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A Performance Index Approach to Aerodynamic Design With the Use of Analysis Codes Only

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Summary

A method is described for designing an aerodynamic configuration for a specified performance vector, based on results from several similar but not identical trial configurations, each defined by a geometry parameter vector. The theory indicates that the method is effective provided that (1) the results for the trial configurations provide sufficient variation so that a linear combination of them approximates the specified performance and (2) the differences between the performance vectors (including the specified performance) are sufficiently small that the linearity assumption of sensitivity analysis applies to the differences. A computed example describes the design of a high supersonic Mach number missile wing-body configuration based on results from a set of four trial configurations.

Introduction

Following the discussion in references 1 and 2 of the problems inherent in most design-by-optimization procedures, the method described in reference 3 was developed to avoid most of these problems. The method of reference 3 was developed specifically for application to problems involving large (in running time and/or storage) analysis codes so that a very limited number of analysis runs are practical. It attempts to take maximum advantage of the designer's experience and intuition. Although the problems discussed relate to aerodynamic design, the method is applicable in a more general context.

The method described in reference 3 is specifically applicable to problems in which a shape function is designed to obtain a specified pressure distribution corresponding to that shape. For many aerodynamic problems, especially for synthesized configurations, the shape functions are not specified directly but by a set of shape geometry parameters. The desired performance may not be specified by the pressure distribution as such, but by integrated quantities such as lift, thrust, moments. Furthermore, it is important not to generate a sensitive "operating-point" design or to generate a design based on optimizing a single payoff quantity, such as lift-drag ratio for a single flight condition without consideration of off-design conditions. It is preferable to specify a reasonable goal in terms of performance parameters designated over some range of flight conditions (i.e., a performance index). This type of design problem is treated in the present analysis.

The method to be described does not generate an optimum configuration in an absolute mathematical sense. However, when the basic assumptions of the theory are met, it makes optimal (in a true mathematical sense) use of the available computed results to generate an improved design. It does not require the expensive calculation of sensitivities for the variation of each individual geometric shape parameter. Finally, it avoids the pitfalls of single-point-design configurations by specifying the goal to be obtained in terms of a performance index, or vector, rather than by a single performance parameter.

Symbols

$a_i, \tilde{a}_i$	coefficients
b_i	coefficients determined by equation (10)
c_i	coefficients or parameters
C_D	drag coefficient based on inviscid drag only
C_L	lift coefficient
$\boldsymbol{d}$	vector denoting difference between two performance parameter vectors
$\hat{e}_i$	set of orthonormal vectors
$\boldsymbol{g}$	vector whose components are an ordered set of parameters prescribing configuration geometry
H_i, h_i	coefficients of polynomial terms
L/D	$= C_L/C_D$
l_i	lengths specified in inlet example (fig. 1)
m	number of vectors
M	free-stream Mach number
n	number of components in each vector
$\boldsymbol{p}$	performance vector
P	predicted (in figures)
S	specified (in figures)
t	normalized geometry variable
w	weight factor
x, y, z	Cartesian coordinates
α	angle of attack
$\lambda, \tilde{\lambda}$	functions of x determining size variation
σ	function of x determining shape variation

Subscripts:

d desired or design

i, j, k indices

Analysis

The term configuration is used in this report to refer not only to a complete vehicle but also to any component whose performance is calculated from its geometry and the existing flight or flow conditions. It is assumed that small changes in the configuration geometry result in small changes in performance. This is the basic assumption of sensitivity analysis. It is noteworthy that this assumption holds even in the presence of shock waves if the shock locations and strengths change slightly with small geometry changes.

The method can be described as a step-by-step procedure. The basic idea of each of the six steps is as follows:

1. Compute the performance quantities for several configurations, each defined by assigning values to a set of geometry parameters. Compute weights for the performance quantities to render them commensurable (comparable in magnitude). The resulting performance vectors for these base configurations are termed the "basic vectors."
2. Establish a desired performance vector as a goal.
3. Approximate the desired performance vector formally as a linear combination of the basic performance vectors. Express this relationship in terms of incremental vectors.
4. Determine the coefficients in this relationship so that the approximation is optimal.
5. Use these coefficients to determine a set of geometry parameters representing the optimal combination of the original configurations in accordance with the local linearity assumption of sensitivity analysis. Then analyze this synthesized configuration.
6. Compare the results of this analysis with the desired results. Then, if necessary, add more configurations to the base, and repeat the procedure.

These steps are now discussed in detail.

Step 1: The first step in the procedure is to analyze several practical, and hopefully good, configurations which possess a degree of geometric similarity. For example, it is not feasible to compare a forward-swept-wing fighter with several swept-back configurations with large sweep angles. Each configuration must satisfy all the required geometric constraints.

The geometry for each configuration is defined by a set of shape parameters which is ordered to form a geometry vector $\boldsymbol{g}$. The corresponding performance parameters (lift, drag, moments, etc., at various operating conditions) are ordered into an array $\boldsymbol{p}$, which is denoted the performance vector.

There are two aspects to weighting performance parameters: importance and magnitude. The possibility of weighting the performance quantities so as to deliberately bias the result in accordance with the relative importance of the various performance criteria is a highly problem-dependent consideration and is to a certain extent subjective, and therefore is not considered here. However, the second aspect of weighting must be considered in multiobjective optimization problems since the various performance parameters are, in general, not commensurable (or comparable); that is, they may be of different orders of magnitude or of different dimension. Thus, the smaller quantities have less influence in the optimization procedure, and the final design is biased toward the larger performance criteria, even though the smaller quantities are of equal importance. An obvious way to deal with this problem is to weight the various components of the performance vector. The weights can be determined as follows: (1) compute the root-mean-square (rms) value of the ith components for all configurations, (2) form an average vector from these rms values, (3) normalize this vector, and (4) take the reciprocals of the ith component as the corresponding weight to be applied to the ith component of each performance vector. Thus, for example, if there are m vectors each with n components, the weight factor for the kth component may be computed by rms averaging. Denoting the ith component of the jth vector by $\boldsymbol{p}_{ij}$,

$$w_k = \frac{\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} \boldsymbol{p}_{ij}^2}{n \sum\limits_{j=1}^{m} \boldsymbol{p}_{kj}^2} \tag{1}$$

Henceforth, it will be assumed that the performance vector $\boldsymbol{p}$ consists of weighted components.

Step 2: The next step in the procedure is to establish a goal in the form of a desired performance vector which represents an improvement over the best of the configurations analyzed. If the configurations which have already been analyzed were actually "good" configurations, then a modest improvement over the best of these should represent a satisfactory goal. In this case, one proceeds to the next step in the procedure.

Suppose, however, that the actual required performance is well beyond that of any of the configurations analyzed. Then, one may proceed by one of several alternatives. If one can surmise from the results of the initial calculations the nature of the defect in the configurations analyzed, this knowledge may lead to the scrapping of the original type of configuration and starting anew with a different class of geometries.

If, on the other hand, a study of the results of the initial calculation indicates a way in which the basic geometry type can be modified to gain large steps toward the performance goal, then these improved versions can be added to the base of configurations to be analyzed.

Finally, if the calculations provide no obvious clue toward making a significant design improvement, one can work gradually toward the goal by first specifying a subgoal representing say 20 percent of the difference between the best of the configurations and the actual required goal. Then after taking this initial step, add more configurations to the base, specify the next subgoal, and continue in this manner until the required goal is achieved.

Step 3: The next step in the procedure is to approximate the desired performance vector by a linear combination of the performance vectors of the basic configurations

$$\boldsymbol{p}_d \approx \sum_1^m \tilde{a}_i \boldsymbol{p}_i \tag{2}$$

subject to the constraint

$$\sum_1^n \tilde{a}_i = 1 \tag{3}$$

where the coefficients are still to be determined. This constraint preserves the scaling; that is, it assures that the components of $\boldsymbol{p}_d$ will be close in magnitude to the corresponding components of $\boldsymbol{p}_i$, as is required for linearity in differences. The significance of this constraint in terms of the geometry parameters is discussed subsequently in step 5.

Equation (3) reduces the number of required coefficients in equation (2) by one, so that equation (2) can be written in terms of $n-1$ coefficients a_i by setting

$$\tilde{a}_n = 1 - \sum_1^{n-1} a_i \tag{4a}$$

$$\tilde{a}_i = a_i \quad (i = 1,\ 2,\ \ldots,\ n-1) \tag{4b}$$

Thus, equation (2) becomes

$$\boldsymbol{p}_d \approx \left(1 - \sum_1^{n-1} a_i\right) \boldsymbol{p}_n + \sum_1^{n-1} a_i \boldsymbol{p}_i \tag{5}$$

In this form, it is easily seen that equation (3) is satisfied. Equation (5) can also be written in the form

$$\boldsymbol{p}_d \approx \boldsymbol{p}_n + \sum_1^{n-1} a_i(\boldsymbol{p}_n - \boldsymbol{p}_i) \tag{6a}$$

or

$$\boldsymbol{p}_d - \boldsymbol{p}_n \approx \sum_1^{n-1} a_i(\boldsymbol{p}_n - \boldsymbol{p}_i) \tag{6b}$$

In this form, $\boldsymbol{p}_n$ is sometimes called a basis vector while the remaining vectors $\boldsymbol{p}_i$ $(i = 1,\ 2,\ \ldots,\ n-1)$ are called comparison vectors.

Equation (6b) is an equation for the approximate representation of the difference $\boldsymbol{p}_d - \boldsymbol{p}_n$ and can be written, with difference notation, as

$$\boldsymbol{d}_d = \sum_1^{n-1} a_i \boldsymbol{d}_i \tag{7}$$

Now since the input geometry parameter vectors $\boldsymbol{g}_i$ define configurations of the same basic type that each satisfies all the prescribed geometric constraints, then the differences $\boldsymbol{g}_n - \boldsymbol{g}_i$ $(i = 1,\ 2,\ \ldots,\ n-1)$ should be small (have small components). Furthermore, since each of the configurations analyzed is assumed to represent a good design, whose performance vector roughly approximates $\boldsymbol{p}_d$, then the difference vectors $\boldsymbol{d}_d$ and $\boldsymbol{d}_i$ $(i = 1,\ 2,\ \ldots,\ n-1)$ should be small. These are simply statements of some of the sensitivity analysis requirements.

Step 4: It yet remains to determine the coefficients a_i in equation (7). An optimal choice of these coefficients would provide the closest approximation (in the least-squares sense) to $\boldsymbol{d}_d$ that is possible with the $n-1$ vectors $\boldsymbol{d}_i$. If the vectors $\boldsymbol{d}_i$ were an orthonormal set, optimal coefficients would be obtained simply by projecting $\boldsymbol{d}_d$ onto each $\boldsymbol{d}_i$. In general, the vectors $\boldsymbol{d}_i$ are not orthonormal, but they can be orthogonalized by the Gram-Schmidt process and normalized. That is, a new set of $n-1$ vectors $\hat{e}_i$ can be defined as a linear combination of $\boldsymbol{d}_i$:

$$\hat{e}_i = \sum_{j=1}^{i} c_{ij} \boldsymbol{d}_j \tag{8}$$

such that the $\hat{e}_i$ vectors form an orthonormal set. The procedure for computing the coefficients c_{ij} in equation (8) is well-known but is given in the appendix in the present notation for the convenience of the reader. Now $\boldsymbol{d}_d$ can be optimally approximated by a linear combination of the orthonormal vectors

$$\boldsymbol{d}_d \approx \sum_1^{n-1} b_i \hat{e}_i \tag{9}$$

with the coefficients b_i determined by

$$b_i = \boldsymbol{d}_d \cdot \hat{e}_i \tag{10}$$

With the coefficients b_i determined in this manner, the right-hand side of equation (9) represents the projection of $\boldsymbol{d}_d$ onto the $n-1$ dimensional linear subspace determined by the vectors $\hat{e}_i$. This projection defines the closest point of the surface to $\boldsymbol{d}_i$. Since the same subspace is determined by the vectors $\boldsymbol{d}_i$, this closest point can also be represented as a linear combination of the vectors $\boldsymbol{d}_i$. This is accomplished by substituting from equation (8) into equation (9):

$$\begin{aligned}
\boldsymbol{d}_d &\approx \sum_1^{n-1} b_i \hat{e}_i \\
&= \sum_1^{n-1} b_i \sum_1^{i} c_{ik} \boldsymbol{d}_k \\
&= \sum_1^{n-1} b_i \sum_1^{n-1} c_{ik} \boldsymbol{d}_k \qquad (c_{ik} = 0 \text{ for } k > i) \\
&= \sum_{k=1}^{n-1} \left(\sum_{i=1}^{n-1} b_i c_{ik} \right) \boldsymbol{d}_k \qquad (c_{ik} = 0 \text{ for } k > i) \\
&= \sum_1^{n-1} a_k \boldsymbol{d}_k
\end{aligned} \tag{11}$$

with

$$a_k = \sum_{i=1}^{k} b_i c_{ik} \qquad (c_{ik} = 0 \text{ for } k > i) \tag{12}$$

These are the optimal coefficients to be used in equation (6b).

Step 5: Now, with these same coefficients we synthesize a configuration geometry from the geometry vectors $\boldsymbol{g}_i$:

$$\boldsymbol{g}_d = \boldsymbol{g}_n - \sum_1^{n-1} a_i (\boldsymbol{g}_n - \boldsymbol{g}_i) \tag{13}$$

With the original assumption that the differences and the sum of the differences are small, it follows from the linearity assumption of sensitivity analysis that the performance of this configuration satisfies equation (6a) within the accuracy of the local linearity assumption.

Equation (13) can also be written

$$\begin{aligned}
\boldsymbol{g}_d &= \left(1 - \sum_1^{n-1} a_i \right) \boldsymbol{g}_n + \sum_1^{n-1} a_i \boldsymbol{g}_i \\
&= \sum_1^{n} \tilde{a}_i \boldsymbol{g}_i
\end{aligned} \tag{14}$$

This configuration represents the best combination of the original configurations that satisfies the constraint of equation (3). At this point, it is appropriate to consider the significance of this constraint. It constrains the scale of the final design to be consistent with the scales of the original configurations. Since these configurations represent small variations of a particular configuration type, they must be nearly the same size also. The constraint assures that the final design will also be of this scale. This, in turn, assures that the quantity $\boldsymbol{g}_d - \boldsymbol{g}_n$ in equation (13) will be small, which is required for the validity of the linearity assumption.

For many problems, the necessity of this constraint is more specific. Consider, for example, the design of a two-dimensional supersonic inlet of the type shown in figure 1. The lower wall is the basic compression surface, whereas the upper wall is the canopy. For this type inlet, it has been specified that the lower surface shall consist of a double wedge compression ramp succeeded by a smooth polynomial fairing to the throat. The wedge-section lengths, l_1 and l_2, and the length of the curved fairing l_3 vary for the individual configurations but the overall length is constrained to be 10 units:

$$l_1 + l_2 + l_3 = 10 \tag{15a}$$

The canopy consists of a single wedge section of length l_5 with a smooth fairing of length l_6 to the throat, with the constant

$$l_5 + l_6 \leq 6 \tag{15b}$$

Since each of the base configurations satisfies equation (15a), the final design also satisfies this constraint because of the condition specified by equation (3). However, even though each base configuration satisfies equation (15b), equation (3) does not prevent the final design from violating the inequality constraint, since some of the coefficients may be negative. Thus, in this method inequality constraints appear as soft constraints. The condition of equation (3) only assures that the violation, if it occurs, is small in magnitude. Such a result would indicate that a longer canopy is superior, and perhaps that a slight relaxing of the constraint would yield aerodynamic advantages that would more than offset the additional weight burden. If, however, it is not feasible to lengthen the canopy, in a future calculation all the base configurations should have canopies with the optimal length of 6.

The calculation of the coefficients $\tilde{a}_i$ requires perhaps 1 second on a modern computer, whereas the performance analysis calculations may require minutes and perhaps hours for complex configurations or computer analysis codes.

The right-hand side of equation (2) with the coefficients given by equations (4) and (12) is called the predicted performance. The left-hand side of equation (2) is the desired or specified performance. The actual design performance is obtained by performing the full analysis calculation for the optimum configuration, defined by equation (14). The extent to which this actual performance approximates $\boldsymbol{p}_d$ depends primarily on two factors. The first of these is the extent to which the original set of configurations provides the types of shapes that are effective in approximating the required result, that is, on the difference between the left- and right-hand sides of relation (2). If the right-hand side does not closely approximate $\boldsymbol{p}_d$, then additional configurations must be analyzed.

The second factor regards the accuracy of the local linearity assumption. This condition is violated if the differences between the data base configurations are too large, if the specified design performance is too far beyond the performances of the analyzed configurations, or if the problem is inherently nonlinear even for small variations.

Sample Problem Calculation

An example that illustrates the previously described procedure is to design a wing-body combination for a high supersonic Mach number rocket-powered missile. Four configurations were analyzed with geometries defined by the method of reference 4. In the equations which follow, the 11 components of the vectors $\boldsymbol{g}_i$ are denoted c_j ($j = 1,\ 2,\ \ldots,\ 11$)

$$\left.\begin{aligned} y(x,t) &= \lambda(x)t \\ z(x,t) &= \lambda(x)\{[1-\sigma(x)]z_1(t) \\ &\quad + \sigma(x)\, z_2(t)\} \end{aligned}\right\} \quad \begin{pmatrix} -1 & \leq & t & \leq & 1 \\ 0 & \leq & x & \leq & 1 \end{pmatrix}$$

where

$$z_1(t) = c_{11}\sqrt{1-t^2} \qquad (c_{11} < 1;\ \text{upper surface})$$

$$z_1(t) = -(1-c_{11})\sqrt{1-t^2} \qquad \text{(Lower surface)}$$

For $|t| \leq c_2 < 1$,

$$z_2(t) = \sqrt{c_2^2 - t^2} \qquad \text{(Upper surface)}$$

$$z_2(t) = -\sqrt{c_2^2 - t^2} \qquad \text{(Lower surface)}$$

For $c_2 < |t| \leq 1$,

$$z_2(t) = 0 \qquad \text{(Both surfaces)}$$

$$\sigma(x) = x^{c_1}$$

For $|t| \leq c_2$,

$$\lambda(x) = \tilde{\lambda}(x) = 0.075 \sum_{i=1}^{4} h_i x^i$$

where the polynomial coefficients h_i are determined by specifying the slope $\lambda' = \frac{d\lambda}{dx}$ at three locations:

$$\lambda' = \begin{cases} c_3 & (x = 0) \\ c_4 & (x = 1) \\ c_5 & (x = c_6) \end{cases}$$

For $1 \geq |t| > c_2$,

$$\lambda(x) = \tilde{\lambda}(x) + 0.1625 \left(\frac{|t| - c_2}{1 - c_2}\right) \sum_{1}^{4} H_i x^i$$

where the polynomial coefficients are determined as for h_i with c_3, c_4, c_5, and c_6 replaced by c_7, c_8, c_9, and c_{10}, respectively.

The four configurations are shown in figure 2. Each configuration was analyzed with the use of the computer code described in reference 5. The inviscid drag and L/D (based on inviscid drag only) were calculated for $\alpha = 1°$ and $3°$ at $M = 3$ and 4. These four values of L/D were taken to be the components of the performance vector. These vectors are shown in bar chart form in figure 3.

Several properties of the results are noteworthy. In terms of these performance criteria, configuration 4 is significantly inferior and configuration 2 is significantly superior to the other configurations. Although the difference between the performance of configurations 1 and 3 is small, the remaining differences are so large that the linearity assumption is not expected to be a good approximation.

Nevertheless, an improved performance vector was specified, and the above procedure was applied to design an improved configuration. Only a very slight improvement over the performance of configuration 2 was specified for two reasons. First, configuration 2 is so clearly superior to the others that it was suspected to be close to optimal, and it appeared that only a slight improvement could be expected. Second, the variation from configuration 2 was kept small to hold the analysis as near the linear range as possible. The specified values of L/D are indicated in figure 3 by an arrow and the letter S.

The value indicated by the letter P in figure 3 denotes the predicted performance of the improved design, that is, the best approximation to the specified performance vector that can be obtained as a linear configuration of the four configuration performance vectors, subject to the constraint of equation (3). Since only four components are specified and four configuration performance vectors are available, the predicted performance would be exactly the same as the specified performance if it were not for the constraint, which reduces the number of degrees of freedom by one.

The actual performance of the improved design indicated by the letter D is given in figure 3. If the calculations were perfectly linear, these values would coincide with the P (predicted) values. However, because of the large differences in the four configuration performance vectors, some nonlinear effects exist. Consequently, the actual performance differs somewhat from the predicted performance and is, in fact, a considerable improvement over the predicted performance. The improved design configuration is shown in figure 4. As expected it varies only slightly from configuration 2.

Curves of L/D versus α for the four base configurations and for the final design are shown in figure 5. The better configurations have curves that peak below $\alpha = 3°$. Since c_2 is a nearly linear function of α for $\alpha > 3°$, this shift in the curve maxima indicates a significantly lower inviscid drag (a flattening of the C_D versus α relation) in the range of $\alpha = 1°\text{–}2°$.

Concluding Remarks

A method has been described for designing a configuration for a specified performance vector, based on results from several similar but not identical trial configurations, each defined by a geometry parameter vector. The theory indicated that the method is effective provided that (1) the results for the trial configurations provide sufficient variation so that a linear combination of them approximates the specified performance and (2) the differences between the performance vectors (including the specified performance) are sufficiently small that the linearity assumption of sensitivity analysis applies to the differences. A computed example described the design of a high supersonic Mach number missile wing-body configuration based on results from a set of four trial configurations.

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Appendix

Gram-Schmidt Orthogonalization and Series Expansion of Difference Vectors

Denote the norm, or length, of the vector $\boldsymbol{d}$ by $|\boldsymbol{d}|$. Then, the orthonormal set $\hat{e}_i$ is defined recursively by the following procedure:

$$\hat{e}_1 = \frac{\boldsymbol{d}_1}{|\boldsymbol{d}_1|} \tag{A1}$$

$$\boldsymbol{e}_2 = \boldsymbol{d}_2 - (\boldsymbol{d}_2, \hat{e}_1)\,\hat{e}_1 \tag{A2a}$$

$$\hat{e}_2 = \frac{\boldsymbol{e}_2}{|\boldsymbol{e}_2|} \tag{A2b}$$

$$\boldsymbol{e}_i = \boldsymbol{d}_i - \sum_{j=1}^{i-1} (\boldsymbol{d}_i, \hat{e}_j)\,\hat{e}_j \tag{A3a}$$

$$\hat{e}_i = \frac{\boldsymbol{e}_i}{|\boldsymbol{e}_i|} \tag{A3b}$$

This procedure is sufficient to derive the $\hat{e}_i$ vectors from the $\boldsymbol{d}_i$ vectors. However, it is advantageous to express the $\hat{e}_i$ vectors as linear combinations of $\boldsymbol{d}_i$:

$$\hat{e}_i = \sum_{j=1}^{i} c_{ij}\boldsymbol{d}_j \tag{A4}$$

The coefficients in equation (A4) are computed as follows. From equation (A1),

$$c_{11} = \frac{1}{|\boldsymbol{d}_1|}$$

From equations (A4),

$$c_{22} = \frac{1}{|\boldsymbol{e}_2|}$$

$$c_{21} = \frac{(\boldsymbol{d}_2, \hat{e}_1)}{|\boldsymbol{e}_2||\boldsymbol{d}_1|}$$

For the general term, we substitute into the summation in equation (A3a) from equation (A4):

$$\boldsymbol{e}_i = \boldsymbol{d}_i - \sum_{j=1}^{i-1} (\boldsymbol{d}_i, \hat{e}_j) \sum_{k=1}^{j} c_{jk}\boldsymbol{d}_k$$

$$= \boldsymbol{d}_i - \sum_{j=1}^{i-1} (\boldsymbol{d}_i, \hat{e}_j) \sum_{k=1}^{i-1} c_{jk}\boldsymbol{d}_k \quad (c_{jk} = 0 \text{ for } k > j)$$

$$= \boldsymbol{d}_i - \sum_{k=1}^{i-1}\sum_{j=1}^{i-1} (\boldsymbol{d}_i, \hat{e}_j)\, c_{jk}\boldsymbol{d}_k \quad (c_{jk} = 0 \text{ for } k > j)$$

$$= \boldsymbol{d}_i - \sum_{k=1}^{i-1} \tilde{c}_{i-1,k}\boldsymbol{d}_k \tag{A5}$$

where

$$\tilde{c}_{i-1,k} = \sum_{j=1}^{i-1} (\boldsymbol{d}_i, \hat{e}_j)\, c_{jk} \quad (c_{jk} = 0 \text{ for } k > j) \tag{A6}$$

Finally, using equations (A5) and (A6) with equation (A3b) in equation (A4) yields

$$c_{ik} = \left\{ \begin{array}{ll} \frac{1}{|\boldsymbol{e}_i|} & (k = i) \\ -\frac{\tilde{c}_{ik}}{|\boldsymbol{e}_i|} & (k < i) \end{array} \right\} \tag{A7}$$

where the coefficients $\tilde{c}_{ik}$ are obtained recursively from equation (A6). Thus, with c_{ik} computed by relation (A7), equation (A4) is the result required in the main text (eq. (8)).

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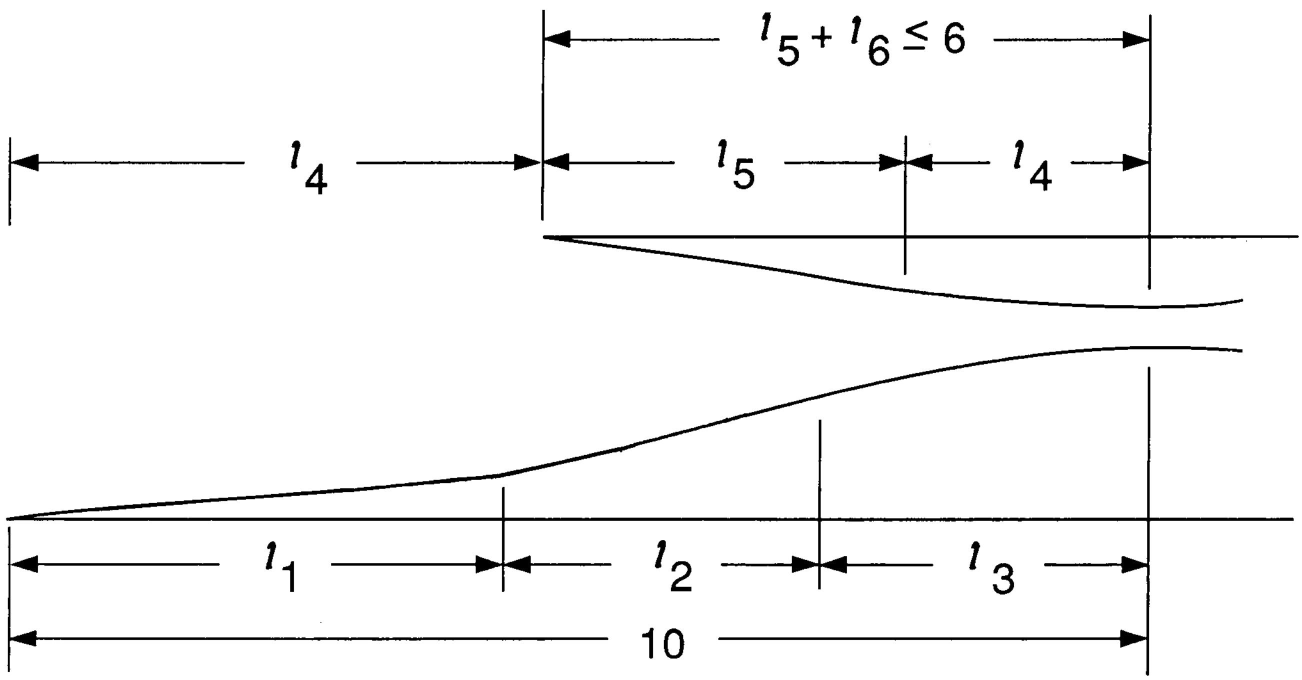
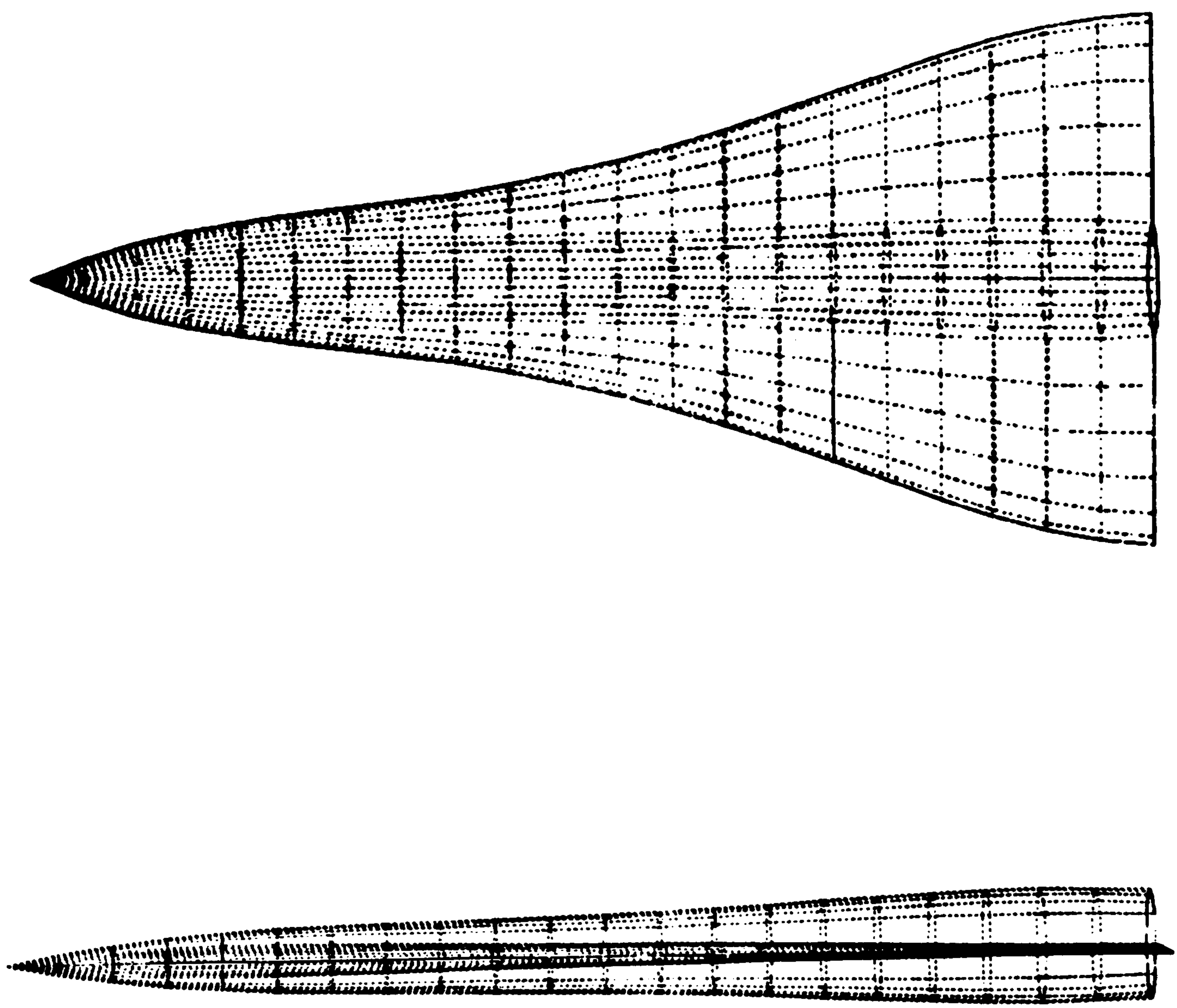
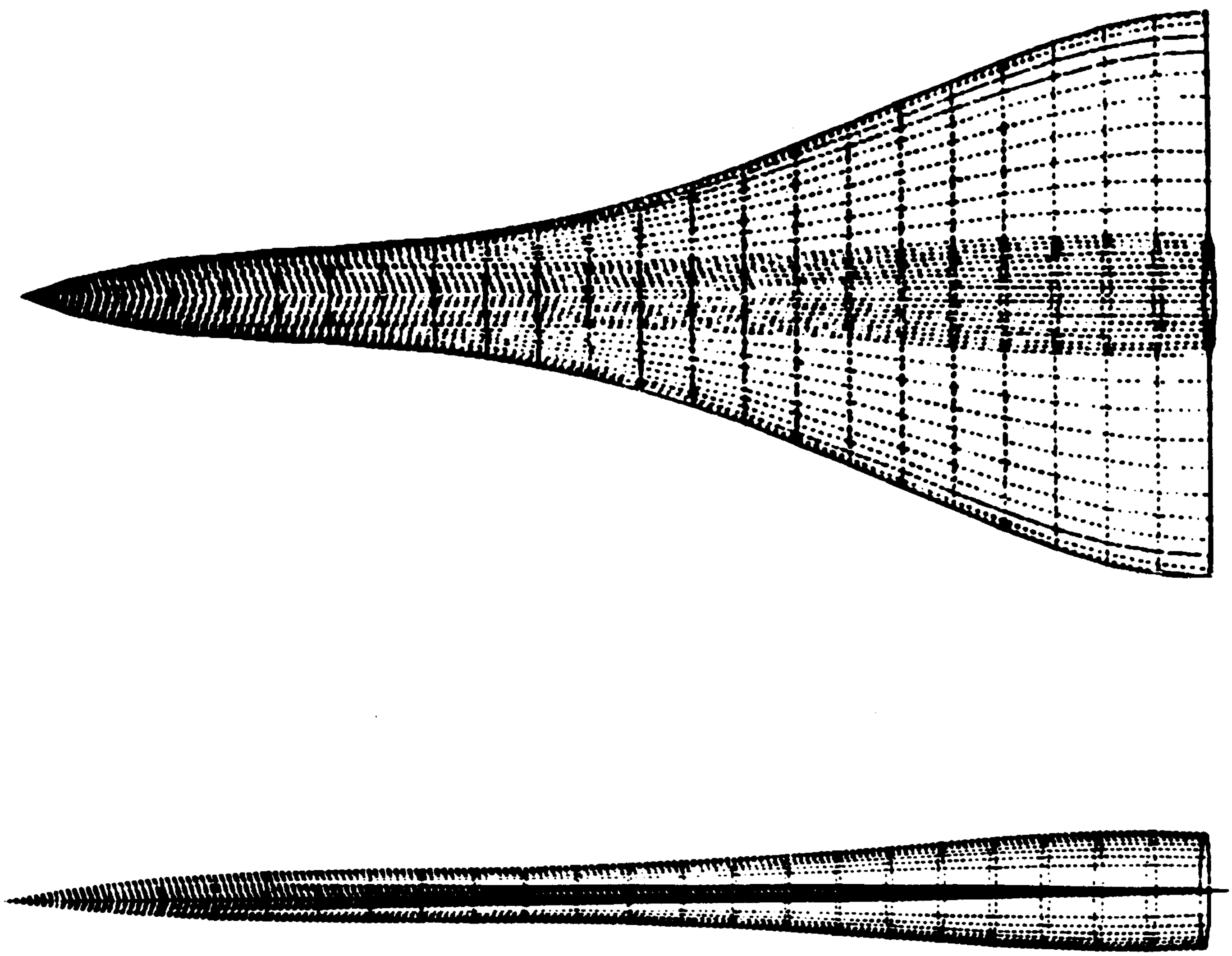


Figure 1. Supersonic inlet example illustrating application of constraint equation.



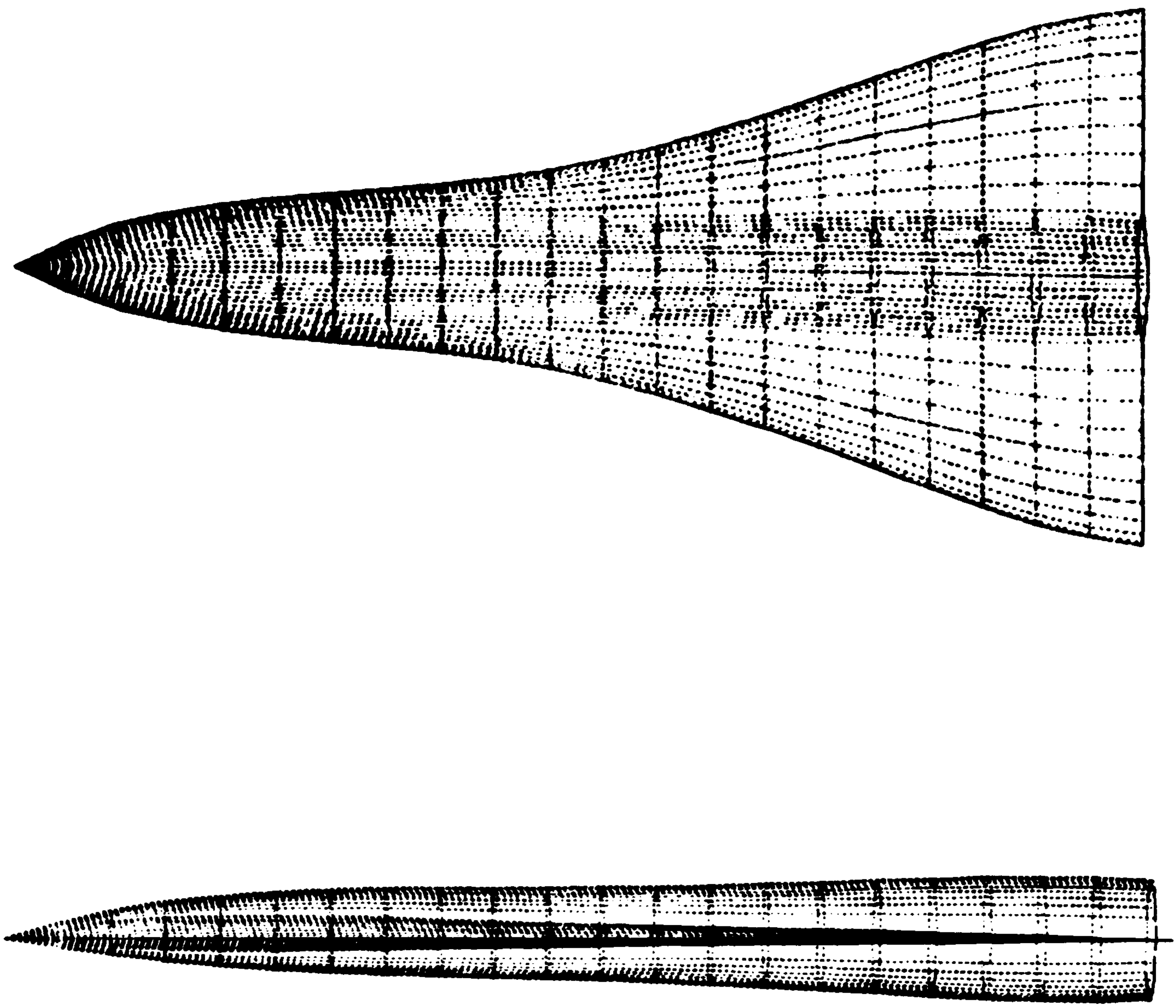
(a) Configuration 1.

Figure 2. Plan and side views of four base wing-body configurations.



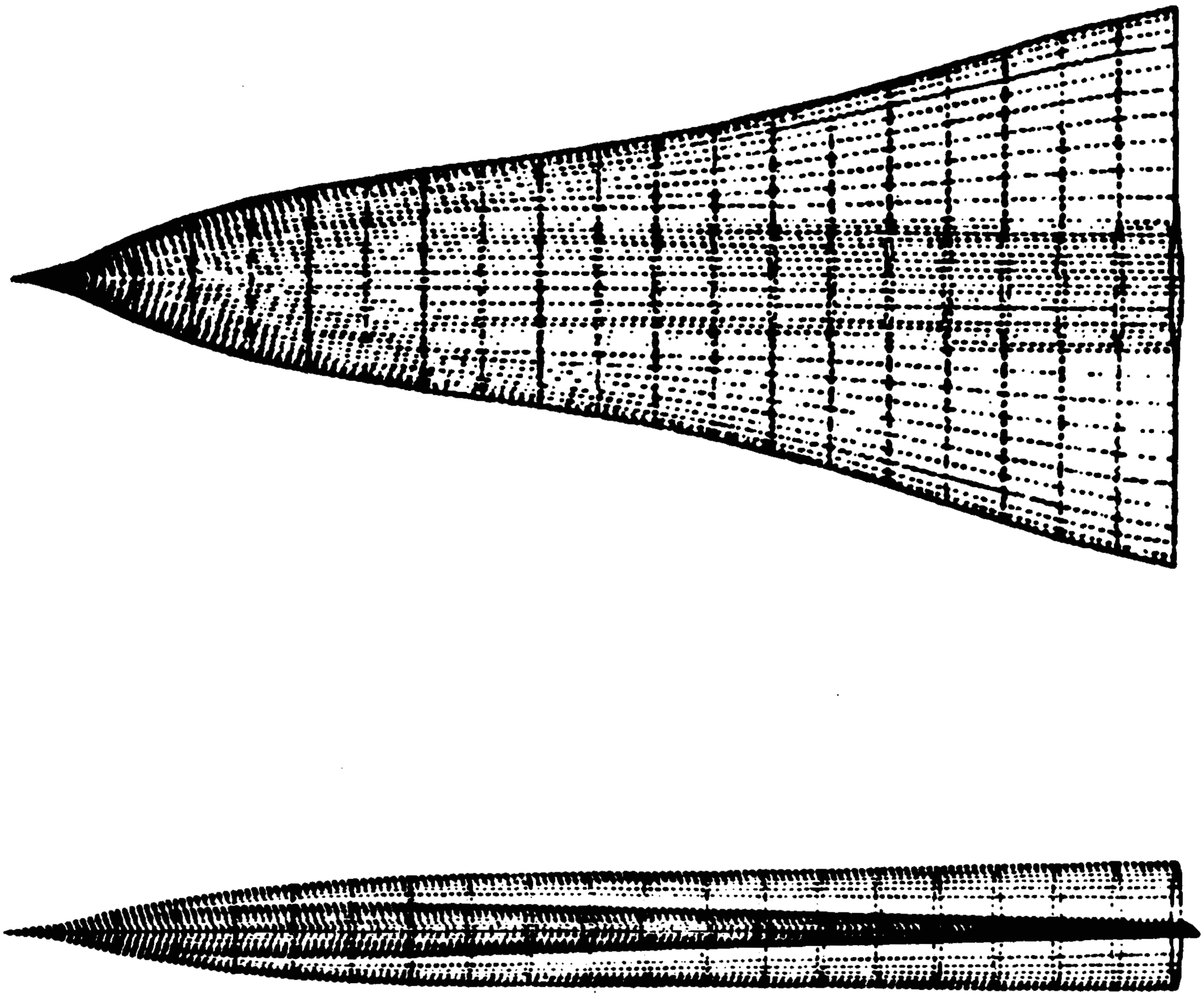
(b) Configuration 2.

Figure 2. Continued.



(c) Configuration 3.

Figure 2. Continued.



(d) Configuration 4.

Figure 2. Concluded.

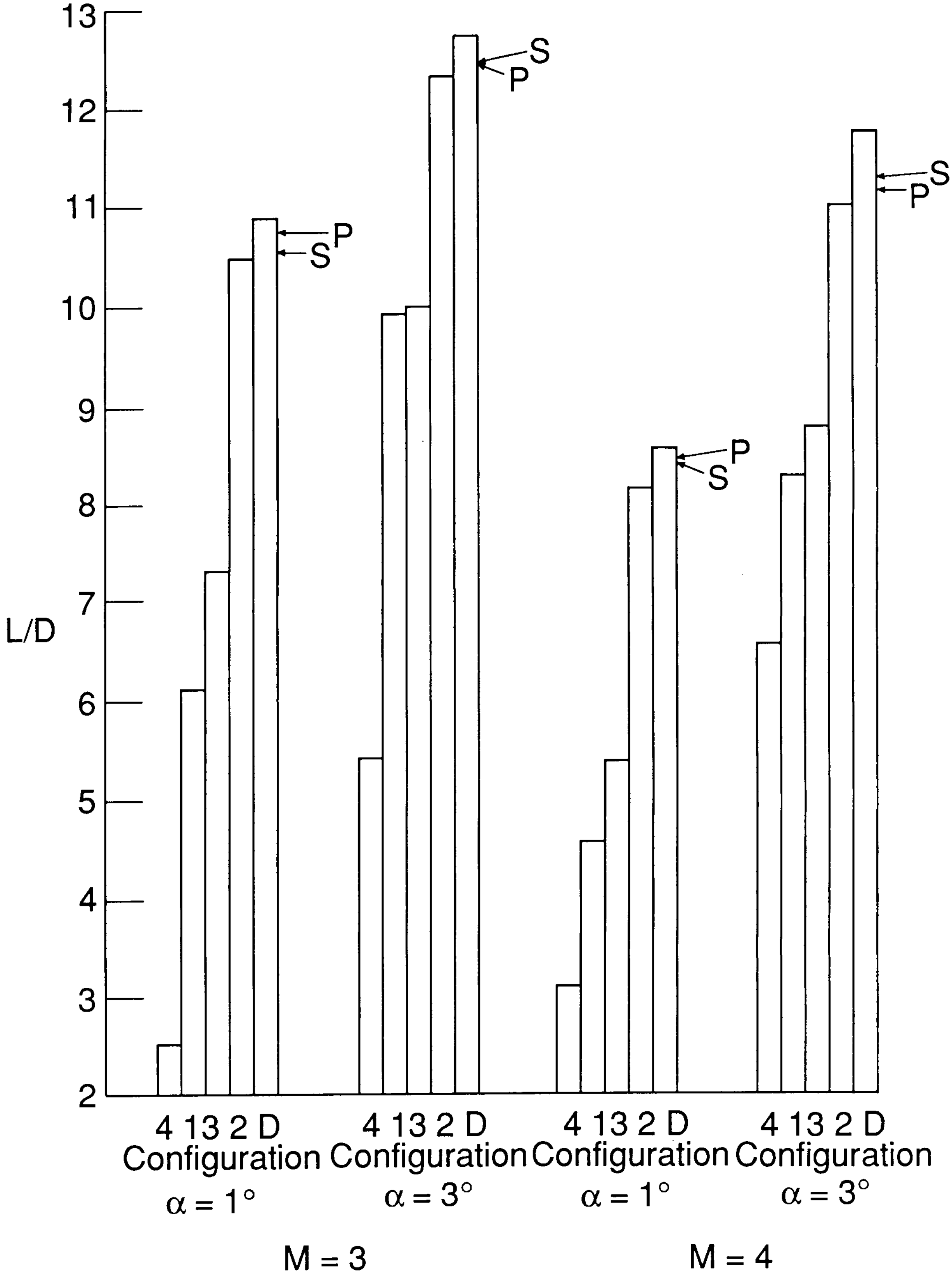


Figure 3. Computed performance of four base configurations and design configuration, together with specified (S) and predicted (P) performance values.

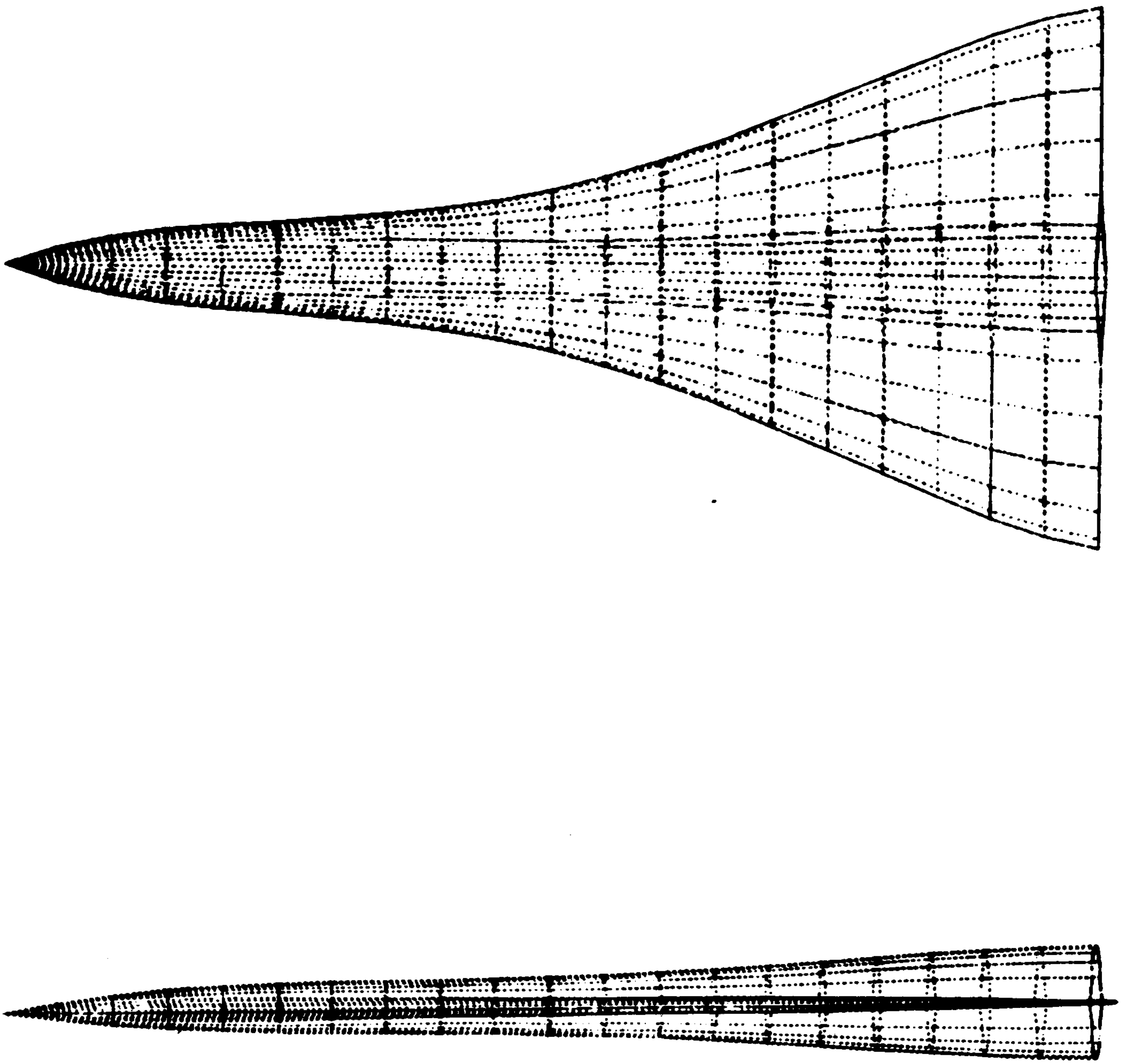
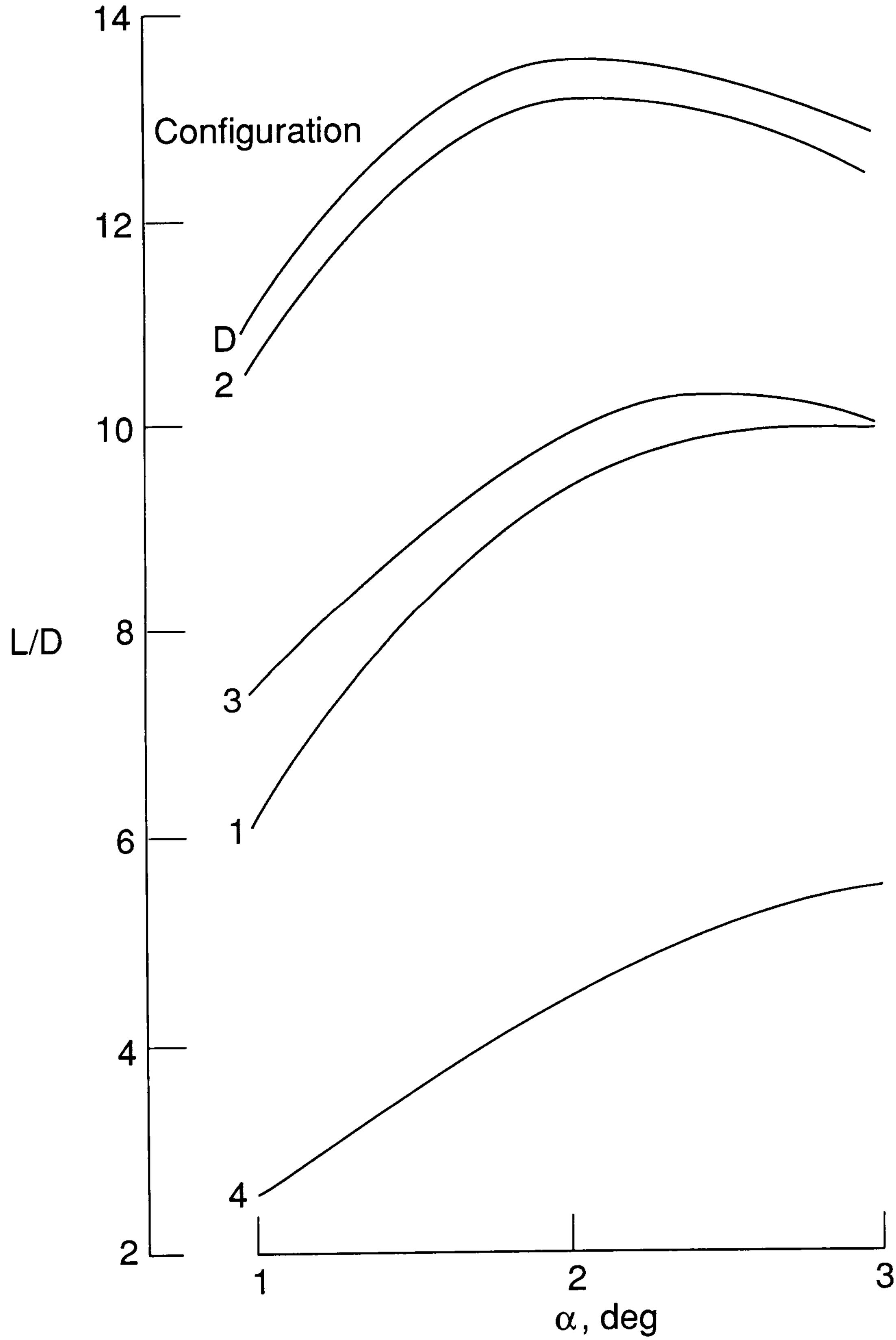
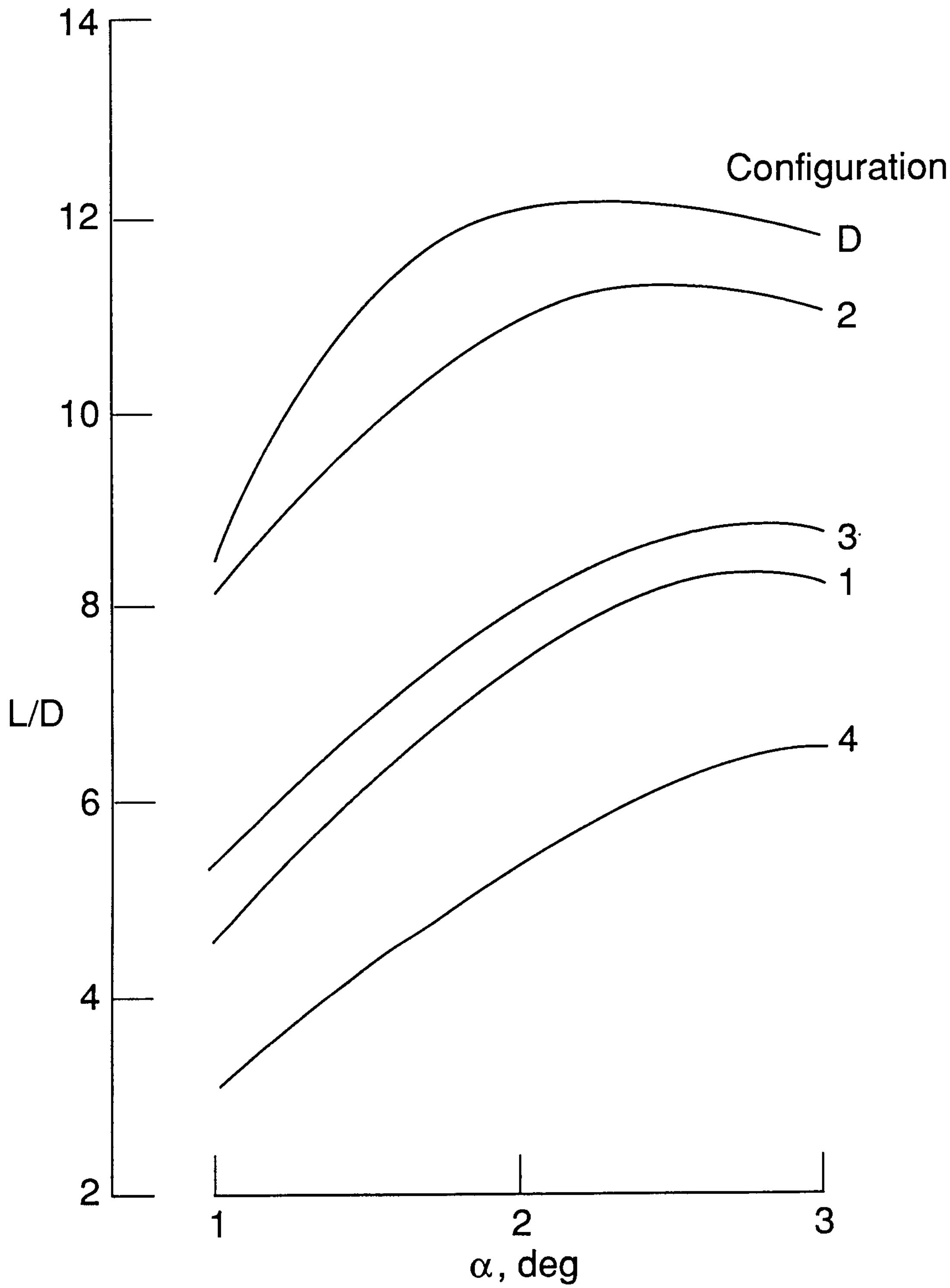


Figure 4. Design configuration.



(a) $M = 3$.

Figure 5. Plots of L/D versus α for four base configurations and design configuration.



(b) $M = 4$.

Figure 5. Concluded.

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16. Abstract
A method is described for designing an aerodynamic configuration for a specified performance vector, based on results from several similar but not identical trial configurations, each defined by a geometry parameter vector. The theory indicates that the method is effective provided that (1) the results for the trial configurations provide sufficient variation so that a linear combination of them approximates the specified performance and (2) the differences between the performance vectors (including the specified performance) are sufficiently small that the linearity assumption of sensitivity analysis applies to the differences. A computed example describes the design of a high supersonic Mach number missile wing-body configuration based on results from a set of four trial configurations.

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