N88-18957

Approximate Method for Predicting the Permanent Set in a Beam in Vacuo and in Water Subject to a Shock Wave

A. L. Stiehl
R. C. Haberman
J. H. Cowles

An approximate method to compute the maximum deformation and permanent set of a beam subjected to a shock wave loading in vacuo and in water was investigated. The method equates the maximum kinetic energy of the beam (and water) to the elastic plastic work done by a static uniform load applied to the beam. Results for the in water case indicate that the plastic deformation is controlled by the kinetic energy of the water. The simplified approach can result in significant savings in computer time or it can expediently be used as a check of results from a more rigorous approach i.e., a finite element solution. The accuracy of the method is demonstrated by various examples of beams with simple support and clamped support boundary conditions.

INTRODUCTION

Computer codes exist today to calculate the elastic-plastic deformation of structures subject to shock wave loading, such as the finite element codes ANSYS reference [1], and STAGS reference [2]. Although these and other codes offer the analyst a large variety of elements and several plasticity theories, the computational effort to calculate the dynamic plastic deformation of a large structure may be substantial. To reduce the computational effort, an approximate method based upon energy considerations was investigated. Calculated quantities of interest are maximum deformation and permanent set. The method equates the maximum kinetic energy to the elastic-plastic work done by a static uniform load applied to the structure. The kinetic energy is calculated from a relatively simple time dependent elastic analysis of the structure and the work is calculated from another relatively simple elastic plastic static analysis. Essential savings in computer time and effort on the part of the analyst can be realized by the simplified method. The method is very similar to approximate energy methods developed over a decade ago for structures in air subjected to impulsive type loadings, references [3], [4] and [5]. The primary differences are, (1) the structure (beam) is in water, and (2) the implementation of the method and validation is carried out with the use of a modern finite element code.

 PRECEDING PAGE BLANK NOT FILMED

To demonstrate the accuracy of the method, several examples are given. They consist of beams of various lengths and boundary conditions subject to shock wave loadings at normal incidence in vacuo and in water. The shock loading is from an exponentially decaying step wave, with decay time much smaller than the period of vibration. Under these conditions, the loading tends to apply an impulse to (at least) the fundamental mode of the structure, with the maximum kinetic energy occurring primarily in the fundamental mode at very early time and after the shock wave pressure has nearly diminished to zero. These are very important considerations in the energy balance method used herein because the method does not account for the additional external work performed by the shock wave pressure after the peak kinetic energy occurs in the structure. Furthermore, the uniform pressure used in the elastic-plastic static analysis exactly simulates the applied load distribution and approximates the inertia load distribution corresponding to the fundamental mode of the beam. For the case of the beam in water, the fluid reaction force applied back onto the beam is mathematically represented by the doubly asymptotic approximation, DAA reference [6]. This approximation is easily introduced into the dynamic elastic-plastic finite element model.

DESCRIPTION OF ANALYTICAL MODELS

Three analytical models were used in this investigation. They were, (1) an ANSYS dynamic elastic plastic finite element model to calculate the beam's deflection and kinetic energy (and water's kinetic energy) due to a shock wave, (2) an ANSYS static elastic plastic finite element model to calculate the beam's deflection and external work performed by a uniform pressure, and (3) a simplified two degree of freedom model to calculate the elastic plastic response of the beam's fundamental mode to the shock wave.

Fig. 1 shows the dynamic finite element model for the simple support condition. Fixed ends were also considered. The model consisted of eight equal length beam elements with dampers attached and mass elements attached to the opposite end of the dampers. The dampers and masses were used to model fluid-structure interaction effects. The dampers represent the radiation term of the DAA and the masses represent the virtual mass term of the DAA. In the case of the beam's response in vacuo, the damping coefficient and water mass were zero. The applied loading consisted of the time dependent blocked pressure of the shock wave.

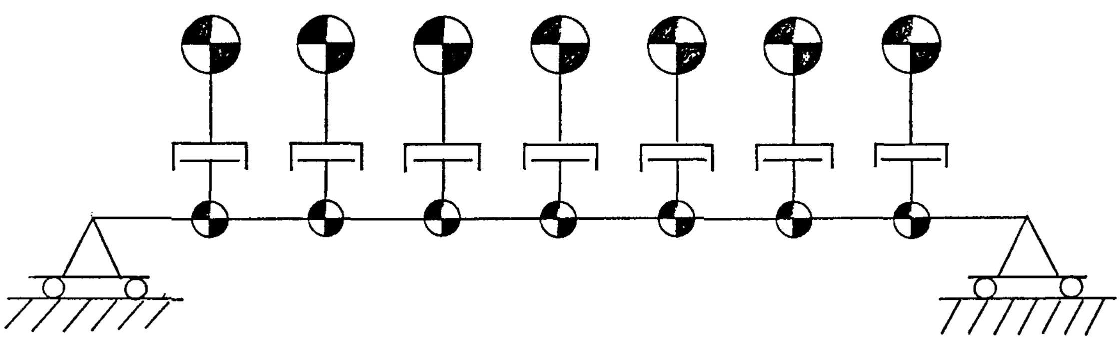

Figure 1. Finite Element Model

The elastic-plastic static finite element model consisted of eight elements also. This beam model was subjected to a statically applied pressure loading. The external work performed by the pressure was calculated with the use of this model by simply integrating the pressure-displacement response.

The simplified spring-mass-damper model is shown in Fig. 2. The spring characteristic is obtained from the elastic-plastic force deflection response of the static model, and the mass, damping, and force terms are obtained by applying Lagrange's equation of motion to the fundamental mode of the beam. This model can be used to calculate the maximum displacement of the beam and the permanent deformation. It will be shown to give very good comparisons with the ANSYS finite element model, thereby demonstrating the dominant contribution of the funadmental mode to the elastic-plastic response.

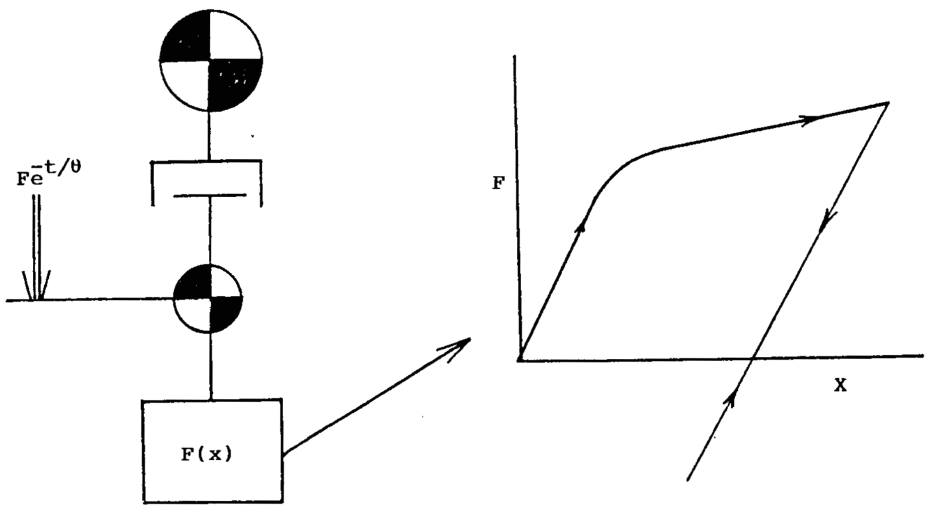

Figure 2. Two DOF Model

RESULTS

Several examples were used to investigate the energy balance method. Beams of 50, 75 and 100 inches long were considered, with simple support and clamped boundary conditions. The width and thickness were 6 inches and 2 inches, respectively. It is to be noted that the width dimension is completely arbitrary. The material was steel with a yield stress of 80000 psi with very little work-hardening. The shock loading pressure on the beam was mathematically expressed as $p = p_o \, e^{t/o}$ where P_o is the blocked pressure. A decay constant of o = .001 seconds was used which for most of the beams was small compared to the fundamental period of vibration. A blocked peak pressure of 2500 psi was used for the water case and for the air case, it was 667 psi. These pressures were chosen for the purpose of producing significant plastic deformation. Cavitation of the water was not accounted for in this investigation.

Fig. 3 illustrates the non-linear force displacement for the simply supported 100 inch long beam, and Fig. 4 illustrates the static work functions for the 100, 75 and 50 inch beams. Similar curves are obtained for clamped support conditions. The beam's dynamic maximum deflection predicted by the energy balance method is obtained by entering into Fig. 4 the peak kinetic energy of the beam (and water) induced by the dynamic load. The peak kinetic energy calculation for this study occurred during the very early time (elastic) response. The permanent deflection is obtained by subtracting from the maximum deflection a recoverable deflection the beam would experience under removal of a static uniform load.

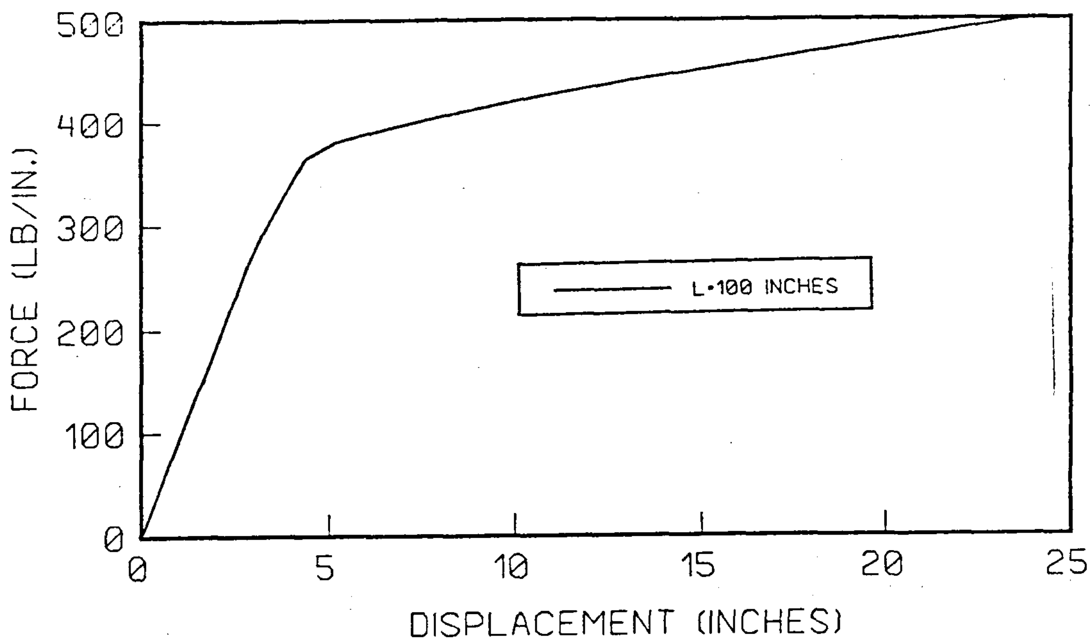

Figure 3. Force-Deflection for S.S. Beam

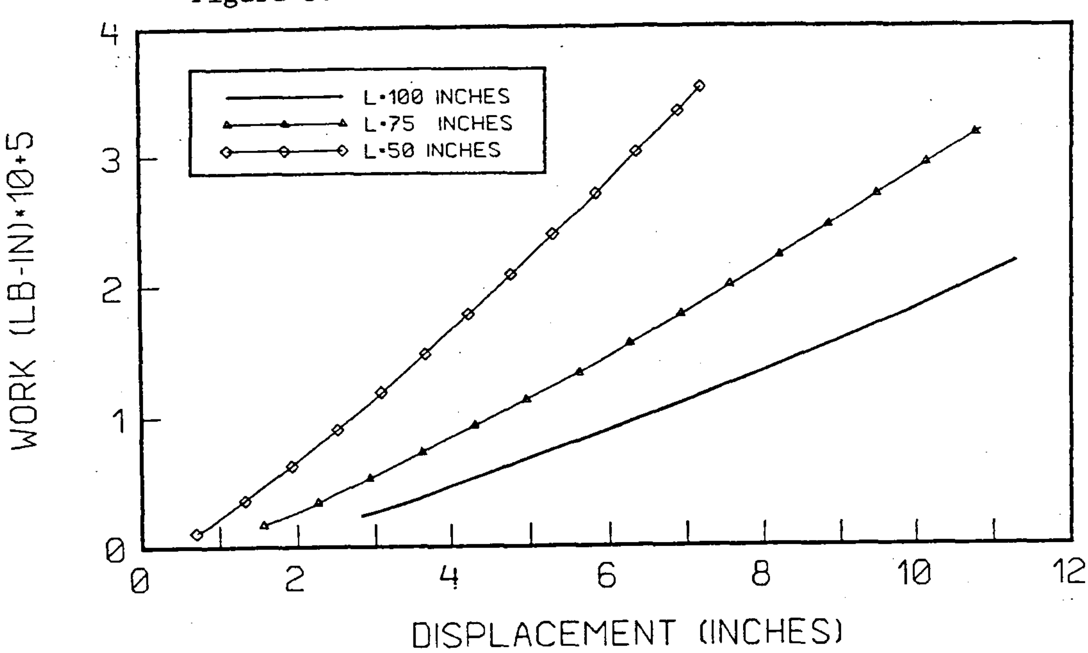

Figure 4. Work Functions for S.S. Beam

Fig. 5 illustrates the actual time dependent in water displacement response at the center of the 100, 75 and 50 inch clamped supported beams. The permanent set was obtained by drawing a mean line through the later time decayed response. The peak kinetic energy occurs at very early time as shown in Fig. 6 for the 100 inch clamped supported beam. This figure also demonstrates that most of the kinetic energy resides in the water (which is primarily converted to plastic work in the beam).

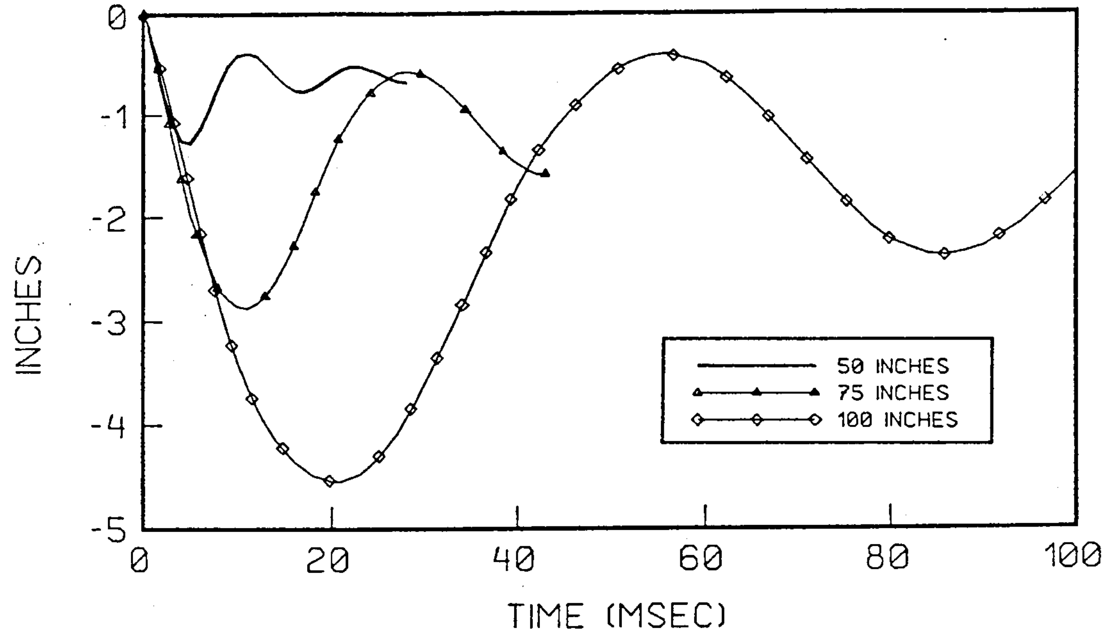

Figure 5. Displacement of C.S. Beam

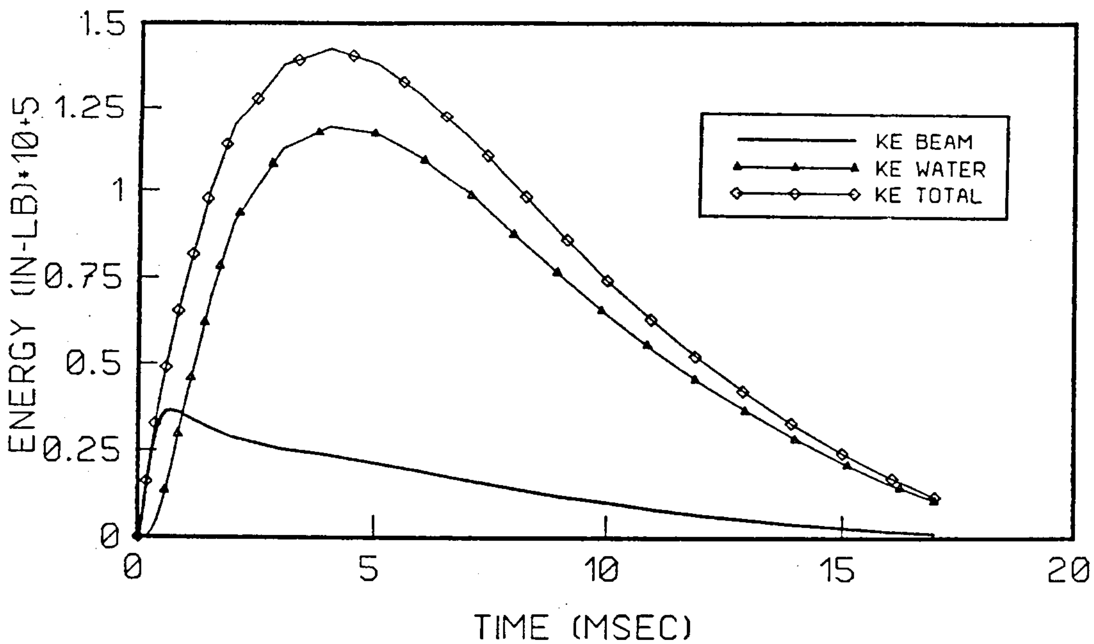

Figure 6. Kinetic Energy of C.S. 100 in. Beam

Table 1 compares the peak deflections and permanent deflections obtained from the ANSYS dynamic response model and the energy balance method, and Table 2 lists the fundamental period of vibration of the six beams. As seen, the comparisons are better for the longer beams. The reason is believed to be the longer period of vibration of the longer beams. As the period approaches, the decay time (.001 seconds), the kinetic energy of the beam reaches a maximum before the shock wave diminishes to zero. Thus there is additional dynamic work performed by the shock wave which is not accounted for in the energy balance method. Another consideration is the influence of higher frequency modes. If they are excited to any appreciable extent, the static work function would therefore not be a good approximation of the strain energy in the dynamically deformed beam. The static work function is only valid for beams responding primarily in their fundamental mode since the work function is based on a uniform pressure applied to the beam. A final consideration is the use of a uniform pressure in the generation of the static work function. A better approximation would have been a distribution corresponding to the fundamental mode of the beam, since it is the kinetic energy associatd with this distribution that is converted to plastic work in the beam.

Fig. 7 compares results of the spring-mass-damper two degree of freedom (DOF) model with the ANSYS finite element model for the clamped supported 50 inch beam in water. As shown, the comparison is reasonably good. Since this simple model is based on the fundamental mode of the beam, it may be concluded that higher order modes do not contribute significantly to the dynamic elastic plastic peak response.

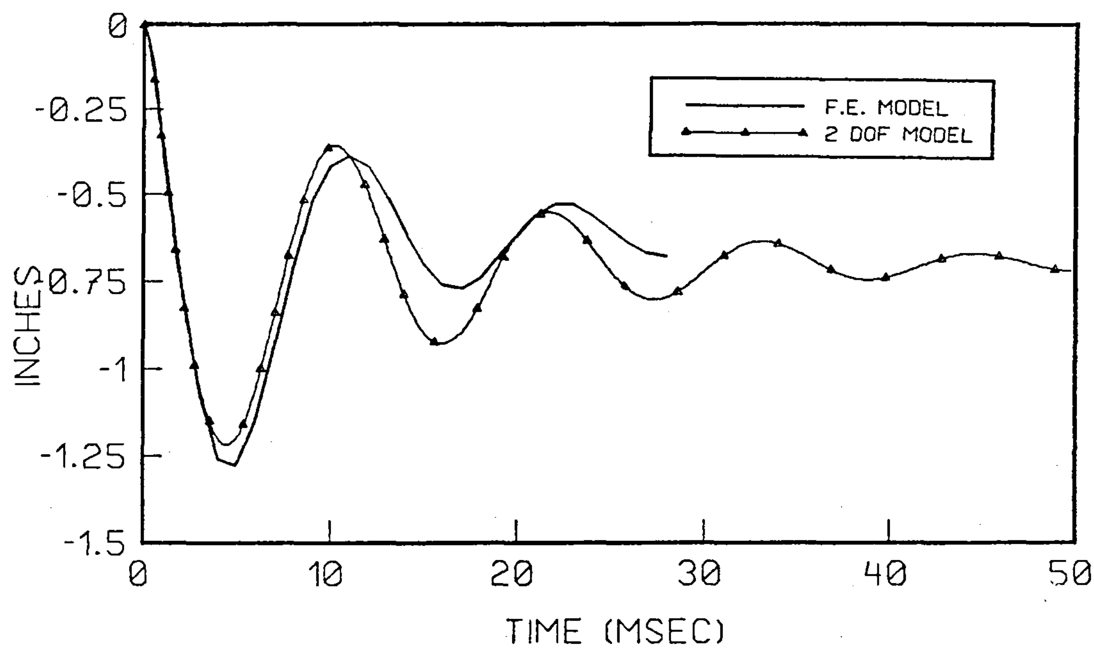

Figure 7. Displacement of 50 in. C.S. Beam

Table 1 Comparison of ANSYS and Simplifed Model

	PEAK DEFLECTION ANSYS (INCHES)	PEAK DEFLECTION ENERGY-BALANCE (INCHES)	PERMANENT DEFLECTION ANSYS (INCHES)	PERMANENT DEFLECTION ENERGY-BALANCE (INCHES)
Air				
100" Long Beam	5.9	5.9	1.8	2.0
Water				
Simple Support				
100"	10.7	10.3	5.5	5.8
75"	6.60	6.4	4.0	4.7
50"	3.58	2.9	2.0	1.7
Fixed Support				
100"	4.52	4.2	1.5	2.3
75"	2.87	2.4	1.2	1.25
50"	1.28	.8	.60	.29

Table 2 Fundamental Periods of Vibration

Boundary Condition	Length (inches)	Period (seconds)
Simple Support	100	.054
	75	.030
	50	.013
Clamped Support	100	.024
	75	.014
	50	.006

Using the two DOF models, Figs. 8 and 9 compare the kinetic energies for the simply supported long beam and clamped supported short beam with the external work of the applied shock wave, the lost work due to acoustic radiation, and the strain energy of deformation. For the energy balance method to be accurate, the peak kinetic energy should occur after most of the external work has been completed and before there is appreciable strain energy in the beam. This occurs in the case of the long simply support beam but not for the clamped short beam, and therefore helps to illustrate why the energy balance method performed better for the longer beam.

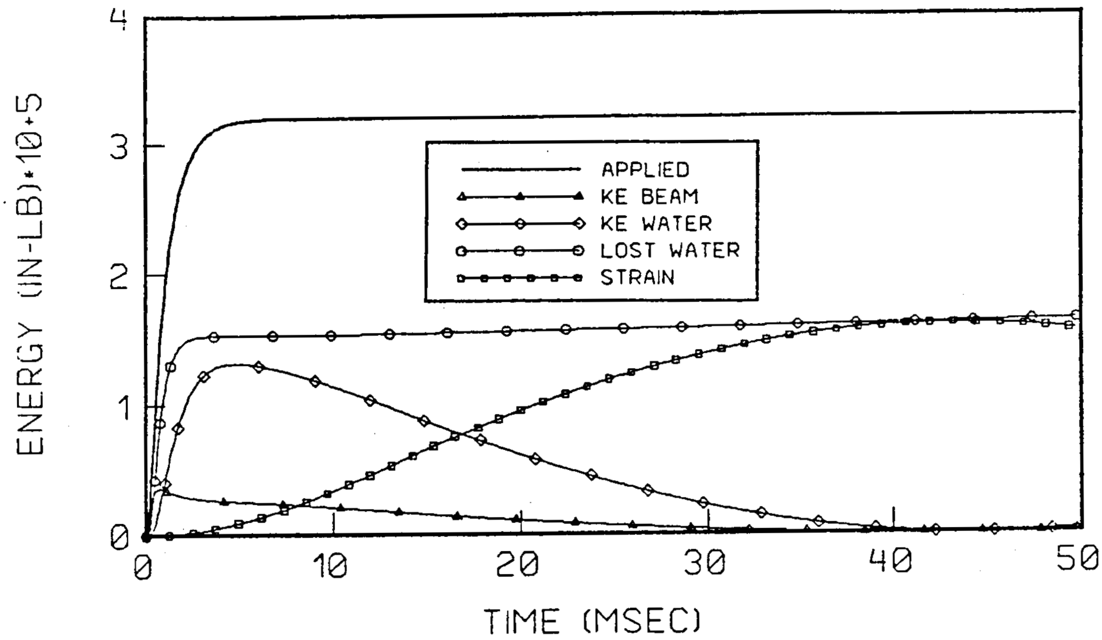

Figure 8. System Energies for 2 DOF Model, S.S. Beam, L = 100 inches

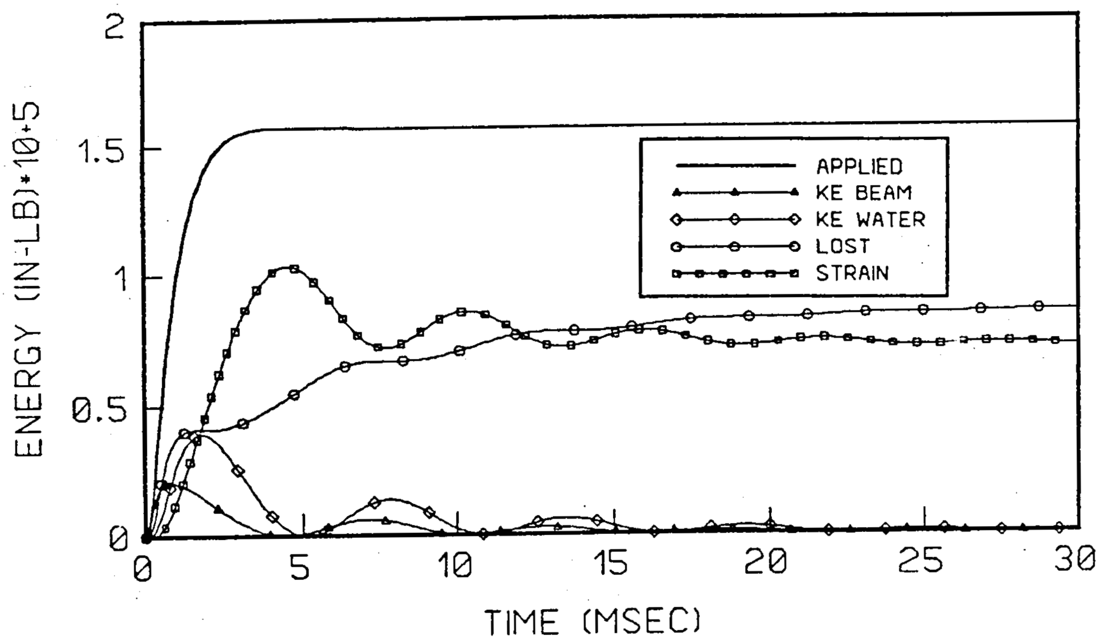

Figure 9. System Energies for 2 DOF Model C.S. Beam, L = 50 inches

CONCLUSION AND RECOMMENDATIONS

The energy balance method has been shown to give reasonably accurate results for the prediction of peak deformation and plastic set in a beam in vacuo and in water subjected to a shock wave whose decay time is small compared to the fundamental period of vibration. Results for the in water case reveal that the plastic deformation is primarily controlled by the kinetic energy of the water. Improvements to the method could be realized by (1) accounting for the actual mode shape in the static work function, and (2) utilizing the improved second order doubly asymptotic approximation, reference [6]. Further validation in predicting the actual in-fluid elastic-plastic response should be carried out by using fluid finite elements to model the fluid. Such a model could also investigate the importance of fluid cavitation. Consideration should also be given to using the energy balance method for other structures, such as plates and cylinders.

REFERENCES

1. "ANSYS FINITE ELEMENT CODE, REVISION 3.2.", Swanson Analysis Systems Inc., Houston, PA

2. B.L. Almroth, F.A. Brogan and G.M. Stanky, Structural Analysis of General Shells Volume II LMSC-D633873, Lockheed Palo Alt-Research Laboratory, January 1983

3. W.E. Baker, "Approximate Techniques for Plastic Deformation of Structures under Impulsive Loading", Shock Vibration Dig, Vol. 7, No. 7, pp 107-117, 1975

4. W.E. Baker, "Approximate Techniques for Plastic Deformation of Structures under Impulsive Loading II", Shock Vibration Dig, Vol. II, No. 7, pp 19-24, 1979

5. W.E. Baker, P.A. Cox, P.S. Westine, J.J. Kulesz, and R.A. Strehlow, Explosion Hazards and Evaluation, pp 300-330. Elsevier Scientific Publishing Co., Amsterdam, 1982.

6. T.L. Geers, "Doubly Asymptotic Approximations for Transient Motions of Submerged Structures", Journal of the Acoustical Society of America, Vol. 64, No. 5, pp 1500-1508, Nov. 1978