NASA Technical Memorandum 100536

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(NASA-TM-100536) A NEW ALGORITHM FOR GENERAL MULTIOBJECTIVE OPTIMIZATION (NASA) 11 p CSCL 12A

N88-20830

Unclas

G3/59 0134768

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MARCH 1988

NASA
National Aeronautics and
Space Administration

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A NEW ALGORITHM FOR GENERAL MULTIOBJECTIVE OPTIMIZATION

by

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ABSTRACT

The paper describes a new technique for converting a constrained optimization problem to an unconstrained one, and a new method for multiobjective optimization based on that technique. The technique transforms the objective functions into goal constraints. The goal constraints are appended to the set of behavior constraints and the envelope of all functions in the set is searched for an unconstrained minimum. The technique may be categorized as a SUMT algorithm. In multi-objective applications, the approach has the advantage of locating a compromise minimum without the need to optimize for each individual objective function separately. The constrained to unconstrained conversion is described, followed by a description of the multiobjective problem. Two example problems are presented to demonstrate the robustness of the method.

NOMENCLATURE

A	cross sectional area
E_i	Youngs Modulus of member i
F_k	k-th objective function
Fe	iron
$\tilde{F}$	global criterion or compromise objective function
F*	reduced objective function or goal constraint
f_j	set of functions
g_i	problem constraints
J	NCON + NOBJ
K-S	Kreisselmeier-Steinhauser function
NCON	number of constraints
NOBJ	number of objective functions
Ti	titanium
X	vector of design variables
ρ	K-S coefficient

Superscripts

cm	constrained minimum
o	initial design point
s	scaled value

Subscripts

i	ith element
j	jth element
k	kth element
max	maximum value
min	minimum value

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INTRODUCTION

The paper has two purposes: to bring to the optimization practitioners' attention a new technique[1] for converting a constrained optimization problem to an unconstrained one, and to present a new method for multiobjective optimization based on that technique.

The conversion technique[1] may be categorized as a "Sequential Unconstrained Minimization Technique" (SUMT) class method[2] but it does not require the use of a draw-down factor, unlike the classical procedure. Also, the unconstrained function it uses to represent the constrained problem at hand is defined over both the feasible and infeasible domains, similar to an extended penalty function[3].

Difficulty in defining a single objective function in many engineering design problems is a motivation for continuing interest in development of techniques for multiobjective optimization applications. Many of the multiobjective optimization methods require either a conversion to a single objective function by means of a composite function with judgmental "weight factors", or separate optimizations for each objective followed by an additional "global" optimization to arrive at a suitable compromise[4]. The technique introduced in this paper, for the constrained-to-unconstrained optimization problem conversion, is also shown to have an intrinsic applicability to multiobjective optimization. Its primary benefit in that application is the elimination of the potentially expensive separate optimizations for each objective.

CONSTRAINED-TO-UNCONSTRAINED CONVERSION

The conversion technique replaces the constraint boundary surfaces and the objective function surface in n-dimensional design space with a single envelope surface constructed using the Kreisselmeier-Steinhauser (K-S) function[5] first introduced for optimization of control systems. That function has subsequently been established in structural optimization as a means to replace many constraints with a single cumulative constraint[6]. The function is a differentiable envelope of a set of functions $f_j(X)$ and it has this form:

$$K\text{-}S = f_{max} + \frac{1}{\rho} \operatorname{Log}\left[\sum_j e^{\rho(f_j - f_{max})}\right], \quad j=1,\ldots,J \tag{1}$$

and a property such that

$$f_{max} + \frac{Log(J)}{\rho} < K\text{-}S \quad (2)$$

where ρ controls the distance of the K-S envelope surface from the f_{max} surface. The K-S function may be regarded as analogous to the MAX function available in many high-order programming languages but, unlike the MAX function, it is differentiable (value- and slope-continuous); therefore, it may be called upon by a gradient-guided optimization algorithm to search for a minimum of the envelope of a set of functions.

The constrained optimization problem to be solved with the aid of the K-S function is, in conventional formulation:

$$\text{minimize } F_k(X), \quad k = 1 \text{ to NOBJ}; \quad (3)$$

such that

$$g_i(X) \leq 0, \quad i = 1 \text{ to NCON};$$

where constraint functions g_i are written in terms of the computable functions, termed DEMAND (X) and CAPACITY (X), that provide the measures, respectively, of what the design is asked to carry versus what it can sustain:

$$g_i(X) = DEMAND(X)/CAPACITY(X) - 1 \quad (4)$$

For introductory purposes, $F_k(X)$ in equation 3 is a single objective (i.e., NOBJ = 1); extension to many objectives will follow later. To formulate the K-S function as an envelope of the objective functions and constraints, one has to normalize the objective function in order to make it comparable to the normalized constraint functions. The normalized objective and constraint functions form a set of functions whose envelope is approximated by the K-S function. An unconstrained minimum (except for the usual side constraints) of the K-S envelope may be found by any suitable search algorithm.

The procedure formulated[1] is illustrated in figures 1 and 2. For graphic simplicity, one design variable is shown. To make this paper self-contained the procedure is restated here in descriptive terms keyed to figures 1 and 2.

A single design variable x is measured on the horizontal axis. The objective function F and contraint functions g_1 and g_2 are represented on the vertical axis. The initial design point is at $x = x^\circ$ where the constraints are violated and the objective has the value F°. By inspection, the constrained minimum lies at x^{cm}. The requirement is to locate that minimum starting from x°.

Referring to figure 1, the objective function in its original form before normalization is labeled F. F is normalized by dividing F by F°. The scaled $F_s = F/|F^\circ|$ is then shifted to intersect the abscissa by subtracting unity and further offset by subtracting g_{max}. The shifting and offsetting is expressed as follows:

$$F^* = (F_s - 1) - g_{max} \quad (5)$$

which moves the objective function to F^* in figure 1. The normalized, shifted, and offset objective function, F^* will be referred to as a reduced objective function. The reduced objective function is included with the constraint functions to form a set of functions $f_j(x)$ whose envelope is approximated by the K-S function shown by the dashed line.

An unconstrained minimum of that K-S function is found at x^1 by means of any search method suitable for unconstrained optimization. Locating that minimum completes one cycle of the procedure. The procedure cycle count should not be confused with the count of the iterations carried out by the unconstrained minimum search algorithm; many of the latter iterations may be needed in one of the former.

Referring to figure 2, the next cycle starts with x^1 and F^1. Equation 5 is used to compute a new F^* using F^1. This formulation takes into account that g_{max} may be a negative value. The K-S function is fitted to the set (g_1, g_2, F_2^*) using equation 1 and its minimum is found at x^2. This completes cycle 2 of the optimization procedure. Successive cycles are continued until convergence.

It is apparent, from the above two cycles, that the unconstrained minima progress from the initial x° to x^1, to x^2, approaching the constrained minimum at x^{cm}. One may also observe that in contrast to cycle 1, cycle 2 starts from a feasible design. This illustrates the capability of proceeding either from an infeasible or a feasible initial design point. Finally, it should be noted that the process of shifting and offsetting changes the position of the normalized objective but perserves its slope for each cycle.

At the initial location x°, the K-S function reflects almost exclusively the geometrical properties of the constraint boundary g_1. In contrast, the objective function dominates the K-S envelope function in the search for a minimum in the case of a feasible design. Consequently, if a design point in the midst of the i-th cycle is infeasible, the search direction toward a smaller K-S envelope will point toward smaller values of the dominant constraint, thus reducing the amount of constraint violation at the possible expense of increasing the objective function. On the other hand, if that design is feasible, the search direction toward reduced K-S values will be equivalent to moving toward lesser values of the objective at the possible expense of increasing the values of the satisfied dominant constraints.

The process converges to the state shown in figure 3 when it is no longer possible to reduce the objective function without violating the constraints. At the constrained minimum point the formula of equation 5 produces no offset because $g_{max} = 0$. The constrained minimum is properly located at the boundary surface of the maximal constraint. Generalized to n-dimensional space with m constraints, the unconstrained minimum of the K-S envelope constructed as above approximates the location of a constrained minimum defined by a full vertex of the design space, or by a point of tangency between the objective function and the dominant constraint, or a mixture of these two extreme conditions.

MULTIOBJECTIVE OPTIMIZATION

The same procedure described above, applied in this case, will establish a sequence of K-S envelope minima with each K-S envelope function containing all of the reduced objective functions. Since all normalized objective functions intersect at the same point located at x°, their values in the interval $x > x^{\circ}$ into which the search will be progressing will rank according to the magnitudes of the corresponding slopes (for a convex problem). The sequence leads to a constrained minimum point, where the configuration of constraint boundaries and reduced objective functions are illustrated in figure 4. The configuration is identical to that shown in figure 3 except for the presence of several objective functions, all of which intersect at the constrained minimum point after scaling and offsetting. The constrained multiobjective minimum point, illustrated in figure 4, has the property that one can not depart from it without either violating the constraint(s) or increasing at least one of the objective functions - the classical definition of a pareto-optimum. It is pointed out in the appendix how the method relates to the goal programming class of algorithms.

NUMERICAL EXAMPLES

The method was tested using a three-bar truss design problem, figure 5, which is a classical test case[7]. The truss is considered symmetric, thus $A_1 = A_3$ and $E_1 = E_3$. This problem considers two distinct load cases P_1 and P_2. The problem is generalized by including the angle α with the cross-sectional areas A_1, A_2 as a third design variable. In addition, two materials are used. A_1 and A_3 are made of the same material and A_2 is constructed of a different material. The goal of the optimization is to minimize combinations of weight, cost and support area width D in figure 5. Steel, with a greater Youngs Modulus and inexpensive, and titanium, with a greater stress allowable, a lower Youngs Modulus, and much more expensive were selected as the two materials to provide for a meaningful trade-off between the objectives of weight and cost. Introduction of the angle α brings in the dimension D, a function of α, as a consideration in design; reduction of D increases the forces in the members and also reduces the length of the outer members, such that a complex coupling of the geometry, weight, cost, and strength is created. The truss analysis is given in the Appendix and the material properties, including costs, are stated in table 1.

Verification of the method with one material common to all the members, the objective of minimum weight, the angle α fixed at 45 degrees, and the two cross-sectional areas A_1 and A_2 as the only two design variables, yielded results which agreed with the classical test case[7]. Subsequently, a number of optimization experiments were carried out with various combinations of material configurations, choice of design variables, and selection of objectives. In all cases the search for the minimum of the K-S function envelope was carried out by the Davidon-Fletcher-Powell algorithm[8].

Table 2A displays a list of cases, showing for each case the objective function(s), material(s) used for the truss members and design variable(s). For example, case 9 in table 2A uses weight and the dimension D as objective functions; steel for the outboard members; titanium for the center member; and A_1, A_2, and α as design variables. Initial conditions of A_1, A_2, and α for all cases of table 2A are given in parenthesis, table 2B.

The results in table 2B, corresponding to the cases of table 2A, show consistently the method's ability to generate optima for single and several objectives for the example problem. It is instructive to observe that when an optimization is executed with a single objective, for example, the weight, and then repeated with additional objectives, the latter are significantly reduced at the price of a relatively small increase of the former - see for instance the cases 10, 12, and 16 for $(T_i/F_e/T_i)$ and cases 9, 11, and 13 for $(F_e/T_i/F_e)$. These cases are also illustrated graphically in figures 6 and 7.

Cases 15 and 18 in table 2B include results obtained by the global criterion formulation[4]. That method solves a multiobjective optimization problem by first executing separate optimizations for each of the objectives F_k to obtain a set of feasible minimum solutions $F_{k_{min}}$. Next, a search is carried out for a constrained minimum of a compromise objective $\tilde{F}(X)$ where

$$\tilde{F}(X) = \sum_{k=1}^{NOBJ} \left[\frac{F_k(X) - F_{k_{min}}}{F_{k_{min}}} \right]^2 \qquad (6)$$

such that

$$g_i(X) \leq 0, \; i = 1 \text{ to NCON;}$$

Comparison with cases 19 and 20 respectively shows a good match between the method reported herein and the global method, figure 7.

Additional test cases, not included in the above, were carried out starting the optimization procedure from a variety of initially feasible and infeasible points. Convergence to the same optimal design was observed, regardless of the initial point, except for cases which clearly indicated existence of local minima. One should note that by adjusting the coefficient ρ in equation 1 toward smaller values, one can keep the K-S function from "dipping" into inferior local minima, thus maintaining the search advance toward the global minimum. A method for controlling ρ has been suggested[9]. Typically ρ may be held constant after a proper value is determined. It should be noted that the coefficient ρ affords the user a degree of control over the procedure. On the other hand, problem dependent sensitivity of the procedure to that coefficient may be a drawback which forces experimentation to find a range of ρ values for the case at hand.

The second example is an optimization of an electric power transmission line shown in figure 8. The nomenclature for this problem is listed in the appendix. The line is assumed to extend over a fixed distance D, over a flat terrain. It comprises equidistant towers, separated by a distance L, assumed to be thin-walled cylindrical columns of radius R, with a constant wall thickness t. All towers are geometrically identical and made of steel. Differences between the weight and cost of the real towers and their idealization as cylindrical columns are represented by shape factors. The towers support three parallel electric cables made of an aluminum alloy. The cable weight and the weight of ice accumulated on the cables are the combined loads q, which put the cables in tension and the towers in compression. The cables are assumed to be infinitely rigid in tension so that they form a catenary curve between the towers. The catenary sag is assumed small relative to the cable length so that a parabolic approximation to the catenary equation is used. Constraints are placed on the stresses in the towers due to yielding, cylinder wall buckling, and column buckling. Column bending is also considered for the case of failure of all cables at a point in the line. Tensile stress constraints are imposed for the electric cables, and the cable-to-ground clearance is constrained. Side constraints are imposed on the cable cross-sectional areas A, the tower geometry, and the distance between towers L. Another side constraint is prescribed for a minimal cable cross sectional area A as required for electric power transmission. Problem design variables are the distance between towers L, tower height H, column radius R, wall thickness t, cable cross-sectional area A, and cable tension, P. The problem objectives are the total weight of steel and aluminum W, the total cost, and the tower height. The cost objective entails the cost of: steel, aluminum, and tower foundations. The height objective is included to reflect environmental specifications, a typical unquantifiable consideration engineers are being confronted with increasingly often.

This case is rich in complex interaction among the design variables. For example, the required cable-to-ground clearance may be attained by at least three different means. Increasing the tower height, cable tension, or decreasing the tower-to-tower distance. However, each of these means has a different influence on each of the objective functions and constraints.

Details of the example analysis are provided in the Appendix. Table 3A displays a list of cases for this example. Optimization results for combinations of the objective functions are given in table 3B. Initial conditions of design variables for all cases are given in parenthesis, table 3B. Figure 9 displays results of several cases of discussion graphically. The results, again, show that the method has the ability to locate compromise designs satisfying all the constraints. The optimization for cost only (case 2) reduces the cost by 28% of the cost of the minimum weight design, case 1, while increasing the weight by only 9%. In contrast, the reduction of the tower height objective is much more expensive in terms of the weight and cost (because the cable tension tends to infinity as the tower height approaches the required cable ground clearance). Thus, the inclusion of the tower height as another objective, along with weight and cost, results in the cost, case 6, nearly 12 times greater and weight nearly 3 times greater than the weight and cost of the minimum cost only design, case 2. In the latter case, the towers are widely separated and made tall which reduces the cost of the tower foundations. On the other hand, the cost and weight of the compromise design, case 6, are 7% and 50% lower than those of the design for minimum height alone, case 3.

CONCLUDING REMARKS

A new technique[1] for converting a constrained minima problem to an unconstrained one was demonstrated to be a useful tool in single objective and multiobjective applications. The technique transforms the objective functions into goal constraints, the goal value for each objective is an adjustable quantity. The objective goal constraints are then appended to the set of behavior constraints and the envelope (cumulative constraint) to all the functions in the set is constructed using the Kreisselmeier-Steinhauser function, whose minimum is searched for by any unconstrained minimization algorithm (the Davidon-Fletcher-Powell method[8] was used in the reported study). Search toward the minimum of the envelope function advances the design toward the compromise constrained minimum. That minimum is reached in an iterative procedure, which updates a set of behavior and objective goal constraints and their envelope function at the outset of each iterative cycle. By representing the objective function(s) as objective goal constraints, the method is related to the goal programming approach[4], and the constrained minimum it attains conforms to the classical pareto-optimum definition.

The method typically converged after 8 to 50 cycles, depending on the mix of design variables, parameters and objective functions. The technique was demonstrated on variable geometry and cross-section trusses built of different materials contributing a wide range of mechanical properties and cost to the design objectives. Objectives also included the amount of space occupied by the truss on its support surface. An example of an electric power transmission line was also optimized for a compromise of objectives. The objectives were material volume, cost, and support tower height.

The method results compared well with those obtained by a goal programming algorithm and the method performance was satisfactory in all of the single and multiobjective test problems. In contrast to other multiobjective optimization procedures, the method showed an ability to locate compromise optimum designs without the expense of having to optimize individual objectives.

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APPENDIX

Step-by-Step Optimization Procedure

1. Set iteration counter K=0; Initialize $x = x^*$;
2. Execute analysis of the problem to obtain F_k and g_i's;
3. Define a reduced objective function per equation 5;
4. Define an envelope function $K\text{-}S(F^*, g_j)$ per equation 1;
5. Find coordinates X_{min} of the unconstrained (except for side constraints) minimum of the K-S function by any suitableunconstrained optimization algorithm.
6. Reset K = K + 1, and reset $x_K = x_{min}$;
7. Repeat from #2 until convergence criteria is satisfied.

A computational cost saving option in step 5 is to execute only a limited number of steps toward the K-S minimum instead of a full fledged function minimization. Under that option, one may begin with a single step and progressively increase the number of steps as the procedure advances. In the case of many objective functions step 3 is carried out for all of them, and the entire set of the reduced objectives F^* are included as arguments in the K-S function in step 4.

Analysis of the Truss Example

Referring to figure 5, the truss is analyzed as a system with two elastic degrees of freedom. Utilizing the stiffness method the following load/deflection equations are obtained:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \{R_1\};\ \{R_2\}; \qquad \text{A1}$$

Due to symmetry $K_{12} = K_{21}$.

Where:

$$K_{11} = \cos^2\alpha(k_3 + k_1);$$
$$K_{12} = -\cos\alpha\ k_3 \sin\alpha + \cos\alpha\ k_1 \sin\alpha; \qquad \text{A2}$$
$$K_{22} = \sin^2\alpha(k_3 + k_1) + k_2;$$

and:

$$DET = K_{11}K_{22} - K_{12}^2;$$

The member stiffness are given as,

$k_j = E_j A_j / \ell_j, \quad j = 1,\ldots 3$

where:

$\ell_1 = h/\sin\alpha$
$\ell_2 = h$
$\ell_3 = \ell_1$

For load case P_1:

$$R_1 = \begin{Bmatrix} -P_1 \cos\beta \\ \hline P_1 \sin\beta \end{Bmatrix};$$

For load case P_2:

$$R_2 = \begin{Bmatrix} P_2 \cos\beta \\ \hline P_2 \sin\beta \end{Bmatrix};$$

The solution of equation A1 for load case P_1 yields:

$$u_1 = DET^{-1}((-P \cos\alpha)k_{22} - (P_1 \sin\alpha)k_{12});$$

$$u_2 = DET^{-1}((P_1 \sin\alpha)k_{11} - (-P \cos\alpha)k_{12});$$

and for load case P_2 yields:

$$u_1 = DET^{-1}((P_2 \cos\alpha)k_{22} - (P_2 \sin\alpha)k_{12});$$

$$u_2 = DET^{-1}((P_2 \sin\alpha)k_{11} - (P_2 \cos\alpha)k_{12}); \qquad \text{A3}$$

From the displacements A3 and Young's moduli E_j for each member the stresses are recovered as follows:

$$\sigma_1 = ((u_1 \cos\alpha + u_2 \sin\alpha) / \ell_1)E_1;$$

$$\sigma_2 = (u_2/\ell_2)E_2;$$

$$\sigma_3 = ((-u_1 \cos\alpha + u_2 \sin\alpha) / \ell_3)E_3; \qquad \text{A4}$$

Analysis of the Electric Power Transmission Line Example

Refering to figure 8, the material used for the cables is 2024 aluminum alloy. The material used for the towers is AISI carbon steel. The following is the nomenclature used in the analysis. The actual data are noted in parentheses, including minimum (lower bounds) values where appropriate.

R - tower mean cylindrical cross-section radius (minimum 5.0 in.);
L - tower-to-tower distance (minimum 300.0 in.);
t - tower wall thickness (minimum 0.125 in.);
b_1 - ratio of R/t (minimum 50);
b_2 - ratio of L/H (minimum 4);
H - tower height;
D - total distance covered by the transmission line (60.0 mi.);
l - true length of the cable between towers;
A - cable cross-section area (minimum 0.20 $in.^2$);
P - cable tension;
f - sag of the catenary between towers;
E - Young's modulus for steel (towers) (3.0×10^6 psi.);
γ_1 - specific weight of aluminum (0.100 $lb/in.^3$);
γ_2 - specific weight of steel (0.284 $lb./in.^3$);
γ_3 - specific weight of ice (0.033 $lb/in.^3$);
m_1 - shape factor for tower weight (0.75);
m_2 - shape factor for tower cost (1.0);
n - number of parallel cables; (3);
s_0 - safety factor for cables (1.5);
s_1 - safety factor for combined bending-compression stress in the tower (1.25);
s_2 - safety factor for cable stress (1.50);
s_3 - safety factor for compressive stress in tower (2.00);
s_4 - buckling factor (0.60);
q - load on the cable including cable weight and ice accumulation;
c_1 - cost of the cable material per unit weight (0.40 \$/lb.);
c_2 - cost of the tower material per unit weight (0.09 \$/lb.);
c_3 - cost of the foundation per tower (\$50,000.00);
k_{vol} - ratio of the volume of ice to the volume of the cable (3);
σ_1 - allowable stress for the cable (44 ksi);
σ_2 - allowable stress for the tower (36 ksi);
h - cable-to-ground clearance (H-f);
h_r - required cable-to-ground clearance (32.8 ft.);

The strength of materials equations for the problem are:

Weight on the unit length of the cable including ice

$$q = \gamma_2 A(1.0 + k_{v01}(\gamma_3/\gamma_2)) \qquad \text{A5}$$

Sag of the cable and its true length (assuming inextensional cable)

$$f = q L^2/(8 P) \qquad \text{A6}$$

$$l = L(1.0 + 2.67(f^2/L)) \qquad \text{A7}$$

Stress in the cable

$$\sigma_c = s_2 P/A \qquad \text{A8}$$

Stress in the tower due to combined compression and bending; the worst case of the latter occurs when the cables break on one side of the tower

$$\sigma_T = (nPH/\pi R^2 t) + (nP(4f/L)) \qquad \text{A9}$$

Critical stress for cylindrical wall buckling (conservatively using a formula for uniform compression although the tower is in the state of combined compression and bending)

$$\sigma_{comp} = s_3\ 8\ P\ n\ f/L \qquad A10$$

Critical force of the tower buckling in the column mode

$$F_{crit} = \pi^3 ER^3 t/(4\ H^2) \qquad A11$$

The total material weight of the towers and cables (neglecting the last tower)

$$W_t = (ALn\gamma_1 + m_1 2\pi RtH\gamma_2)(D/L) \qquad A12$$

The total cost of the transmission line and towers including the foundation costs

$$Cost = ((c_1 Al\gamma_1 n(1.0 + 2.67(f^2/L))) +$$

$$(m_2 2\pi c_2 RtH\gamma_2) + c_3)(D/L) \qquad A13$$

The objectives of the problem are contributed by eq. A12, A13, and the tower height H. The constraints are:

cable to ground clearance

$$h > h_r \qquad A14$$

strength of the tower

$$\sigma_T < \sigma_2/s_1 \qquad A15$$

buckling of the tower in the cylindrical wall and column modes, respectively

$$\sigma_{comp} < \sigma_{cyl} 2\pi Rt/s_2 \qquad A16$$

where: $\sigma_{cyl} = s_4 E(t/R)/s_2$

$$\sigma_{comp} < F_{CRIT}/s_3 \qquad A17$$

strength of the cable

$$\sigma_c \leq \sigma_1/s_0 \qquad A18$$

Ratio of R/t

$$R/t \geq b_1 \qquad A19$$

Ratio of L/H

$$L/H \geq b_2 \qquad A20$$

The design variables are A, R, t, H, L and P.

The Method's Relationship to Goal Programming

It can be shown that the method described in the report relates to the methods of the goal programming category. Indeed, by including coefficients β and η in equation 5

$$F^*_k = \left(\frac{F_k}{|F^\circ_k|\beta_k} - \eta_k\right) - q_{max} \qquad A21$$

where: F°_k are previously computed feasable solutions, it becomes possible to assign priorities to the objective functions by controlling the shifted distance and slopes (i.e., the relative magnitudes of the normalized objectives away from their intersection point). One may observe that at the multiobjective constrained minimum point we have

$$\frac{F_k}{|F^\circ_k|\ \beta_k} - \eta_k = 0 \qquad A22$$

for all the objective functions F_k. By definition of F_k, the expression in parentheses may be regarded as a constraint imposed on the objective F^*_k. If the value $F^\circ_{k_{min}}$ is known, one could replace $F^\circ_{k_{min}}$ for $|F^\circ_k|$ β_k and 1 for η_k in equation A22 and, thus, formulate the optimization problem as follows.

Find X such that

$$F_k/F_{k_{min}} - 1 \leq 0; \quad k = 1,\ldots NOBJ \qquad A23$$

$$g_i \leq 0, \quad i = 1,\ldots NCON;$$

This formulation states a solution to a set of inequalities, expressing the intent to modify the design to a state in which all the behavioral constraints are satisfied and all objectives are maintained below or at their target $F_{k_{min}}$. Thus the procedure may also be categorized as a goal programming approach.

Table 1. Material Properties and Costs

Property	MATERIAL	
	STEEL (Fe)	TITANIUM (Ti)
Specific weight (lbm/in^3)	0.282	0.160
Cost ($/lbm)	0.41	25.00
Allowable stress-tension (ksi)	36	110
stress-compression (ksi)	27	82.5
Young's modulus (psi)	30 x 10^6	15.5 x 10^6

Table 2A. Cases for Three-Bar Truss Problem

CASE	OBJECTIVE FUNCTION(S)	MATERIAL(S)	DESIGN VARIABLE(S)
1	Weight	Fe/Fe/Fe	A
2	Weight	Fe/Ti/Fe	A
3	Weight	Ti/Fe/Ti	A
4	Cost	Fe/Ti/Fe	A
5	Weight/Cost	Fe/Ti/Fe	A
6	Cost	Ti/Fe/Ti	A
7	Weight/Cost	Ti/Fe/Ti	A
8	Weight/D	Fe/Fe/Fe	A/α
9	Weight/D	Fe/Ti/Fe	A/α
10	Weight/D	Ti/Fe/Ti	A/α
11	Wt/Cost/D	Fe/Ti/Fe	A/α
12	Wt/Cost/D	Ti/Fe/Ti	A/α
13	Weight	Fe/Ti/Fe	A/α
14	Cost	Fe/Ti/Fe	A/α
15	Goal	Fe/Ti/Fe	A/α
16	Weight	Ti/Fe/Ti	A/α
17	Cost	Ti/Fe/Ti	A/α
18	Goal	Ti/Fe/Ti	A/α
19	Weight/Cost	Fe/Ti/Fe	A/α
20	Weight/Cost	Ti/Fe/Ti	A/α

Table 2B. Results for Three-Bar Truss Problem

CASE	WEIGHT (lb)	COST ($)	A_1 (in^2)	A_2 (in^2)	α (deg)
Initial Conditions of Design Variables			(1.0)	(1.0)	(45.0)
1	4.18	1.71	0.441	0.2341	45.00
2	4.20	17.56	0.450	0.402	45.00
3	0.805	14.05	0.123	0.088	45.00
4	4.54	2.36	0.563	0.013	45.00
5	4.43	2.65	0.548	0.011	45.00
6	1.09	12.21	.108	.211	45.00
7	0.936	12.44	.106	.201	45.00
8	6.091	2.50	1.003	0.029	75.64
9	6.332	5.23	1.074	0.067	76.89
10	3.782	63.20	0.779	0.452	84.02
11	4.881	8.640	0.767	0.169	69.67
12	1.153	14.69	0.157	0.204	60.15
13	4.117	10.20	0.536	0.2161	53.35
14	4.213	2.19	0.633	0.012	58.27
15	4.148	2.12	0.602	0.011	55.24
16	0.864	17.89	0.199	0.054	63.65
17	1.092	12.20	0.105	0.218	44.43
18	1.142	12.20	0.103	0.236	43.79
19	4.006	2.05	0.592	0.010	56.75
20	0.984	12.42	0.102	0.176	42.11

Table 3A. Cases for Transmission Tower and Cables

CASE	OBJECTIVE FUNCTION(S)	DESIGN VARIABLE(S)
1	Weight	R/H/L/A/P/t
2	Cost	R/H/L/A/P/t
3	H	R/H/L/A/P/t
4	Weight/Cost	R/H/L/A/P/t
5	Weight/H	R/H/L/A/P/t
6	Weight/Cost/H	R/H/L/A/P/t

Table 3B. Results for Transmission Tower and Cables

CASE	Weight (lbm.) (X10^7)	Cost ($) (X10^6)	R (in.)	H (in.)	L (in.)	A (in.^{2})	P (lb.)	t (in.)
Initial Conditions of Design Variables			(15.00)	(310.00)	(2500.0)	(0.60)	(3500.00)	(0.40)
1	0.18491	0.34687	27.05	445.24	5824.00	0.37	6141.00	0.13
2	0.20176	0.098592	29.81	1019.00	21000.00	0.28	5030.50	0.22
3	1.0686	1.2734	20.92	397.20	1589.00	0.43	7827.50	0.37
4	0.37654	0.75959	17.12	435.00	2655.50	0.56	4171.50	0.21
5	0.42284	1.1982	21.67	411.00	1678.00	0.54	4136.50	0.13
6	0.53838	1.1862	14.35	402.20	1696.50	0.25	4281.00	0.28

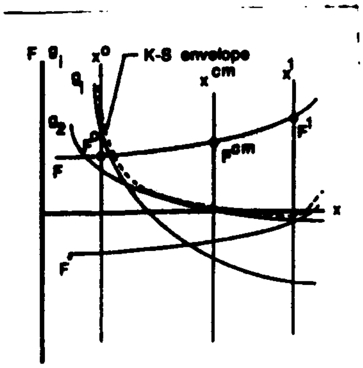

Figure 1. Constrained to Unconstrained Conversion, Infeasable Initial Design x^0

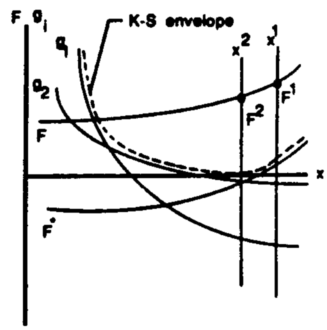

Figure 2. Constrained to Unconstrained Conversion, Feasable Initial Design x^1

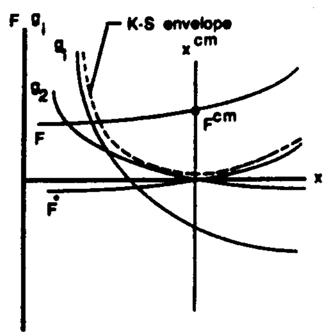

Figure 3. Constrained to Unconstrained Conversion, Constrained Minimum x^{cm}

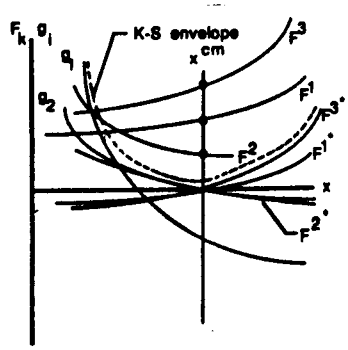

Figure 4. Constrained to Unconstrained Conversion with Multiple Objectives at Constrained Minimum

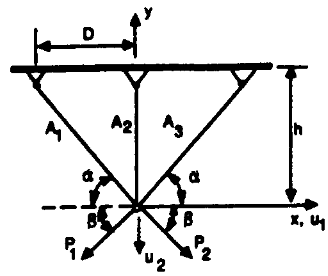

Figure 5. Symetric Three Bar Truss

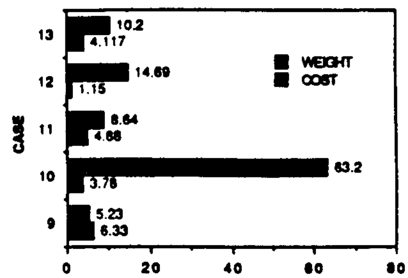

Figure 6. Weight/Cost Comparison, Three Bar Truss .

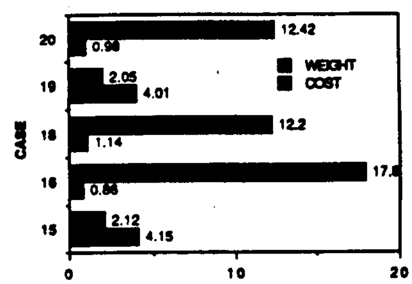

Figure 7. Weight/Cost Comparison, Three Bar Truss

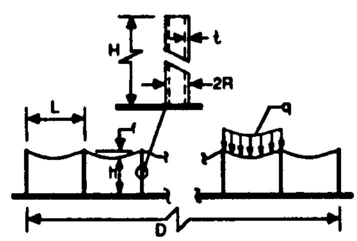

Figure 8. Transmission Tower with Cables

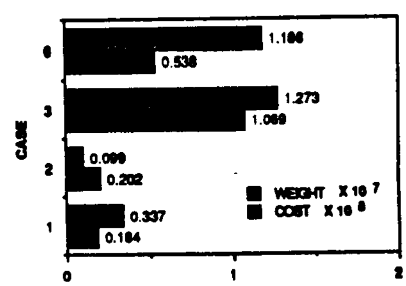

Figure 9. Weight/Cost Comparison, Transmission Tower with Cables

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-100536		

4. Title and Subtitle

A New Algorithm for General Multiobjective Optimization

5. Report Date

March 1988

6. Performing Organization Code

7. Author(s)

Jaroslaw Sobieszczanski-Sobieski, Augustine R. Dovi, and Gregory A. Wrenn

8. Performing Organization Report No.

10. Work Unit No.

505-63-11

9. Performing Organization Name and Address

NASA Langley Research Center
Hampton, VA 23665-5225

11. Contract or Grant No.

13. Type of Report and Period Covered

Technical Memorandum

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, DC 20546-0001

14. Sponsoring Agency Code

15. Supplementary Notes

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Augustine R. Dovi and Gregory A. Wrenn, Planning Research Corp., Hampton, VA
Presented as AIAA Paper No. 88-2434 at 29th AIAA ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20,1988

16. Abstract

The paper describes a new technique for converting a constrained optimization problem to an unconstrained one, and a new method for multiobjective optimization based on that technique. The technique transforms the objective functions into goal constraints. The goal constraints are appended to the set of behavior constraints and the envelope of all functions in the set is searched for an unconsttained minimum. The technique may be categorized as a SUMT algorithm. In multiobjective applications, the approach has the advantage of locating a compromise minimum without the need to optimize for each individual objective function separately. The constrained to unconstrained conversion is described, followed by a description of the multiobjective problem. Two example problems are presented to demonstrate the robustness of the method.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Multiobjective optimization Objective function Design variables Constraints	Unclassified - Unlimited Subject Category 59

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
UNCLASSIFIED	UNCLASSIFIED	10	A02

NASA FORM 1626 OCT 86