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Frame Synchronization Performance and Analysis

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This article describes the analysis used to generate the theoretical models showing the performance of the frame synchronizer for various frame lengths and marker lengths at various signal-to-noise ratios and bit error tolerances.

I. Introduction

The telemetry data stream from a deep space mission, usually many hundreds of thousands of bits long, is divided into units called "frames." Each frame is introduced by a "marker" (generally 32 bits long, but not always) announcing the frame boundary. Until now, the Deep Space Network (DSN) has sent the data stream to the projects without regard to frame boundaries.

The DSN is beginning to install "frame synchronizers" which look for markers and divide the data into frames. Beginning with Magellan, the DSN will deliver data to the projects already divided into frames. The currently planned configuration is for these frame synchronizers to work on decoded information bits; therefore, they must follow the Viterbi decoder (see Fig. 1). See [1] for a discussion of the relative merits of frame synchronization before and after Viterbi decoding.

The technical problems of dividing a noisy data stream into frames are probabilistic, depending on the channel. When the data stream arrives, a search for the marker is conducted. However, because of channel noise, the marker might be corrupted; on the other hand, the actual data within the frame might include 32 bits that are exactly the same as the marker. Thus, when looking at the data, the frame synchronizer needs to decide which bits most likely represent the marker in order to decide where the frame begins.

II. The Frame Synchronizer

Before describing the Frame Synchronizer Subassembly (FSS), what is meant by "finding" the marker must be established.

Since it is likely that a marker may contain errors due to normal deep space communications noise, a match is "found" when a string of bits disagrees with the marker in no more than T places, where T is the Bit Error Tolerance (BET) threshold [2]. If more than T errors are made in the frame sync word, then it will be "missed." The "best match" is the string of decoded bits disagreeing with the marker in the fewest places. During the search for the frame sync marker (see below), if the minimum error detector (MED) is enabled, then the best match identified in one frame length is used as the marker. Otherwise, the first match is used. (If errors in the decoded bit error stream were independent, then the best match would most likely be the marker. See [2] for another method taking Viterbi decoder error statistics into account.)

After a possible marker is identified, the data stream is examined at a point one frame length away for another marker. If a match is found, then the marker is considered verified once. This process may be repeated for various numbers of verifications.

The frame synchronizers in the DSN use the following algorithm (see Figs. 2 and 3), given T = BET, K = the number of verifies, and N = the number of flywheels:

(1) SEARCH: Search the data stream until a possible marker is found with fewer than T disagreements with the marker. If the MED is enabled, continue the search for one complete frame length.

(2) VERIFY: Examine the data stream at a point one frame length away for another marker. If the match is within T of the marker, continue the verification; otherwise return to step 1. If K verifications are successful, then "frame in lock," or sync, is established.

(3) LOCK: Continue testing the marker. If more than T errors are found, enter the flywheel mode. FSS has sync while in this mode.

(4) FLYWHEEL: Determine whether the sync is lost by testing up to N consecutive frames. If the marker is found within the N frames, then sync is reestablished and the FSS returns to step 3. Otherwise, return to step 1.

III. Analysis and Probabilities

In analyzing the probability of finding the sync marker, only two cases were considered: (1) the probability of frame acquisition within F = 4 frames with K verifications (K = 3, 2, or 1) and the MED enabled; and (2) the probability of frame acquisition within F frames (F = 1, 2, 3, or 4) with no verifications and the MED enabled.

Using data from Table C-1 of [3] (Viterbi Decoder Burst Statistics: 3233013 [7, 1/2] Convolutional Code), error distribution data was generated by simulation and used to determine the probabilities for the number of errors made by the Viterbi decoder during a 32-bit span of bits, and thus the probabilities of different numbers of disagreements between the decoded marker and the true marker. Below, D is used for the number of disagreements between the decoded marker and the true marker, so $P(D = j)$ is the probability that the Viterbi decoder makes j errors in the marker.

The binomial probability distribution with $p = 1/2$ was used to determine the probabilities for the number of disagreements between 32 random bits and the marker. R is the number of disagreements between 32 random bits and the markers, so

$$P(R = j) = \binom{32}{j} 2^{-32}$$

The analysis below assumes that in all cases of "ties," i.e., the same number of disagreements in two places, the frame synchronizer makes the wrong decision. This means that actual performance will be slightly better than that predicted below.

Variable	Typical values
T = threshold	$0 \leqslant T \leqslant 32$
L = frame transport length	L = 5120, 6720, 480, 16320
F = number of frames	F = 1, 2, 3, 4
K = number of times sync is verified after finding the initial marker	K = 1, 2, 3

A. Probability of Frame Acquisition Within F = 4 Frames With K Verifications

With K = 3 verifications and F = 4 frames, the marker must be found correctly in each of the frames:

P(sync found correctly and verified 3 additional times)

$= P$(sync found correctly in first frame and verified 3 times)

Assuming independence between frames, this is

$$P(\text{sync found correctly}) \cdot [P(\text{verified})]^3$$

$$= \sum_{j=0}^{T} P(D = j) \cdot [P(R > j)]^L \cdot [P(D < T)]^3$$

A large assumption is implicit in the use of $[P(R > j)]^L$ for the probability that the marker is never found in random data. Finding the marker in different places in random data is not independent unless the places are at least a marker length apart. The actual probability that the marker is found in random data is at least $1 - 32(1 - [P(R > j)]^{L/32})$, which is close to $[P(R > j)]^L$ for small j. As j increases, these quantities begin to differ, but the contribution to the sum becomes small. An actual calculation of the probability of finding the marker with up to j disagreements somewhere in a frame is analytically intractable, and approximation by simulation uses a great deal of computer time (see [1]).

The probability that fewer than T errors are found in the sync word 3 times in a row is $[P(D<T)]^3$, assuming independence between different frames.

With $K = 2$ verifications and $F = 4$ frames, sync can be declared correctly in two ways:

P(sync found correctly and verified 2 additional times)

$=P$(found correctly in 1st frame and verified twice)
$+P$(found correctly in 2nd frame and verified twice, not found in 1st frame)

$=P$(found correctly in 1st frame and verified twice)
$+P$(found correctly in 2nd frame and verified twice)
$\cdot P$(not found anywhere in 1st frame)

$=P$(found correctly and verified twice)
$\cdot$ {1 + P(not found anywhere in 1st frame)}

$$=\sum_{j=0}^{T} P(D=j)\cdot[P(R>j)]^L\cdot[P(D<T)]^2$$

$$\cdot\left\{1+P(D>T)\cdot[P(R>T)]^L\right\}$$

With $K = 1$ verification and $F = 4$ frames, sync can be declared correctly in five ways:

P(sync found correctly and verified 1 additional time)

$=P$(found correctly in 1st frame and verified once)
$+P$(found correctly in 2nd frame and verified once, not found in 1st frame)
$+P$(found correctly in 3rd frame and verified once, not found in first two frames)
$+P$(found correctly in 3rd frame and verified once, found correctly in 1st frame, but not verified)
$+P$(found correctly in 3rd frame and verified once, found incorrectly in 1st frame, but not verified)

$=P$(found correctly and verified once)
$\cdot$ {1 + P(not found anywhere in 1st frame)
$+P$(not found anywhere in 1st and 2nd frames)
$+P$(found correctly in 1st frame, but not verified)
$+P$(found incorrectly in 1st frame, but not verified)}

$$=\sum_{j=0}^{T} P(D=j)[P(R>j)]^L\cdot[P(D<T)]$$

$$\cdot\Big\{1+P(D>T)\cdot[P(R>T)]^L$$

$$+[P(D>T)\cdot[P(R>T)^L]^2$$

$$+\sum_{j=0}^{T} P(D=j)\cdot[P(R>j)]^L\cdot P(D>T)$$

$$+P(D>T)\cdot(1-[P(R>T)]^L)\cdot P(R>T)\Big\}$$

Two sets of graphs can be generated from the data. One set is a function of SNR, the other a function of BET.

B. Probability of Frame Acquisition Within *F* Frames With No Verifications

With $F = 1$ frame:

P(sync found correctly in 1 frame)

$$=\sum_{j=0}^{T} P(D=j)\cdot[P(R>j)]^L$$

With $F = 2$ frames:

P(sync found correctly in 2 frames)

$=P$(found correctly in 1st frame)
$+P$(found correctly in 2nd frame, not found in 1st frame)

$$=\sum_{j=0}^{T} P(D=j)\cdot[P(R>j)]^L$$

$$\cdot\left\{1+P(D>T)[P(R>T)]^L\right\}$$

With $F = 3$ frames:

P(sync found correctly in 3 frames)

$=P$(found correctly in 1st frame)
$+P$(found correctly in 2nd frame, not found in 1st frame)
$+P$(found correctly in 3rd frame, not found in first two frames)

$$= \sum_{j=0}^{T} P(D=j) \cdot [P(R>j)]^L$$

$$\left\{ 1 + P(D>T)[P(R>T)]^L \right.$$

$$\left. + [P(D>T)[P(R>T)]^L]^2 \right\}$$

With $F = 4$ frames:

P(sync found correctly in 4 frames)

= P(found correctly in 1st frame)
+ P(found correctly in 2nd frame, not found in 1st frame)
+ P(found correctly in 3rd frame, not found in first two frames)
+ P(found correctly in 4th frame, not found in first three frames)

$$= \sum_{j=0}^{T} P(D=j) \cdot [P(R>j)]^L$$

$$\cdot \left\{ 1 + P(D>T)[P(R>T)]^L \right.$$

$$+ [P(D>T)[P(R>T)]^L]^2$$

$$\left. + [P(D>T)[P(R>T)]^L]^3 \right\}$$

IV. Frame out of Lock

The probability of Frame out of Lock occurring at least N consecutive times is as follows:

Frame Error Distribution: $P(D > T)$

Frame out of Lock: $[P(D > T)]^N$ $\quad N = 1, 2, 3, 4, 5$

V. Numerical Results

Figures 4 through 6 generated by the models represent the two main operating FSS modes: (1) "Frame Acquisition," comprising the Search and Verify modes; and (2) "Frame in Lock," comprising the Lock and Flywheel modes.

Many curves, using different parameters, were generated, but only three representative curves will be included here. Figure 4 shows the probability of frame acquisition within $F = 4$ frames with two verifications versus bit signal-to-noise ratios (SNR or E_b/N_0) for various BETs. Figure 5 shows the probability of frame acquisition within $F = 4$ frames with SNR = 2.1 dB versus the BET for various numbers of verifications. Figure 6 shows the probability that the marker is corrupted by more than BET errors $N = 2$ times in a row versus the bit SNRs for various BETs.

Figures 4 and 5 give probabilities of "Frame Acquisition," while Fig. 6 gives probabilities of "Frame out of Lock," the opposite of "Frame in Lock." It is easy to see that the number of combinations of parameters is far too great to allow a full set of graphs to be included here.

All of these graphs are based on the "geometric model" for Viterbi decoder burst errors, and on the statistics in [3].

VI. Conclusions

Several parameters may be chosen when operating the Frame Synchronizer Subassembly. Some of these are T, the bit error threshold which specifies the maximum number of differences allowable when declaring a set of bits the frame sync marker, K; the number of verifications needed before declaring the stream of bits in lock; and N, the number of flywheels to go through before declaring a stream out of lock. The operator must also set the length of the marker and the length of the frame.

There is no obviously best choice for T, K, and N because each one gives a trade-off between the probability of incorrectly declaring lock and incorrectly not declaring lock; the merits of the two competing factors must be weighed. However, the graphs at the end of this report should help in making that decision by giving probability estimates in several cases. The results of this study are summarized in the graphs.

References

[1] L. Swanson, *A Comparison of Frame Synchronization Methods*, JPL Publication 82-100, Jet Propulsion Laboratory, Pasadena, California, 1982.

[2] M. Shahshahani and L. Swanson, "A New Method for Frame Synchronization," *TDA Progress Report 42-90*, vol. April–June 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 111–123, August 15, 1986.

[3] R. L. Miller, L. J. Deutsch, and S. A. Butman, *On the Error Statistics of Viterbi Decoding and the Performance of Concatenated Codes*, JPL Publication 81-9, Jet Propulsion Laboratory, Pasadena, California, 1981.

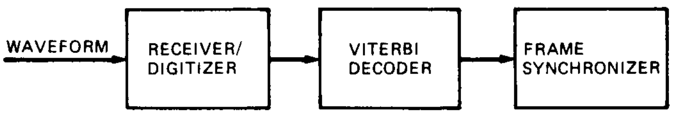

Fig. 1. Proposed telemetry system, including the FSS

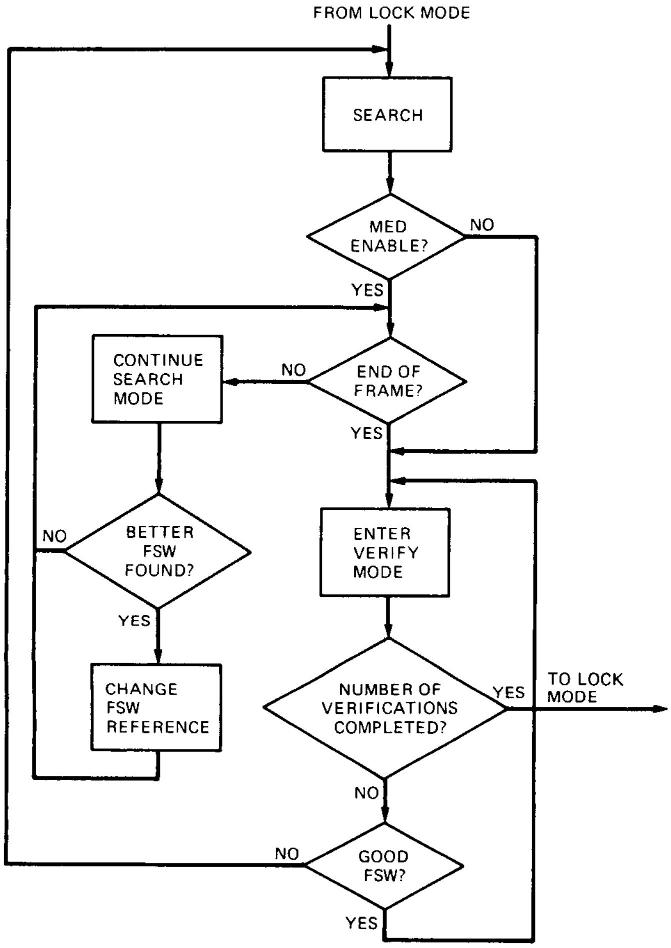

Fig. 2. Flowchart for search and verify modes

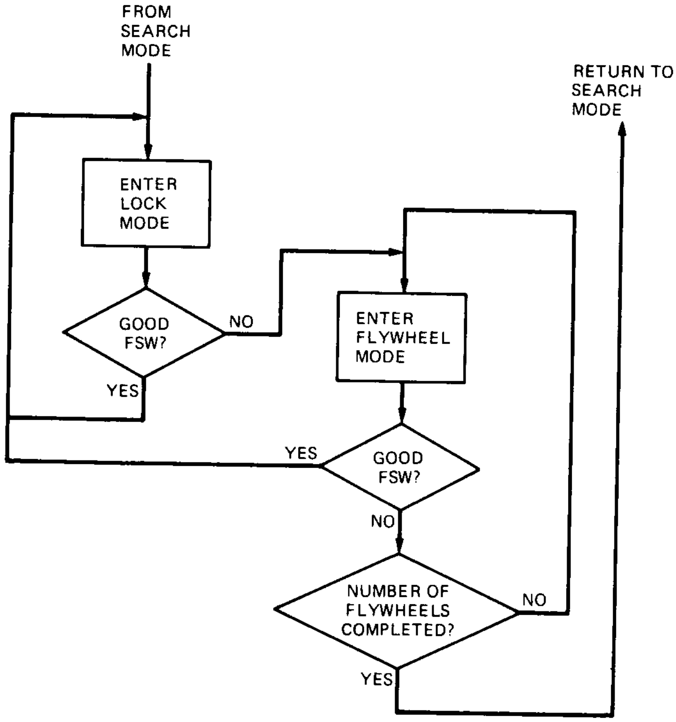

Fig. 3. Flowchart for lock and flywheel modes

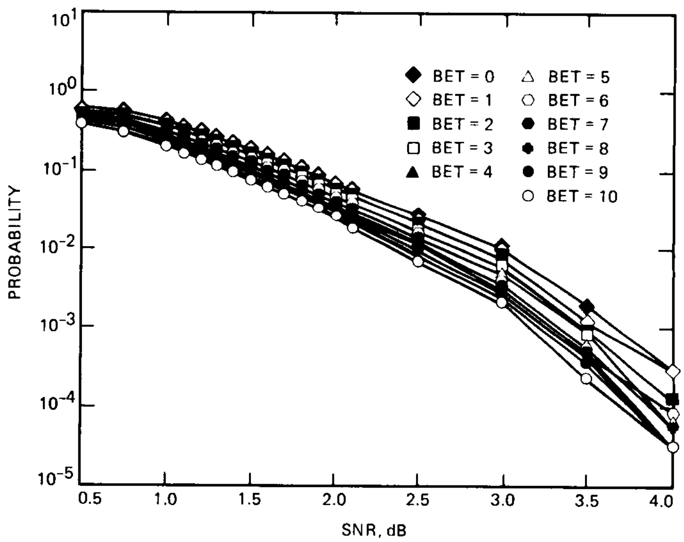

Fig. 4. Frame acquisition within a fixed time with a fixed number of verifications. This chart shows 1 minus the probability that correct sync is found within four frames with two verifications and a frame transport length of 5120 (from computations using the analysis above) versus the SNR.

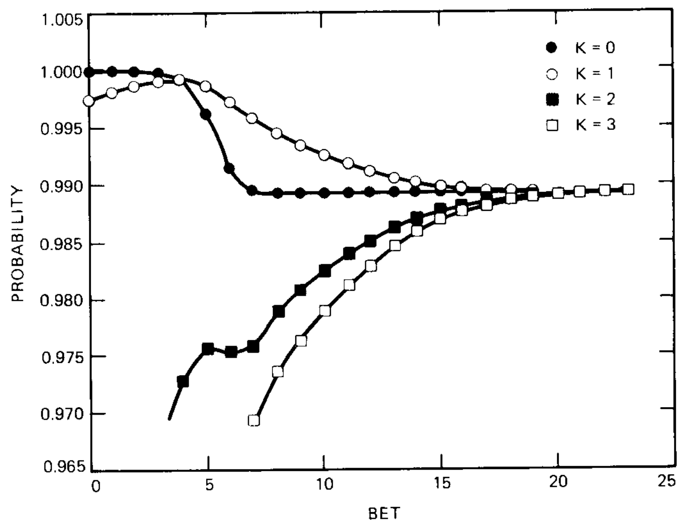

Fig. 5. Frame acquisition within a fixed time with a fixed SNR. This chart shows the probability that correct sync is found within four frames at 2.1 dB and a frame transport length of 5120 (from computations using the analysis above) versus the bit error threshold. *K* is the number of verifications.

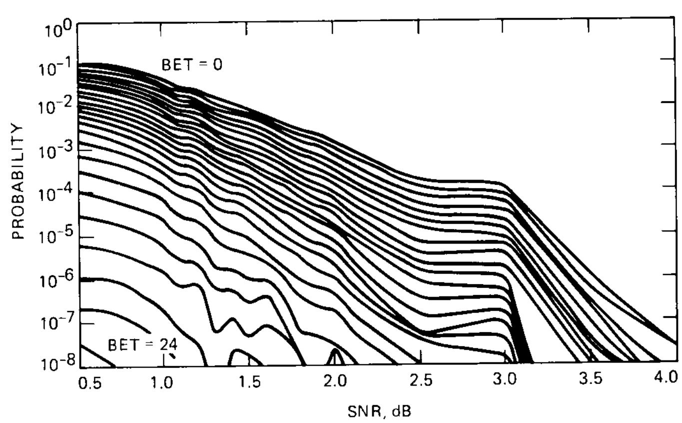

Fig. 6. Frame out of lock with a fixed number of flywheels. This chart shows 1 minus the probability that correct sync is lost at a given time with two flywheels (1−*P*[*D* > BET two times in a row]) versus the SNR.