NASA Technical Memorandum 100586

SLIP VELOCITY METHOD FOR THREE-DIMENSIONAL COMPRESSIBLE TURBULENT BOUNDARY LAYERS

(NASA-TM-100586) SLIP VELOCITY METHOD FOR THREE-DIMENSIONAL COMPRESSIBLE TURBULENT BOUNDARY LAYERS (NASA) 22 p CSCL 20D
N88-20592
Unclas
G3/34 0135348

Richard W. Barnwell and Richard A. Wahls

MARCH 1988

NASA
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665-5225

A slip velocity method for two-dimensional incompressible turbulent boundary layers was presented in reference 1. The inner part of the boundary layer was characterized by a law of the wall and law of the wake, and the outer part was characterized by an arbitrary eddy viscosity model. In the present study for compressible flows, only a law of the wall is considered. The problem of two-dimensional compressible flow is treated first; then the extension to three-dimensional flow is addressed.

Two-Dimensional Compressible Flow

Basic Equations

The governing equations for compressible boundary layer flow are

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad (1)$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} - \rho_e U\frac{dU}{dx} = \frac{\partial}{\partial y}\left\{(\mu+\mu_t)\frac{\partial u}{\partial y}\right\} \qquad (2)$$

where x and y are the normal and tangential coordinates, u and v are the respective velocity components, ρ is the density, ρ_e and U are the density and fluid speed at the outer edge of the boundary layer, and μ is the dynamic viscosity. The sum of the dynamic viscosity and the turbulent eddy viscosity μ_t is defined as

$$\mu + \mu_t = K(x,y)\ \rho_e U\delta_i^* \qquad (3)$$

where K is a general nondimensional function of x and y, and δ_i^* is the incompressible displacement thickness.

In this treatment, the defect stream function of Clauser (ref. 2) is used. This function is defined as

$$f'(\xi,\eta) = \frac{u-U}{u^*} \qquad (4)$$

where

$$\xi = x, \ \eta = \frac{y}{\Delta}$$

The prime denotes differentiation with respect to η, and the shear stress velocity u^* is defined as

$$u^* = \left[\frac{\tau_w}{\rho_w} \right]^{1/2}$$

where τ_w and ρ_w are the wall shear stress and density. The boundary layer thickness parameter Δ is defined as

$$\Delta = \frac{U\delta_i^*}{u^*}$$

or

$$u^*\Delta = U\delta_i^* \qquad (5)$$

Partial derivatives with respect to x and y are of the form

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} - \eta \frac{\dot{\Delta}}{\Delta} \frac{\partial}{\partial \eta} \quad , \quad \frac{\partial}{\partial y} = \frac{1}{\Delta} \frac{\partial}{\partial \eta} \tag{6}$$

where the dot denotes differentiation with respect to ξ.

Law of the Wall

It is assumed that the flow is adiabatic, and the gas is calorically perfect. The law of the wall for this flow is obtained with a treatment similar to that of Van Driest (ref. 3) as

$$u = \left[\frac{2}{r(\bar{\gamma}-1)}\right]^{1/2} a_{aw} \sin\left\{\left[\frac{r(\bar{\gamma}-1)}{2}\right]^{1/2} \frac{u^*}{a_{aw}} \left[\frac{1}{\kappa} \ln\left(\frac{\rho_w u^* y}{\mu_{aw}}\right) + B\right]\right\}$$

where $\bar{\gamma}$ is the ratio of specific heats, κ and B are the von Karman constants, and a_{aw} and μ_{aw} are the adiabatic-wall speed of sound and dynamic viscosity, respectively. The quantity r is the recovery factor, which is typically evaluated as $Pr^{1/3}$, where Pr is the Prandtl number. The equation for the density for this flow is

$$\rho = \frac{\rho_e}{1 + \frac{r(\bar{\gamma}-1)}{2} M_e^2 \left(1 - \frac{u^2}{U^2}\right)} \tag{7}$$

where M_e is the edge Mach number. Since the ratio u^*/a_{aw} is small, the equation for u can be written as

$$u = u^* \left\{\frac{1}{\kappa} \ln\left(\frac{\rho_w u^* y}{\mu_{aw}}\right) + B\right\} + 0\left(\left[u^*/a_{aw}\right]^3\right) \tag{8}$$

Note that the lowest order term in this equation is the same as the law of the wall for incompressible flow.

The nondimensional shear stress velocity ratio is

$$\gamma = \frac{u^*}{U} \qquad (9)$$

and where $m(\xi)$ is defined as

$$\frac{1}{m} = \frac{U}{\dot{U}} \frac{\dot{\Delta}}{\Delta} \qquad (10)$$

The Clauser pressure gradient parameter β is defined as

$$\beta = \frac{\delta_i^*}{\tau_w} \frac{\partial p}{\partial x} = - \frac{\rho_e}{\rho_w} \frac{\Delta}{\gamma} \frac{\dot{U}}{U} \qquad (11)$$

where p is the pressure. The dot can be used to represent the gradient of x as well as ξ since U is a function only of $x = \xi$.

An important relationship for the lowest order form of the stream function $f(\xi,\eta)$ and its derivative f' is obtained from the law of the wall. Equation (8) can be written in terms of η and ξ as

$$u = u^* \{\frac{1}{\kappa} \ln (Re_{\delta}{}^* \eta) + B\} + 0 ([u^*/a_{aw}]^3) \qquad (12)$$

where the Reynolds number based on the incompressible displacement thickness δ_i^*, the wall properties μ_{aw} and ρ_w, and the edge velocity U is

$$Re_{\delta}{}^* = \frac{\rho_w U \delta_i^*}{\mu_{aw}}$$

The stream function f can be expanded in terms of the small parameter γ as

$$f = f_o + \gamma f_1 + \ldots$$

With this expansion and equation (4), equation (12) can be written to lowest order as

$$f_o = \eta \{f_o' - \frac{1}{\kappa}\} \tag{13}$$

This equation pertains throughout the inner region of the boundary layer.

Governing Equation for f_o

The governing equation for f_o is obtained from equation (2), the tangential momentum equation. To establish the equation for f_o, the mass flux components ρu and ρv and the partial derivatives of u with respect to x and y must be expressed in terms of f_o. With equations (4) and (7), the u component of velocity and the density can be expressed to first order in γ as

$$u = U \{1 + \gamma f_o'\} \tag{14}$$

$$\rho = \rho_e \{1 + 2\gamma \frac{(\rho_e - \rho_w)}{\rho_w} f_o'\}$$

With these equations the mass flux ρu can be approximated as

$$\rho u = \rho_e U \{1 + \gamma (2 \frac{\rho_e}{\rho_w} - 1) f_o'\} \tag{15}$$

From equations (6) and (14) the derivative $\partial u/\partial x$ is obtained as

$$\frac{\partial u}{\partial x} = \dot{U} \{1 + \gamma [f_o' - \frac{1}{\beta} \frac{\rho_e}{\rho_w} \frac{\partial f_o'}{\partial s} - \frac{\eta}{m} f_o'']\} \qquad (16)$$

where the nondimensional tangential coordinate s is defined as

$$s = \int^{\xi} \frac{\gamma}{\Delta} d\xi$$

The zero-order approximation for the mass flux component ρv is obtained from equations (1), (6), and (15) as

$$\rho v = - \int^{y} \frac{\partial (\rho u)}{\partial x} dy = - \frac{\partial (\rho_e U)}{\partial x} \Delta \eta = -\Delta \dot{U} \rho_e (1 - M_e^2) \eta \qquad (17)$$

Finally, the normal derivative of u is

$$\frac{\partial u}{\partial y} = \frac{U\gamma}{\Delta} f_o'' \qquad (18)$$

With equations (15), (16), (17), and (18), the tangential momentum equation can be written as

$$\frac{1}{\beta} \frac{\rho_e}{\rho_w} \frac{\partial f_o'}{\partial s} = \frac{1}{\beta} (\frac{\rho_e}{\rho_w} K f_o'')' - (1 - M_e^2 + \frac{1}{m}) \eta f_o'' + 2 \frac{\rho_e}{\rho_w} f_o' \qquad (19)$$

It should be noted that this stream function treatment is patterned after that of Mellor and Gibson (ref. 4) for incompressible equilibrium boundary layers. In particular, it is patterned after the lowest-order treatment of Mellor and Gibson.

The three boundary conditions for f_o involve the values of f_o at the wall and in the free stream and the value of the shear stress at the wall. Since

With boundary condition (22), this equation can be integrated across the boundary layer to obtain

$$\frac{1}{m} = M_e^2 - 1 - 2\frac{\rho_e}{\rho_w} - \frac{1}{\beta}$$

With this value, the first integral of equation (19) for arbitrary η is written as

$$\frac{\rho_e}{\rho_w}\frac{\partial f_o}{\partial s} = \frac{\rho_e}{\rho_w} K f_o'' + \left[1 + 2\frac{\rho_e}{\rho_w}\beta\right]\eta f_o' - f_o - 1 \qquad (23)$$

Note that no assumptions have been made which limit the arbitrariness of the nondimensional viscosity coefficient K.

Match Point Location

The match point divides the outer and the inner regions of the boundary layer. In the outer part of the boundary layer the viscosity coefficient K is arbitrary. In the inner region, the flow is governed by an empirical law of the wall. There is one point, the match point, at which both the arbitrary coefficient of viscosity and the law of the wall pertain. At this point the stream function f_o and its first three derivatives with respect to η are continuous. Note that in the parlance of asymptotic expansions, the "match point" would be properly termed a "patch point."

It is assumed that the flow in the inner region and hence at the match point is essentially in equilibrium. The quantities f_o'' and f_o in the inner region are evaluated with the law of the wall as

$$f_o'' = \frac{1}{\kappa\eta} \quad , \quad f_o = \eta\{f_o' - \frac{1}{\kappa}\}$$

the mass flux component ρv must vanish at the wall, the defect stream function f_o must also vanish at the wall:

$$\lim_{\eta \to 0} f_o(\xi, \eta) = 0 \tag{20}$$

The free stream boundary condition is obtained from the definition of the incompressible displacement thickness as

$$\int_o^\infty \{u - U\}dy = u^* \Delta \int_o^\infty f_o'(\eta)d\eta = u^* \Delta \{f_{o,\infty} - f_o(o)\} = -U\delta_i^*$$

or

$$f_{o,\infty} = -1 \tag{21}$$

With equations (4), (5), (9), and (18), the shear stress and shear stress at the wall can be written as

$$\tau = \gamma^2 U^2 \rho_e K f_o'' , \quad \tau_w = \gamma^2 U^2 \rho_w$$

From these equations, it is seen that the shear stress boundary condition is

$$\lim_{\eta \to o} K f_o'' = \frac{\rho_w}{\rho_e} \tag{22}$$

The governing equation for f_o has a first integral. Equation (19) can be written as

$$\{\frac{1}{\beta} \frac{\rho_e}{\rho_w} K f_o'' - (1 - M_e^2 + \frac{1}{m}) (\eta f_o' - f_o) + 2 \frac{\rho_e}{\rho_w} f_o - \frac{1}{\beta} \frac{\rho_e}{\rho_w} \frac{\partial f_o}{\partial s}\}' = 0$$

Thus at the match point the governing equation (23) can be written as

$$\frac{\rho_e}{\rho_w}\frac{K}{\kappa\eta_m} = 1 - \{\frac{1}{\kappa} + 2\frac{\rho_e}{\rho_w}\beta f'_{o,m}\}\eta_m$$

Let

$$A = \frac{1}{\kappa} + 2\frac{\rho_e}{\rho_w}\beta f'_{o,m}$$

and note that

$$\eta_m = \frac{y_m}{\Delta}$$

The governing equation is

$$A\left(\frac{y_m}{\Delta}\right)^2 - \frac{y_m}{\Delta} + \frac{\rho_e}{\rho_w}\frac{K}{\kappa} = 0$$

The solution for the match point is

$$\frac{y_m}{\Delta} = \frac{1 - \left[1 - 4A\frac{\rho_e}{\rho_w}\frac{K}{\kappa}\right]^{1/2}}{2A}$$

This solution depends strongly on the parameter $\frac{\rho_e}{\rho_w}\frac{K}{\kappa}$ and relatively weakly on the function A:

$$\frac{y_m}{\Delta} = \frac{\rho_e}{\rho_w}\frac{K}{\kappa} + A\left(\frac{\rho_e}{\rho_w}\frac{K}{\kappa}\right)^2 + 2A^2\left(\frac{\rho_e}{\rho_w}\frac{K}{\kappa}\right)^3 \ldots$$

Shear Stress

The governing equation in the inner region can be written as

$$(\mu + \mu_t)\ (\frac{\partial u}{\partial y}) = \rho_w (u^*)^2 \{1 - A\ (\frac{y}{\Delta})\}$$

If u and $\partial u/\partial y$ are known at some point y, the shear stress can be determined as

$$u^* = \left[\frac{(\mu+\mu_t)\ (\frac{\partial u}{\partial y})}{\rho_w \{1 - A\ \frac{y}{\Delta}\}} \right]^{1/2}$$

Edge Velocity Determination

If the total pressure at the edge of the boundary layer can be defined, it can be used to determine the edge velocity U. The tangential momentum equation at edge of the boundary layer is

$$\rho_e U\ \frac{dU}{dx} + \frac{dp}{dx} = 0$$

which can be written as

$$\frac{1}{p}\ \frac{dp}{dx} = -\ \frac{\bar{\gamma}}{a_{aw}^2}\ \frac{U\ \frac{dU}{dx}}{1 - r\ (\frac{\bar{\gamma} - 1}{2})\ (\frac{U}{a_{aw}})^2}$$

The solution is

$$p = p_t\ [1 - r(\frac{\bar{\gamma} - 1}{2})\ (\frac{U}{a_{aw}})^2]^{\frac{\bar{\gamma}}{\bar{\gamma} - 1}}$$

$$U = a_{aw} \left\{ \frac{2}{r(\bar{\gamma} - 1)}\ [1 - (\frac{p}{p_t})^{(\bar{\gamma} - 1)/\bar{\gamma}}] \right\}^{1/2}$$

where p_t is the total pressure at the boundary layer edge.

A second approach is to define the boundary layer edge in terms of the deviation of the total enthalpy from the freestream value.

Solution Algorithm

This solution process is for either an iterative solution of the viscous-inviscid problem or a marching solution of the viscous problem with the inviscid solution known. In either case, approximate values for the solution u(x,y), v(x,y), p(x,y), and ρ(x,y) are known. Also, the parameter u* is known. The six steps for one iteration are:

(1) From the turbulence model compute the edge velocity U and the product $U\delta_i^*$

(2) Calculate

$$\delta_i^* = \frac{U\delta_i^*}{U}, \quad r = Pr^{1/3}, \quad \frac{\rho_e}{\rho_w} = 1 + \frac{r(\bar{\gamma}-1)}{2} M_e^2, \quad a_{aw} = [\frac{\bar{\gamma} p}{\rho_w}]^{1/2}$$

$$\beta = \frac{\delta_i^*}{\rho_w (u^*)^2} \frac{\partial p_w}{\partial x}, \quad Re_{\delta^*} = \frac{\rho_w U \delta_i^*}{\mu_{aw}}$$

(3) Compute iteratively

$$A = \frac{1}{\kappa} + 2\frac{\rho_e}{\rho_w} \beta (\frac{u_m - U}{u^*})$$

$$\frac{y_m}{\Delta} = \frac{1 - [\, 1 - 4A \frac{\rho_e}{\rho_w} \frac{K}{\kappa}\,]^{1/2}}{2A}$$

$$u_m = a_{aw} \left[\frac{2}{r(\bar{\gamma}-1)} \right]^{1/2} \sin\{ \left[\frac{r(\bar{\gamma}-1)}{2} \right]^{1/2} \frac{u^*}{a_{aw}} [\frac{1}{\kappa} \ln (Re_{\delta^*} \frac{y_m}{\Delta}) + B]\}$$

(4) Calculate

$$v_m = \beta u^* \frac{y_m}{\Delta} (1 - M_e^2)\frac{\rho_w}{\rho_e}$$

$$T_m = T_e \left\{1 + \frac{r(\bar{\gamma} - 1)}{2} M_e^2 (1 - \frac{u_m^2}{U^2})\right\}$$

(5) Compute the new solution $u(x,y)$, $v(x,y)$, $p(x,y)$, and $\rho(x,y)$ using u_m and v_m as boundary conditions.

(6) Calculate

$$u^* = \left(\frac{(\mu + \mu_t)_m \; (\frac{\partial u}{\partial y})_m}{\rho_w \{1 - A \frac{y_m}{\Delta}\}} \right)^{1/2}$$

Note that the boundary conditions u_m and v_m can be applied at the model surface as a first approximation.

The main difficulty with the present method is the need to specify the edge velocity U. For hypersonic flow and slender body flow U can be replaced with u_∞ .

Cross-Flow Effects

Initially, at least, it will be assumed that a small crossflow approach can be used. Cooke (ref. 5) has shown that the equation for the streamwise velocity component, measured relative to the velocity vector at the outer edge of the boundary layer, for incompressible flow is independent of cross-flow effects and hence is similar to the equation for two-dimensional flow. It will be assumed that the same is true for compressible flow.

In this treatment, it is assumed that the turning angle near the wall is essentially linear with respect to distance from the wall so that it can be evaluated by extrapolation. The linear behavior of the turning angle near the wall is supported by the data of Johnston (ref.6) and Van Den Berg, Elsenaar, Lindhout, and Wesseling (ref. 7).

An additional arguement for linear extrapolation comes from the results of Mellor (ref. 8) for two-dimensional, incompressible, high Reynolds number flow and the three-dimensional extension of Goldberg and Reshotko(ref. 9). Both of these treatments show that, to lowest order, the inner layer flow is determined by viscous forces; pressure-gradient forces do not appear. Since turning is the result of the interplay of pressure and viscous forces within the boundary layer, it is reasonable to assume that turning is complete by the time the inner region is reached and only viscous forces remain.

For purposes of illustration, let x,y, and z be Cartesian boundary layer coordinates with y the normal coordinate. The respective velocity components are u, v, and w. The turning angle ϕ and tangential velocity u_{tan} are defined as

$$\phi = \tan^{-1} \left(\frac{w}{u}\right) , \quad u_{tan} = (u^2 + w^2)^{1/2}$$

Let y_1 and y_2 be the two locations immediately above the match point y_m. The match point turning angle is

$$\phi_m = \frac{y_2 - y_m}{y_2 - y_1} \phi_1 - \frac{y_1 - y_m}{y_2 - y_1} \phi_2$$

and the law of the wall is expressed in terms of u_{tan} as

$$u_{tan} = u^{*}\{\frac{1}{\kappa} \ln (\frac{\rho_w u^{*} y}{\mu_w}) + B\}$$

The velocity components u_m and w_m, which are needed as slip velocity boundary conditions, are obtained from ϕ_m and $u_{tan,m}$ with the equations

$$u_m = u_{tan,m} \cos \phi_m$$

$$w_m = u_{tan,m} \sin \phi_m$$

The equation for u^{*} is

$$u^{*} = \left(\frac{(\mu + \mu_t) \quad (\frac{\partial u}{\partial y} tan)_m}{\rho_w\{1 - A \frac{y_m}{\Delta}\}} \right)^{1/2}$$

<u>Implementation</u>

Consider an x,y, z Cartesian coordinate system fixed to the three-dimensional configuration with the x coordinate in the axial direction. Let the unit normal to the surface be

$$\vec{n} = \vec{i}\, n_x + \vec{j}\, n_y + \vec{k}\, n_z$$

The flow angle ϕ will be measured in the tangent plane (the plane normal to $\vec{n}$). The angle ϕ will be measured from the line where a reference plane intersects the tangent plane. The reference plane will be either the x-y plane or the x-z plane, depending upon whether the projection of the unit vector $\vec{n}$ on the y-z plane is more closely aligned with the unit vectors $\vec{j}$ or $\vec{k}$.

The unit vector $\vec{n}_p$ in the direction of the projection of $\vec{n}$ on the y-z plane is

$$\vec{n}_p = \vec{j}\,\frac{n_y}{(n_y^2 + n_z^2)^{1/2}} + \vec{k}\,\frac{n_z}{(n_y^2 + n_z^2)^{1/2}}$$

Thus the direction cosines between $\vec{n}_p$ and the $\vec{j}$ and $\vec{k}$ axes are

$$\cos\theta_{py} = \frac{n_y}{(n_y^2 + n_z^2)^{1/2}}, \quad \cos\theta_{pz} = \frac{n_z}{(n_y^2 + n_z^2)^{1/2}}$$

The conditions for choosing the reference planes are:

$$-\frac{\sqrt{2}}{2} < \frac{n_y}{(n_y^2 + n_z^2)^{1/2}} < \frac{\sqrt{2}}{2}, \quad \text{Use x-z plane}$$

$$\frac{n_y}{(n_y^2 + n_z^2)^{1/2}} > \frac{\sqrt{2}}{2}, \quad \frac{n_y}{(n_y^2 + n_z^2)^{1/2}} < -\frac{\sqrt{2}}{2}, \quad \text{Use x-y plane}$$

x-y Plane as Reference

A vector in the x-y plane normal to $\vec{n}$ is

$$\vec{e}_{xy} = \vec{n} \times \vec{k} = \vec{i} \times \vec{k}\, n_x + \vec{j} \times \vec{k}\, n_y = \vec{i} n_y - \vec{j} n_x$$

The unit vector in the x-y plane normal to $\vec{n}$ is

$$\vec{i}_{xy} = \vec{i}\,\frac{n_y}{(n_x^2 + n_y^2)^{1/2}} - \vec{j}\,\frac{n_x}{(n_x^2 + n_y^2)^{1/2}}$$

The unit vector normal to $\vec{i}_{xy}$ and $\vec{n}$ is

$$\vec{i}_c = \vec{i}_{xy} \times \vec{n} = -\vec{i}\,\frac{n_x n_z}{(n_x^2 + n_y^2)^{1/2}} - \vec{j}\,\frac{n_y n_z}{(n_x^2 + n_y^2)^{1/2}} + \vec{k}\,(n_x^2 + n_y^2)^{1/2}$$

The velocity component in the x-y plane tangent to the surface is

$$U_s = \vec{V} \cdot \vec{i}_{xy} = \frac{un_y}{(n_x^2 + n_y^2)^{1/2}} - \frac{vn_x}{(n_x^2 + n_y^2)^{1/2}} = \frac{(un_y - vn_x)}{(n_x^2 + n_y^2)^{1/2}}$$

where the velocity vector is

$$\vec{V} = u\vec{i} + v\vec{j} + w\,\vec{k}$$

The velocity component in the $\vec{i}_c$ direction tangent to the surface is

$$W_s = \vec{V} \cdot \vec{i}_c = \frac{-\,un_x\,n_z - vn_y n_z \;+ w\,(n_x^2 + n_y^2)}{(n_x^2 + n_y^2)^{1/2}}$$

The crossflow angle ϕ is defined as the angle in the tangent plane between the velocity vector and the intersection of the tangent plane with the x-y plane:

$$\phi = \tan^{-1}\left\{\frac{W_s}{U_s}\right\} = \tan^{-1}\left\{\frac{-\,un_x n_z - vn_y n_z + w(n_x^2 + n_y^2)}{un_y - vn_x}\right\}$$

Let n be the coordinate in the normal direction. The angle ϕ_m at the surface is obtained by linear extrapolation of the values ϕ_1 at n_1 and ϕ_2 at n_2 :

$$\phi_m = \frac{n_2 - n_0}{n_2 - n_1}\,\phi_1 - \frac{n_1 - n_0}{n_2 - n_1}\,\phi_2$$

Now assume that the new "slip velocity" U_{tan}, the angle ϕ_m, and the "inner layer transpiration velocity" V_n have been determined. The values u, v, and w are needed as boundary conditions:

$U_c = U_{tan} \cos \phi_m, \quad W_c = U_{tan} \sin \phi_m$

The velocity vector can be written as

$\vec{V} = \vec{i}_{xy} U_c + \vec{n} V_n + \vec{i}_c W_c$

The Cartesian velocity components are

$$u = \vec{V} \cdot \vec{i} = \frac{n_y U_c}{(n_x^2 + n_y^2)^{1/2}} + n_x V_n - \frac{n_x n_z W_c}{(n_x^2 + n_y^2)^{1/2}}$$

$$v = \vec{V} \cdot \vec{j} = - \frac{n_x U_c}{(n_x^2 + n_y^2)^{1/2}} + n_y V_n - \frac{n_y n_z W_c}{(n_x^2 + n_y^2)^{1/2}}$$

$$w = \vec{V} \cdot \vec{k} = n_z V_n + (n_x^2 + n_y^2)^{1/2} W_c$$

x-z Plane as Reference

Now use the x-z plane for reference. A vector in the x-z plane normal to $\vec{n}$ is

$$\vec{e}_{xz} = \vec{n} \times (-\vec{j}) = -\vec{i} \times \vec{j}\, n_x - \vec{k} \times \vec{j}\, n_z = \vec{i} n_z - \vec{k} n_x$$

The unit vector in the x-z plane normal to $\vec{n}$ is

$$\vec{i}_{xz} = \vec{i} \frac{n_z}{(n_x^2 + n_z^2)^{1/2}} - \vec{k} \frac{n_x}{(n_x^2 + n_z^2)^{1/2}}$$

The unit vector normal to $\vec{i}_{xz}$ and $\vec{n}$ is

$$\vec{i}_c = \vec{i}_{xz} \times \vec{n} = \vec{i} \frac{n_x n_y}{(n_x^2 + n_z^2)^{1/2}} - \vec{j} (n_x^2 + n_z^2)^{1/2} + \vec{k} \frac{n_z n_y}{(n_x^2 + n_z^2)^{1/2}}$$

The velocity component in the x-z plane tangent to the surface is

$$U_s = \vec{V} \cdot \vec{i}_{xz} = \frac{un_z}{(n_x^2 + n_z^2)^{1/2}} - \frac{wn_x}{(n_x^2 + n_z^2)^{1/2}} = \frac{(un_z - wn_x)}{(n_x^2 + n_z^2)^{1/2}}$$

The velocity component in the $\vec{i}_{\bar{c}}$ direction tangent to the surface is

$$W_s = \vec{V} \cdot \vec{i}_{\bar{c}} = \frac{un_x n_y - v(n_x^2 + n_z^2) + wn_z n_y}{(n_x^2 + n_z^2)^{1/2}}$$

The crossflow angle ϕ is defined as the angle in the tangent plane between the velocity vector and the intersection of the tangent plane with the x-z plane:

$$\phi = \tan^{-1}\{\frac{W_s}{U_s}\} = \tan^{-1}\{\frac{un_x n_y - v(n_x^2 + n_z^2) + w\, n_z n_y}{un_z - wn_x}\}$$

The value of ϕ at the surface is determined by extrapolation as before.

Now assume that ϕ_m and U_{tan} are known. It follows that

$$U_c = U_{tan} \cos \phi_m \quad , \quad W_c = U_{tan} \sin \phi_m$$

Also the inner layer transpiration velocity V_n is known. The velocity vector can be written as:

$$\vec{V} = \vec{i}_{xz}\, U_c + \vec{n}\, V_n + \vec{i}_{\bar{c}}\, W_c$$

The Cartesian velocity components are:

$$u = \vec{V} \cdot \vec{i} = \frac{n_z U_c}{(n_x^2 + n_z^2)^{1/2}} + n_x V_n + \frac{n_x n_y W_c}{(n_x^2 + n_z^2)^{1/2}}$$

$$v = \vec{V} \cdot \vec{j} = n_y V_n - (n_x^2 + n_z^2)^{1/2} W_c$$

$$w = \vec{V} \cdot \vec{k} = -\frac{n_x U_c}{(n_x^2 + n_z^2)^{1/2}} + n_z V_n + \frac{n_z n_y W_c}{(n_x^2 + n_z^2)^{1/2}}$$

References

1. Barnwell, Richard W.; Wahls, Richard A.; and DeJarnette, Fred R.: A Defect, Stream Function, Law of the Wall/Wake Method for Turbulent Boundary Layers. AIAA Paper No. 88-137, 1988.
2. Clauser, Francis H.: The Turbulent Boundary Layer. Advances in Applied Mechanics, vol. 4, 1956, pp. 1-51.
3. Van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids. J. Aeronaut. Sci., vol. 18, no. 3, 1951, pp. 145-160, 216.
4. Mellor, G. L. and Gibson, D. M.: Equilibrium Turbulent Boundary Layers. J. Fluid Mech., vol. 24, part 2, 1966, pp. 225-253.
5. Cooke, J. C.: An Axially Symmetric Analogue for General Three-Dimensional Boundary Layers. RAE R and M, no. 3200, 1959.
6. Johnston, J. P.: On the Three-Dimensional Turbulent Boundary Layer Generated by Secondary Flow. Journal of Basic Engineering, vol. 82, 1960, pp. 233-248.
7. Van Den Berg, B.; Elsenaar, A.; Lindhout, J. P. F.; and Wesseling, P.: Measurements in an Incompressible Three-Dimensional Turbulent Boundary Layer, Under Infinite Swept-Wing Conditions, and Comparison with Theory. J. Fluid Mech., vol. 70, part 1, 1975, pp. 127-148.

8. Mellor, George L.: The Large Reynolds Number, Asymptotic Theory of Turbulent Boundary Layers. International Journal of Engineering Science, vol. 10, 1972, pp. 851-873.

9. Goldberg, V. and Reshotko, E.: Scaling and Modeling of Three-Dimensional Pressure-Driven Turbulent Boundary Layers. AIAA Journal, vol. 22, no. 7, 1984, pp. 914-920.

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-100586		

4. Title and Subtitle	5. Report Date
Slip Velocity Method for Three-Dimensional Compressible Turbulent Boundary Layers	March 1988
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Richard W. Barnwell and Richard A. Wahls	
	10. Work Unit No. 763-01-21-10
9. Performing Organization Name and Address	11. Contract or Grant No.
NASA Langley Research Center Hampton, VA 23665-5225	
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Memorandum
National Aeronautics and Space Administration Washington, DC 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Richard W. Barnwell, Langley Research Center, Hampton, Virginia
Richard A. Wahls, North Carolina State University, Raleigh, North Carolina

16. Abstract

A slip velocity method for two-dimensional incompressible turbulent boundary layers was presented in AIAA Paper 88-0137. The inner part of the boundary layer was characterized by a law of the wall and law of the wake, and the outer part was characterized by an arbitrary eddy viscosity model. In the present study for compressible flows, only a law of the wall is considered. The problem of two-dimensional compressible flow is treated first; then the extension to three-dimensional flow is addressed. A formulation for primitive variables is presented.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Compressible flow Turbulent boundary layer Slip boundary condition	Unclassified - Unlimited Subject Category 34

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	21	A02