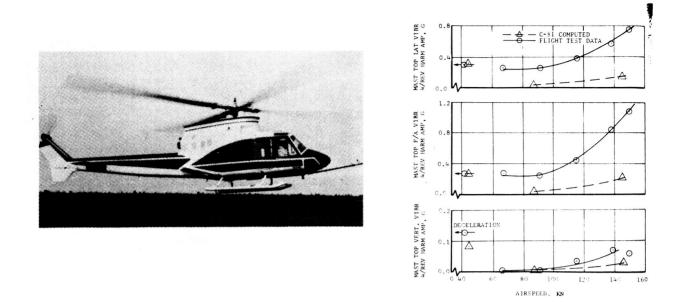


Figure 31.- Comparison of computed and measured hub vibrations for a helicopter with a four-bladed hingeless rotor. (From ref. 63.)

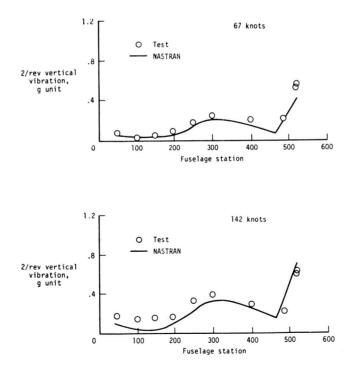


Figure 32.- Comparison of measured 2/rev vertical airframe vibrations with C-81 analysis using measured hub accelerations for a two-bladed teetering rotor helicopter. (From ref. 37.)

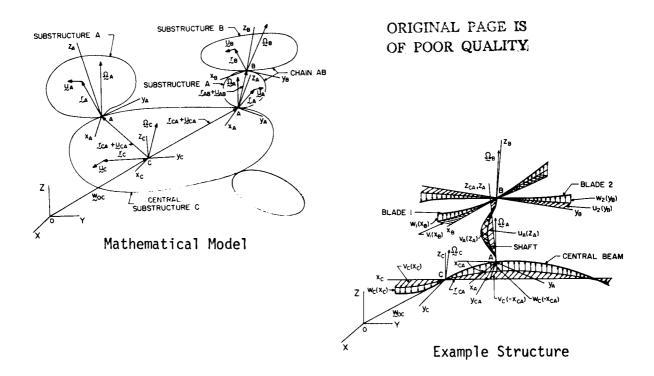


Figure 33.- A formulation for dynamic synthesis of gyroscopic structures consisting of point-connected substructures. (From ref. 75.)

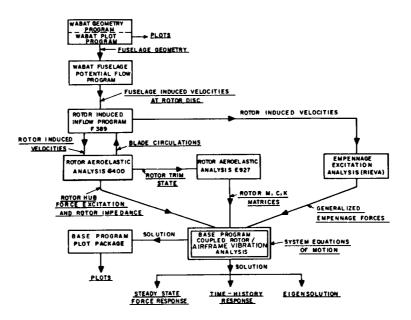


Figure 34.- Arrangement of program blocks in SIMVIB coupled rotor-airframe vibration analysis program. (From ref. 82.)

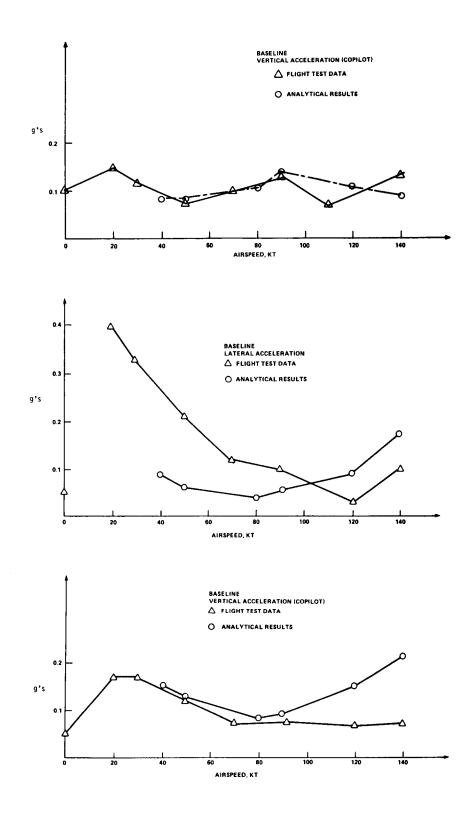
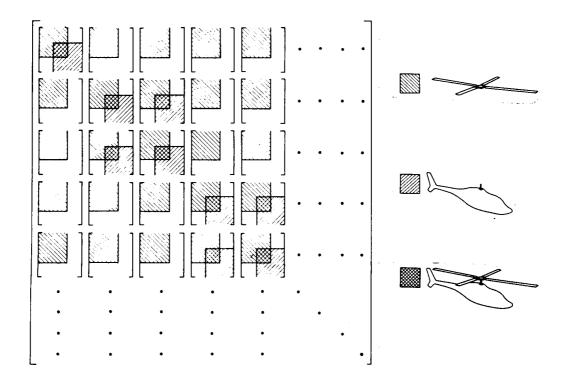
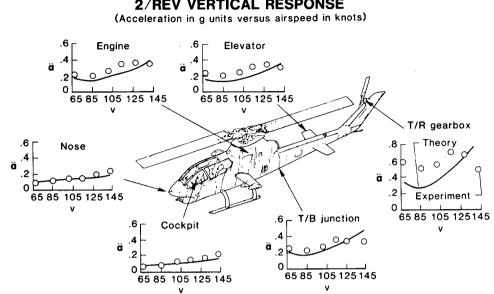


Figure 35.- Comparison of measured 4/rev vibrations for SH-60B with SIMVIB analysis using hub loads measured in flight. (From ref. 83.)



$$= (UOK)[KROC] + \frac{1}{2}(ULK)[KRLP] + \frac{1}{2}(UHK)[KRHP]$$
$$+ X_{3}\Omega((UOC)[CROC] + \frac{1}{2}(ULC)[CRLQ] + \frac{1}{2}(UHC)[CRHQ])$$
$$- (X_{3}\Omega)^{2}((UOM)[MROC] + \frac{1}{2}(ULM)[MRLP] + \frac{1}{2}(UHM)[MRHP])$$

Figure 36.- Illustrative example showing manner in which matrices and equations are depicted in a formulation of rotor-airframe coupling aimed at providing a basis for implementation of a design analysis capability for airframe vibrations. (Adapted from ref. 84.)



AH-1G FLIGHT VIBRATION ANALYSIS/EXPERIMENT CORRELATION 2/REV VERTICAL RESPONSE

AH-1G FLIGHT VIBRATION ANALYSIS/EXPERIMENT CORRELATION 4/REV VERTICAL RESPONSE

(Acceleration in g units versus airspeed in knots)

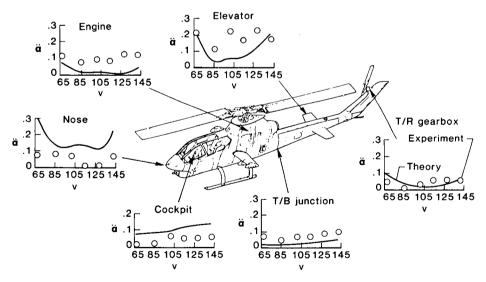


Figure 37.- Comparison of measured AH-1G 2/rev and 4/rev vertical vibrations with results of C-81 analysis. (Based on results presented in ref. 88)

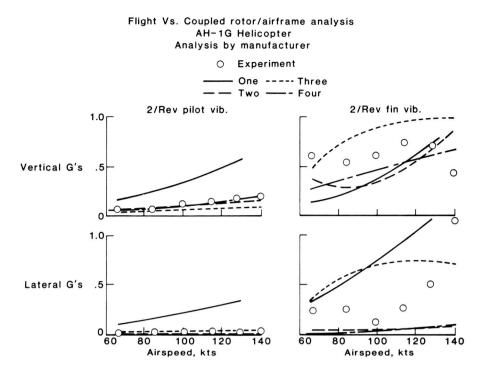


Figure 38.- Typical comparisons of measured AH-1G 2/rev vibrations with analysis by each of four manufacturers. (From results presented in draft final reports submitted under DAMVIBS program.)



Figure 39.- OH-6A helicopter used in NASA/Army higher harmonic control flight test program.

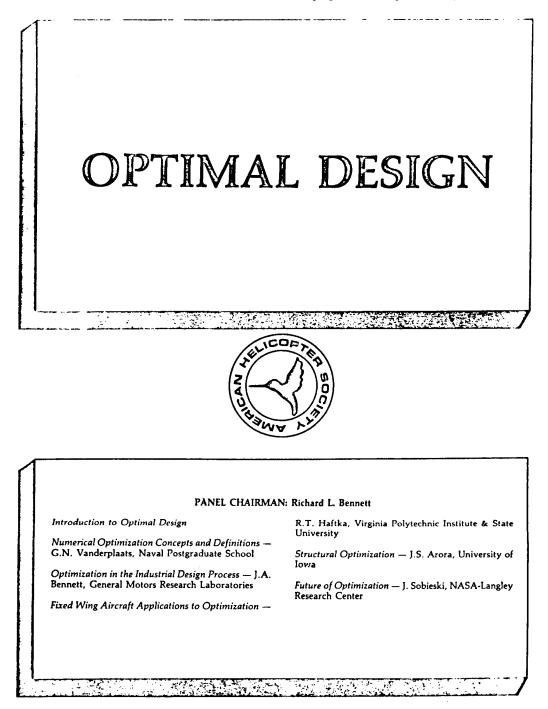


Figure 40.- Panel on Optimal Design held at 39th Annual National Forum of the American Helicopter Society. (From ref. 94.)

#### <u>Part 2</u>

| SESSION 5A: ROTORCRAFT APPLICATIONS<br>Co-Chairmen: C. E. Hammond and R. J. Huston        |
|---|
| OVERVIEW: APPLICATIONS OF NUMERICAL OPTIMIZATION METHODS TO<br>HELICOPTER DESIGN PROBLEMS |
| APPLICATION OF MODERN STRUCURAL OPTIMIZATION TO VIBRATION REDUCTION<br>IN ROTORCRAFT      |
| HELICOPTER ROTOR BLADE AERODYNAMIC OPTIMIZATION BY MATHEMATICAL<br>PROGRAMING             |
| REGRESSION ANALYSIS AS A DESIGN OPTIMIZATION TOOL   |

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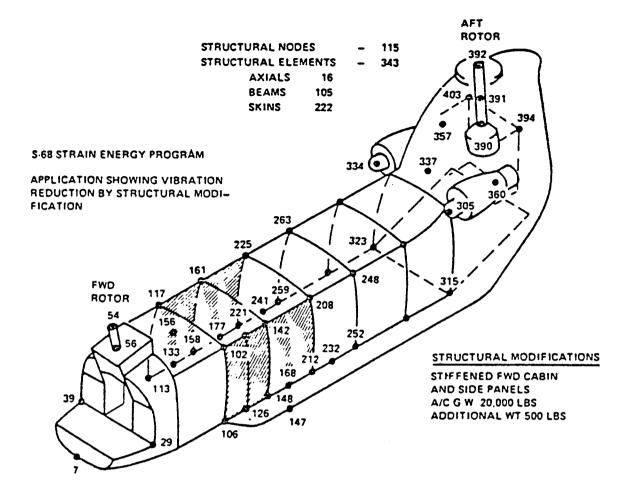
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| A ROTOR OPTIMIZATION USING REGRESSION ANALYSIS   | 590 |
| OPTIMIZATION OF HELICOPTER ROTOR BLADE DESIGN FOR MINIMUM<br>VIBRATION                                       | 509 |
| APPLICATION OF NUMERICAL OPTIMIZATION TO ROTOR AERODYNAMIC DESIGN 6<br>W. A. Pleasants III and T. J. Wiggins | 527 |
| AEROELASTIC/AERODYNAMIC OPTIMIZATION OF HIGH SPEED HELICOPTER/<br>COMPOUND ROTOR                             | 543 |
| THE STRUCTURAL OPTIMIZATION OF A SPREADER BAR FOR TWIN LIFT<br>HELICOPTER OPERATIONS                         | 63  |
| MULTIOBJECTIVE OPTIMIZATION TECHNIQUES FOR STRUCTURAL DESIGN 6<br>S. S. Rao                                  | i75 |

Figure 41.- Rotorcraft session at Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization. (From ref. 97.)



| Mode  | Original<br>Fuselage<br>Hz  | Modified<br>Fuselage<br>Ez   | Excitation Frequency<br>11.45 Hz   |
|---|---|--|--|
| 7<br>8<br>9<br>10<br>11<br>12<br>13<br>14<br>15 | 6.687<br>7.879<br>8.442<br>12.03<br>12.52<br>14.48<br>15.69<br>16.95<br>18.94 | $\begin{array}{c} 6.703 \\ 8.160 \\ 8.494 \\ 12.74 \\ 12.79 \\ 14.82 \\ 16.02 \\ 17.22 \\ 19.06 \end{array}$ | Natural Frequency<br>Changed (Mode No. 10,<br>4th Elastic Mode)-<br>Mode Amplification<br>Factor Down From<br>9.1 to 5.0 |

Figure 42.- CH-47A vibration reduction by structural modification based on modal strain energy. (From ref. 102.)

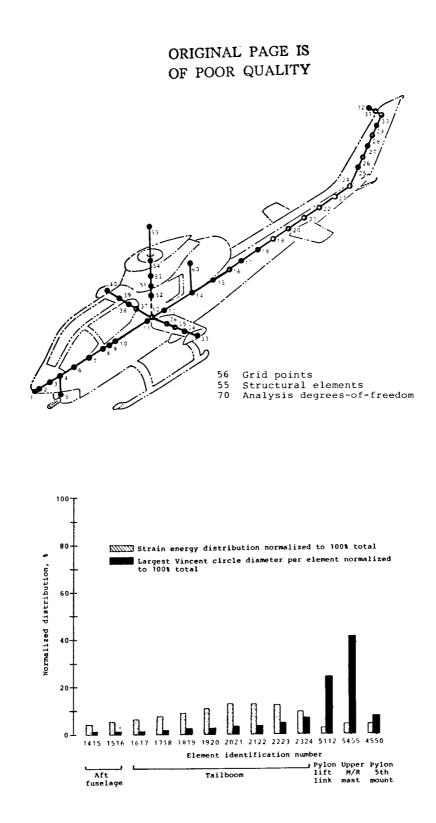
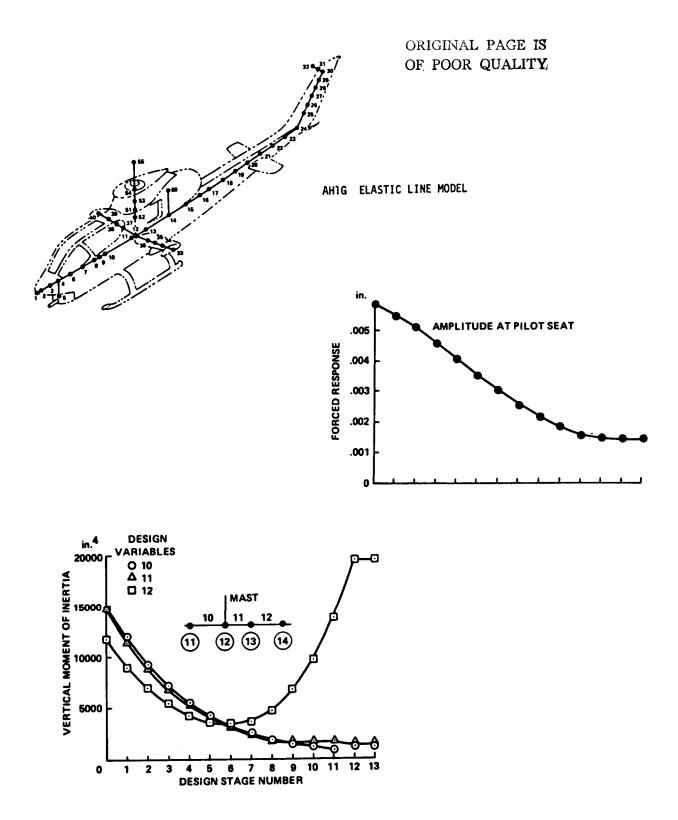
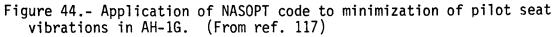
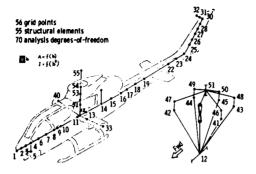


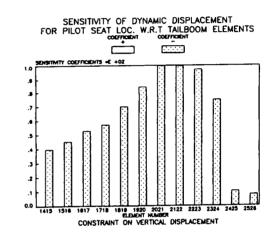
Figure 43.- Comparison of forced response strain energy and Vincent Circle methods as indicators of elements having the most potential for reducing vibrations. (From ref. 105.)











ELEMENT STRAIN ENERGY DENSITIES IN TAILBOOM FOR FORCED RESPONSE

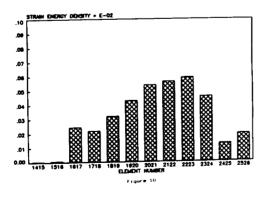


Figure 45.- Distribution of sensitivity of dynamic displacement and strain energy in tail-boom elements of AH-1G. (From ref. 122.)

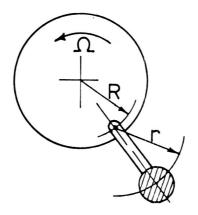
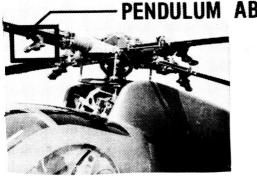


Figure 46.- Simple Pendulum Absorber.



PENDULUM ABSORBER

EFFECT OF PENDULUM FREQUENCY 10 r NO PENDULUM 1 ABSORBER RELATIVE VERTICAL HUB FORCE 3.87 PER REV .1 .01 4.3 3.9 4.0 4.1 4.2 3.8 FREQUENCY

Figure 47.- Response of Blade Mounted Pendulum Absorber. (From ref. 126.)

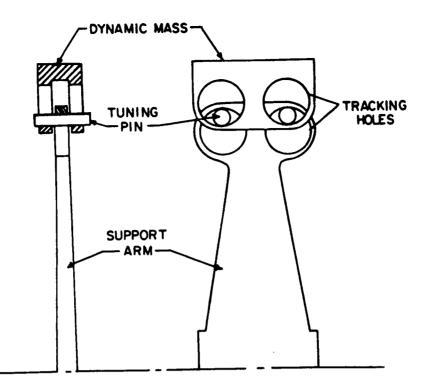


Figure 48.- Schematic of Bifilar Assembly. (From ref. 127.)

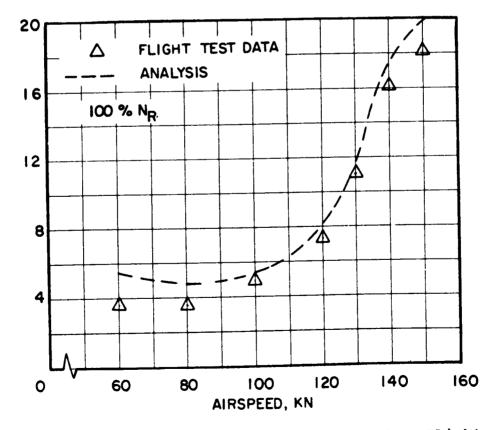


Figure 49.- Correlation of UH-60 Bifilar Absorber Flight Test Data and Analysis. (From ref. 128.)

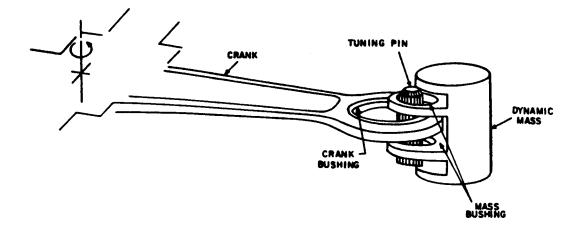
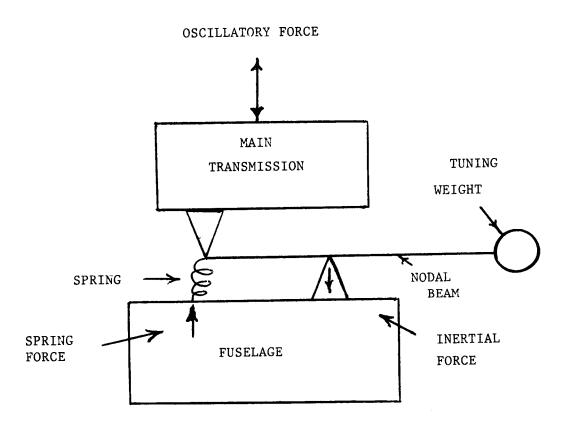
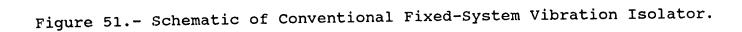


Figure 50.- Monofilar Schematic. (From ref. 129.)





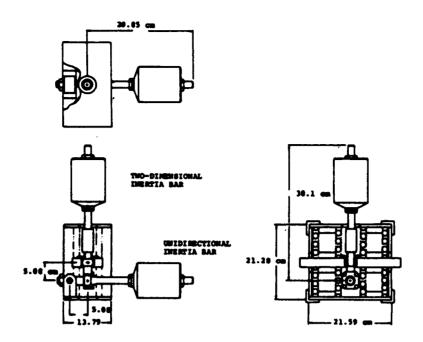


Figure 52.- Three-Dimensional DAVI. (From ref. 131.)

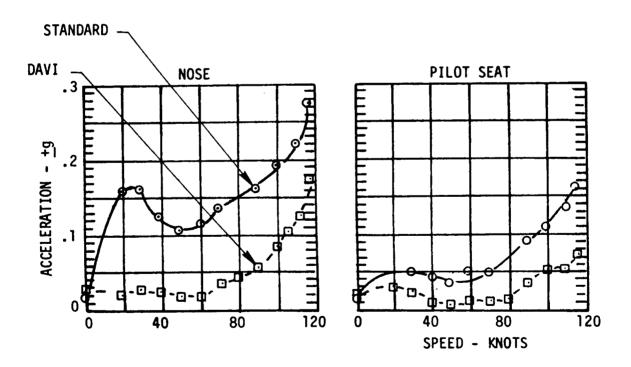


Figure 53.- DAVI Flight Test Data. (From ref. 132.)

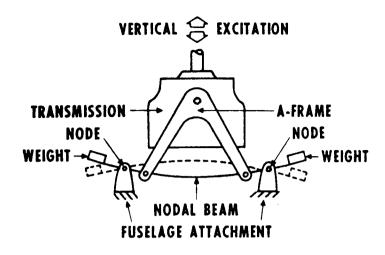


Figure 54.- Isolation by "Nodalized" Beam. (From ref. 133.)

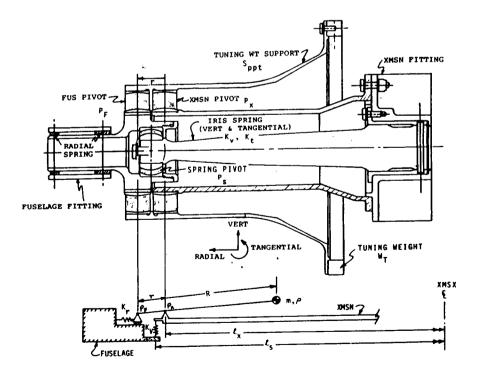


Figure 55.- Bidirectional IRIS Unit. (From ref. 136.)

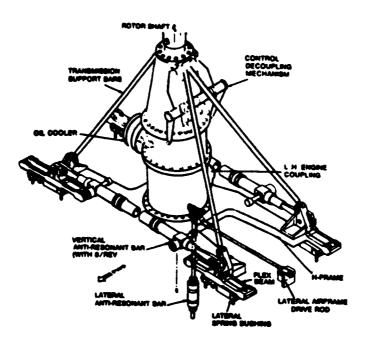


Figure 56.- BO-105 IRIS Installation. (From ref. 137.)

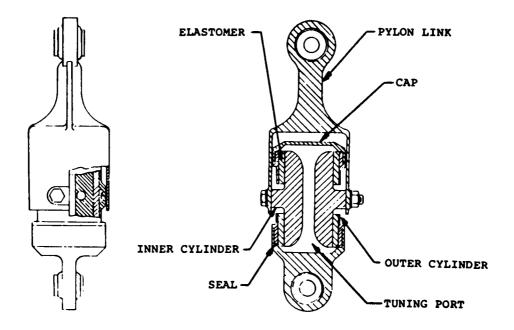
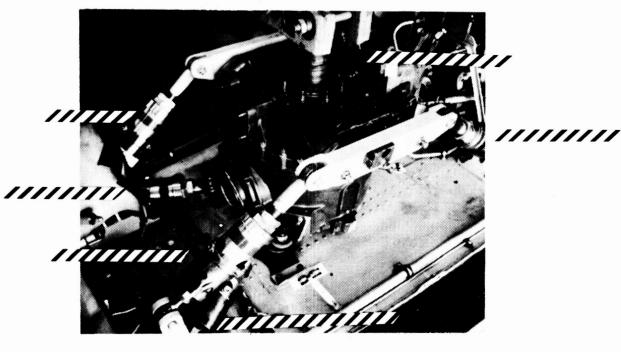


Figure 57.- Pinned-Pinned LIVE Unit. (From ref. 139.)

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////// Six LIVE Units

Figure 58.- Total Rotor Isolation System Aircraft and LIVE Installation.

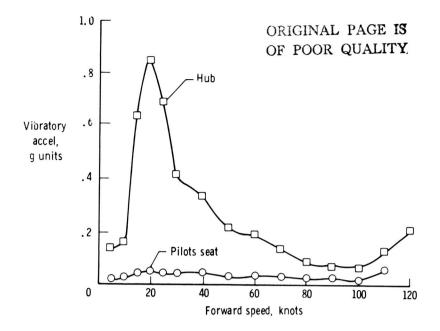


Figure 59.- TRIS Flight Test Data. (From ref. 141.)



Figure 60. - Rotor Systems Research Aircraft - RSRA.

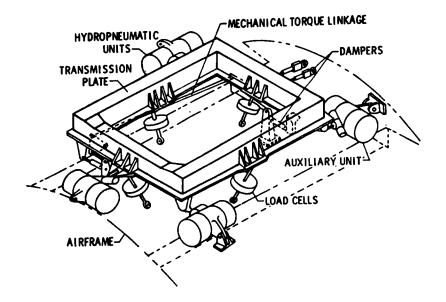


Figure 61.- RSRA Aircraft AIBS Schematic. (From ref. 142.)

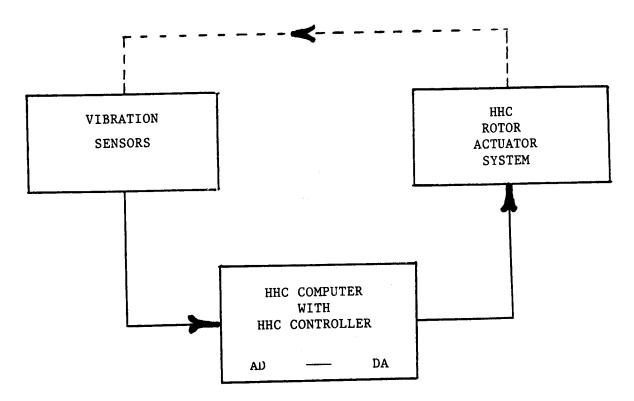


Figure 62.- HHC Feedback System.

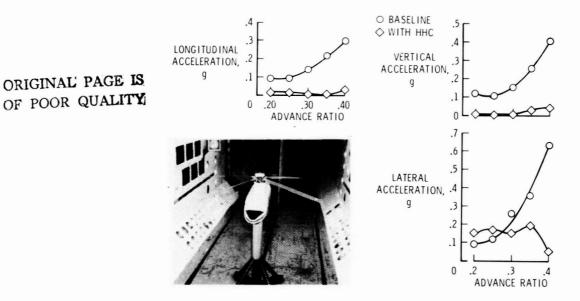


Figure 63.- Higher Harmonic Control Closed-Loop Wind Tunnel Test Results. (From ref. 150.)



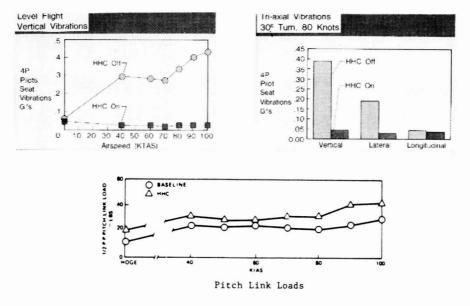
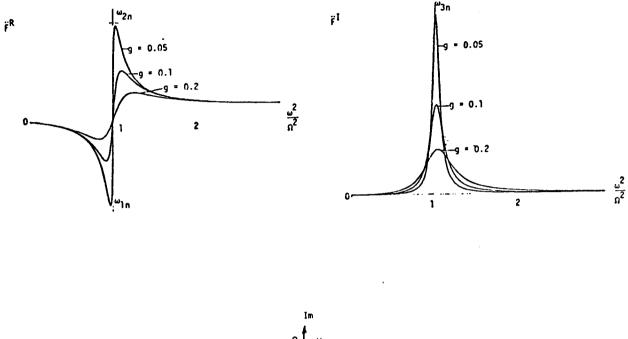


Figure 64.- OH-6A HHC Flight Test Results.



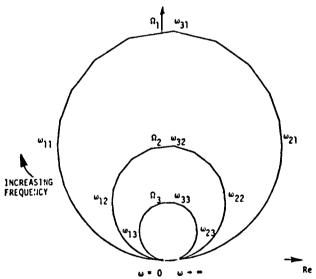
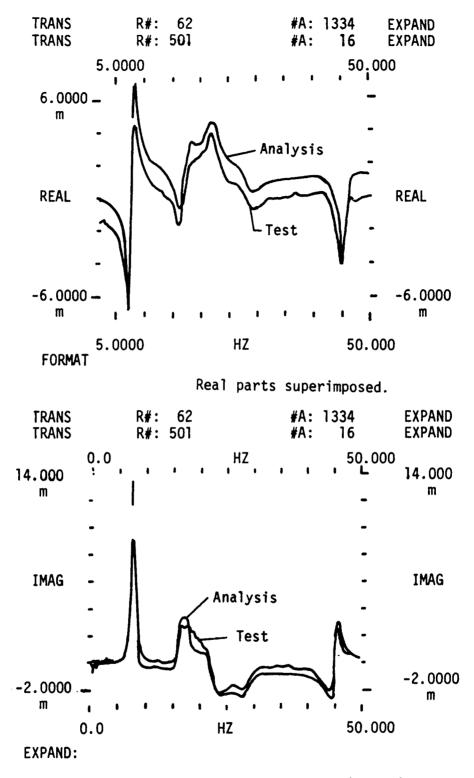


Figure 65.- Typical frequency domain representations of dynamic response. (From ref. 168.)

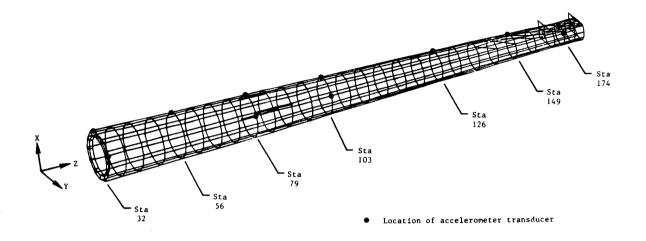


Imaginary parts superimposed.

Figure 66.- AH-1G modal analysis correlation. (From ref. 168.)

L

L



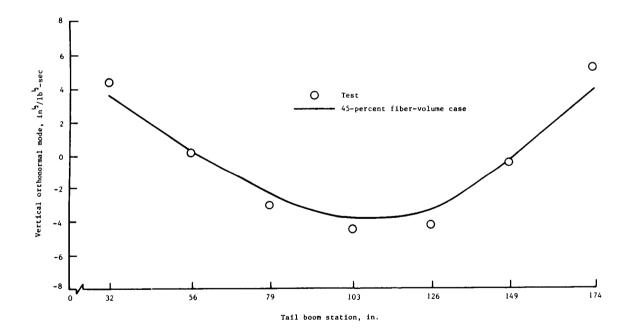


Figure 67.- OH-58 composite tail boom mode shape correlation. (From ref. 48.)

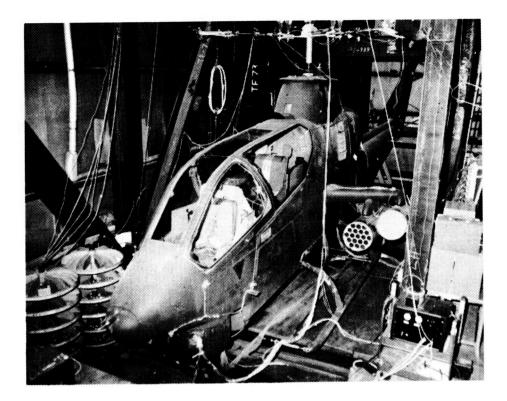


Figure 68.- AH-1G shake test configuration.

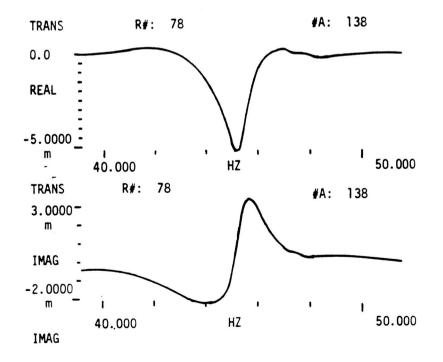


Figure 69.- Frequency response measurement of complex mode. (From ref. 168.)

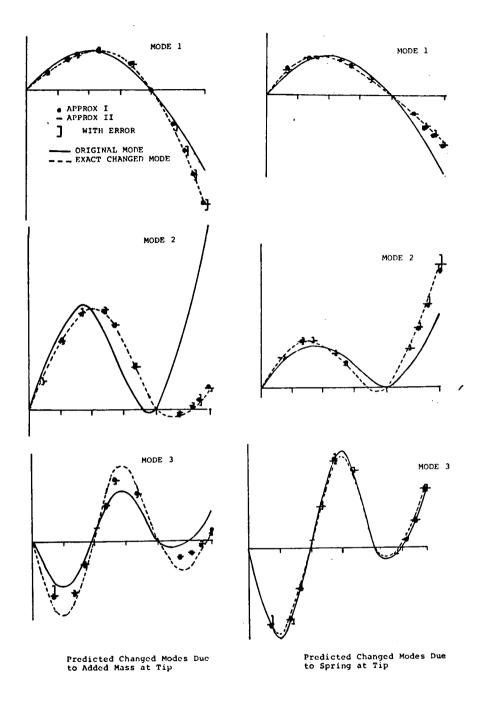


Figure 70.- Application of incomplete model theory. (From ref. 178.)

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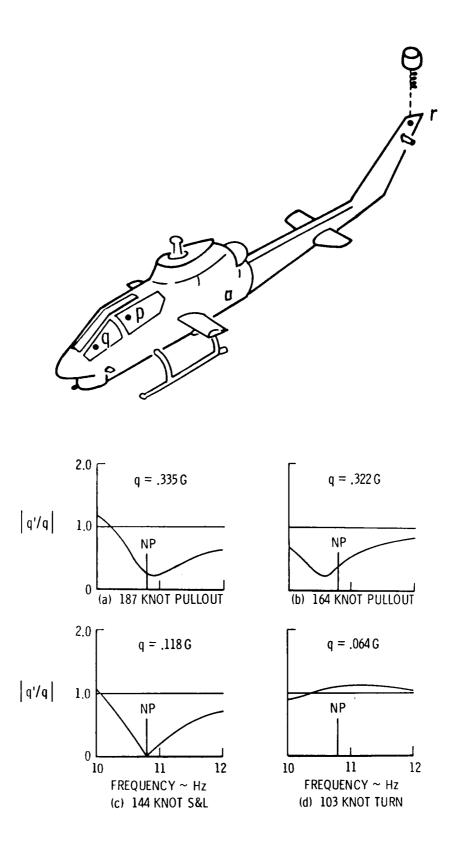
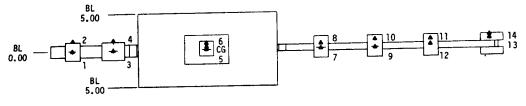
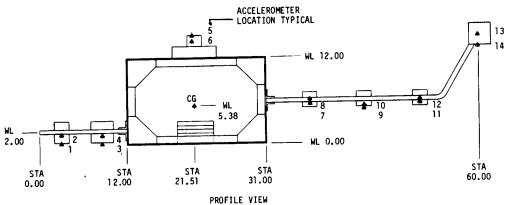


Figure 71.- Simulated effect of vertical fin absorber on cockpit vibration. (From ref. 198)







Schematic of Dynamic Test Model

#### DYNAMIC MODEL VIBRATIONS DUE TO LATERAL NOSE REMOTE ABSORBER

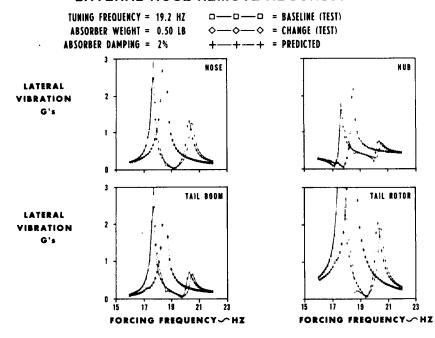
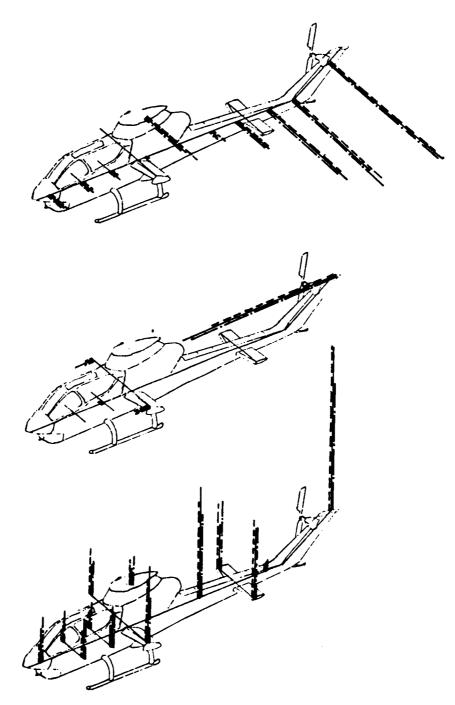
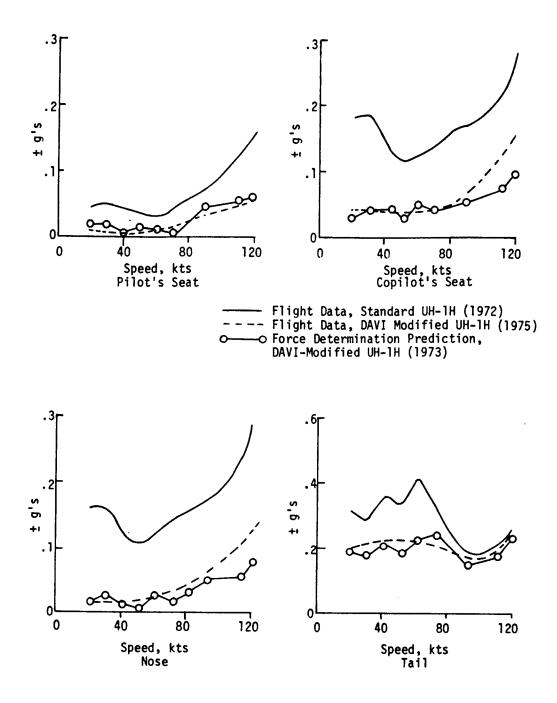


Figure 72.- Analytical Testing verification on generic helicopter dynamic model.



Left rolling pullout at a gross weight of 8465 pounds.

Figure 73. - AH-1G Force Determination results. (From ref. 205.)



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Two-Per-Rev Vertical Vibration on the UH-1H

