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Exact Image Method for Gaussian Beam Problems Involving a Planar Interface

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ABSTRACT

Exact image method, recently introduced for the solution of electromagnetic field problems involving sources above a planar interface of two homogeneous media, is shown to be valid also for sources located in complex space, which makes its application possible for Gaussian beam analysis. It is demonstrated that the Goos-Hänchen shift and the angular shift of a TE polarized beam are correctly given as asymptotic results by the exact reflection image theory. Also, the apparent image location giving the correct Gaussian beam transmitted through the interface is obtained as another asymptotic check. The present theory makes it possible to calculate the exact coupling from the Gaussian beam to the reflected and refracted beams as well as to the surface wave.

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INTRODUCTION

The Gaussian beam is an important model for radiation from large aperture antennas and laser sources, from which the energy is sent in a relatively narrow space angle. Introduction of a complex space dipole to mathematically represent a source for the Gaussian beam by Deschamps [1] made it possible to extend analytical results, derived earlier for real space sources, to Gaussian beam excitations. The basic Gaussian beam originates from a dipole in complex space, while multipoles of higher order can be shown to produce more complex Hermite-Gaussian and Laguerre-Gaussian beams [2], [3], [4].

The basic problem in Gaussian beam analysis is the problem of reflection and refraction of the beam at a planar interface of two homogeneous dielectric media, where losses might be present. This problem has been treated both through complex space dipole method [5] and other methods [6], [7], [8]. The general field expressions in the exact formulations are involved, but asymptotic considerations for narrow beams allow simpler analysis of reflection and transmission problems and lead to the well-known shift phenomena of the reflecting beam. The most famous of these was first demonstrated by Goos and Hänchen [9] in 1947 and analyzed by Artman [10] in 1948, involving a parallel shift of the beam reflected from an interface in denser of two lossless dielectric media. Other shifts which can be defined for narrow beams are the angular shift [5], [11] and the focal shift [12]. Infinities arising in the idealized theory for the Goos-Hänchen shift can be avoided through a more realistic analysis [6], [13], [17].

The purpose of the present paper is to give a more general account of the exact image theory, introduced elsewhere for sources in the air above a planar interface of a homogeneous medium [14], [15] (the Sommerfeld problem) and allow sources to be in complex space. Thus, the theory is applicable to global Gaussian beam analysis in exact form. In comparison with other exact methods applying Sommerfeld integrals, the present theory, dealing with sources and their integration, gives more physical insight and does not resort to special integration procedures dependent on the field point. For the application of the exact image theory, functions characterizing the image sources are needed and methods for their computation have been presented in [14], [15]. These functions need only be calculated once in the computer memory, after which the field calculation involvs converging well-b haved integrals. The exact image of a point source is a line source, which is normally in complex space. As checks to the theory, known asymptotic expressions for the reflected beam shifts and the apparent transmission image location are shown to arise as special cases when the exact line image is approximated by a point image.

REFLECTION IMAGE THEORY

The notation applied here is based on Ref. [14] except that we consider two half spaces with respective parameters $\mu_1\mu_0$, $\epsilon_1\epsilon_0$ for $\bar{u}\cdot\bar{r} > 0$ and $\mu_2\mu_0$, $\epsilon_2\epsilon_0$ for $\bar{u}\cdot\bar{r} < 0$. In [14], [15], the medium $\bar{u}\cdot\bar{r} > 0$ was assumed to be air, but generalization for any medium pair is straightforward, if instead of k we write $k_1 = k\sqrt{\mu_1\epsilon_1}$, where $k = \omega\sqrt{\mu_0\epsilon_0}$, and everywhere understand that $\mu = \mu_2/\mu_1, \epsilon = \epsilon_2/\epsilon_1$. To avoid unnecessary, although not excessive, complication, the theory here is limited to dielectric media only with $\mu_1 = \mu_2 = 1$.

As another generalization, the original source is taken to be in complex space. Because the theory in [14], [15] was analytically derived and no use of the tacit assumption of real source location was made, the final expressions can also be applied for complex source locations. The same idea has been applied earlier with success when field expressions, derived for problems with sources in real space, have been generalized for sources in complex space [16], resulting in solutions for problems with Gaussian beam excitation.

Let us consider, for simplicity, the source $\bar{J}(\bar{r}) = \bar{v} IL\delta(\bar{r} - (\bar{u}h + j\bar{b}))$, which is a dipole with the direction of the unit vector $\bar{v}$ and location at the real height h from the interface $\bar{u} \cdot \bar{r} = 0$ and imaginary depth $j\bar{b}$. When calculating the fields in the half space 1, the half space 2 can be replaced by the exact image source, as was shown in [14]:

$$\bar{J}_i(\bar{r}, p) = f_1(p)\bar{J}_{ct}(\bar{r}) - \left(f_\epsilon(p) + \frac{\epsilon - 1}{\epsilon + 1}\delta_+(p)\right) \bar{u}\bar{u}\cdot\bar{J}_c(\bar{r}) - \bar{u}\frac{1}{k_1^2}\frac{\epsilon + 1}{\epsilon} f_\epsilon(p)\bar{u}\cdot\nabla(\nabla_t\cdot\bar{J}_c(\bar{r})). \quad (1)$$

Here p is an integration variable and the image function $f_\epsilon(p)$ can most conveniently be calculated through the following Bessel function series [15]:

$$f_\epsilon(p) = \frac{-8\epsilon}{\epsilon^2 - 1}\sum_{n=1}^{\infty} n \left(\frac{\epsilon - 1}{\epsilon + 1}\right)^n \frac{J_{2n}(p)}{p}, \quad f_1(p) = \frac{-2J_2(p)}{p}. \quad (2)$$

Further, $\delta_+(p)$ denotes $\delta(p - 0_+)$, t a component transverse to $\bar{u}$, and c the reflection operation

$$\bar{a}_c = \overline{\overline{C}} \cdot \bar{a}, \quad \bar{J}_c(\bar{r}) = \overline{\overline{C}} \cdot \bar{J}(\overline{\overline{C}} \cdot \bar{r}), \quad \overline{\overline{C}} = \overline{\overline{I}} - 2\bar{u}\bar{u}. \quad (3)$$

The field from the image source can be written as a fourfold integral over space and the parameter p:

$$\bar{E}(\bar{r}) = -j\omega\mu_0 \int_V \int_0^\infty \overline{\overline{G}}_1(D) \cdot \bar{J}_i(\bar{r}', p) dV' dp, \quad (4)$$

with

$$\overline{\overline{G}}_1(D) = \left(\overline{\overline{I}} + \frac{1}{k_1^2}\nabla\nabla\right)\frac{e^{-jk_1 D}}{4\pi D}, \quad (5)$$

$$D(\bar{r}, \bar{r}', p) = \sqrt{(\bar{r} - \bar{r}' + \bar{u}p/jB)\cdot(\bar{r} - \bar{r}' + \bar{u}p/jB)}, \quad B = k\sqrt{\epsilon_2 - \epsilon_1}.$$

The image current expression contains Bessel functions $J_n(p)$, which are convergent only if the integration parameter p is real. For a point source, (1) can also be written so that the integration parameter p is absent, because in the field integral (4), the coordinate $z = \bar{u} \cdot \bar{r}$ and p are related through the distance function D in the argument of the Green function. The expression (1) can thus be written

$$\bar{J}_i(\bar{r}) = -IL\frac{\epsilon - 1}{\epsilon + 1}\bar{u}(\bar{u}\cdot\bar{v})\delta_+(\bar{r} + \bar{u}h - j\bar{b}_c) - jk_1\sqrt{\epsilon - 1}IL\Big[\bar{v}_c f_1(p(z))\delta(\bar{\rho} - j\bar{b}_t) -$$

$$-\bar{u}(\bar{u}\cdot\bar{v})f_\epsilon(p(z))\delta(\bar{\rho} - j\bar{b}_t) - \bar{u}\frac{1}{k_1^2}\frac{\epsilon + 1}{\epsilon} f_\epsilon(p(z))\bar{u}\cdot\nabla(\bar{v}_c\cdot\nabla)\delta(\bar{r} + \bar{u}h - j\bar{b}_c)\Big], \quad (6)$$

with

$$p(z) = -jB(z + h + j\bar{u}\cdot\bar{b}). \quad (7)$$

To obtain an exponentially converging Green function, we must select the branch of the distance function D so that $Im[k_1 D] < 0$. Also, to obtain a converging image current function, the path of z integration must be chosen so that p in (7) is real and positive, in which case all the Bessel functions in (2) converge. This means that the image line must start at the point $\bar{r} = -\bar{u}h + \bar{b}_c$ in complex space and lie parallel to the complex z plane with

$$arg(z + h + j\bar{u} \cdot \bar{b}) = arg(jB). \tag{8}$$

Requiring that the additional condition $Re[z] \not> 0$ be valid, the complex distance function satisfies $D \neq 0$ for all field points in $Re[z] > 0$ and the field integrand in (4) is nonsingular [14]. This condition defines the branch of B, or the square root $\sqrt{\epsilon_2 - \epsilon_1}$ through

$$Re[j\sqrt{\epsilon_2 - \epsilon_1}] = Re[\sqrt{\epsilon_1 - \epsilon_2}] \geq 0, \tag{9}$$

implying for example, that if $\epsilon_1, \epsilon_2 = \epsilon\epsilon_1 \leq \epsilon_1$ are real, then $\sqrt{\epsilon - 1} = -j\sqrt{1 - \epsilon}$, provided $\sqrt{1 - \epsilon} > 0$. For some combination of lossy medium parameters there may be doubt of choosing the correct branch of the distance function D to obtain the best convergence [18]. A close study of the integration path on the complex z' plane will reveal that a branch cut line, starting from the branch points at $z' = z \pm j\rho$, defining the converging branch of the Green function, may be crossed for some combination of parameter values ϵ_1, ϵ_2, when p moves from 0 to ∞. In this case it is possible to take the image current line in the 'wrong' half space with $Re[z'] > 0$, which makes the integrand converging again. This question is the subject of a forthcoming paper.

REFLECTED GAUSSIAN BEAM

There does not seem to exist a global definition for the Gaussian beam field, instead, the term "Gaussian beam" is understood as an asymptotic property of radiation fields close to the axis of the main radiation. Thus, many fields with the same asymptotic quadratic exponential behavior are called Gaussian beams. One example is the field from a point source in complex space with the imaginary part of the position vector, $\bar{b} = \bar{u}bcos\theta - \bar{u}_y bsin\theta$.

The radiation beam of the point source is obtained in the direction of $-\bar{b}$. The reflected beam is obtained from the image source (6), and, for points far enough from the interface, the field can be approximated by a Gaussian beam, if the image source can be approximated by a point source. To demostrate the validity of the exact image theory in this generalized sense, we show that the Goos-Hänchen shift as well as the angular shift of the reflected beam are obtained as asymptotic results from the exact image field expression. To keep notation simple, let us consider a two-dimensional problem with the line source $\bar{J}(\bar{r}) = \bar{u}_x I\delta(y + jbsin\theta)\delta(z - h - jbcos\theta)$, where θ is the angle between $\bar{u}$ and $\bar{b}$. This assumption simplifies the image source (1) into

$$\bar{J}_i(\bar{r}, p) = f_1(p)\bar{J}_c(\bar{r}) = f_1(p)\bar{u}_x I\delta(y + jbsin\theta)\delta(z + h + jbcos\theta). \tag{10}$$

The Green function (5) integrated in x direction produces the two-dimensional Green function in the medium 1,

$$\overline{\overline{G}}_1(D) = (\overline{\overline{I}} + \frac{1}{k_1^2}\nabla\nabla)G_1(D), \quad G_1(D) = \frac{1}{4j}H_0^{(2)}(k_1 D), \tag{11}$$

where $H_0^{(2)}(x)$ is the Hankel function.

The field calculated from the exact image source is valid in any point in the upper half space, but since we are interested in Gaussian beam properties, let us consider the far field with $|k_1 D| >> 1$, in which case the image can be approximated by a point source. Also the asymptotic expression for the Hankel function can now be used:

$$H_0^{(2)}(k_1 D) \approx \sqrt{\frac{2j}{\pi k_1 D}} e^{-jk_1 D}. \tag{12}$$

D here represents the complex distance between the field point $\bar{r}$ and the complex integration point $\bar{r}' + j\bar{u}p/k_1\sqrt{\epsilon - 1}$, where $\bar{r}' = -\bar{u}h + j\bar{b}_c$, if p integration is done separately. Because the image function $f_1(p)$ is decaying, the effective p integration range can be regarded as small with respect to $|\bar{r} - \bar{r}'|$ and we can write

$$D \approx D_0 - \frac{jpq}{k_1}, \quad D_0 = \sqrt{(\bar{r} - \bar{r}') \cdot (\bar{r} - \bar{r}')}, \quad q = \frac{\bar{u} \cdot \bar{w}}{\sqrt{\epsilon - 1}}, \quad \bar{w} = \frac{\bar{r} - \bar{r}'}{D_0}. \tag{13}$$

Applying (12), the field integral can be written on the plane $x = 0$ as

$$\bar{E}(\bar{r}) \approx -j\omega\mu_0 \bar{u}_x I G_1(D_o) \int_0^\infty f_1(p) e^{-pq} dp. \tag{14}$$

If (14) can be written in the form

$$\bar{E}(\bar{r}) \approx -j\omega\mu_0 \bar{u}_x I G_1(D_0) e^{jk_1 \bar{w} \cdot \bar{s}} e^{-jk_1 \bar{w}_0 \cdot \bar{s}} \int_0^\infty f_1(p) e^{-pq_0} dp, \tag{15}$$

in the vicinity of the radiating direction $\bar{w} \approx \bar{w}_o$, where $\bar{w}_o$ corresponds to a field point $\bar{r}_o$ on the axis of the reflected beam, the field can be thought of as arising from the current line shifted by the vector $\bar{s}$ from the mirror image location $\bar{r}'$. Approximation (15) is obviously possible, if (14) can be expanded as a power series in terms of the small difference vector $\bar{w} - \bar{w}_o$. This is possible only for narrow enough beamwidths. After some steps of Taylor expansion, the expression for $\bar{s}$ can be written in the form

$$\bar{s} = \frac{j\bar{u}}{k_1\sqrt{\epsilon - 1}} \frac{\int_0^\infty p f_1(p) e^{-pq_0} dp}{\int_0^\infty f_1(p) e^{-pq_0} dp}, \tag{16}$$

where $q_0 = \bar{u} \cdot \bar{w}_0/\sqrt{\epsilon - 1}$. To obtain an expression for $\bar{s}$, the following integral identities are needed:

$$\int_0^\infty f_1(p) e^{-pq} dp = \frac{q - \sqrt{q^2 + 1}}{q + \sqrt{q^2 + 1}}, \tag{17}$$

$$\int_0^\infty p f_1(p) e^{-pq} dp = \frac{2}{\sqrt{q^2 + 1}} \frac{q - \sqrt{q^2 + 1}}{q + \sqrt{q^2 + 1}}. \tag{18}$$

Thus, we have

$$\bar{s} = \frac{2j\bar{u}}{k_1\sqrt{\epsilon - 1}\sqrt{q_0^2 + 1}} = \frac{2j\bar{u}}{k\sqrt{\epsilon_2 - \epsilon_1 + \epsilon_1(\bar{u} \cdot \bar{w}_o)^2}} = \frac{2j\bar{u}}{k_1\sqrt{\epsilon - sin^2\theta}}, \tag{19}$$

with the branch of the square root so defined that $Im[\sqrt{\epsilon - sin^2\theta}] \leq 0$, [14]. (19) explains the Goos-Hänchen shift and the angular shift when the Taylor approximation condition is valid. In fact, because $\bar{w}_o = -\bar{b}_c/b$, we have $q_0 = cos\theta/\sqrt{\epsilon - 1}$, and (17) represents the TE reflection coefficient of a plane wave coming at the angle θ. Let us consider the two possibilities with real ϵ:

1) $\epsilon < sin^2\theta$, which can only happen for $\epsilon_1 > \epsilon_2$. In this case, $\bar{s}$ is real and thus the real part of the location vector is shifted by

$$\bar{s} = -\frac{2\bar{u}}{k_1\sqrt{sin^2\theta - \epsilon}}, \tag{20}$$

which means a parallel shift of the reflecting beam, Fig. 1. This is the well-known Goos-Hänchen shift. Because $\bar{s}$ is in $-\bar{u}$ direction, the parallel shift is $ssin\theta$.

2) $\epsilon > sin^2\theta$. In this case, the square root is real and the shift $\bar{s}$ is imaginary $= js\bar{u}$, which means that the image line is shifted an imaginary distance in the $\bar{u}$ direction from the original location. Because the imaginary part of the position vector determines the direction of the beam, the beam is shifted angularly from its mirror image direction $\bar{w}_o$ to the direction determined by $\bar{b}_c - j\bar{s}$, Fig.2. If the shift angle $\Delta\theta$ is small, it satisfies

$$tan(\Delta\theta) \approx -\frac{s}{b}cos\theta = \frac{2cos\theta}{k_1 b\sqrt{\epsilon - sin^2\theta}}. \tag{21}$$

The previous expressions are only valid for sufficiently narrow beams, which presumes sufficiently large values for b. They are not valid if the Taylor expansion is not applicable, which happens at and close to the branch point of the square root function $\sqrt{q_0^2 + 1}$, i.e., at $sin^2\theta \approx \sin^2\theta_c = \epsilon$, where (19) would predict an infinite shift. This is the definition of the critical angle θ_c, for which $\sqrt{q_0^2 + 1} = 0$. For angles $\theta \approx \theta_c$ we can write from (14), (17)

$$\bar{E}(\bar{r}) \approx -j\omega\mu_0\bar{u}_x I G_1(D_0)e^{-2j\sqrt{2(cos\theta - cos\theta_c)/\sqrt{1-\epsilon}}}, \tag{22}$$

which cannot be described by a simple shift of image source, because the reflecting beam is distorted and not exactly of Gaussian form. To find out the shift of the maximum of the reflecting beam, some numerical analysis must be made, as in [6], [13], [17].

For complex ϵ values, there are both real and imaginary parts for the vector $\bar{s}$, which means a combined Goos-Hänchen and angular shift. In this case, the beam does not enter at the critical angle, because it is now complex. For beams narrow enough, the real and imaginary parts of $\bar{s}$ of (19) determine the Goos-Hänchen and angular shifts, respectively.

This simple test demostrates the applicability of the exact image theory for the Gaussian beam analysis. The field of the reflected beam can be calculated at any point from the exact formulation (4). This also includes the coupling to the surface wave, which however is small when θ is not ≈ 0.

TRANSMISSION IMAGE THEORY

The image theory for fields transmitted through a planar interface was developed in ref. [15] and the generalization discussed for reflection image theory also applies here. To express the result, the upper dielectric half space is replaced by the transmission image source:

$$\bar{J}_i(\bar{r},p) = \bar{u}\bar{u}\cdot\bar{J}(\bar{r})\left(\frac{2\epsilon}{\epsilon+1}\delta_+(p) + F'_\epsilon(Bz,p)\right) + \bar{J}_t(\bar{r})\Big(\delta_+(p) + F'_1(Bz,p)\Big) +$$

$$\bar{u}[F_\epsilon(Bz,p) - F_1(Bz,p)]H'(z,p)\nabla_t\cdot\bar{J}(\bar{r}). \qquad (23)$$

Here, the original source is $\bar{J}(\bar{r})$ and z denotes its coordinate. Further, we have

$$B = k_1\sqrt{\epsilon-1} = k\sqrt{\epsilon_2-\epsilon_1}, \qquad (24)$$

$$H(z,p) = \sqrt{z^2+(p/B)^2}, \qquad (25)$$

$$F_\epsilon(\tau,p) = \frac{2\epsilon}{\epsilon+1}J_0(p) - \frac{4\epsilon}{\epsilon^2-1}\sum_{m=1}^{\infty}\left(\frac{\epsilon-1}{\epsilon+1}\frac{1-\sqrt{(p/\tau)^2+1}}{1+\sqrt{(p/\tau)^2+1}}\right)^m J_{2m}(p), \qquad (26)$$

and $'$ in functions H and F denotes differentiation with respect to p.

The field is obtained from an integral similar to (4) but the medium is now ϵ_2. Also, the distance function D is here defined to be

$$D(\bar{r},\bar{r}',p) = \sqrt{(\bar{r}-\bar{\rho}'-\bar{u}H(z',p))\cdot(\bar{r}-\bar{\rho}'-\bar{u}H(z',p))}. \qquad (27)$$

Again, to obtain a converging image function, the parameter p must be real. To obtain a converging Green function, $Im[k_2D]$ must be nonpositive, which defines the branch of the D function. Also in this case, for certain lossy media, the branch cut of the Green function may be crossed if the image is restricted to the half space $Re[z'] > 0$, leading to nonconverging Green function. This problem can be overcome by taking the other branch of the $H(z,p)$ function.

TRANSMITTED GAUSSIAN BEAM

Without delving more into the theory itself, which can be found in [15], let us apply it to the same line source as in the previous Section. The transmission image source can be written in the form

$$\bar{J}_i(\bar{r},p) = \bar{u}_x I\Big(\delta_+(p) + F'_1(Bh',p)\Big)\delta(y + jb sin\theta)\delta(z-h'), \qquad (28)$$

where the function F'_1 is defined as

$$F'_1(Bh',p) = \frac{-1}{H(H+h')}\Big((H+h')^2J_1(p) + (H-h')^2J_3(p)\Big), \qquad (29)$$

and

$$H \equiv H(h',p) = \sqrt{h'^2+(p/B)^2}, \quad h' = h + jb cos\theta. \qquad (30)$$

The convergence condition $Im[p] = 0$ defines a path $z = H(p)$, which consists of a part of hyperbolic curve in the complex z plane [15]. To obtain an asymptotic result from the exact image field solution, let us once again study the far field of the beam. Making use of (12), the following approximation for $|\bar{r}|$ large must now be applied instead of (13), for $x = 0, z < 0$:

$$D \approx D_0 + qH(p), \quad D_0 = \sqrt{(y + jbsin\theta)^2 + z^2}, \quad q = |z|/D_0 \approx cos\theta_2. \qquad (31)$$

Thus, the field in medium 2 can be written in the approximative form

$$\bar{E}(\bar{r}) \approx -j\omega\mu_0\bar{u}_x IG_2(D_0)\left(1 + \int_0^\infty F_1'(Bh', p)e^{-jk_2qH(p)}dp\right). \qquad (32)$$

Here, the Green function is defined similarly as in (5), for the medium 2 with k_2 replacing k_1. Applying an integral identity [15, eq. (17)], we can write for the expression in braces in (32):

$$1 + \int_0^\infty F_1'(Bh', p)e^{-j\gamma H(p)}dp = \frac{2\gamma}{\gamma + \sqrt{\gamma^2 - B^2}}e^{-j\sqrt{\gamma^2 - B^2}h'}. \qquad (33)$$

Use of this and (see Fig. 3 for the different parameters)

$$D_0 \approx r + jbsin\theta sin\theta_2 \approx r_2 + d' sin\theta_2, \qquad (34)$$

with

$$d' = d + jbsin\theta, \qquad (35)$$

makes it possible to write (32) in the following form:

$$\bar{E}(\bar{r}) \approx -\frac{1}{4}\omega\mu_0\bar{u}_x I\sqrt{\frac{2j}{\pi k_2 r_2}}\frac{2k_2cos\theta_2}{k_1cos\theta_1 + k_2cos\theta_2}e^{-jk_2r_2}e^{-jk_1sin\theta_1 d'}e^{-jk_1cos\theta_1 h'}. \qquad (36)$$

Here, we have made the far field assumption $r_2 \gg h, d, b$, otherwise the expressions would have been much more complicated. (36) can be interpreted in terms of geometrical optics rays of the Gaussian beam, with a phase shift in each medium and a transmission coefficient at the interface. From this, an equivalent source point location can be defined from the divergence of two adjacent rays close to the beam axis $\theta_1 = \theta$. Taking the combination of the last two exponentials in (36), we may require, that the corresponding expression for the image case has the same change when the angle θ_1 differs from the axial angle θ. Writing for the exponential expressions

$$\Psi_1 = -jk_1(d' sin\theta_1 + h' cos\theta_1) = -j\frac{k_1 h}{cos\theta_1} + k_1 bcos(\theta_1 - \theta), \qquad (37)$$

$$\Psi_2 = -jk_2(d_2' sin\theta_2 + h_2' cos\theta_2) = -j\frac{k_2 h_2}{cos\theta_2} + k_2 b_2 cos(\theta_2 - \theta_0), \qquad (38)$$

$$k_1 sin\theta = k_2 sin\theta_0, \qquad (39)$$

we may require that phase and amplitude changes along the plane $z = 0$ be the same for the original and the image case. This implies that the differentials of Ψ_1 and Ψ_2 are the same:

$$d\Psi_1 = d\Psi_2. \tag{40}$$

When supplemented by the condition between the differentials $d\theta_1$, $d\theta_2$, arising from Snell's law $k_1 sin\theta_1 = k_2 sin\theta_2$, (40) gives us the following expression for the unknown h_2:

$$h_2 = h\frac{k_2 cos^3\theta_2}{k_1 cos^3\theta_1}, \ at \ \theta_1 = \theta. \tag{41}$$

Because of $L_1 = h/cos\theta_1$, $L_2 = h_2/cos\theta_2$, the corresponding result for L_2 is

$$L_2 = L_1\frac{k_2 cos^2\theta_2}{k_1 cos^2\theta_1}, \tag{42}$$

which was also given in [5]. To obtain an expression for $\bar{b}_2$, defining the apparent source location in complex space, we must study the real parts of (37), (38). The direction of the vector is known from the main beam direction in the half space 2. The magnitude b_2 is not obtained from (40), since the first differentials vanish identically at $\theta_1 = \theta$. Taking the second differentials of Ψ_1 and Ψ_2 and equating the real parts gives us the result

$$b_2 = b\frac{k_2 cos^2\theta_2}{k_1 cos^2\theta_1}, \ for \ \theta_1 = \theta, \tag{43}$$

which condition was also given in [5].

Thus, the location of the approximate image source can be obtained through two different approaches: the present exact image theory and the saddle-point asymptotic analysis of Fourier integal representation of the transmitted field in [5]. The present approach has the advantage of working with sources, which gives a more physical insight for the problem in addition to being exact.

CONCLUSION

In this paper, the exact image method, previously introduced for problems involving sources in real space with two homogeneous media and a planar interface, is extended to problems involving sources in complex space. This is possible because the analytic functions of real space source position can be extended to functions of complex space source position, which idea has been applied before with success in diffraction problems. The complex source point theory gives a possibility to analyze Gaussian beam problems with a simple source instead of a complicated source in real space. The method has been tested in this paper by analyzing asymptotic (far field) expressions for a reflected and a transmitted Gaussian beam, resulting in well known Goos-Hänchen and angular shifts for the reflected beam and apparent position for the transmitted beam. The present method suggests itself for more exact calculation of the fields in these beams.

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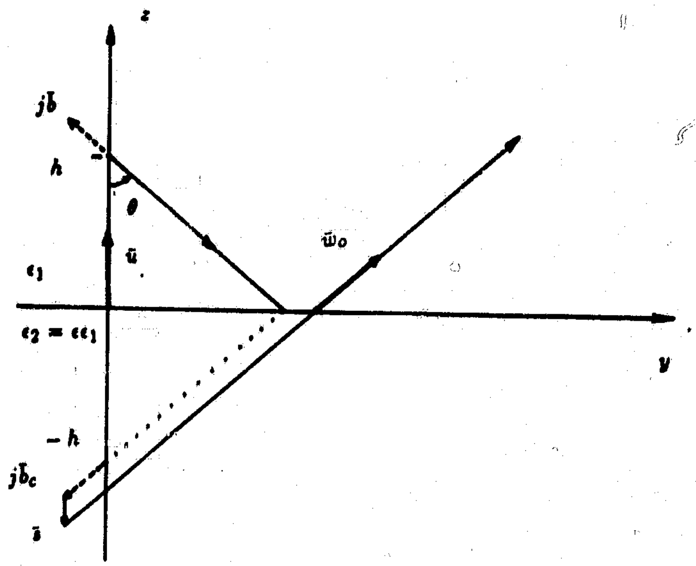

Figure 1.
Goos-Hänchen shift of the Gaussian beam arising from an approximate image point source with a real shift vector $\bar{s}$ from the mirror image point $-\bar{u}h + j\bar{b}_c$.

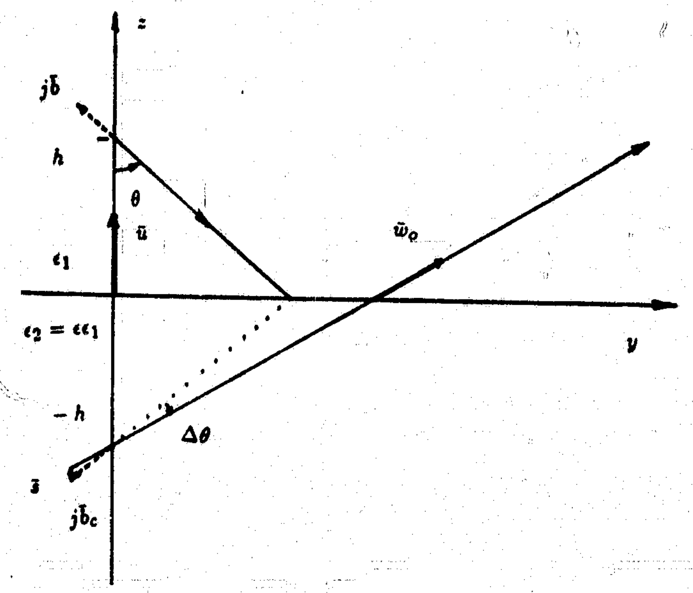

Figure 2.
Angular shift of the Gaussian beam arising from an approximate image point source with an imaginary shift vector $\bar{s}$ from the mirror image point $-\bar{u}h + j\bar{b}_c$.

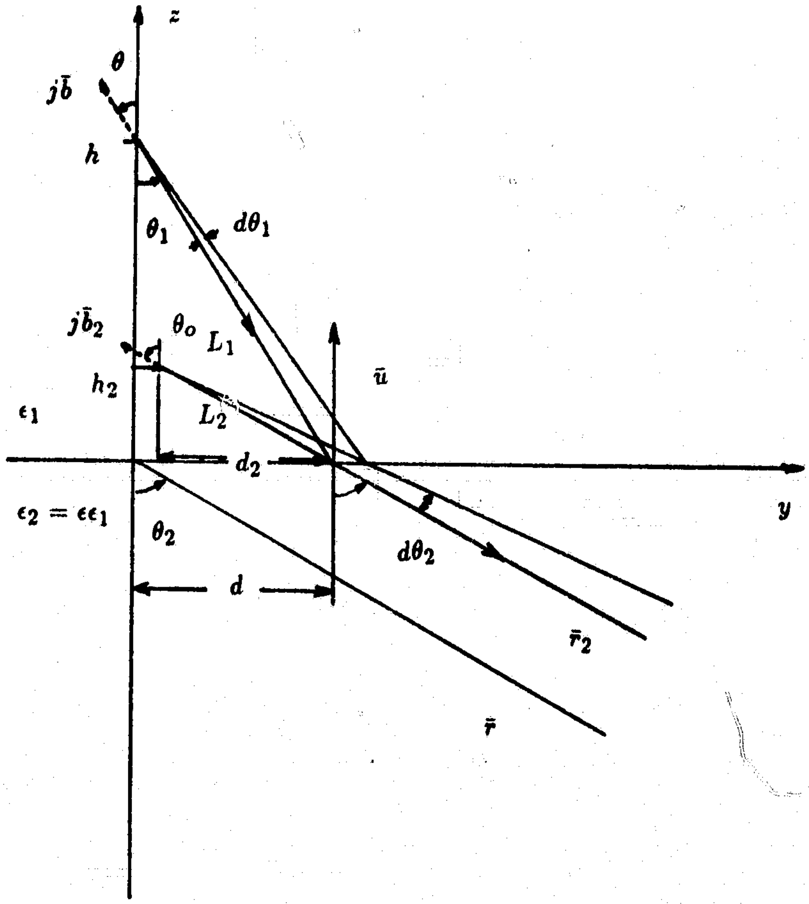

Figure 3.
Gaussian beam transmitted through the interface seen as arising from an approximating point source at point $\bar{u}h_2 + j\bar{b}_2$.

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