## NASA Technical Memorandum 100696

## Deformations in VLBI Antennas

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## Part I

## Structural Analysis with GIFTS on an Antenna

## 1 Introduction

This is the first part in the report on the project that has studied antenna structures. The aim of the project has been to estimate deformations in the large antennnas that are used in the VLBI project. As a test antenna the 85 foot dish in Fairbanks, Alaska was chosen.

It was immediately clear that a finite element code was needed. Such codes are now available for PCs. Several different packages exist and these were studied in detail, and it was easy to get free demos. The one I found most impressive was GIFTS. It's an interactive program and very easy to learn. The graphics is very nice and without it the task would have been much more difficult. GIFTS was installed on June 1, 1987. It's a program for both mainframes, such as IBM and DEC, and PCs. The version used in this project is installed on a PC - a COMPAQ/386. The micro processor is an Intel 80386 clocked at 16 MHz , and the coprocessor is a 80287 . The PC also has a 70 Mb hard disk drive.

The aim of this part is to describe the use of GIFTS and especially its application on an antenna model and to communicate some tricks of the trade. However, to actually do work with GIFTS the GIFTS Users Reference Manual (Ref. 1.) will have to be consulted for the exact syntax of these commands. The part only presents the most used commands and when to use them. This means that a lot of the capabilities will go unmentioned. This report also describes some programs written especially for this project and how to use them.

## 2 Drawings

Good drawings are of such importance that a section on it is warranted. To make a model and an analysis of an existing antenna a lot of information
is needed. First one needs to know the material used in various parts of the antenna. The material is often steel and sometimes aluminum. A code for the material according to ASTM (American Society for Testing and Materials) is very useful: one can then use tables listing the properties of this particular steel. Secondly, one needs to know the dimensions of the structure. Furthermore, one needs to know the coordinates for every point where two beams are joined, the so called nodes. An antenna is a very complex structure with thousands of different beams joined together. All these beams have different crossections according to their place in the antenna. Finally, one needs information on these crossections.

The best source of information is the actual design drawings. However, the antennas are sometimes more than twenty years old and it can be very hard to obtain drawings on these. Furthermore, the antennas sometimes have undergone major renovations that have led to modifications of the structure. Drawings of these are then also needed.

The alternatives are to use photos, which will at least give approximate coordinates of the nodes, and meaurements on the actual antenna. This will, however, result in a less accurate model, because a lot of information can not be obtained this way.

## 3 An overview of GIFTS

The GIFTS program consists of several different processors. Each processor performs a special task (e.g. applying loads to a structure or defining boundary conditions). These processors have to be run in a certain order in a complete analysis.

A brief description of an analysis of an antenna is as follows. From a detailed set of drawings crossections of the different beams are defined. The coordinates of the nodes (the point where beams are joined together) are entered. The beams and their orientation with the correct crossection are specified - in the language of finite element analysis: the elements are defined. Then boundary conditions are entered (e.g. nodes that are not moving because they are rigidly linked to the ground). Forces are applied. This can be for example gravity or thermal loads. GIFTS then solves a large matrix equation and the result is deformation (and stresses) at all
nodes.
Now all of the different steps described above are performed by a specific processor, some by the same. The analysis is simplified by the use of batch files that call different processors automatically. These batch files are easy to modify for the application that one is interested in. However, the actual specification of the model has to be done "by hand", but there are a lot of tricks that simplify this work.

Aside from calculating deformations there is also the possibility of doing dynamic analysis, extracting eigenfrequencies in the structure, calculating heat transfer and doing gap analysis. Larger models are made easier to handle by the use of substructures.

All relevant processors needed for the study of an antenna are installed on the D: drive in directory GIFTS. (There are still more processors on the floppy disks that contains the complete program, e.g. solid elements, but these are not installed to save disk space.) The work on models is recommended to be done in subdirectory JOBS to protect the program. Each job has many files with the same name - the jobname one chooses - but with different extentions (e.g. dish.blk, dish.ptr ...). To enter a processor one just types its name, and one will then be promted for the name of the job one wishes to work on; to exit a processor one types QUIT.

There are two types of processors: interactive and computational. The interactive are used to build the model, define loads and boundary conditions; in each interactive processor there is a set of commands one can use. The computational processors do not accept commands from the user, they just perform a part of the calculation needed to compute the deflections. To some extent it's possible to go back and forth between the different interactive processors, but some require that certain tasks have been accomplished before they are used. The computational processors can only be run in a specified sequence.

A big model requires a lot of commands in the interactive processors. GIFTS has the capability of using what GIFTS calls "ON-LINE-BATCH", command OLB, which makes it possible to make a file (with for example EDLIN or WordStar) and then run it through the processor. These files should have the extension .SRC. It's a very nice feature; it's possible to insert comments about the model in these files which gives them structure and makes them easy to understand. The file also saves all the work one
has made on the model and can at a later time be changed and improved. In the following section the most used processors will be described.

### 3.1 BEAMCS

BEAMCS is a processor that one uses when one wants to specify the crosssections of the beams in the antenna. In this connection it's useful to have a copy of the AISC (American Institute of Steel Construction) Steel Construction Manual that contains the dimensions and codes for all standard beam crossections.

There are of course different types of crossections. The most common in antennas have been tbeams, angles, ibeams and circular and rectangular tubes. The commands used are TBEAM, ANGLE, IBEAM, CIRCS, CIRCH and RECTS. Each crossection one enters is identified with a number that later is used to refer to it. After the crossection is defined it's useful to plot it, see Figure 1. Note the P and Q axes that define the principal axis of the moment of inertia. Also note the direction that helps to orient the crossection in the model. For further discussion on this see the next section on EDITM. In the figure the point $A$ which is the attachment point to the node. The attachment point can through special commands be moved to suit the use.

### 3.2 EDITM

The processor EDITM is the one that is used the most while working on an antenna model. With this processor one specfies the nodes and how they are connected with each other through different beams.

But first of all, it's important to know what coordinate system one uses. The global one - the one used if no commands specifying another are present - is a cartesian coordinate system. With the command LOCSYS it's possible to change the coordinate system to cylidrical, spherical or a different cartesian coordinate system with different orientation and origon. These coordinate systems can be used on different parts of the model and are a great help to simplify the work. For example, when working on an antenna dish a cylindrical coordinate system is excellent.


Figure 1: A plot of a crossection (ANGLE) defined in processor BEAMCS. Note the principal axes $P$ and Q and the direction NP3. The latter is important when one defines the orientation of the crossection in the model. The table contains information on the dimensions and the mechanical properties.

After a suitable system has been choosen the nodes can be specified. This is done with the command POINT. Each point is defined by a number that is used to refer to it and the coordinates. Command POINT/11/1,2,3/ defines point 11 at $(1,2,3)$. One can also use a repetitive command
POINT/11,13,3/1,2,3/3,6,9/
which defines points 11,12 and 13 at $(1,2,3),(2,4,6)$ and $(3,6,9)$. This is very useful in a dish defined in a cylindrical coordinate system were there is a lot of repetition around the axis of symmetry.

It's also important how one numbers the nodes, see Figure 2. for a useful way to number nodes in a dish. The trick is to have some logic or system in the numbering that makes it easy to remember where different nodes belongs. Believe me, when the number of points gets large it gets really hard to keep track of them. Preferably, one should also keep some notes for later reference on how the numbering was done.

When all nodes are specified it's time to start connecting them with beams. First one chooses the material in the beam with the command PTRM and the crossection with the command PTRTH. Finally, the command BEAM2 is used to specify the beam. BEAM2/1,2,3/ defines a beam between nodes 1 and 2 with the reference point in the crossection NP3 in the plane spanned by nodes 1, 2 and 3. See Figure 3. and note the orientation of the beam crossection and the command sequence.

The command PLOT plots the model and it's possible to plot different crossections in different colours. WI lets the user zoom in on a desired point. There are commands that rotate the model ROTV and introduce perspective (VDIST). All these are wonderful when one wants to check the model.

### 3.3 LOADBC

After one is finished with BEAMS and EDITM the model is ready for application of loads and boundary conditions. This is done in processor LOADBC.

Certain points in the model have to be attached rigidly to the reference frame, i.e. their deflections are prescribed to be zero. If not, the matrix in


Figure 2: This is a useful way to number the nodes in an antenna dish. (One is looking into the dish from the receiver.) It's important to have some system in the numbering; it makes it easier to remember where different nodes belong.


Figure 3: There is an important relationship between the command sequence BEAM2/N1,N2,N3/ and how the beam is oriented. This is illustrated in the figure. The beam is connected between nodes N1 and N2 and the direction NP3 (see Figure 1.) points towards node N3.
the equation is singular. This means that the model collapses or that it is possible to rotate or translate the whole or some part of the model without resistance. The points that are chosen to be fixed are suppressed with the command SUPP. Other more selective commands that only suppress some of the six degrees of freedom in the nodes (three rotations and three translations) also exist.

Gravity loads are applied by using the command TRANACC that applies an acceleration of specified magnitude and direction to the model. Before this is done the mass of the model has to be calculated, done with command MASS. This command uses the density in the specified material and the lengths and crossections in the beams. It's also possible to apply point masses with command PMASS.

Loads due to changes in temperature are applied in two steps. The command PTEMP specifies the temperature in the nodes. The beam between two temperatures is assumed to have the average of the two temperatures. To get the actual loads in the structure one has to exit LOADBC and use prossesor THERLD which calculates the forces due to the thermal expansion.

Finally, the command LOADP makes it possible to directly apply a load at a desired node.

### 3.4 OPTIM

As mentioned before the deformations in the structure is calculated by solving a matrix equation. Let's write it

$$
\begin{equation*}
F=\mathbf{K} \delta \tag{3.1}
\end{equation*}
$$

where $\delta$ is a vector of the deformations that one wishes to calculate, $K$ the matrix representing the stiffness of the model - called the stiffness matrix and $F$ a vector containing the loads applied to the model. The purpose of OPTIM is to reduce the time it takes to solve this equation to a minimum.

The time it takes to solve Eq. 3.1 is proportional to the square of the matrix bandwidth and to the number of unknowns. The number of unknowns is the number of nodes times the number of degrees of freedom of deformation at each node (as mentioned before: three rotations and three translations). The number of unknowns cannot be changed. However, it's
possible by renumbering the nodes in a smart way to reduce the bandwidth, and this is exactly what OPTIM does.

The bandwidth of a matrix is small when the nonzero elements are close to the diagonal. One achieves this by numbering nodes that are connected to each othe with numbers that are close. OPTIM requires a starting "wavefront" a set of nodes to start renumbering from. Then OPTIM works its way through the model like a wave to the connecting nodes and renumbers them sequentially.

Trial and error is the best way to minimize the bandwidth; one can enter several wavefronts in OPTIM and then OPTIM chooses the best one. One general rule one can use is that wavefronts at short edges of the model are generally good.

### 3.5 Macro EASY

EASY is a batch file that calls the processors ADSTIF, ELSTFF and STASS that calculates the stiffness matrix K. Then it calls DECOM the processor that inverts the matrix, and finally it calls the processor DEFL that calculates the deflections. All these processors are computational.

There is the possibility of using other macros, e.g. MSTATIC that in addition the above also calculates the stresses in the beams.

### 3.6 RESULT

This processor is used to view the results of the calculation. One uses the same plotting commands mentioned in the section about EDITM. The structure is plotted with the deformations exaggerated but still in scale. PLOT, 1 plots the deformed structure with the undeformed structure outlined, and INFDN lists the deflections at desired nodes.

A nice feature when studying antennas is the possibility of making contour plots of deformation in the dish surface. This is done with the command CONTOUR. This requires however that one has specified plate elements, e.g. TB3, in the surface, because these are needed to calculate the contours.

## 4 More useful features

This section contains comments on some more processors and commands that are useful.

After a complete analysis all the files can take up a lot of disk space, sometimes the whole disk. If one wants to save the results for later use one can use the processor DBCOND that compresses the files (e.g. 20 Mb to to 1 Mb ).

In EDITM, where as mentioned before the major part on the antenna is done, there are some more useful commands. When the model gets large it's sometimes useful to plot the outline of the structure. These lines are defined with the command PLOTLINE. One can switch between plotting lines or elements with the commands LINES and ELE. Another way to get a better view of the model is to use the command sequence ELE, 25 that shrinks the elements 25 percent in the plots.

In LOADBC there are also a variety of useful commands. When the model has symmetry it's not neccesary to model the complete structure. It's sufficient to use just one part of the model, see Figure 4. The command SYM or ASYM applies the correct boundary conditions at the cut. Which one one should use depends on the loads. In the situation in Figure 4. one should use SYM because the forces are also symmetric with respect to the cut. If the forces were horizontal then one should use ASYM. Another very useful command is LDCASE. This makes it possible to switch to a different loading case and to calculate the deflections due to different load situations in the same analysis. The results can then later be combined. For example, an antenna dish moves through different elevations. If one defines load case 1 to be when the antenna points at the zenith and load case 2 to be when it points at the horizon, see Figure 5, then forces at different elevations are a linear combination of these two cases. And because Eq. 1 is a matrix equation, the deflections are also the same linear combination of the calculated deflections for the two loading cases. The combination of the deflections from the different load cases is done in RESULT with command COMP.

One can plot the models and print information with special commands. To plot a model one selects a nice view of the model and then types HCON. One is then prompted for a name of the output file; let's call it PIC. Then


Figure 4: When there is symmetry in the model only part of it needs to be modeled. It's enough to model just half an antenna dish. The gravity loads, direction downward, are also symmetric around the cut, and boundary conditions are applied along the cut with command SYM.


Figure 5: The forces due to gravity in an antenna at different elevations are a combination of two load cases. The force at $\alpha$ elevation is $\sin \alpha * L C 1+\cos \alpha * L C 2$. Call the deflections calculated for the two load cases $D F 1$ and $D F 2$. Then the deflection at $\alpha$ elevation is $\sin \alpha * D F 1+\cos \alpha * D F 2$.
one types PLOT and the information is written to PIC.HCY. When one is finnished one types HCOFF. GIFTS has several processors that transform PIC.HCY into a file PIC.P01 that can be sent to a plotter. DRHP is used for the HP 7475 in the computer room; DRHI is used for the Houston Instruments DMP-40 in Bill Boyers office. Copy PIC.P01 on a floppy and put it in drive A of the computer that is connected to the desired plotter. To send the file to the plotter type: COPY A:PIC.P01 COM2.

If one wants to list information to the printer type LPON. All the commands thereafter that list information will send the information to the printer connected to the COMPAQ. When one is finished one types LPOFF.

## 5 Substructures

The model of the Fairbanks antenna was done in several steps. Each major part of the antenna (pedestal, $x$-wheel, $y$-wheel, dish and quadrupod) was modeled separately. It's possible to use some of these src-files in the same job - the files have to be carefully done so there are no overlapping numbering of points or crossections, and the coordinate systems have to be carefully chosen. This is like running a very large src-file in small steps. The model becomes quite big, sometimes too big for the PC to handle.

The other alternative is building the total model with substructures. The substructure idea is to make the inversion of matrix $K$ in several minor steps, or to consider the major parts (consisting of a lot of elements) as one big element. Take for instance the $x$-wheel; it's connected to the pedestal at two points and to the $y$-wheel at another two points. It can be thought of as an element having four points where it can be connected to nodes. Calculating the properties of this element is nearly the same as doing a part of the big matrix inversion.

Suppose the model of the $x$-wheel is finished and that one wants to make a substructure out of it. Here's the way. Build the model with BEAMCS and EDITM. Apply loads but no boundary conditions. Run OPTIM and then DEFCS; this is the correct order and not mentioned in the manuals. This processor allows one to give the substructure a name, e.g. xwheel, and to define the external points. External points are those points one later wants to connect to other elements. Then calculate the stiffness matrix and
decompose it. Finally run REDCS. This processor calculates the properties of the substructure and transformations between the reduced properties and the full model. This makes it possible, when one knows the deformations in the external nodes, to calculate the deformations in the whole structure of the x -wheel.

The substructure can now be used as a regular element in other models. Figure 6. shows a plot of the antenna consisting of several substrutures. The substructures are plotted with lines between their external points. Instead of BEAM2 one uses the command COSUB to define the position of the substructure - element. Gravitational loads and masses should be computed before the substructure is calculated, otherwise the results get very strange. In the main model where the substructure is used the forces are transferred to it with the command LOADCS.

When the main model is finished and all forces and boundary conditions applied it is analysed as a regular model. As a result one gets the deflections in the external nodes. If one wants to examine the substructure in more detail one can use the processor LOCAL that calculates the deflections in the internal points. These are then examined in the processor RESULT.

The one major difficulty is keeping track of the orientation of the substructures and the numbering of the external points. Detailed notes are the best help.

## 6 Program PARABOL

Program PARABOL is a program that fits a paraboloid to a deformed antenna dish. The code was originated at JPL by S. Katow (Ref. 2-3). A listing of this program adapted in BASIC for the COMPAQ/386 is found in Appendix A, and in Appendix B there is a listing of a full run of the program. The program is installed in the same diretory as GIFTS (drive D:, directory GIFTS). PARABOL.EXE contains the executable file and PARABOL.BAS contains the code. In addition to the code for the best fit paraboloid, code has been added to manage files and to calculate the change in average pathlength to the receiver.

To calculate the best fit paraboloid PARABOL needs the calculated deformations at the nodes that lie at the dish surface. This is done in two


Figure 6: This is a plot of a model consisting of four substructures (pedestal, x -wheel, y -wheel and quadrupod). These substructures internally consist of many elements. They are plotted with lines connecting their external nodes - the nodes that can be connected to other elements and substructures. Note the forces plotted as arrows.
steps. First one has to use the processor GFTOUT in the GIFTS package. GFTOUT is a processor that writes data from the GIFTS files to ASCII files. It's possible to select a lot of different data and formats with the processors commands. Program PARABOL is written in such a way that it requires the following steps in GFTOUT. First the nodes have to be sorted in sequential order according to their numbering. This is done with the command SORT/POINT/1. Then these need to be written to the ASCII file together with the global coordinates of the points, command $\mathrm{PW} / \mathrm{NU}, \mathrm{XC}, \mathrm{YC}, \mathrm{ZC} / 5,-2,10,-2,10,-2,10 /$ where the string of numbers formats the output. Finally, the deformations are written to the file with the command DNSW/NU,XD,YD,ZD/5,-2,10,-2,10,-2,10/. All these data are written in the same file name JOBNAME.OUT, where JOBNAME is the name of the model and job one is working on.

This file may need some editing (e.g. with WordStar). As mentioned above the file contains first the coordinates of the points in sequential order followed by the deflections. Now, GFTOUT has the feature that it lists all the points, but it only lists the deflections of those points that are not suppressed in LOADBC. For PARABOL to work the coordinates of the these points have to be deleted. This is easy, just look at the deflections and note which are missing and then delete these in the coordinate listing.

The program is run by typing PARABOL. The first time it is used on a job one has to chose the options NEW JOB and NEW AREAS.

For PARABOL it's also important that the nodes at the dish surface are numbered in a certain way, see Figure 2. The first rib in the dish should be numbered in the 100 's, $101,102,103, \ldots$ and then the following rib 200 and so on. This is needed because in PARABOL one needs to know which nodes are on the surface. Suppose the model contains 25 ribs and that points $101,201, \ldots, 2501$ are on the surface and so on for all points up to $110,210, \ldots, 2510$. Then this is specified in PARABOL with the boundaries 1,10 and maximum point 2600. These numbers are asked for when the program is run. PARABOL then ignores all points less than 100 or greater than 2600 and only takes into account those that end with $01, \ldots, 10$. The program can handle the two first loading cases. These are written into the files JOBNMAME.DF1 and JOBNAME.DF2. The selected coordinates are written into JOBNAME.COR

The program also asks for the last point number in the listing of the
points and the focal length of the undeformed dish. The program then asks for the areas at the nodes that are of interest. These have to be calculated in advance, and weighted with the illumination over the dish, i.e. how the receiver responds to the different parts of the dish surface. This information is written into file JOBNAME.DAT .

At a later time, if one saves the above files, it's possible to run PARABOL and use option OLD JOB, then one doesn't have to enter all this information again.

The program is then ready to calculate the best fit paraboloid. This is done by minimizing the integrated square of the surface error over the dish. The parameters that are optimized are: focal length, vertex postion and rotation about the x and y -axis. The original paraboloid is assumed to have its focus on the $z$-axis and the vertex in the origo. There is the possibility supressing some of these parameters. Suppose the model is half a dish, see Figure 4., using the symmetry in the calculations of the deflections with the part that has negative $y$-coordinates cut away, and the deflections as a result of forces paralell to the xz-plane. Then one doesn't expect the the vertex to move in the $y$-coordinate and any rotation around the $x$-axis, these two parameters have to be set to zero. In addition to the calculation of the best fit paraboloid the the surface rms error is also calculated and presented. The two different load cases can be combined to yield the deflections at different elevations, and thus the corresponding best fit paraboloid.

The other calculation the program can do is to calculate the change in average pathlength to the receiver compared to the undeformed dish. This is also called time delay in the program and is calculated by dividing the change in path length with the speed of light (metric units used, $c=$ $3 \cdot 10^{8} \mathrm{~m} / \mathrm{s}$ ). The deflected postion of the receiver has to be entered. Again the load cases can be combined. The change is presented in two different ways of calculation: linearized and "exact". They usually differ in the third digit.

## 7 Program WINDLOAD

The program windload calculates forces due to wind. It uses results from wind tunnel tests performed at JPL (Ref. 4.). These tests have resulted
in pressure coefficients over the anntenna dish for different elevations of the dish. These pressure coefficients are in files WINDXXX.PRS where XXX is the elevation in degrees. Appendix C contains a list of this program; Appendix D contains the coefficient files and a listing of a run with this program. The program is installed in the same directory as GIFTS and PARABOL. WINDLOAD.EXE is the executable file and WINDLOAD.BAS contains the code.

The program assumes half a dish oriented as in Figure 4., note the coordinat system in the lower left corner. To run this program one again has to use GFTOUT; this time it's sufficient to list only the coordinates of the nodes in a sorted sequence.The commands are SORT/POINT/1/ and PW/NU,XC,YC,ZC/5,-2,10,-2,10,-2,10/. One has to select one of the above files and also, as in program PARABOL, select the nodes at the surface. The program needs the air density, the wind speed and the areas at the selected nodes. The program calculates the wind forces (normal to the surface) at the nodes and writes them to file WINDLOAD.SRC. This file can then be used in LOADBC together with the OLB command to apply the wind loads to the structure.

The program uses the following formula to calculate the wind pressure p :

$$
\begin{equation*}
p=C \frac{\rho}{2} v^{2} \tag{2}
\end{equation*}
$$

where $C$ is the pressure coefficient (depends upon the surface position of the node), $v$ the wind speed and $\rho$ the air density. The pressure coefficients are interpolated from the values in the files and the coordinates of its position. It's thus important that one has used a consistent set of units.

The program only calculates the forces on the dish surface. The loads on the backup structure or the different beams are not taken into account. Neither is the influence of these on the wind flow taken into account. The coefficient from the wind tunnel measurement are for a dish with no backup structure and $f / D=0.33$. The antenna at Fairbanks has a large backup structure and $f / D=0.42$. Thus the calculated forces are rather appoximate in this case.

## References

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## 8 APPENDIX A: Listing of program PARABOL

```
1000 OPTION BASE I
1010 DIM A# (6,6), B# (6),AR#(100): FOR M=1 TO 100: AR# (M)=1#: NEXT M
1020 DIM R#(100),N#(100),A(100), B(100),NCO(100)
```



```
1040 PRINT " PROGRAM PARABOL"
1050 PRINT " FITS A PARABOLOID TO A DEFORMED DISH"
1060 PRINT " OR CALCULATES AVRERAGE TIME DELAY TO RECEIVER"
```



```
1080
1090
1100
1110
1120
1130
1130
1140
1150 INPUT R
1160 IF R<I
1160 IF R<1 OR R>6 THEN GOTO 1150
1170 ON R GOSUB 1400,1980,2220,2360,3890,1210
1180
1190'
1200'
1210 'Subroutine updates JOBNAME.par and ends program
1220 OPEN JN$+".PAR" FOR OUTPUT AS 5
1230 PRINT #5,F#
1240 PRINT #5,D
1250 FOR I=1 TO D
1260 PRINT #5,A(I),B(I)
1270 NEXT I
1280 PRINT #5,MAX
1290 PRINT #5,MP
1300 FOR I=1 TO D
1310 FOR J=A(I) TO B(I)
1320 PRINT #5,AR#(J)
1330 NEXT J
1340 NEXT I
1350 CLOSE #5
1360 END
1370,
1380 '
1390 '
1400 'Subroutine edits JOBNAME.out from GIFTS
```



```
1420 PRINT " NEW JOB"
1430 PRINT "====================================
1440 PRINT * NAME OF GIFTS JOB": INPUT JNS
1450 PRINT "-------------------------------------------------------------
1460 PRINT " ORIGINAL FOCAL LENGTH":INPUT F#
```



```
1480 PRINT " BOUNDARIES FOR NODAL POINTS MIN,MAX (0,0=S
1490 I=0
1500 I=I+1:INPUT A(I),B(I)
1510 IF A(I)=0 AND B(I)=0 THEN I=I-1 : GOTO 1540
1520 IF A(I)<0 OR B(I)<0 OR A(I)>B(I) THEN I=I-1:PRINT "TRY AGAIN"
1530 GOTO 1500
1540 PRINT " AND LESS THAN":INPUT MAX
```

```
1550
1560
1570 D=
1580
1590
1600
1610
1620
1630
1640
1650
1660
1670
1680
1690
1700
1710
1720
1730 INPUT #1,QS:INPUT #1,QS:INPUT #1,Q$
1740 K=0
1750 INPUT ##,NU,DX#,DY#,DZ #
1760 NUR=NU-100*INT (NU/100)
1770 FOR J=1 TO I
1780 IF NU<100 OR NUR<A(J) OR NUR>B(J) OR NU>MAX THEN GOTO 1800
1790 PRINT #3,NU,DX#,DY#,DZ#
1800 NEXT J
1810 IF NU=MP THEN GOTO 1830
1820 GOTO 1750
1830 INPUT #1,QS:INPUT #1,QS
1840 K=0
1850 INPUT #1,NU,DX#,DY#,DZ#
1860 NUR=NU-100*INT (NU/100)
1870 FOR J=1 TO I
1880 IF NU<100 OR NUR<A(J) OR NUR>B(J) OR NU>MAX THEN GOTO 1900
1890 PRINT #4,NU,DX#,DY#,DZ#
1900 NEXT J
1910 IF NU=MP THEN GOTO }193
1920 GOTO 1850
1930 CLOSE #1,#3,#4
1940 FOR M=2 TO 20000: NEXT M: RETURN 1030
1950 '
1960
1970'
1980 'Subroutine reads parameters from JOBNAME.par
```



```
2000
2010
2020
2030
2040
2050 INPUT #5,F
2060 INPUT #5,D
2070 FOR I=1 TO D
2080 INPUT #5,A(I),B(I)
2090 NEXT I
```

```
2100 INPUT #5,MAX
2110 INPUT #5,NP
2120 FOR I=1 TO D
2130 FOR J=A(I) TO B(I)
2140 INPUT #5,AR#(J)
2150 NEXT J
2160 NEXT I
2170 CLOSE #5
2180 FOR M=1 TO 20000: NEXT M:RETURN 1030
2190 '
2200 '
2210 '
2220 'Subroutine reads areas
2230
2240
2250
2260
2270
2280
2290
2300
2310
2320 F
2330
2340'
2350 '
2360 'Subroutine fits paraboloid to deformed dish
2370
2380
2390
2400
2410
2420
2430
2440
2450
2460
2470
2480
2490
2500
2510
2520
2530
2540
2550
2560
2570 'readi
2580 OPEN JN$+".COR" FOR INPUT AS 1
2590 OPEN JN$+".DF1" FOR INPUT AS 2: OPEN JN$+".DF2" FOR INPUT AS 3
2600 IF EOF(1) THEN CLOSE: GOTO 2860
2610 INPGT *1,NP,X#,Y#,Z#: NPR=NP-100*INT(NP/100)
2620 INPUT #2,MP,U1#,V1#,W1#: INPUT #3,OP,U2#,V2#,W2#
2630 'compute partials and gradient
2640 U# maL#*U1#+BE#*U2#:V #mAL#*V1#+BE#*V2#:W#=AL#*W1#+BE#*W2#
```

```
2650 Z|=(X##X#+Y#*Y#)/(4##F#)
2660 TA=SQR(X#*X#+Y#*Y#+4#*F#*F#)
2670 R年(1) = -X#/T#
2680 R#(2) =-Y#/T#
2690 R|(3)=2#*F#/T#
2700 WT#=AR# (NPR)*R# (3)*R# (3)
2710 R*(4)=R#(3)*Z#
2720 R|(5) =Y #*R# (3)-Z#*R# (2)
2730 R#(6)=Z#*R#(1)-X#*R# (3)
2740 computing matrix A and vector B
2750 FOR I=1 TO 6
2760 FOR J=1 TO 6
2770 A# (I,J) =A# (I,J) +WT#*R# (I) *R#(J)
2780 NEXT J
2790 NEXT I
2800 WT#=WT#*(R#(1)*U#+R#(2)*V#+R#(3)*W#)
2810 FOR I=1 TO 6
2820 B# (I) =B# (I) +WT##R#(I)
2830 NEXT I
2840 GOTO 2600
2850 'constraining equations
2860 FOR J=1 TO 6
2870
2880
2890
2900
2910
2920 N
2930 'solve A(6,6)X(6)=B(6); X put in B
2940 SING#=.00001
2950 RMIN**=1.00001
2960 FOR I=2 TO 6
2970
2980
2990
3000
3010
3020
3030
3040
3050
3060
3070
3080
3090
3100
3110 NEXT
3120 IF RMIN#<SING# THEN PRINT "MATRIX SINGULAR": PRINT "-m--------------
3130 'forward substitution
3140 FOR I=2 TO }
3150 M=I-1
3160 FOR J=1 TO M
3170 B#(I)=B#(I)-A#(I,J)*B#(J)
3180 NEXT J
3190 NEXT I
```

```
3200 'backward substitution
3210 B4(6)=B#(6)/A# (6,6)
3220 FOR I=5 TO 1 STEP -1
3230 SUM#=0#
3240 M=I+1
3250 FOR J=M TO 6
3260
3270
3280
3290
3300
3310'
3320 'computation of rms errors
3330 RNS#=0: ATOT#=0#
3340 OPEN JN$+".COR" FOR INPUT AS 1
3350 OPEN JN$+".DF1" FOR INPUT AS 2: OPEN JN$+".DF2" FOR INPUT AS 3
3360 IF EOF(1) THEN CLOSE:GOTO 3670
3370 INPUT #1,NP,X#,Y#,2#: NPR=NP-100*INT(NP/100)
3380 INPUT #2,MP,U1#,V1#,W1#: INPUT #3,OP,U2#,V2#,W2#
3390 U#=AL#*U1#+BE**U2#: V#=AL#*V1#+BE#*V2#: W#=AL#*W1#+BE#*W2#
3400 'compute normal errors
3410 FF#=F#/(1#+B#(4)): Z#=(X#*X#+Y#*Y#)/(4#*F#)
3420 T#=SQR(X#*X#+Y#*Y#+4#*FF#*FF#)
3430 D1&=-X*/T#
3440 D2&=-Y//T*
3450 D3#=2#*FF#/T#
3460 CX1#=-D1#*B#(1)
3470 CY2#=-D2#*B#(2)
3480 C23*=-D3**B#(3)
3490 CFOC#=-B#(4)*D3#*Z#
3500 TT&=SQR(X#*X#+Y#*Y#+4##F#*F#)
3510 DDI#=-X#/TT#
3520 DD2*=-Y#/TT%
3530 DD3#=2#*F#/TT#
3540 CXX#=B#(6)*Z#
3550 CX|=D1#*(U#-CXX#)+DD1#*CXX#
3560 CYY#=-B#(5)*2#
3570 CY*=D2**(V#-CYY#)+DD2#*CYY#
3580 CZ2*=B#(5)*Y*-B#(6)*X#
3590 CZ|=D3**(W*-CZZ#)+DD3**C2Z#
3600 CXR#=B#(5)*(2#*DD2#-Y#*DD3#)
3610 CYR#=B#(6)*(X**DD3#-Z#*DD1#)
3620 'G& total normal error at each node
3630 G#=CX#+CY#+CZ#+CFOC#+CXR#+CYR*+CX1#+CY2#+CZ3#
3640 H*=G音*D3#
3650 RMS#=RMS##+AR# (NPR) *H#*H#: ATOT#=ATOT#+AR# (NPR)
3660 GOTO 3360
3670 RMS=SQR(RMS#/ATOT#): ATOT=ATOT#
3680 'presentation of results
```



```
3700
3710
3720
3730
3740
    PRINT " RESULTS OF " T$
```



```
    PRINT " VERTEX COORDINATES XO:",B1
    PRINT " Yo:",B2
    PRINT "" zo:",B3
```

```
PRINT＂
PRINT＂
PRINT＂
PRINT＂
RETURN 1080
!
3860
3870
3880
3890
3900
3910
3920
3930
3940
3950
3960
3970
3980
3990
4000
4 0 1 0
4020 TFL#m0#:TLL##0菑:ATOT##0%
4030 'reading files
4040 OPEN JNS+".COR" FOR INPUT'AS 1
4050 OPEN JN$+".DF1" FOR INPUT AS 2: OPEN JNS+".DF2" FOR INPUT AS 3
4060 IF EOF(1) THEN CLOSE: GOTO 4290
4070 INPUT #1,NP,X*,Y*,Z#:NPR=NP-100*INT (NP/100)
4080 INPUT *2,MP,U1*,V1#,W1#: INPUT #3,OP,U2#,V2#,W2#
4090 'compute partials and gradient
```



```
4110 Z䔨=(X**X#+Y##Y#)/(4**F|)
```



```
4130 N*(1)=-X*/T*
```



```
4150 N*(3)=(F|-Z#)/T*
4160 'Euli computation
```




```
4190 YFMm(F#+2#)*N*(2)-V粪+FX|
```



```
4210 FL*=AR|(NPR)*(FL|+SQR(XF**XF*+YF**YF#+ZF#*ZF#)-F#-Z#)
4220 TFL#=TPL*+FL*
4230 '1inearized computation
```




```
4260 TLL**TLL&*LL*
4270 ATOT*=ATOT#+AR萛(NPR)
4280 GOTO 4060
4290 FLmTFL*/ATOT#: LL=TLL#/ATOT##:ATOT=ATOT*
```



## 9 APPENDIX B: A full run of PARABOL

```
D: \GIFTS\JOBS\GFTOUT
GFTOLIT VER. 6.2.2
TYFE JOE NAME: STOF
* SORT/FOINT/1/
* FW/NU, XC, YC, ZC/E, -2,10,-2,10,-2,10/
* DNSW/NU, XD, YD,ZD/=,-2,10,-2,10,-2,10/
*
```

Comment. Run GFTOUT and write coordinates and deflections to file STOR.OUT. Exit with QUIT and then run PARABOL.

```
FFOGFAM FARABDL
FITS A FAFAEOLDID TO A DEFOFMED DISH OF CALCULATES AVFEFAGE TIME DELAY TO RECEIVER
```



```
NEW JOE: 1
OLD JUE: 2
NEW AREAS: 3
FIT A FAFAEDLOID: 4
AVERAGE TIME DELAY: 5
END: 6
```

? 1

Comment. Select NEW JOB by entering 1.


```
NEW JOE
```



```
    NAME OF GIFTS JOB
? STOF
    GRIGINAL FOCAL LENGTH
? 10.9728
    BOUNDARIES FOR NODAL FOINTS MIN,MAX (O,O=STOF)
? 1,10
?0,0
    AND LESS THAN
? 2600
    MAXIMLMM NUMEEF FOR NODAL FOINT
? S1301
```

Comment. Enter data about the dish and select nodes at the dish surface.


```
                    FROGFAM FAFAEOL
                FITS A FAFABOLOID TO A DEFOFMED DISH
    OF CALEULATES AVFEFAGE TIME DELAY TO FECEIVER
```



```
                    NEW JOE: 1
                    OLD JOE: 2
                    NEW AREAS: 3
            FIT A F'AFAEOLOID: 4
            AVEFAGE TIME DELAY: 5
                        END: b
? 
```

Comment. Select NEW AREAS.

| NODE | 1 |
| :---: | :---: |
| ?.566 |  |
| NODE | 2 |
| $? 1.067$ |  |
| NODE | 3 |
| $? 1.268$ |  |
| NODE | 4 |
| 71.454 |  |
| NODE | 5 |
| ?.688 |  |
| NODE | 6 |
| $? .706$ |  |
| NODE | 7 |
| ?.899 |  |
| NODE | B |
| ?.945 |  |
| NODE | 9 |
| ?.639 |  |
| NODE | 10 |
| ?.159 |  |

Comment. Enter areas (weighted with the illumination over the dish). Then select FIT A PARABOLOID from the menue.


```
                                    LEAST SQUAFE FIT
```



```
    CASE TITLE
? \mp@code { H O F I I Z O N }
    COMFOSITE LOAD A*(LDCASE 1) + E*(LDCASE 2) A,B
? 1,-1
    FAFAMETEFS TO EE CONSTRAINED (YES/NO) 1/O
    XO,YO,ZO
?0,1,0
    FOTATION AEOUT X AND Y-AXIS
? 1,0
    FOCAL LENGTH
70
```

Comment. Select composite load and constraints.

| RESULTS OF HORIZON |  |
| :---: | :---: |
| VEFTEX CODRDINATES | -7.671075E-02 |
|  | 0 |
|  | 1.490902E-08 |
| FOTATION AEOUT | 0 |
|  | 1.618997E-03 |
| NEW FOCAL | 10.96856 |
|  | -4.240478E-03 |
| FIMS EFFIOF: TOTAL AREA: | $1.495526 E-6 \%$ |
|  | 150.018 |
| NEW JOE: 1 |  |
| OLD JOE: | 2 |
| NEW AREAS: | 3 |
| FIT A FAFIAEOLOID: | 4 |
| AVEFAGE | 5 |
|  | 6 |

Comment. The result of the calculation of the best fit paraboloid. Select AVERAGE TIME DELAY.


```
    COMFLITATION DF AVEFAGE TIME DELAAY
```



```
    CASE TITLE
? HOFIZON
    DEFOFMATION OF FEED SUFFORT
    CHANGE IN FECIEVEF FOINT DX,DY,DZ
?.01,0,0
    DEFOFMATION OF DISH
    COMFOSITE LOAD A*(LDCASE 1) + E*(LDCASE 2) A,B
* 1,-1
```

Comment. Enter the new position of the receiver and the composite load.


```
    FESLLTS OF HOFIZON
```



```
    FULL CALCULATION OF CHANGE
        FATH LENGTH: -3.628727E-0S
        TIME DELAY: -1.209576E-11
    LINEAFIZED CALCULATION OF CHANGE
        FATH LENGTH: --3n647771E-OS
        TIME DELAY: -1.215924E-11
                            TOTAL AREA: 150.018
                            NEW JOB: 1
                            OLD JOE: 2
            NEW AFEAS: 
        FIT A FAFAEOLOID: 4
        AVEFAGE TIME DELAY: }
            END: 6
?6
```

Comment. The result of the calculation. Exit by entering 6 .

\section*{10 APPENDIX C: Listing of program WINDLOAD <br> ' Program WINDLOAD computes windforces on a parabola <br> ' Description. <br> ' Run GFTOUT on the job. Use: SORT/POINTS/1 and PW/NU,XC,YC,ZC/5,-2,10,.. <br> 'Run WINDLOAD. <br> ' Program reads preasure coefficients from file <file name>.prs. <br> 'This file should contain on the first line the number of radial data an <br> 'the number of angular data. <br> 'Thereafter should follow radius/diameter, angle and preasure coeffcient; <br> 'lowest radius first with increasing angle. <br> 1 Example: 22 <br> | $:$ | 0.1 | 15 | 1.31 |
| :--- | :--- | :--- | :--- |
| $:$ | 0.1 | 45 | 1.41 |
| $:$ | 0.3 | 15 | 1.35 |
| $:$ | 0.3 | 45 | 1.42 | <br> ' Program then interpolates the coefficients for the selected nodes. 'WINDLOAD.SRC contains the forces. In LOADBC OLB/WINDLOAD. <br> 'End of description.}

,
pi=3.14159265
dim pla(10, 20), plr(10, 20), $\operatorname{cof}(10,20), a(10), b(10), \operatorname{ar}(99)$
 print " PROGRAM WINDLOAD" print " computes windloads on a antenna dish"
print $"====\pi=================================================11$
print " Name of preasure coefficient file ?": input wns
print " Windspeed and air density ?": input v,ro

print " Name of GIFTS job ?": input jns
print " Focal length and diameter of antenna": input foc, diam print " Boundaries for nodal point min,max (0,0=stop)"
1
'Selecting nodes
$i=0$
$100 i=i+1:$ input $a(i), b(i)$
if $a(i)=0$ and $b(i)=0$ then $i=i-1: d=i$ : goto 200
if $a(i)<0$ or $b(i)<0$ or $a(i)>b(i)$ then $i=i-1:$ print "Try again" goto 100
200 print " And less than": input max

,
'Reading areas
print ${ }^{(1)}$ Area at"
for $i=1$ to d
for $j=a$ (i) to $b$ (i)
print " node",j: input ar(j)
next $j$
next $i$

'Reading presure coefficients
open wn\$+".prs" for input as 1
input \#1, imax,jmax
for $i=1$ to imax
for $j=1$ to jmax

```
                    input #1,plr(i,j),pla(i,j), cof(i,j)
        next j
    next i
    close
I
'Reading coordinates
    open jn$+".out" for input as 1
    open "windload.src" for output as 2
300 if eof(1) then print "2, "END":close: end
    input #1,nu,xc,yc,zc: nur=nu-100*int(nu/100)
    for i=1 to d
        if nu>max then print #2, "END": close: end
        if nu<100 or nur<a(i) or nur>b(i) then goto 400
        gosub force
4 0 0 ~ n e x t ~ i ~
    goto 300
    end
'
force:
'Subroutine calculates wind force at nodes
'Calculating gradient
    t=sqr (xC*xc+yc*yc+4*foc*foc)
    rl=xc/t
    r2=yc/t
    r3=-2*foc/t
    rad=sqr(xc*xc+yc*yc)/diam
    ang=90
    if xc>0 and abs (xc)>0.0001 then ang=180*(1-atn (yc/xc)/pi)
    if xc<0 and abs (xc)>0.0001 then ang=-180*atn(yc/xc)/pi
'
'Get interpolated coefficient
    gosub interpol
*
'Print forces to src file
    frcmro*V*v/2*coff*ar(nur)
    print #2,"LOADP,1/"N,NU, "/", frc*r1,"/"
    print *2, "LOADP, 2/'N,NU, "/N, frC*r2, "/N
    print *2,"LOADP,3/",NU,"/n, frc*r3,N/"
    return 300
'
interpol:
'Subroutine interpolates preasure coefficient
'Finding surounding coordinates
    i=2
50 if plr(i,1)<rad or plr(i,i)=rad then i=i+1
    if i=imax or plr(i,i)>rad then goto }6
    goto 50
60 Ih=i: il=ih-1
!
    j=2
70 if pla(in,j)<ang or pla(in,j)=ang then j=j+1
    if j*jmax or pla(ih,j)>ang then goto 80
    goto }7
80 jh=j: jl=jh-1
```

```
'Interpolation
    cl=(cof(ih,jl)-cof(11,jl))*(rad-plr(il,jl))
    cl-cl/(plr(in,jl)-plr(il,jl))+cof(il,jl)
1
    ch=(cof(in,jh)-cof(il,jh))*(rad-plr(il,jh))
    ch-ch/(plr(ih,jh)-plr(il,jh))+cof(il,jh)
,
    coff=cl+(ch-cl)*(ang-pla(il,jl))/(pla(il,jh)-pla(il,jl))
'
'Diagnostics section to be turned on
1 print
' print nu,rad,ang
- print plr(in,jl),plr(il,jh)
p print cof(ih,jl),cof(il,jl),pla(ih,jl)
', print cof(ih,jh),cof(il,jh),pla(ih,jh)
- print cl,ch,coff
! print frc*r1,frc*r2,frc*r3
    return
```


## 11 APPENDIX D: A full run of WINDLOAD

| FILESFEC:EXT |  | EYTES | ATF | ---LS ${ }^{\text {L }}$ | E.-. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| WINDOOO | ${ }^{\text {.FFS }}$ | 977 |  | 09/22/87 | 12:57 |
| WINDO60 | . FHE | 871 |  | 09/22/87 | 13:\% |
| WINDO90 | ${ }^{\text {®FRS }}$ | 939 |  | 09/22/87 | 13:05 |
| WTND 520 | . FRS | 1066 |  | 09/22/87 | 13:2 |
| WTND180 | . FFS | 985 |  | 99/22/87 | 1ヵ: |

Comment. The available files with preasure coefficients. WIND060.PRS contains the coefficients for the antenna at 060 degrees elevation.

```
                    PROGRAM WINDLOAD
                            computes windloads on a antenna dish
```



```
    Name of preasure coefficient file?
? wind090
    Windspeed and air density?
? 10,1.29
    Name of GIFTS job?
? stor
    Focal length and diameter of antenma
? 10.9728,26
    Boundaries for nodal point min,max (0,0=stop)
? 1.10
?0,0
    And less than
?2600
    Area at. 
?.5658
    node 2
?1.112
    node 3
71.409
    node - 4
? 1.780
    node 5
?.982
node 6
? 1.0.88
    node 7
? 1.605
    node &
? 1.8.17
    node 9
?2.060
    node 10
? 1.134
```

Comment. A full run of the program. The units used are metric. The forces are written to file WINDLOAD.SRC and can be used in processor LOADBC.

## Part II

## Path Length Changes in an Antenna

## 12 Introduction

This part contains some basic facts about how an antenna deforms. Section 14 identifies some important parameters that can be used to characterize the behaviour of the dish; Section 15 triea to characterize the behaviour of the antenna when it deforms due to gravity loads; and Section 16 studies the effects of temperature. In this report the average path length to the receiver is used to estimate the time delay experienced in the antenna; a more detailed analysis would use diffraction theory.

## 13 The paraboloid

The equation for a paraboloid in a cartesian coordinate system with the vertex at $(0,0,0)$ and the focus at $(0,0, f)$ is

$$
\begin{equation*}
z=\frac{x^{2}+y^{2}}{4 f} \tag{13.1}
\end{equation*}
$$

or if we put $x^{2}+y^{2}=r^{2}$

$$
\begin{equation*}
z=\frac{r^{2}}{4 f} \tag{13.2}
\end{equation*}
$$

In Figure 7 there is a reference plane through ( $0,0, a$ ). If one traces a ray parallel to the $z$-axis from this plane and reflects it in the paraboloid, it passes through the focal point. What's more, the distance from the plane to the focal point is constant. The distance from A a point on the reference plane to P a point on the parabola is

$$
a-\frac{r^{2}}{4 f}
$$



Figure 7: In the figure the paraboloid is cut along the $y$-axis. A ray is traced from the reference plane parallel to the $z$-axis and passes through the focus when reflected in the paraboloid. The equation for the paraboloid is $z=\left(x^{2}+y^{2}\right) / 4 f$, where $f$ is the focal length.
and the square of the distance from P to F is

$$
r^{2}+\left(f-\frac{r^{2}}{4 f}\right)^{2}=r^{2}+f^{2}+\frac{r^{4}}{16 f^{2}}-\frac{r^{2}}{2}=\left(f+\frac{r^{2}}{4 f}\right)^{2}
$$

and thus the sum of these two distances is $a+f$, independent of $r$.

## 14 Changing the paraboloid

Now if the antenna deforms due to external forces the paraboloid changes shape, positon and orientation. The receiver also changes position. Then the average path length to the receiver from the reference plane changes. See Ref. 1. Call this change $\Delta L$. Let the change in focal length be $\Delta f$ and the change in the position of the vertex in $z$-coordinate be $\Delta V$. The changes in x and y -coordinates do not change the path length because they are parallel to the reference plane; assume these coordinates to be unchanged. Let the rotation about the $x$ and $y$-axis be $\theta$ and $\phi$ and the change in the position of the receiver be $\Delta P, \Delta Q$ and $\Delta R$. Then $\Delta L$ is a function of these variables; call this funtion $F$.

$$
\Delta L=F(\Delta f, \Delta V, \theta, \phi, \Delta P, \Delta Q, \Delta R)
$$

The changes in the variables are small and $F(0, \ldots, 0)=0$ so to first order

$$
\Delta L=F_{\Delta f}^{\prime} \Delta f+F_{\Delta V}^{\prime} \Delta V+F_{\theta}^{\prime} \theta+F_{\phi}^{\prime} \phi+F_{\Delta P}^{\prime} \Delta P+F_{\Delta Q}^{\prime} \Delta Q+F_{\Delta R}^{\prime} \Delta R
$$

where the partial derivatives are taken at $(0, \ldots, 0)$. It's obvious that

$$
F(0,0, \theta, 0,0,0,0)=F(0,0,-\theta, 0,0,0,0)
$$

from the symmetry of the geometry. (The illumination over the dish is assumed to depend only on r.) Thus $F_{\theta}^{\prime}=0$ and in the same way $F_{\phi}^{\prime}=0$. The same argument also shows that $F_{\Delta P}^{\prime}=0$ and $F_{\Delta Q}^{\prime}=0$. Now put $F_{\Delta f}^{\prime}=\alpha_{f}, F_{\Delta V}^{\prime}=\alpha_{V}$ and $F_{\Delta R}^{\prime}=\alpha_{R}$. Then we have

$$
\begin{equation*}
\Delta L=\alpha_{f} \Delta f+\alpha_{V} \Delta V+\alpha_{R} \Delta R \tag{14.1}
\end{equation*}
$$

There exist, some simple relations between the different coefficients in this equation. For example, if the paraboloid is translated along the the $z$-axis
a distance $d$ towards the reference plane and the receiver is moved to the new focus, then $\Delta L=-d, \Delta f=0, \Delta V=d$ and $\Delta R=d$. From this one gets

$$
\begin{equation*}
\alpha_{V}+\alpha_{R}=-1 \tag{14.2}
\end{equation*}
$$

If the paraboloid changes focal length $\Delta f=l$, the vertex is kept fixed $\Delta V=0$ and the receiver is moved to the new focus $\Delta R=l$, then $\Delta L=l$. Now we get

$$
\begin{equation*}
\alpha_{f}+\alpha_{R}=1 \tag{14.3}
\end{equation*}
$$

### 14.1 Estimation of $\alpha_{R}$

Ref 1. shows how to estimate changes in a Cassegrainian antenna. A similar technique is used below. According to Figure 1, the vector from $P$ to $F$, call it PF, is

$$
\mathbf{P F}=\left(-x,-y, f-\frac{r^{2}}{4 f}\right)
$$

or if we normalize and call this vector $\mathbf{n}$ and put $r / 2 f=t$ then

$$
\mathbf{n}=\left(\frac{-x / f}{1+t^{2}}, \frac{-y / f}{1+t^{2}}, \frac{1-t^{2}}{1+t^{2}}\right)
$$

From Figure 8, we see that the change in pathlength due to a change in the receiver position $\Delta R$ to first order is

$$
\mathbf{n} \cdot \mathbf{e}_{z} \Delta R=\frac{1-t^{2}}{1+t^{2}} \Delta R
$$

This is just the change for one ray and to get $\alpha_{R}$ we have to average $\mathbf{n} \cdot \mathbf{e}_{z}$ over the dish. We assume that the illumination over the dish only depends on the radial distance $r$ (or $t$ ). Call this function $I$. The illumination usually has a maximum in the center and decreases towards the rim. If we normalize this function so that

$$
\int_{0}^{r_{0}} I 2 \pi r d r=1
$$

where $r_{0}$ is the radius of the dish, then the function can be used for averaging. To estimate $\alpha_{R}$ we choose the function to be constant: $I=1 / \pi r_{0}^{2}$.


Figure 8: In the figure the receiver is moved along the $z$-axis a distance $\Delta R$. This changes the path length, and one sees that this change is $\mathbf{n} \cdot \mathbf{e}_{z} \Delta R$, where $\mathbf{n}$ is a unit vector along the reflected ray and $\mathbf{e}_{z}$ is a unit vector along the z -axis.

Finally we get

$$
\alpha_{R}=\int_{0}^{r_{0}} \frac{1-t^{2}}{1+t^{2}} I 2 \pi r d r=\frac{4 f^{2}}{r_{0}^{2}} \int_{0}^{t_{0}} \frac{1-t^{2}}{1+t^{2}} 2 t d t
$$

Evaluation of the integral yields

$$
\begin{equation*}
\alpha_{R}=\frac{2}{t_{0}^{2}} \ln \left(1+t_{0}^{2}\right)-1 \tag{14.4}
\end{equation*}
$$

where $t_{0}=r_{0} / 2 f$. The parameter $t_{0}$ is easy to relate to an actual dish, because $t_{0}=D / 4 f$ and $f / D$ is well known parameter for all dishes. Eq. 14.4 is plotted in Figure 9.

A typical value, $f / D=0.4$, yields $\alpha_{R}=0.7$. Calculations on a computer for a more realistic illumination function with the illumination at the rim one tenth of the value at the center yields $\alpha_{R}=0.8$. What happens is that the central part, which is more heavily illuminated and where the change is greater, has more influence. Using this value we can now use Eqs. 14.2-3 and get $\alpha_{f}=0.2$ and $\alpha_{V}=-1.8$.

### 14.2 Deviations from a paraboloid

The discussion so far has assumed a perfect paraboloid. When a dish deforms due to external forces its, shape will not be that of a paraboloid. We can however fit a paraboloid to the deformed dish in such a way that the rms error in path length to the best fit focus is minimum. See Ref. 2. Figure 10 defines this error $\delta$. It is a function of $x, y$ and some set of parameters $\mathbf{q}$ that determines the change in the paraboloid to achieve the best fit. A choice of parameters is the change in focal length, the change in position of the vertex and the rotations about the $x$ and $y$-axis. Thus we have

$$
\delta=\delta(x, y ; \mathbf{q})
$$

The function $R(\mathbf{q})=\int \delta^{2}(x, y ; \mathbf{q}) I d x d y$, where $I$ is the the illumination as before, is minimized to yield the values $\mathrm{q}_{0}$ for the best fit paraboloid. This means that $R_{q_{i}}^{\prime}=0$ for $\mathbf{q}=\mathbf{q}_{0}$. In the following let the partial derivatives be evaluated at this point. An important result is that

$$
\begin{equation*}
\int \delta\left(x, y ; \mathbf{q}_{\mathbf{0}}\right) I d x d y=0 \tag{14.5}
\end{equation*}
$$



Figure 9: $\alpha_{R}$ as a function of $f / D$ according to Eq. 14.4.


Figure 10: Definition of the error $\delta: \delta=l_{1}-l_{2}$. This error is minimized when calculating the best fit paraboloid. Both $l_{1}$ and $l_{2}$ run from the reference plane to the paraboloid and the deformed dish and finally to the focus. Note that the reference plane is orthogonal to the optical axis of the paraboloid one wishes to fit.

This means average path length to the best fit focus is the same for the deformed dish as for the best fit paraboloid. To see this, note that

$$
R_{q_{i}}^{\prime}=\int 2 \delta_{q_{i}}^{\prime} \delta I d x d y=0
$$

and then for a set of constants $a_{i}$

$$
\sum a_{i} R_{q_{i}}^{\prime}=\int 2\left(\sum a_{i} \delta_{q_{i}}^{\prime}\right) \delta I d x d y=0
$$

Now if $\sum a_{i} \delta_{q_{i}}^{\prime}$ is independent of x and y for some choice of $a_{i}$ then Eq. 14.5 is true. From Figure 11 we have that

$$
\begin{gathered}
\delta\left(x, y ; f_{0}, z_{0}\right)=l_{1}-l_{2} \\
\delta\left(x, y ; f_{0}+\Delta f, z_{0}-\Delta f\right)=l_{1}-l_{3}
\end{gathered}
$$

and this means that

$$
\delta\left(x, y ; f_{0}+\Delta f, z_{0}-\Delta f\right)-\delta\left(x, y ; f_{0}, z_{0}\right)=l_{2}-l_{3}=-2 \Delta f
$$

and we then have $\delta_{f}^{\prime}-\delta_{z}^{\prime}=-2$, independent of x and y just as required.
But the receiver is not necessarily at the best fit focus. We have to study the difference in path length to the receiver between the deformed dish and the best fit paraboloid. (The difference to the best fit focus is as just shown zero). We assume as before that the deformations in the antenna are small. This means that the difference in position between the best fit focus and the receiver $\Delta R$ and the error from the best fit paraboloid $\Delta d$ are small. See Figure 12. Then a simple calculation shows that the change in $l_{1}-l_{2}$ is on the order of $\Delta d \Delta R / f$, where $f$ again is the focal length. Here $\Delta d$ and $\Delta R$ are much less than $f$ and thus the error from the deformed dish is negligible compared to the changes calculated in Section 14. (on the same order as $\Delta R$ ). Furthermore, this error should be averaged over the dish which will reduce it even further. As an example: with GIFTS and PARABOL, mentioned in Part I, typical calculated values for the Fairbanks antenna are $\Delta L \sim 10^{-2} m, \Delta R \sim 10^{-2} m$ and $\Delta d \sim 10^{-2} m$; for this antenna $f \sim 10 \mathrm{~m}$; this yields an error of $10^{-5} \mathrm{~m}$ or 0.1 percent of the calculated change.


Figure 11: In the figure $l_{2}$ is reflected in the best fit paraboloid, and $l_{3}$ is reflected in a paraboloid with longer focal length but translated along the optical axis in such a way that $l_{3}$ also passes through the best fit focus.


Figure 12: When the receiver is moved $l_{1}-l_{2}$ changes. From the figure one sees that the change is $\delta_{2}-\delta_{1}$. One also gets $\gamma \approx \Delta R / f$. Furthermore, $\delta_{1}<\Delta d \tan \gamma \approx \Delta d \gamma \approx \Delta d \frac{\Delta R}{f}$ and $\delta_{2} \approx \Delta d \tan \gamma \tan \beta \approx \Delta d \gamma \tan \beta$. By taking the derivative of Eq. 13.2 we get $\tan \beta=r / 2 f<1$. To sum it up: the change is on the order of $\Delta d \Delta R / f$.


Figure 13: The incomming wave front - the reference plane - is not orthogonal to the optical axis. The angle is $\beta$. From the figure one sees that $A=\Delta d \sin (\alpha)$ and thus $\Delta A \approx \Delta d \cos (\alpha) \beta$.

Furthermore, the incomming wavefront does not have to be orthogonal with the optical axis. This error is illustrated in Figure 11. And the calculation yields an error on the order of $\Delta d \cos (\alpha) \beta$. A typical value for $\beta$ is $10^{-2} \mathrm{rad}$ and thus an error of $10^{-4} \mathrm{~m}$ or about 1 percent of the calculated change.

To sum it up: the changes in the average path length to the receiver when a dish deforms can be approximated with the change to the receiver from the best fit paraboloid. This change, to first order Eqs. 14.1-3, is much greater than the error made in the approximation.

## 15 Changes due to gravity

The deformations in an antenna due to gravity depend upon the position of the antenna. These deformations change the path length to the receiver. Numerical results will be given in Part III of this report; this section tries to motivate the behaviour of the change at different positions of the antenna. Again call the change in path length $\Delta L$.

For an AZ-EL antenna $\Delta L$ by symmetry depends only on the elevation EL:

$$
\Delta L=G(E L)
$$

The function G can be expanded in a Fourier series

$$
\begin{equation*}
\Delta L=a_{0}+a_{1} \cos E L+b_{1} \sin E L+\ldots \tag{15.1}
\end{equation*}
$$

where the sine and cosine terms are closely related to the components of the gravity vector shown in Figure 14. The component along the optical axis is proportional to $\sin E L$. This component also determines the deflection in this direction, and thus this term should dominate in the series.

For a X-Y mounted antenna:

$$
\Delta L=H(X, Y)
$$

with the obvious symmetries for H

$$
H(X, Y)=H(-X, Y)=H(X,-Y)=H(-X,-Y)
$$



Figure 14: The components of the gravity vector. The forces along the optical axis are proportional to $\sin E L$.

Expanding this function into a two variable Fourier series and using the symmetries yields only terms with cosines

$$
\Delta L=\sum_{n, m=0}^{\infty} a_{n m} \cos n X \cos m Y
$$

and if we write the first few terms

$$
\begin{equation*}
\Delta L=a_{00}+a_{10} \cos X+a_{01} \cos Y+a_{11} \cos X \cos Y+\ldots \tag{15.2}
\end{equation*}
$$

Here we note that $\cos X \cos Y=\sin E L$. See Ref. 3. The same argument can be used to show that $\Delta V, \Delta R$ and $\Delta F$ (deformations along the optical axis defined in Sec. 14) should have the same type of expansion.

The same argument that was used to motivate the $\sin E L$ term in the AZ-EL antenna can be used again. However, the distance between the Xand $Y$-axis will also change. The component of the gravity vecto along this distance is proportional to $\cos X$, and then also the change in the distance D. See Figure 15. From the figure we see that the change in distance the wave front has to travel to the reference point is again proportional to $\cos X \cos Y$ or $\sin E L$. And again this term should dominate in the series.

## 16 Changes due to temperature

For a beam with original lenth $l$ the change in length $\Delta l$ due to a change in temperature $\Delta T$ is

$$
\begin{equation*}
\Delta l=\alpha \Delta T l \tag{16.1}
\end{equation*}
$$

where $\alpha$ is the thermal expansion coefficient for the material in the beam.
Thus the parameters $\Delta V, \Delta f$ and $\Delta R$ also should be proportional to $\Delta T$. The same is true for $\Delta D$, the change in distance between x and y -axis. The first three gives rise to a change in path length through the antenna that is proportional to $\Delta T$ according to Eq. 14.1, and the last one to a change that is also proportional to $\cos Y$ according to Figure 15.

And thus the change in average path length through the antenna to the reference point should be

$$
\begin{equation*}
\Delta L=\Delta T\left(c_{1}+c_{2} \cos Y\right) \tag{16.2}
\end{equation*}
$$



Figure 15: The geometry of the $\mathrm{X}-\mathrm{Y}$ antenna. The distance between the two axes is $D$. The distance the incoming wavefront has to travel from the Y -axis to the reference point is $D \cos Y$.
where $c_{1}$ and $c_{2}$ are constants if we asume that $\Delta V, \Delta f, \Delta R$ and $\Delta D$ are independent of X and Y . Calculations presented in Part III will confirm this.

## References

[1] Cha A. G. Phase and frequency stability of Cassegrainian antennas Radio Science, Vol. 22, Number 1, p. 156, 1987
[2] Katow M. S. Antenna Structures: Evaluation of Reflector Surface Distorsions JPL Technical Report 32-1562, VOL. 1, p. 76, JPL Laboratory, Pasadena, Calif.
[3] Salzberg I. M. Mathematical Relationships of the MFOD Antenna Axes Goddard Space Flight Center (X-553-67-213), Greenbelt, Maryland, May 1967

## Part III

## The Antenna at Fairbanks, Alaska

## 17 Introduction

This part presents the numerical results, calculated deformations in the VLBI antenna at Fairbanks, Alaska. It also presents the results from measurements of these deformations as a comparison, and gives a presentation of the different models used and how the calculations were performed on these models.

Many of the ideas and calculations presented were prompted by my discussions with Dr. Lee King at NRAO in Socorro, New Mexico during my visit there. I am grateful for his help. The actual data on the antenna structure was collected during several visits to Bendix Field Engineering in Columbia and on a visit to the station in Alaska. During the latter visit several measurements were also made.

The focus of the part is on deformations that affect the accuray of the VLBI experiments: the movement of the reference point is studied; the time delay through the antenna to the receiver is also studied. The latter was discussed in Part II of this report. The forces acting on the antenna are gravity, temperature and wind loads.

## 18 Description of the antenna

### 18.1 Drawings

The antenna was built in the begining of the 60's by Rohr Corporation. At the same site a similar antenna was built by Blaw Knox Company a short time before. The engineers that built this antenna left Blaw Knox, keeping the drawings, made modifications on these and built the antenna that is of interest for us. Let us call it Fairbanks II. Both companies still exist but neither are involved with building antennas anymore. The companies have
since long lost track of their drawings of these antennas. At Goddard Space Flight Center there was a division that used to keep a set of drawings of this antenna, but the set together with a lot of other drawings was transfered to the archives at Bendix Field Engineering in Columbia, MD. An additional set complementing the first exists at the station itself. The drawings are marked Rohr Corporation and have the drawing numbers in the range of 930 to 980 .

### 18.2 Material

Nearly all of the material used in the antenna is steel A-203. The exceptions are the surface panels of the dish and some few beams in the quadrupod. Neither has great influence on the stiffness of the structure. The code A-203 is according to ASTM (American Society for Testing and Materials) and with the help of material tables one gets the properties of the steel. The relevant properties are Young's Modulus $E=2.0 \cdot 10^{11} \mathrm{~N} / \mathrm{m}^{2}$, Poisson's ratio $\nu=0.3$, density $\rho=7900 \mathrm{~kg} / \mathrm{m}^{3}$ and thermal expansion coefficient $\alpha=1.2 \cdot 10^{-5} K^{-1}$. These are fairly constant for different types of steels.

### 18.3 The major structural parts of the antenna

The antenna is by its design naturally divided into five major parts: pedestal, x-wheel, y-wheel, dish and quadrupod. See Figures 16-20.

The pedestal rises in two pyramid shaped towers to 13 m above the ground. About 3 m up under the southern tower there is a house that contains hydraulic motors that drive the $x$-wheel. The $x$-wheel rests at the top of the two towers and its axis is oriented in the south to north direction. Because of the house and the weight of the hydraulic motors there is additional structural support under the house. However, above the floor of the house the two towers are identical.

The $x$-wheel has a radius of 7.6 m and the two arms that hold the $y$-axis have a height of 7.3 m . At the rim of the wheel large blocks of lead are attached that balance the wheel and the structure it supports around the x -axis. Just below the axis there is a platform that contains the hydraulic drives for the $y$-wheel. Throughout the wheel there are other plattforms and ladders but these have little effect on the structure.


Figure 16: The pedestal. Note the additional structure in the southern tower. It supports the house (not in the figure) that contains the hydraulic drive for the x -wheel.


Figure 17: The $x$-wheel. The axis rests between the two towers of the pedestal in Figure 16. Note the platform under the axis. It contains the drive for the $y$-wheel.


Figure 18: The y-wheel. The axis is attached to the two arms in the $x$-wheel. It is also attached to the dish in the square structure just above the axis.


Figure 19: The dish. Only half of it is modeled to save time and disk space. The use of symmetry commands make it possible to do calculations as if a full model was constructed.


Figure 20: The quadrupod. It is attached to the dish and holds the receiver at its top in the focus of the dish.

The $y$-wheel has a radius of 6.7 m and is in the same way as the $y$-wheel balanced around its axis. The axis is oriented east to west when the antenna points at the zenith. The $y$-wheel is shaped as a square just above its axis with the diameter as a diagonal in the square. This square structure is then attached to the dish.

The dish is built around a square structure to which the $y$-wheel is attached; it is called the square girder. It supports radial ribs that shape the dish in to a paraboloid with a focal length of 11 m and a diameter of 26 m , thus $\mathrm{f} / \mathrm{D}=0.42$. There are three kinds of ribs: in the drawings called A, B and C-ribs. In the dish there are eight of each rib and thus a total of 24 ribs. Connecting the ribs are beams outlining circles around the center. The surface is covered with aluminum panels adjusted to paraboloidal shape. This has been done several times with the help of theodelites.

From the four corners of the square girder the quadrupod rises 15 m . The receiver is located in the box at the top. Halfway up along the legs four circular beams are attached to the center of the dish.

## 19 Forces and deformations

Three different kinds of forces act on the antenna that are of interest: gravity, thermal and wind loads.

Gravity is the easiest to handle because it has a constant magnitude and direction independent of time, whereas the other two depend on the season, the weather and the time of day. Gravity can be studied with good accuracy and the deformation depends only upon where the antenna is pointing. For the other two, values of deformation can be given only if additional parameters are given, such as temperature and wind speed.

The weather at Fairbanks reaches lows around - 30 degrees Celsius during the winter and highs around 30 degrees in the summer. Figure 21 shows a plot of the lowest and highest temperature during the day for the last two years. Furthermore, temperature differences in the antenna have to be taken into account. Some parts may be in the sun whereas others are in the shadow.

Then there is the wind that has an even more random nature. For Fairbanks II the station crew say that the wind speeds are generally low
(less than $10 \mathrm{~m} / \mathrm{s}$ ). The station is located in a valley running east to west which are also the most common wind directions. However, more precise data is not available because the wind meter is presently disconnected.

All these forces give rise to deformations in the antenna and it is important to sort out those deflections that are of interest and influence the accuracy in the VLBI measurements.

The VLBI reference point of the antenna is located in the center of the x -axis, deflections at this point are of interest. But there is also a time delay associated with the time it takes the incoming wave front to reach the receiver. Part II of this report dicussed this and noted that deformations along the optical axis for the receiver and vertex of dish were of interest in this context, and so was also the change in focal length and the distance between the x and y -axis. This is the bare minimum that has to be studied. Other items such as pointing errors in the antenna will also be touched upon.

## 20 Models

The model of the antenna closely follows the five major parts described in Section 18. They can of course be analysed separately but to get more accurate results they have to be joined together. Figure 22 shows the $x$ wheel, $y$-wheel and dish joined together. The model is cut along the $x$-axis and can be used for different elevations of the y-wheel. Figure 23 shows the same parts joined together but now cut along the $y$-axis. This model is used to study the antenna at different X angles with the Y angle zero.

Calculations with these models requires $16-18 \mathrm{Mb}$ of disk space. The quadrupod has not been added because it cannot be cut along the same boundaries as the other parts. (There are diagonal beams in the legs that crosses the boundary.) The absence of the quadrupod introduces an error but the stiffness that the quadrupod should add is much less than the stiffness in the massive square girder that supports the dish and the quadrupod. Furthermore, the calculated deformations in the square girder at the points that are attached to the quadrupod can be transfered to the quadrupod model as boundary conditions. This would then give higher accuracy in the calculated deformations in the quadrupod.


Figure 21: The high and low temperatures at Fairbanks II during the day for the last two years.


Figure 22: The $x$-wheel, $y$-wheel and dish joined together. The model is cut along the x -axis and is used to study different Y angles.


Figure 23: The same parts as in Figure 7. but this time the model is cut along the y -axis. The model is used to study different X angles.

Another way to assemble the parts is to use substructures. In Figures 24-26 the different substructures of the pedestal, $x$ and $y$-wheel and the quadrupod are assembled at different $X$ and $Y$ angles. In these models the dish has been included as loads at the $y$-wheel. The model of the dish is too large for the COMPAQ to make a substructure that is useful.

## 21 Calculated deformations

### 21.1 Gravity

When presenting deformations calculated for gravity one has to take into account that the gravity forces are always acting on the antenna (they cannot be turned off like wind loads). This is especially important for the dish, it was adjusted to a paraboloid when pointing at zenith with gravity acting on it. This means that the deformations one is interested in are changes in the shape at other configurations relative to the zenith configuration. (Technically this is done by calculating deflections at different configurations and from them subtracting the deformations calculated at zenith; thereby the relative deformation at zenith becomes zero, just as required.) Thus what is of interest is changes in deformation from one configuartion to another, not the absolute deformations from when the beams lay on the ground before they were put up in the structure.

### 21.1.1 The quadrupod

The quadrupod can be analysed by itself. The boundary condition then have to be that the ends of the four legs and the points at the center at the end of the four beams supporting the legs are not allowed to move. This is of course not correct but the analysis tells us about the stiffness in this structure. In Figure 27 the value of the deformation along the optical axis away from the dish is plotted at different elevations; in Figure 20 this is in the negative $z$ direction. The deformation can be approximated by $0.15(1-\sin E L) \mathrm{mm}$ where $E L$ is the angle of elevation. This is discussed in Part II, Sec. 15. Figure 28 shows the deflections orthogonal to the optical axis at different elevations. At zero elevation the feed box drops about 4.3 mm towards the ground with these boundary conditions. This time the


Figure 24: The pedestal, $x$-wheel, $y$-wheel and quadrupod assembled as substructures. The angles are: $\mathrm{X}=0$ and $\mathrm{Y}=90$.


Figure 25: The substructures assembled with $\mathrm{X}=90$ and $\mathrm{Y}=0$.


Figure 26: The substructures assembled with $\mathrm{X}=45$ and $\mathrm{Y}=45$.
deformation goes like the cosine of the elevation. Both these calculations were made with the gravity vector in the xz-plane in Figure 20. Figure 29 shows the deflection orthogonal to the optical axis (as in Figure 28) at the horizon when the gravity vector has different angles to this plane. The calculated value of it is nearly constant at 4.3 mm .

### 21.1.2 Results from the substructure model

The substructures have been assembled and analysed for the antenna pointing at different X and Y angles with the increment of 45 degrees. This means a total of three times three, that is nine, different structures that have to be analysed. As mentioned earlier, changes in the position of the reference point and distance between the x and y -axis are of interest. The reference point does not move very much due to the fact that the antenna is well balanced around the x -axis. The counterload is a few hundred pounds more than needed, so that the antenna will move to stowed position if the drive fails. This small force is negligable compared to other forces and results in deformations much less than 1 mm . So the net effect of the antenna on the pedestal is approximately two vertical and identical loads in the two towers.

But the distance between the x and y -axis changes and so does the distance to the receiver. Table 1 summarizes the results for different $X$ and $Y$ angles. Here we note that the distance between $x$ and $y$ axis is 0.7 mm shorter at $\mathrm{X}=0$ than at $\mathrm{X}=90$. Here the deformation is not set to zero at zenith. This way it is easier to see the behaviour of the deformations. There is also some dependence in Y angle but much less than compared to the above deflections. In Part II, Sec. 15 it was argued that the deflection should mainly be proportional to cosine of the X angle. This agrees reasonably with the results.

Also in Table 1 is the change in distance from the square girder to the feed/receiver. At zenith it is 0.15 mm shorter than when the antenna points at the horizon. Part II, Sec. 15 noted that this deflection should mainly be proportional to $\cos X \cos Y$ and again the calculated values agree well with this.


Figure 27: The deflection of the feed box away from the dish at different elevations. Note that the deflection at zenith is set to zero and that it moves away from the dish at lower elevations.


Figure 28: Deflection orthogonal to the optical axis in the feed box at different elevations. When the antenna points at the horizon the feed box drops 4.3 mm towards the ground.


Figure 29: Deflections towards the ground in the feed box when the antenna points at the horizon. The deflections are given for different values of the angle between the gravity vector and the x -axis (see Figure 20). The gravity vector is orthogonal to the $z$-axis when the antenna points towards the horizon.

| $x$ <br> $y$$\| 0$ | 45 | 90 |  |
| :---: | :---: | :---: | :---: |
| 0 | -6.8 | -4.1 | 0.0 |
| 45 | -6.4 | -4.6 | 0.0 |
| 90 | -7.6 | -5.1 | 0.0 |
| $\times$ to y axis |  |  |  |


feed to square girder

Table 1: The change in distance between $x$ and $y$-axis, and the distance to the receiver from the square girder as a function of X and Y angle. Deformations given in millimeters.

### 21.1.3 The dish

To study the effects on the dish the models presented in Figures 22-23 were used. In this context the program PARABOL presented in Part I can be used to calculate the best fit paraboloid and changes in the average path length to the receiver.

The model presented in Figure 22 was used to study the deformation in the antenna with $\mathrm{X}=0$ and different Y angles. The results are presented in Figure 30 . We see that the focal length at horizon is 4.5 mm shorter than it was at zenith. We also note that the change follows the sine of elevation pretty well, just as mentioned in Part II. In the same figure the change in the position of the best fit vertex along the optical axis is also presented. It follows the same behaviour as the change in focal length, but with opposite sign.

In Figure 31 the same data is presented but now for $\mathrm{Y}=0$ but X changing. Again the focal length is 4.5 mm shorter at the horizon than at zenith.

In Table 2 there is a comparison between the change in average pathlength at different X and Y angles with the results presented in Part II, Sec. 14. According to this section the change should be

$$
\begin{equation*}
\Delta L=\alpha_{V} \Delta V+\alpha_{f} \Delta f+\alpha_{R} \Delta R \tag{21.1}
\end{equation*}
$$

The comparison uses the results in Figures 30-31, and thus do not take into account the change in the receiver position. If one uses $\alpha_{V}=-1.8$ and $\alpha_{f}=0.2$ the agreement is very good. If we as boundary conditions in the quadrupod model use the calculated deformations in the square girder, we find that the receiver has moved away 1.3 mm along the optical axis at the horizon. (The distance to the dish has still increased by 0.15 mm because the dish has also moved.) The movement of the receiver follows the same behaviour as in Figure 27. If we use the above data and correct the data in Table 2 with $\alpha_{R}=0.8$ we get the results presented in Table 3. We see that the change is approximately $-2.4(1-\cos X \cos Y) \mathrm{mm}$, that is $-2.4(1-\sin E L) \mathrm{mm}$.


Figure 30: The change in focal length and postion of the best fit vertex along the optical axis as a function of the Y angle when $\mathrm{X}=0$.


Figure 31: The change in focal length and position of the best fit vertex along the optical axis as a function of the X angle when $\mathrm{Y}=0$.


Table 2: Comparison between the calculated average path length change (in mm ) with PARABOL and $\Delta L=-1.8 \Delta V+0.2 \Delta f$. Here the change in receiver position along the optical axis $\Delta V$ and change in focal length $\Delta f$ are taken from Figures 30-31. Note that the change in distance between $x$ and y -axis is included in the change of vertex position.

| $Y$ | 0 | 45 | 90 |
| :---: | :---: | :---: | :---: |
| 0 | 0 | -0.8 | -2.3 |
| 45 | -1.1 | - | - |
| 90 | -2.6 | - | - |

Table 3: The results in Table 2 adjusted for the movement of the receiver.

### 21.1.4 Pointing corrections

Due to deformations in the dish and the structure that supports the receiver corrections in the X and Y angle have to be added to get the optimum pointing on a source. Ed Himwich (Interferometrics Inc./GSFC, Code 621.9) has the literature and knows a lot about pointing and we have discussed the results in this section. The presented pointing parameters have been determined by Ed.

The terms in the correction that corrects for deformation are for the X angle $-0.00968 \sin X \sec Y$. This is the value that should be added in degrees to the angle to point on source. For $Y$ angle the term is $0.153 \cos X \sin Y$. From this we see that when $\mathrm{X}=0$ and $\mathrm{Y}=90$ the correction in X angle is 0 and the correction in Y angle is 0.153 degrees. For the opposite situation $\mathrm{X}=90$ and $\mathrm{Y}=0$ the correction in X angle is -0.00968 and for Y angle 0.

Figure 32 illustrate how the correction $\Delta$ can be calculated. From the figure we get that $\Delta=(V+R) / f-2 \alpha$. We get $V$ and $\alpha$ from the program PARABOL and $R$ is calculated by transfering the deflections in the connection points in the dish to the quadrupod calculated with the models in Figures 22-23 and applying gravity.

For $\mathrm{X}=0$ and $\mathrm{Y}=90$ the calculated values are $V=6.16 \cdot 10^{-2} \mathrm{~m}, R=$ $2.62 \cdot 10^{-2} \mathrm{~m}$ and $\alpha=2.97 \cdot 10^{-3} \mathrm{rad}$. With $f=10.97 \mathrm{~m}$ this gives $\Delta=$ $2.07 \cdot 10^{-3} \mathrm{rad}$, that is 0.12 deg . This agrees well with the experimental value.

For $\mathrm{X}=90$ and $\mathrm{Y}=0$ the calculated values are $V=6.09 \cdot 10^{-2} \mathrm{~m}, R=$ $1.60 \cdot 10^{-2} \mathrm{~m}$ and $\alpha=3.05 \cdot 10^{-3} \mathrm{rad}$. This gives a correction $\Delta=0.051 \mathrm{deg}$, which has the opposite sign of the experimentally determined correction. But note that we subtract two values of equal magnitude to calculate the correction and thus errors have great influence.

### 21.2 Temperature

When metal is heated or cooled it expands or contracts. The basic relationship for a simple beam is

$$
\begin{equation*}
\Delta L=\alpha \Delta T L \tag{21.2}
\end{equation*}
$$



Figure 32: The figure illustrates the pointing correction. The vertex moves $V$, the receiver moves $R$ and the optical axis is rotated an angle $\alpha$. If gravity were turned off all of these would be zero and the antenna would point at the horizon. But with gravity the antenna has to add $\Delta$ to its zenith angle for a wavefront coming from the horizon to reach the receiver. From the figure we get $\alpha+\gamma=(V+R) / f$ and $\Delta=\gamma-\alpha$. Thus we have $\Delta=(V+R) / f-2 \alpha$.
where $\Delta L$ is the change in length, $\alpha$ the thermal expansion coefficient, $\Delta T$ the change in temperature and $L$ the length of the beam. The calculated deformations agree with this formula and indicate that metal is metal and that changes in height and distances do not depend to a high degree on the structure when it comes to changes in the temperature.

### 21.2.1 Substructures

The model consisting of four substructures has been used to calculate the effect of a 10 K temperature rise. This has been done for the nine combinations of the x and y -wheel mentioned in Sec. 21.1.2. It turns out that the change in height of the pedestal is 1.8 mm , the distance between x -axis and $y$-axis is 0.8 mm and the distance from the feedbox to the square girder is 1.6 mm independent on the configuration of the antenna. Table 4 summarizes these results. The values one gets if one uses Eq. 21.2 and the rough dimensions in the antenna ( $13 \mathrm{~m}, 7.6 \mathrm{~m}$ and 13 m ) are $1.6 \mathrm{~mm}, 0.9 \mathrm{~mm}$ and 1.6 mm . Furthermore, the change for any other temperature is easy to calculate because the deformations are proportional to the change in temperature. For example, if the change is 20 degrees Celsius the distance between the x and y -axis should change $2 * 0.8=1.6 \mathrm{~mm}$. For seasonal changes of 50 K the change in height of the pedestal, the distance between x and y -axis, and height of quadrupod should be $9 \mathrm{~mm}, 4 \mathrm{~mm}$ and 8 mm .

The effects of differential heating of the pedestal when parts of it are in the shadow and other parts of it are in the sun has also been studied. The pedestal tilts away from the sun and a rough calculation with a temperature difference of 5 degrees between sun and shadow results in a maximum tilt of 1.2 mm . The result is schematically presented in Figure 33.

### 21.2.2 The dish

The same models as used in Section 21.1.3 were used to study the effects of a 10 K temperature rise in the structure. The results for the focal length are presented in Table 5. We see that the change in focal length is independent of either X or Y angle. The change in distance between vertex and $y$-axis is 0.4 mm . And if we combine this with the change in height of


| $y \times$ | 0 | 45 | 90 |
| :---: | :---: | :---: | :---: |
| 0 | 1.6 | 1.6 | 1.6 |
| 45 | 1.6 | 1.6 | 1.6 |
| 90 | 1.6 | 1.6 | 1.6 |

feed to square girder

Table 4: Changes in the antenna for different X and Y angles for a 10 K rise in temperature. Deflections are in millimeters.


Figure 33: The pedstal tilts away from the sun; a differential temperature of 5 K is assumed. The figure schematically presents the behaviour in relation to the orientation of the pedestal.
the quadrupod we can calculate the change in path length through the dish with Eq. 21.1 and the same values for the coefficients as in Sec. 21.1.3: $\Delta L=-1.8 \cdot 0.4+0.2 \cdot 1.3+0.8 \cdot 1.6=0.8 \mathrm{~mm}$. Here the change in distance between x and y -axis is not included.

### 21.3 Wind

When studying the wind loads the program WINDLOAD, described in Part I, Sec. 7, was used. Only wind forces acting on the dish surface are taken into account. The deformation were calculated with the models presented in Figures $22-23$ with a wind of $10 \mathrm{~m} / \mathrm{s}$. Note that the forces are proportional to the square of the wind speed, and thus also the deformations. In Table 6 the changes in focal length in the antenna at different configurations are presented.

We see that the maximum change is around 60 degree elevation with the wind blowing into the dish. This has to do withthe fact that the dish at this elevation has the leading edge nearly parallel to the ground and acts as a wing. Around this elevation the wing stalls and pressure maximas occur.

The change in the position of the vertex along the axis is at most 0.1 mm and can for an accuracy in millimeters be neglected. Likewise, changes in the position of the center of the x -axis are on the same order.

## 22 Measurements

Two different measurements have been done to verify that the calculated measurements agree with actual deformations. At Fairbanks it is possible to optimize the position of the receiver by moving it along the optical axis. Figure 34. shows the result of these measurements. The optimum position is 7 mm closer to the dish at the horizon. The calculated value is 4.5 mm . The fitted curve is proportional to $\sin E L$ just as expected.

The other measurement is a good test of the substructure model. When the antenna was in service position, the configuration presented in Figure 25 , it was loaded with 175 lb in the receiver box at the top of the quadrupod. With a 5 arcsec level the deformation was measured to be 0.14 mm . The calculated value is 0.19 mm .


Table 5: The change in focal length (in mm) at different $X$ and $Y$ angles for a 10 K rise in temperature.


Table 6: The changes in focal length (in mm ) for a $10 \mathrm{~m} / \mathrm{s}$ wind blowing at the antenna into the dish (front) and from behind (back). Note the maximum around 60 degrees of elevation when the wind blows into the dish.


Figure 34: The measured optimum focus position as a function of zenith angle.

Discrepancies between calculated and measured values can be explained by inaccuracies in the models. First some structural data might not be correct. When I visited Fairbanks, I made measurements of the dimensions in the antenna, but this just covered a small part of the antenna. During the work on the models just notes taken from the drawings were available and all details were in no way covered. There is a possibillity for errors. Furthermore, it is known that the bolted joints in an antenna start to loosen up and members start to be able to move in the joints when the antenna has been moved back and forth a couple of years. Bolts then have to be replaced.

## 23 Conclusions for the Fairbanks antenna

The deformations in the VLBI antenna at Fairbanks are on the order of a few millimeters. The calculated values agree reasonably well with the measured values. At present these deformations are smaller than the errors that the present VLBI measurements have. But if millimeter accuracy which is the stated goal - is to be achieved these deformations have to be taken into account.

The importance of the various deformations are in decreasing order: temperaure, gravity and wind. A first rough model of the change in path length through the antenna is

$$
\begin{equation*}
\Delta L=\Delta T\left(a_{0}+a_{1} \cos X\right)+a_{2}(1-\cos X \cos Y) \tag{23.1}
\end{equation*}
$$

where $\Delta T$ is the change in temperature from some reference temperature. Here $\Delta L=0$ at zenith for $\Delta T=0$. The change in distance between x and y -axis was 0.8 mm for 10 K . This means that $a_{1}$ is on the order of -0.1 $\mathrm{mm} / \mathrm{K}$. From Section 21.2 .2 we can in the same way see that $a_{0}$, the change through the dish, should be on the order of $.1 \mathrm{~mm} / \mathrm{K}$ and from for example Table 3 we see that $a_{2}$ should be on the order of -2 mm .

Furthermore the height of the pedestal changes with temperature, about 0.2 mm per K .

If the VLBI measurements reaches millimeter accuracy these effects should be visible and thus these coefficients could be determined experimentally. This also requires some sort of temperature sensor on the antenna.

Several sensors may make it possible to predict the tilt from differential heating.

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| 16. Abstract <br> The authors present a study of deformations in antennas with the emphasis on the influence on VLBI measurements. GIFTS structural analysis program has been used to model the VLBI antenna in Fairbanks, Alaska. The report identifies key deformations and studies the effect of gravity, wind and temperature. Estimates of expected deformations are given. |  |  |
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