

Dynamic Substructuring for Shock Spectrum Analysis Using Component Mode Synthesis

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Component mode synthesis was used to analyze three different types of structures with MSC NASTRAN. The theory and technique of using Multipoint Constraint Equations (MPCs) to connect substructures to each other or to a common foundation is presented. Computation of the dynamic response of the system from shock spectrum inputs was automated using the DMAP programming language of the MSC NASTRAN finite element code.

INTRODUCTION

Computation time in dynamic analyses increases with either the square or cube of the number of degrees-of-freedom. As a result, there can be substantial savings in computer costs by using dynamic substructuring methods in large dynamic problems. In some cases, finite element models become so large that only a few supercomputers can be used to solve the resulting dynamics problem. In these cases, dynamic substructuring may be required.

Component mode synthesis methods, like those introduced in 1971 by MacNeal [1], are often used in dynamic substructuring. These techniques have been applied to superelement analyses in MSC NASTRAN. Most classes of problems can be solved with this procedure. However, response spectrum analysis problems using model synthesis methods have not been previously analyzed [2,3] with MSC NASTRAN. In this effort, both basic response spectrum methods and U.S. Navy DDAM shock analysis methods that require an evaluation of modal masses in the specification of the shock spectrum input are analyzed. In this paper, the theoretical background for the application of modal synthesis to shock spectrum problems and a numerical evaluation of the methods are presented.

THEORY

Three cases are considered in the theoretical development:

Case I: Single substructure attached to a residual structure (foundation).

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Case II: A series of substructures attached to a residual structure (foundation).

Case III: Parallel substructures connected to each other and to a common residual structure (foundation).

Using these three cases, any linear structure can be divided into a number of substructures, analyzed, and combined using multipoint constraint (MPC) equations.

In all cases the eigenvalue analysis of the substructures to find mode shapes $\{\phi_i\}$ and frequencies $\{\omega_i\}$ is to be determined assuming a free-free substructure.

The equation of motion of a linear system can be written as:

$$[M]\{\ddot{U}\} + [K]\{U\} = \{f(t)\} \quad (1)$$

Expanding the displacement in terms of the free-free mode shapes:

$$\{U(t)\} = \sum_j \{\phi_j\} \xi_j(t) \quad (2)$$

Substituting into Equation (1):

$$[M] \sum_j \{\phi_j\} \xi_j + [K] \sum_j \{\phi_j\} \xi_j = \{F(t)\} \quad (3)$$

Premultiply by $\{\phi_i\}^T$,

$$\{\phi_i\}^T [M] \sum_j \{\phi_j\} \xi_j + \{\phi_i\}^T [K] \sum_j \{\phi_j\} \xi_j = \{\phi_i\}^T \{F(t)\} \quad (4)$$

By virtue of the orthogonality property of vibration modes,

$$\{\phi_i\}^T [M] \{\phi_j\} = m_i \delta_{ij} \quad (5)$$

$$\{\phi_i\}^T [K] \{\phi_j\} = K_i \delta_{ij}$$

and

$$f_i(t) = \{\phi_i\}^T \{F(t)\} \quad (6)$$

where:

m_i is the generalized mass of the i^{th} mode

K_i is the generalized stiffness of the i^{th} mode

f_i is the generalized force for the i^{th} mode

and

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad (7)$$

Using these definitions, the equation of motion in generalized coordinates becomes:

$$m_i \ddot{\xi}_i + K_i \xi_i = f_i \quad (8)$$

for each mode, i .

Case I: Single Substructure

Assume that the connection points shown in Figure 1 represent four degrees-of-freedom designated 997, 998, 999, and 1000.

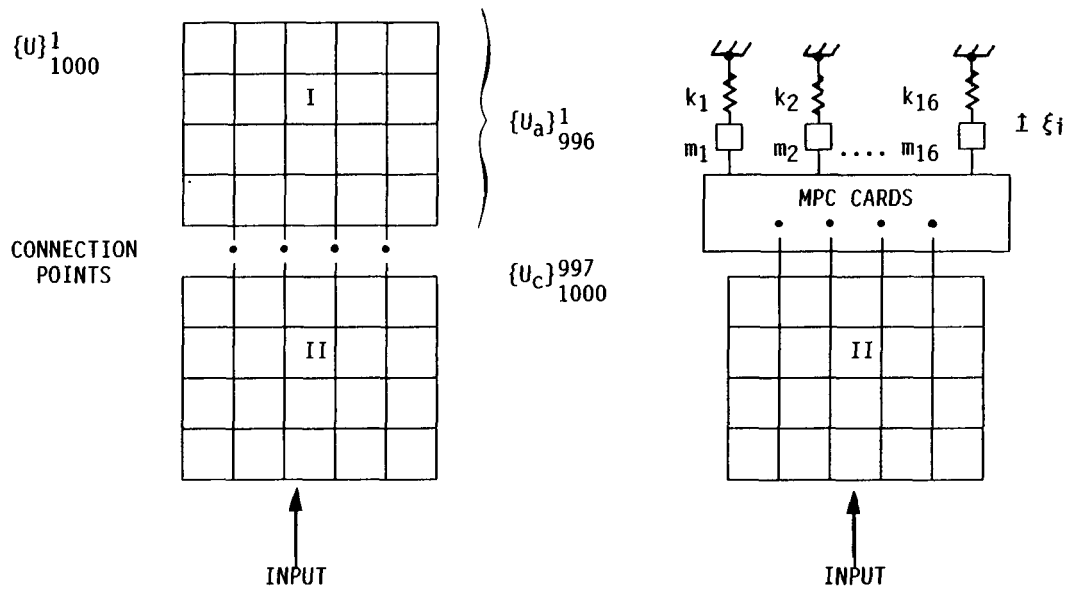


Figure 1. Case I: Substructure and Residual or Foundation Structure

Furthermore, it is assumed that 16 modes can be used to represent the substructure. Generalized stiffness, K_i , and generalized mass, m_i , are connected to each scalar mass point. The displacement of the scalar point is the modal displacement, ξ_i . The MPC cards which relate the connection degrees-of-freedom (997-1000) to the 16 modal displacements, ξ_i , can be written as:

$$\begin{aligned}
 U^{997} &= \phi_1^{997} \xi_1 + \phi_2^{997} \xi_2 + \dots + \phi_{16}^{997} \xi_{16} \\
 U^{998} &= \phi_1^{998} \xi_1 + \phi_2^{998} \xi_2 + \dots + \phi_{16}^{998} \xi_{16} \\
 U^{999} &= \phi_1^{999} \xi_1 + \phi_2^{999} \xi_2 + \dots + \phi_{16}^{999} \xi_{16} \\
 U^{1000} &= \phi_1^{1000} \xi_1 + \phi_2^{1000} \xi_2 + \dots + \phi_{16}^{1000} \xi_{16}
 \end{aligned} \tag{9}$$

In defining the constraint equations above, it was discovered that fewer modes were required for an accurate solution if only translational degrees are included in the constraint equations at the connection points. After a response spectrum analysis has been run on the reduced structure connected to the residual structure by multipoint constraint, Equation (9), it is simple to recover the internal component substructures' displacement. For substructure I shown schematically in Figure 1 with 1000 displacement degrees-of-freedom, the internal displacements for the j th mode of the reduced dynamic model are given in terms of the 16 scalar point (generalized) displacements.

$$\left\{ U_j \right\}_{1000}^1 = \sum_{i=1}^{16} \left\{ \phi_i \right\}_{1000}^1 \cdot \xi_{ij} = \begin{matrix} 1. \\ \vdots \\ 16 \end{matrix} \left[\Phi \right]^{16} \left\{ \xi_j \right\}_{16}^1 \tag{10}$$

Once the displacements are known, stress, forces, etc., are easily determined in NASTRAN using the element stiffnesses matrices. All that remains is to sum the desired quantities over the modes with either the NRL sum (Navy DDAM) or SRSS sum (Earthquake Spectrum Analysis). Thus, the steps needed to use the component mode synthesis method are as follows:

- (1) Determine the free-free mode shapes, ϕ_i , generalized mass, m_i , and generalized stiffness, K_i , for the substructure.
- (2) Choose a sufficient number of modes of the substructure to cover the frequency range of interest.
- (3) Model the residual structure (foundation) and scalar points for each mode using grounded generalized masses and stiffnesses.
- (4) Connect the component mode representation of the substructure to the residual structure with multipoint constraint equations (MPCs) as defined in Equation (9).
- (5) Determine the modal mass of the combined structure to ensure that a sufficient percentage of the total mass is included in the model. If the total mass is less than required, the number of modes in the substructure must be increased.
- (6) If the modal mass requirement is satisfied, determine the dynamic stresses on the residual structure and

recover the stresses in the component substructures with Equation (10).

Case II: Series of Substructures

For this case the method is almost identical with Case I. The free-free modes of vibration must be determined for each substructure. The residual structure is then modeled; each substructure is modeled with scalar points, generalized stiffnesses, and generalized masses for the modes to be considered. MPC cards, as defined previously in Equation (9), relate the connection points of the residual structure to the first substructure. Each substructure is related to the next substructure through a different set of MPC cards based on compatibility of the deflections of the physical connection points between the substructures. This will be shown in detail below. In defining these new MPCs that connect substructures, the dependent degree of freedom relating substructure, S_i , to substructure, S_{i+1} , must be selected from S_{i+1} (see Figure 2). Otherwise, a mode is lost in the resulting combined structure.

As an example of this method, assume that each substructure, S_i , in Figure 2 can be represented by four modes. Mode shapes for the first substructure, S_1 , are designated $\{\phi_i\}$ and for the second substructure, S_2 , the modes are designated $\{\bar{\phi}_i\}$, etc.

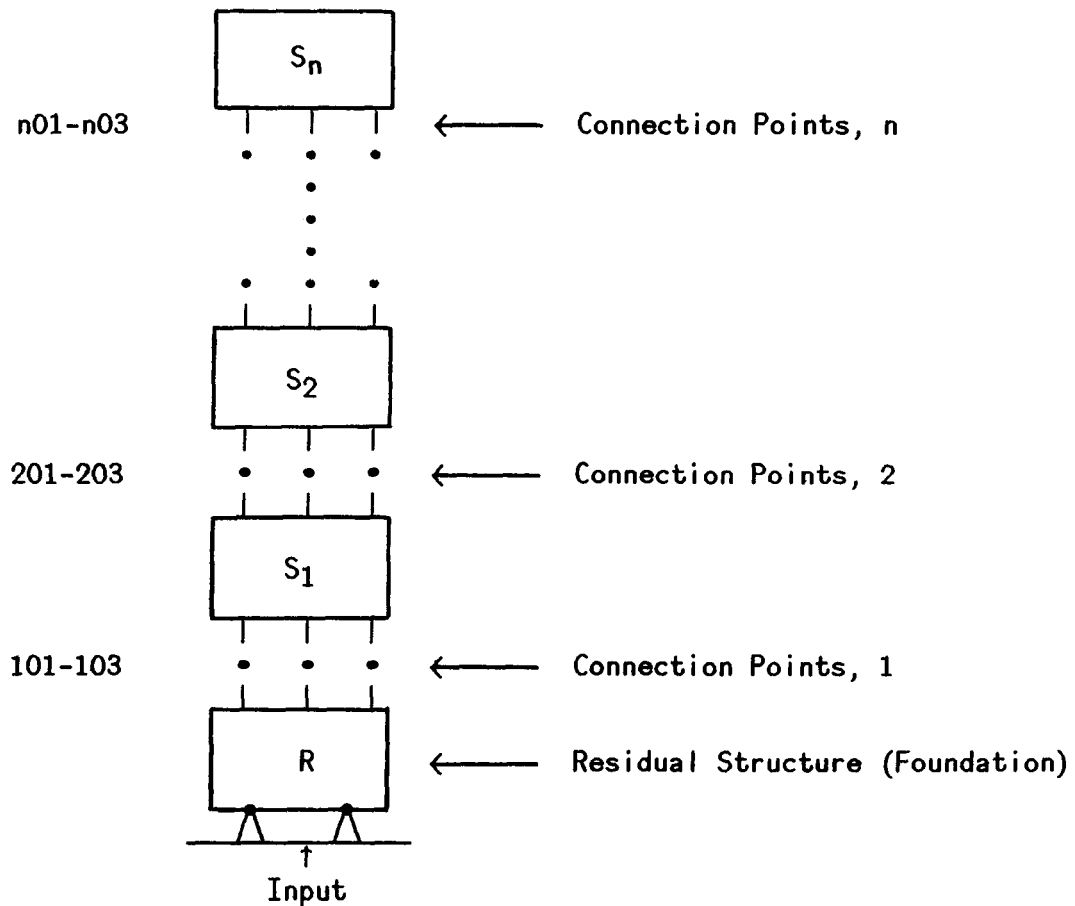


Figure 2. Case II: Series of Substructure S_1 to S_n

structure (to ensure a '0' deflection value). MPC equations can be written with respect to this extra point. This scalar point is the dependent point.

$$U_A + \phi_1^{21} \xi_1 + \dots + \phi_n^{21} \xi_n - \bar{\phi}_1^{21} \bar{\xi}_1 \dots - \bar{\phi}_n^{21} \bar{\xi}_n = 0 \quad (15)$$

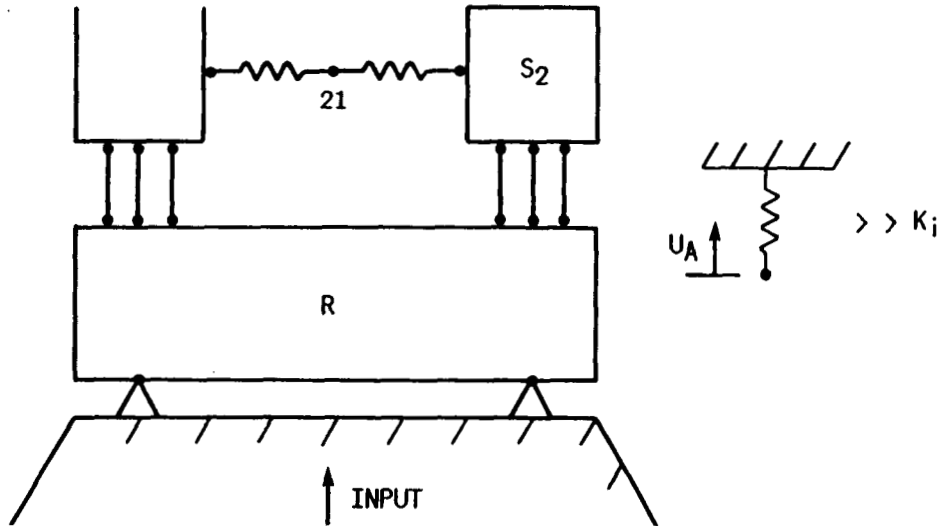


Figure 3. Case III: Substructures Connect to Each Other and to a Residual Structure, R

Since the stiffness of the extra scalar point A is high, $U_A \approx 0$, and

$$\sum \phi_i^{21} \xi_i \approx \sum \bar{\phi}_i^{21} \bar{\xi}_i \quad (16)$$

Thus, this approach results in approximately the correct MPC relationship defined in Equation (14) and retains all modes designated in the substructure.

DISCUSSION OF RESULTS

In order to test the theory developed for Cases I, II, and III, a series of test problems has been developed. In this section the models and results are described.

Case I: Single Substructure Attached to a Foundation

The basic theory was tested on the model described below and shown in Figure 4.

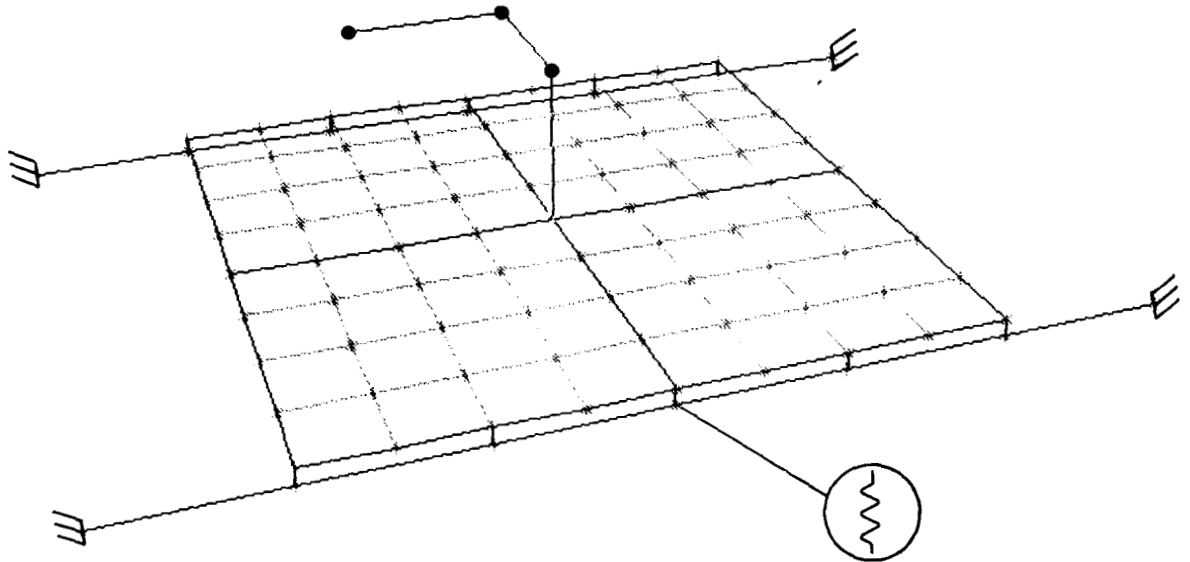


Figure 4. Case I: Basic Model of Single Substructure Attached to a Foundation

A beam stiffened flat plate and three directional cantilevered beam with a concentrated mass at the end were supported on three directional translational springs supported at each of 10 locations. The support locations are on the girders which are fixed at both ends. First, a response spectrum analysis was run on the complete problem with an arbitrary shock response spectrum providing a uniform base motion. Next, the structure was divided. The stiffened flat plate and cantilevered beam became the substructure while the springs and girders became the residual structure. An eigenvalue analysis was then performed on the unsupported substructure. Free-free modes, generalized masses, and stiffnesses were determined for 50 plate modes out of a possible 400 in one case and 25 plate modes out of a possible 400 in a second case. These, in turn, were used to represent the component substructure. The appropriate scalar points (generalized coordinates), generalized masses (concentrated mass cards), and generalized stiffnesses (spring cards) were built into the NASTRAN model of the residual structure. The connection to the residual structure which consisted of multipoint constraint equations were incorporated into the NASTRAN model via MPC cards. Programs have been written to automate the inclusion of the component modes representation of the substructure into the residual structure.

Next, an eigenvalue analysis of the reduced structure (see Figure 5) was performed.

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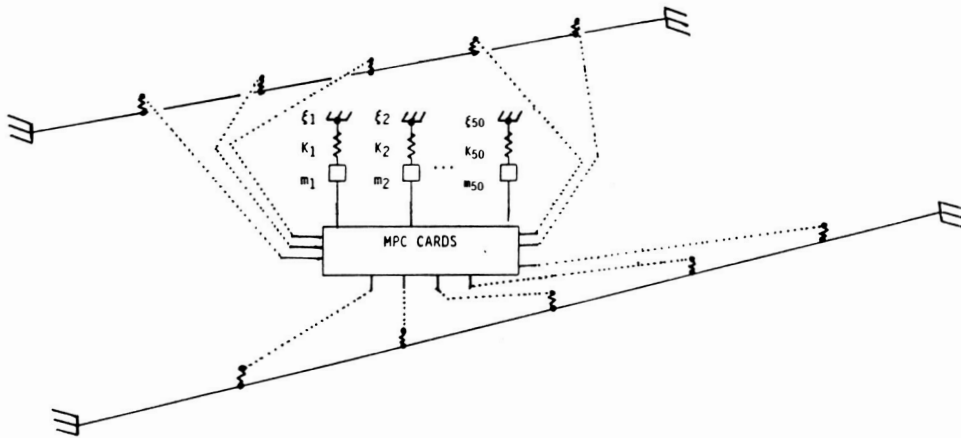


Figure 5. Case I: Reduced Dynamics Model of Single Substructure/Foundation

The eigenvalue quantities for the complete model and the reduced component mode representations are shown in Tables 1, 2, and 3, respectively.

The final step is to recover the displacements, forces, and stresses internal to the substructure. This calculation has not been possible to date in MSC NASTRAN for response spectrum analysis and was accomplished by using Equation 10. In response spectrum analysis, inputs are specified on a mode-by-mode basis. For U.S. Navy DDAM analysis, the total stress is determined from the NRL sum of the modal stresses. NRL contour plots for maximum principle stress due to vertical uniform base acceleration for the plate substructure for both the complete model and CMS models are shown in Figure 6.

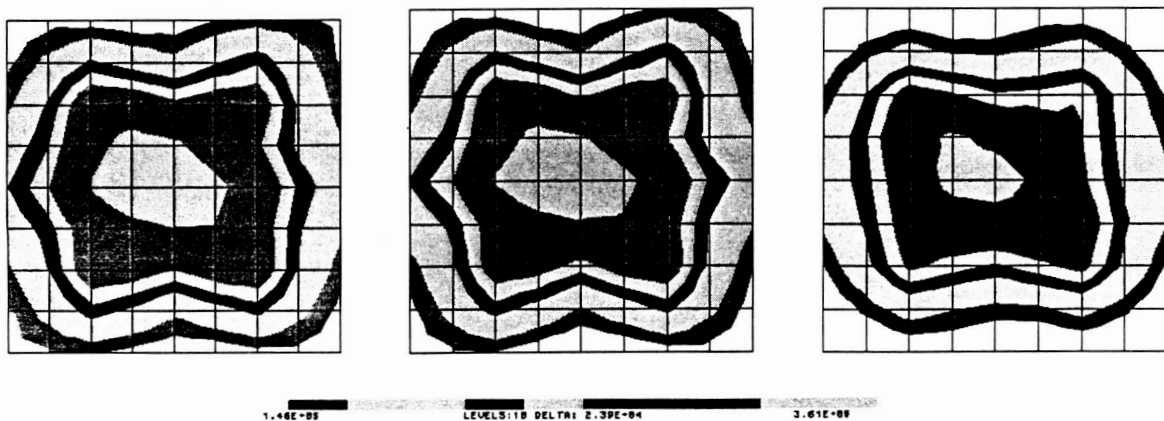


Figure 6. Case I: NRL Contours of Maximum Principal Stress for Complete Model, CMS Model (50 Modes) and CMS Model (25 Modes)

Table 1. Case I: Complete Model Eigenvalue Analysis Results

MODAL EFFECTIVE MASS TABLE

MODE	FREQ(HZ)	MODAL WEIGHT		CUMULATIVE WEIGHT		PARTIC.
		POUNDS	%	POUNDS	%	
1	0.832	50402.78	38.79	50402.78	38.79	1.14E+01
2	1.115	2629.47	2.02	53032.25	40.81	2.61E+00
3	1.464	31946.30	24.59	84978.55	65.40	-9.09E+00
4	2.358	470.23	0.36	85448.78	65.76	-1.10E+00
5	3.370	1038.63	0.80	86487.41	66.56	-1.64E+00
6	5.804	106.24	0.08	86593.65	66.64	5.24E-01
7	6.955	0.14	0.00	86593.78	66.64	-1.88E-02
8	9.022	0.36	0.00	86594.14	66.64	-3.06E-02
9	9.853	2681.81	2.06	89275.95	68.71	-2.63E+00
10	13.837	0.00	0.00	89275.95	68.71	5.01E-13
11	14.035	0.00	0.00	89275.95	68.71	-3.36E-04
12	17.952	18062.16	13.90	107338.12	82.61	6.84E+00

Table 2. Case I: CMS Model Eigenvalue Analysis Results (50 Modes)

MODAL EFFECTIVE MASS TABLE

MODE	FREQ(HZ)	MODAL WEIGHT		CUMULATIVE WEIGHT		PARTIC.
		POUNDS	%	POUNDS	%	
1	0.833	50179.50	38.62	50179.50	38.62	1.14E+01
2	1.116	2666.01	2.05	52845.51	40.67	2.63E+00
3	1.469	32074.36	24.68	84919.88	65.35	-9.11E+00
4	2.360	484.04	0.37	85403.91	65.73	-1.12E+00
5	3.376	1048.68	0.81	86452.59	66.53	1.65E+00
6	5.813	106.81	0.08	86559.41	66.62	5.26E-01
7	6.960	0.13	0.00	86559.53	66.62	-1.82E-02
8	9.061	0.35	0.00	86559.88	66.62	-3.01E-02
9	9.922	2731.86	2.10	89291.74	68.72	-2.66E+00
10	13.859	0.00	0.00	89291.74	68.72	-9.06E-09
11	14.051	0.00	0.00	89291.74	68.72	-1.00E-05
12	18.035	18966.68	14.60	108258.42	83.32	7.01E+00

Table 3. Case I: CMS Model Eigenvalue Analysis Results (25 Modes)

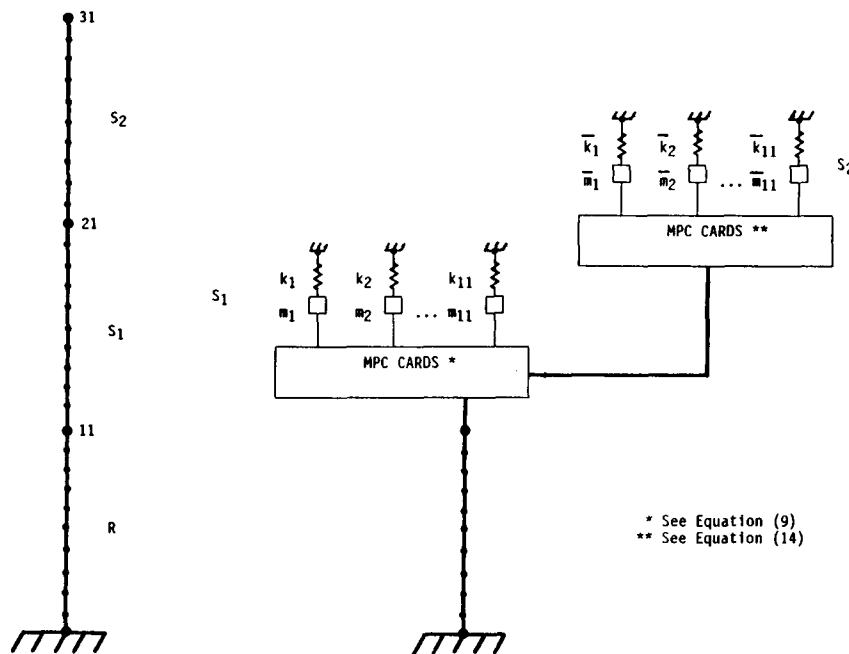
MODAL EFFECTIVE MASS TABLE

MODE	FREQ(HZ)	MODAL WEIGHT		CUMULATIVE WEIGHT		PARTIC.
		POUNDS	%	POUNDS	%	
1	0.839	50573.37	38.92	50573.37	38.92	-1.14E+01
2	1.128	2334.33	1.80	52907.70	40.72	2.46E+00
3	1.478	31913.13	24.56	84820.83	65.28	-9.09E+00
4	2.365	486.54	0.37	85307.37	65.65	-1.12E+00
5	3.382	1051.63	0.81	86358.99	66.46	-1.65E+00
6	5.823	103.41	0.08	86462.40	66.54	-5.17E-01
7	7.005	0.06	0.00	86462.45	66.54	1.19E-02
8	9.531	0.25	0.00	86462.70	66.54	-2.54E-02
9	10.352	2831.56	2.18	89294.27	68.72	-2.71E+00
10	14.269	0.00	0.00	89294.27	68.72	-2.83E-03
11	14.386	0.00	0.00	89294.27	68.72	-1.91E-08
12	18.481	24091.79	18.54	113386.06	87.26	-7.90E+00

The stresses and stress distribution show excellent agreement when 50 plate modes (up to 90 Hz) are used. Some accuracy is lost when only 25 modes (up to 30 Hz) are chosen, but the pattern is still similar, and peaks are in the same locations. It is, therefore, necessary to choose enough modes to ensure inclusion of the important ones.

Case II: A Series of Substructures Attached to a Residual Structure (Foundation)

Case I covered the methodology involved in connecting a substructure (generalized coordinates) to the residual (physical coordinates). Case II demonstrates the situation where two substructures are connected directly to each other. The technique, in the absence of physical points, is described in the section on theoretical development. The test problem for this case consists of a simple beam with one degree of freedom per grid point in the lateral direction (see Figure 7).



* See Equation (9)
 ** See Equation (14)

Figure 7. Case II: Basic Model of a Series of Substructures Connected to Each Other and to a Residual Structure

In this case, all 30 modes were considered in the complete model and all 11 modes in each substructure were retained in their component modes' representation. All output results, both in the eigenvalue analysis and internal displacements recovered in the substructures, agreed to nine significant figures.

Case III: Parallel Substructures Connected to Each Other and to a Common Residual Structure

The sample problem for this case is similar to that considered in Case I except that two identical spring supported plate structures are connected to each other with very stiff springs and supported on extended common girders. A plot of the model is shown in Figure 8.

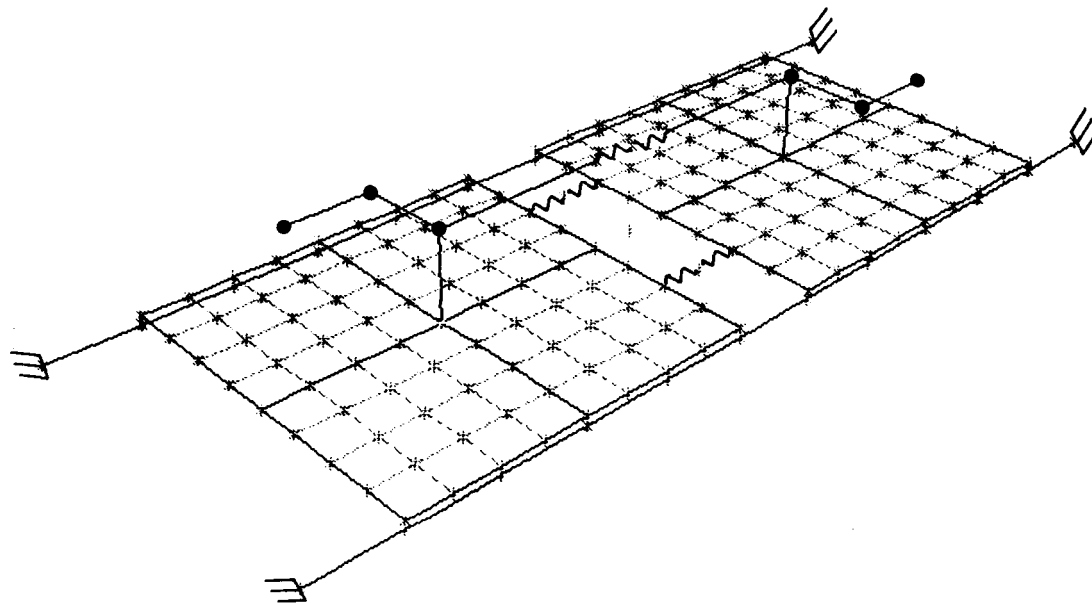


Figure 8. Case II: Model of Parallel Substructures Connected to Each Other and to a Common Foundation

The shock response spectrum analysis was again run for the complete model and free-free component modes were determined for the two substructures. The CMS analysis is similar to that of Case I with the extra constraint equation between substructures as in Case II, with the additional modeling technique described in the theoretical development of Case III. A new scalar point was defined and connected to a very stiff spring and used as the dependent degree of freedom in the constraint equations relating the two substructures. This allowed all other terms in the constraint equation to be independent which, for unknown reasons, was required in MSC NASTRAN for good agreement with the complete model (exact case). Eigenvalue results for both cases are shown in Tables 4 and 5. Stress contours are shown in Figure 9. Fifty modes were considered of a possible 400 in each plate. The agreement, again, is excellent.

Table 4. Case III: Complete Model Eigenvalue Analysis Results

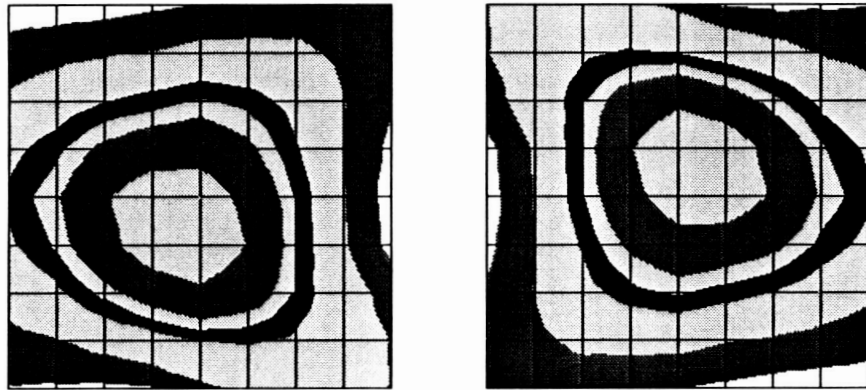
MODAL EFFECTIVE MASS TABLE

MODE	FREQ(HZ)	MODAL WEIGHT		CUMULATIVE WEIGHT		PARTIC.
		POUNDS	%	POUNDS	%	
1	0.842	6528.56	2.57	6528.56	2.57	-4.11E+00
2	0.902	0.00	0.00	6528.56	2.57	3.75E-08
3	1.275	44744.48	17.65	51273.04	20.22	1.08E+01
4	1.409	0.00	0.00	51273.04	20.22	2.78E-08
5	1.985	50777.22	20.03	102050.27	40.25	1.15E+01
6	1.985	0.00	0.00	102050.27	40.25	4.94E-06
7	2.603	42138.27	16.62	144188.53	56.86	1.04E+01
8	3.176	0.00	0.00	144188.53	56.86	-1.74E-09
9	3.566	2623.81	1.03	146812.34	57.90	2.61E+00
10	3.593	0.00	0.00	146812.34	57.90	-1.11E-08
11	5.728	0.00	0.00	146812.34	57.90	2.07E-09
12	5.815	4647.64	1.83	151459.98	59.73	3.47E+00
13	6.221	478.06	0.19	151938.05	59.92	1.11E+00
14	6.223	0.00	0.00	151938.05	59.92	5.47E-09
15	7.075	2392.79	0.94	154330.84	60.86	2.49E+00
16	7.683	0.00	0.00	154330.84	60.86	2.85E-10
17	9.271	0.00	0.00	154330.84	60.86	-5.53E-10
18	9.391	2050.21	0.81	156381.05	61.67	2.30E+00
19	10.232	0.00	0.00	156381.05	61.67	2.34E-12
20	12.625	0.30	0.00	156381.36	61.67	2.81E-02
21	13.756	0.00	0.00	156381.36	61.67	-1.02E-11
22	13.856	0.00	0.00	156381.36	61.67	1.15E-03
23	14.176	0.00	0.00	156381.36	61.67	3.48E-12
24	14.690	197.13	0.08	156578.50	61.75	-7.14E-01
25	14.828	3.84	0.00	156582.34	61.75	9.97E-02

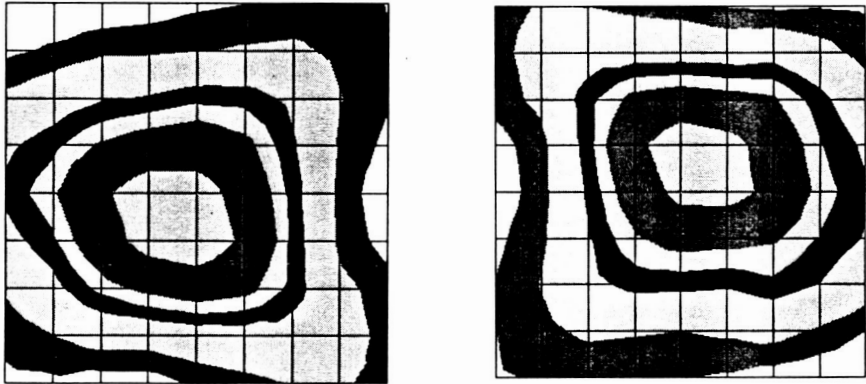
Table 5. Case III: CMS Model Eigenvalue Analysis Results

MODAL EFFECTIVE MASS TABLE

MODE	FREQ(HZ)	MODAL WEIGHT		CUMULATIVE WEIGHT		PARTIC.
		POUNDS	%	POUNDS	%	
1	0.844	6692.56	2.64	6692.56	2.64	4.16E+00
2	0.909	1.06	0.00	6693.62	2.64	5.25E-02
3	1.279	43532.91	17.17	50226.53	19.81	-1.06E+01
4	1.424	0.02	0.00	50226.55	19.81	7.57E-03
5	2.030	44787.91	17.66	95014.47	37.47	1.08E+01
6	2.161	18.66	0.01	95033.13	37.48	2.20E-01
7	2.650	49267.34	19.43	144300.47	56.91	1.13E+01
8	3.378	24.65	0.01	144325.13	56.92	-2.53E-01
9	3.611	1694.85	0.67	146019.98	57.59	2.09E+00
10	5.234	3.33	0.00	146023.31	57.59	-9.28E-02
11	5.830	3100.32	1.22	149123.64	58.81	-2.83E+00
12	5.836	1483.05	0.58	150606.69	59.40	1.96E+00
13	6.230	549.92	0.22	151156.61	59.61	-1.19E+00
14	6.242	17.05	0.01	151173.66	59.62	-2.10E-01
15	7.100	2620.00	1.03	153793.66	60.65	-2.60E+00
16	8.974	0.98	0.00	153794.64	60.65	-5.03E-02
17	9.465	2012.26	0.79	155806.89	61.45	-2.28E+00
18	10.303	3.59	0.00	155810.48	61.45	-9.63E-02
19	13.785	0.00	0.00	155810.48	61.45	2.53E-03
20	13.849	0.00	0.00	155810.48	61.45	-5.91E-04
21	14.355	0.01	0.00	155810.48	61.45	-3.72E-03
22	14.533	0.07	0.00	155810.56	61.45	1.38E-02
23	15.096	15.09	0.01	155825.66	61.45	-1.98E-01
24	15.148	85.75	0.03	155911.41	61.49	4.71E-01
25	16.808	41.10	0.02	155952.50	61.50	3.26E-01



3.70E+04 LEVELS:10 DELTA: 1.12E-04 1.38E+05



3.70E+04 LEVELS:10 DELTA: 1.13E-04 1.38E+05

Figure 9. Case III: Contour of Maximum Principal Stress for Complete and CMS Models

OTHER APPLICATIONS

The sample problems discussed in the previous section are very basic, but illustrate the effectiveness and accuracy of Component Mode Synthesis. In addition to the response spectrum analyses discussed, the methodology was also applied to transient analysis. The theory is the same. Recovery of displacements, forces, and stresses in the component substructure is done per timestep rather than per mode. Implementation in MSC NASTRAN required a different set of DMAP instructions consistent with the transient rigid formats. Preliminary results for Case I were excellent, within 2 percent of the complete case for component substructure principal stresses for all timesteps checked. Work in this area is ongoing.

Additionally, the technique is very useful in problems where reanalysis of large models would be prohibitive. One example is a system in the design phase where model changes are being made with each new analysis, particularly if one component of a system is changing frequently and the component mode representation of the other components remains unchanged. Eigenvalue analysis need only be performed for the changing component and the resulting system dynamic model is very small compared to a complete physical model of the system. For large problems, CMS can help analysts realize a significant savings in time and cost.

Another example where an even greater economy can be realized is that of a large nonlinear problem where only a portion of the system is nonlinear. This would be the case for resiliently mounted shipboard propulsion or generator equipment where the mounts exhibit nonlinear characteristics. The residual structure would contain the nonlinear part of the system. In the transient analysis where frequent stiffness matrix updates may be required, the matrix sizes are small because of the Component Mode Synthesis. Recovery of data for the component structures is still linear and based on the connection point displacement time histories from the nonlinear analysis.

CONCLUSIONS

Dynamic substructuring by component mode synthesis is an effective and accurate way to simulate the dynamic characteristics of large systems. If enough modes are considered, there is relatively little loss in accuracy, unlike simple dynamic reduction methods (i.e., static condensation, Guyan reduction). Work is ongoing to determine a concept (analogous to the modal mass for fixed base structures) to evaluate relative importance of modes for free/free structures. The reduction of model size afforded by CMS makes possible on moderate sized mainframes (such as a VAX 11/780) analyses that otherwise might require a supercomputer.

REFERENCES

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