

**TWO GIMBAL BEARING CASE STUDIES:
SOME LESSONS LEARNED**

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ABSTRACT

In this investigation, two troublesome, torque related problems associated with gimbal actuators are discussed. Large, thin-section angular contact bearings can have a surprisingly high torque sensitivity to radial thermal gradients. A predictive thermal-mechanical bearing analysis, as described, was helpful in establishing a safe temperature operating envelope. In the second example, end-of-travel torque limits of an oscillatory gimbal bearing approached motor stall during limit cycling life tests. Bearing modifications required to restore acceptable torque performance are described. The lessons learned from these case studies should benefit designers of precision gimbals where similar bearing torque related problems are not uncommon.

INTRODUCTION

It is clear to the experienced space actuator designer that meager torque margins necessitate an accurate prediction of bearing drag torque. Furthermore, it is important that these torque levels remain stable with time.

Torque anomalies are among the most common problems for space mechanism bearings and among the most debilitating. This is because of the

- need to keep motor torque margins, hence motor weight, low.
- use of lightweight structural materials with coefficients of thermal expansion (CTEs) different than bearing steels and thus, sensitive to both thermal effects and thermally induced mechanical deflections.
- wide temperature ranges, often 100°C or more, both during qualification tests and in flight.
- likelihood of significant temperature gradients due to the difficulty of heat transfer and cooling within the mechanisms.
- extremely low operation speeds, typically 1 rev/hr or less, which pushes the bearing lubrication mode into the boundary regime, thus, amplifying both the magnitude of the friction coefficient and the uncertainty in its prediction.

This paper attempts to illuminate some of these special problems unique to space mechanism bearings by reviewing two not uncommon, torque related problems encountered on gimbals of different designs.

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LARGE GIMBAL ACTUATOR

In the first example, the need to maintain the precise location of a high precision encoder under ascent loads necessitated a very stiff bearing mount for a Large Gimbal Actuator (LGA). Hard-preloaded, thin-section, duplex bearings (see Figures 1 and 2) were mounted

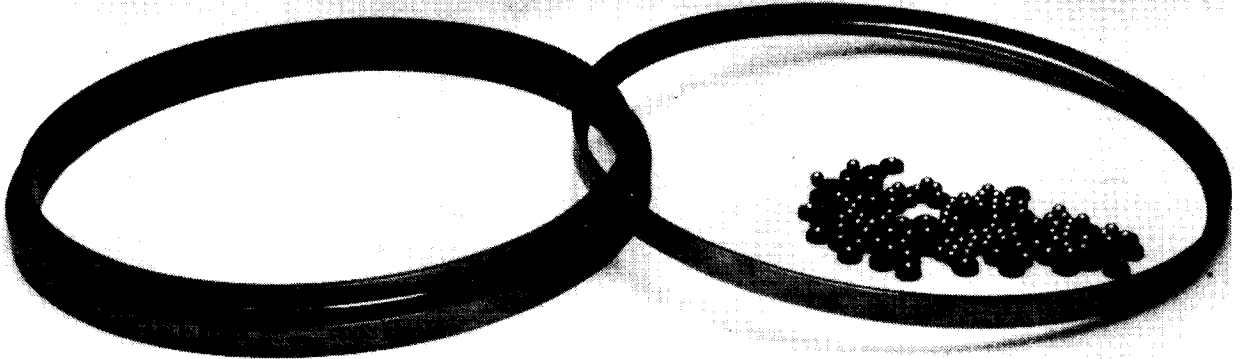


Fig. 1 Large Gimbal Actuator Bearing

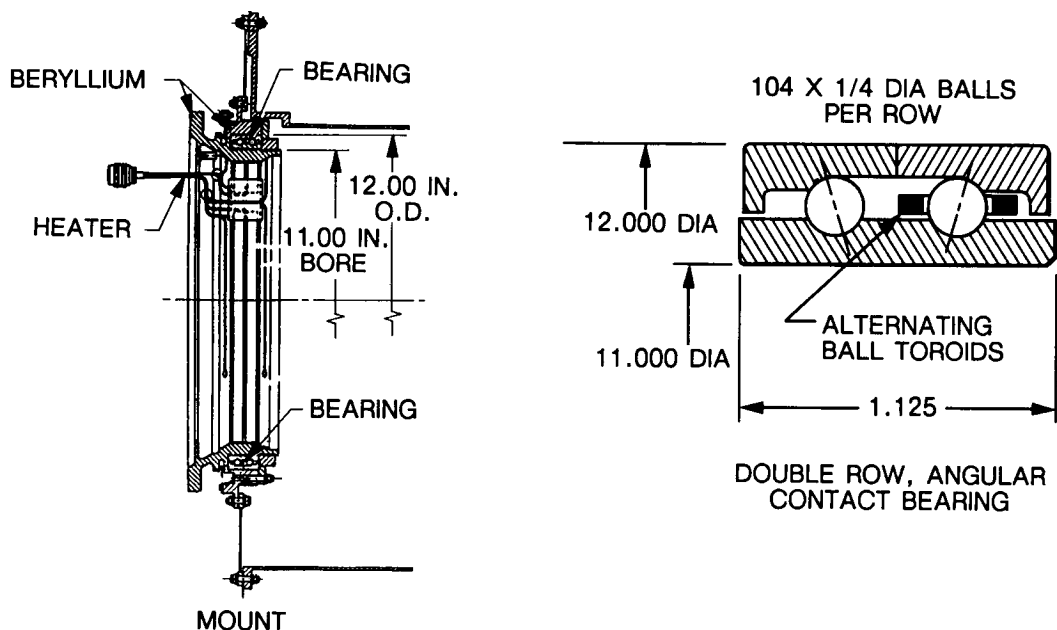


Fig. 2 Geometry of LGA Bearing and Mounting to Beryllium Structure

between a beryllium shaft and housing for maximum stiffness on both the yaw and pitch axes. Internal preload sensitivity, hence torque sensitivity of these bearings to temperature gradients was particularly great due to their hard mount. The coefficient (CTE) of thermal expansion disparity between the steel bearing races and the beryllium mounting structure in combination with the bearing's large size, also contributed to this sensitivity. The immediate concern was the effects that bulk temperature and radial thermal gradients may have on the magnitude of bearing preload and drag torque. Loss of significant bearing preload might permit a shift in encoder position, causing some count error while increased torque might exceed the bearing's allocated torque budget.

Thermal-Mechanical Bearing Analysis

The author found it more expedient to construct a special purpose thermal-mechanical bearing code to solve this class of problem than modify existing general purpose, commercial bearing codes. This also overcame the maximum 30/40 rolling-element limitation of these commercial codes and the need to scale the results to the 104 ball bearing at hand.

The developed program, BRGFIT, calculates the ball contact stresses, bearing displacements and stiffnesses of a thrust loaded, single or duplex angular or deep groove ball bearing. It accounts for the operating bulk temperature, temperature gradients across the bearing and either hollow or solid shafts and housings made of materials dissimilar to the bearing. Furthermore, it predicts bearing drag torque based on bearing internal geometry and user supplied friction coefficients.

The method of solution is based on Jone's bearing theory (reference [1]). The program first calculates the mounted contact angle and axial race displacements (bearing "stickout") for the given desired preload at room temperature and zero temperature gradient. The mounted contact angle is always less than the free contact angle for an interference fit between the inner race and the shaft or between the outer race and housing, due to a reduction in diametral internal clearance. The relationship between the mounted contact angle β_0 and the free contact angle β is given by:

$$\text{COS}(\beta_0) = \text{COS}(\beta) + \text{PD}^*/2Ad \quad (1)$$

where

PD^* = reduction in diametral clearance due to press fit or due to a thermal gradient where $T_{i/r} > T_{o/r}$

A = (inner race conformity + outer race conformity - 1)

d = ball diameter

The reduction in diametral clearance term PD^* is not directly equal to the race interference fit. It must account for the relative-contraction of the hollow shaft under interference pressure and/or the relative expansion of the hollow housing. Both tend to mitigate the magnitude of the interference fit passed on to the bearing.

Next, the bearing is taken to operating temperatures with the specified temperature difference between the inner and outer race. If the inner race temperature is increased relative to the outer race then the internal preload within the bearing is increased since the races are rigidly clamped in the axial direction. The program iteratively solves for the internal preload and race axial deflection

based on the radial deflection of the shaft/inner race and housing/outer race under changing internal ball load. Axial stiffness (or spring rate) is taken as the derivative of the force-deflection curve at the preload point.

Thermal Effects

The effect of radial thermal gradients on LGA bearing internal preload and contact stress can be significant, as shown in Figure 3. This is because the bearing is large (280 mm [11 inch] bore diameter), the Hertzian deformations are small due to the small balls, and the races are rigidly clamped in the axial direction. A relative modest 5.6°C (10°F) increase in inner race temperature will raise the internal preload from 1100 N (250 lbs) with a **nominal race fit** to well over 4400 N (1000 lbs). This, of course, will also increase bearing drag torque, as will be discussed later.

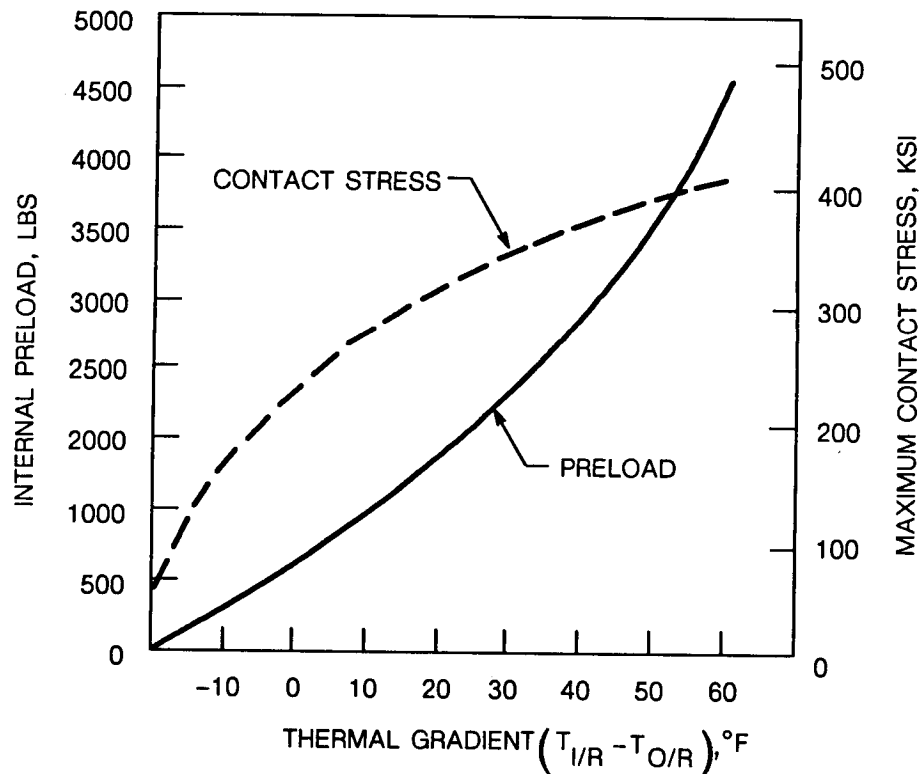


Fig. 3 Effect of Thermal Gradient on Bearing Internal Preload and Contact Stress (Race fits = 0.0002 inch Interference)

The bulk temperature effect is not as dramatic. This is illustrated in Figure 4, which shows the "safe" temperature operating region for the LGA bearings. The beryllium shaft and housing have a slightly larger CTE (by 7×10^{-7} in/in °F) than the stainless steel bearing. As the bulk temperature increases, the housing expands faster than the outer race, creating a potential gap as illustrated by the upper right boundary in Figure 4. This gapping will take place at a higher temperature than one might initially expect, since the outer race will mechanically expand as the support containment of the housing is removed. Thus, the outer race will try to "catch" the expanding housing until the bearing internal preload has dropped completely to zero. At this point, the bearing has become unloaded as defined by the right most boundary in Figure 4.

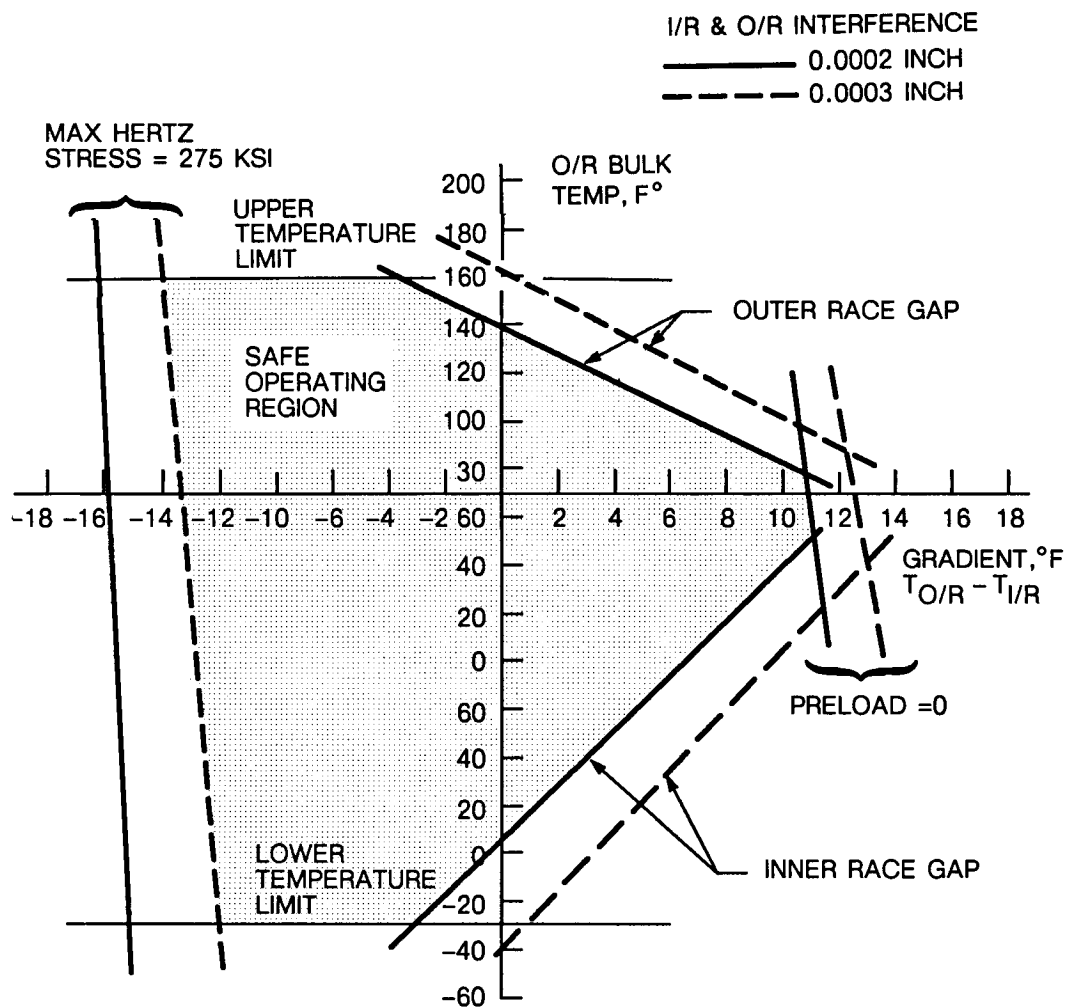


Fig. 4 Bearing Predicted Safe Temperature Operating Region

At cold temperature, the beryllium shaft contracts faster than the steel inner race and a similar gap at the inner race can occur. The upper and lower operation temperature limits for the LGA naturally bound these temperature excursions.

On the other hand, if the inner race is expanding faster than the outer race, the internal preload will increase, as mentioned. There reaches a point where the ball/race contact stresses become unacceptably high as indicated by the left boundary in Figure 4. Thus, all of the above constraints form a thermal operating window that the bearing must operate within.

Drag Torque

Perhaps the most sensitive (and most difficult to predict) parameter to thermal effects is the bearing drag torque. The analysis, due to Jones [2], considers the "scrubbing" friction between the balls and the sides of the tightly conforming raceway ("Heathcoate" slip). It also considers the "spin" friction due to circumferential slip within the contact due to the angular contact between the

ball and the race. (*The origins and implications of spin friction will be covered in greater depth in the second part of this paper.*) This analysis is quite rigorous and has been omitted here but can be found in reference [2].

Applying the thermal analysis to arrive at internal ball loads and using the torque analysis, as described, provides a prediction of the LGA bearing's drag torque as a function of temperature gradient as shown in Figure 5. Test data from our bearings have been included for comparison. A prerequisite for all bearing torque analysis is knowledge about the operating sliding coefficient of friction between the ball and race. For this analysis, the sliding friction coefficient was selected to be 0.15, which turns out to provide acceptable results. If room temperature bearing torque results are available beforehand, which wasn't the case here, the friction coefficient can then be adjusted to give proper torque predictions at different thermal conditions.

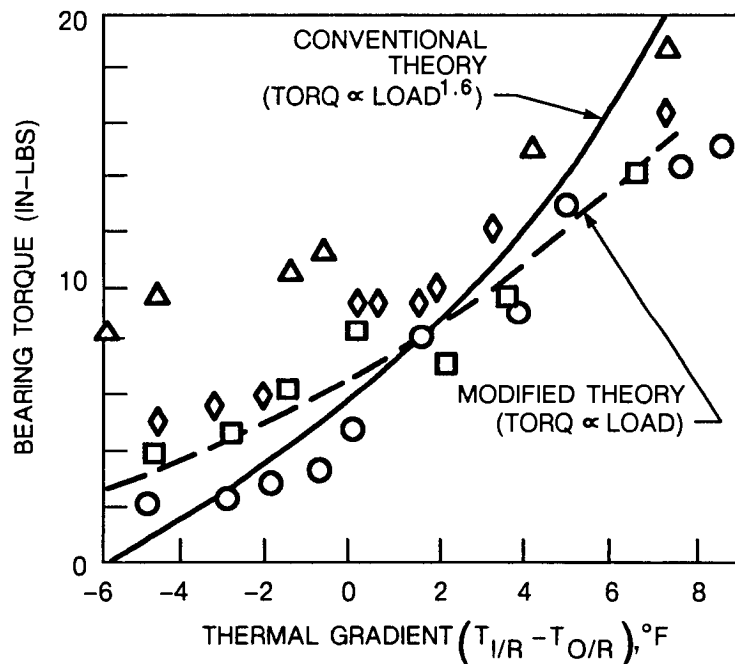


Fig. 5 Comparison Between Predicted and Measured Torque

Two predictive lines appear in Figure 5. The one labelled conventional theory makes use of the Jones method in which torque is proportional to the 1.6 power of load. This exponent considers that not only that the friction forces increase directly with the load, but that the rolling friction and spinning friction are also proportional to the load-expanded contact area as well. One can see from Figure 5 that conventional theory tends to overpredict the torque sensitivity to thermal gradient, although the general magnitude agreement is reasonable. Other low speed, large bore diameter angular contact bearing torque data in the open literature (for example, see references [3] and [4]) also show significantly lower torque sensitivity to thrust load than predicted by conventional theory. In reference [3], Burgmeier measured torque load exponents on the order of 0.7 to 1.0 for a thin-section bearing of similar size to the LGA bearing. Using a relationship that torque is directly proportional to load, indicated by the modified theory line in Figure 5, gives much better trend agreement than before.

CONTAINS USED IN
OF POOR QUALITY

Lessons Learned

The torque sensitivity of the LGA bearing to thermal gradients is quite large, as evidenced by the torque doubling in just a 3.3°C (6°F) change, as shown by the data of Figure 5. *In contrast, only a 10% torque change resulted from more than a 63°C (100°F) change in bulk operating temperature [not shown].* This exceptional torque sensitivity to thermal gradients is a consequence of the bearing's large bore and thin section and the need to rigidly clamp the bearing to ensure encoder position. While wavy spring washers or Belleville springs are often used to decrease thermal sensitivity, axial stiffness is compromised and the position of a position-critical sensor, as in this case, cannot be guaranteed. To insure positive torque margins throughout the flight envelope with a hard mounted bearing, it may be necessary to thermally manage bearing race temperature via active heater control. Such was the case for the LGA bearings (see the heater element in Figure 2).

SPACE TELESCOPE GIMBAL BEARINGS

The Hubble Space Telescope will be the largest and most powerful astronomical instrument ever placed in orbit. It will allow cosmologists to view objects 14 billion light-years away. These targets can be tracked with a pointing stability of 0.007 arc-seconds provided by four reaction wheels which comprise the Pointing Control System (PSC). Ground station communication, via the satellite TDRS, is by way of low gain antennas mounted on the aft shroud and a pair of high gain antennas mounted on the forward shell as shown in Figure 6.

Two High Gain Antenna (HGA) drives, pictured in Figure 7, are cross-mounted to provide pitch and yaw positioning over a hemispherical field of view in excess of 180 degrees. The gimbal contains redundant torque motors and resolvers together with appropriate electrical and data transfer devices.

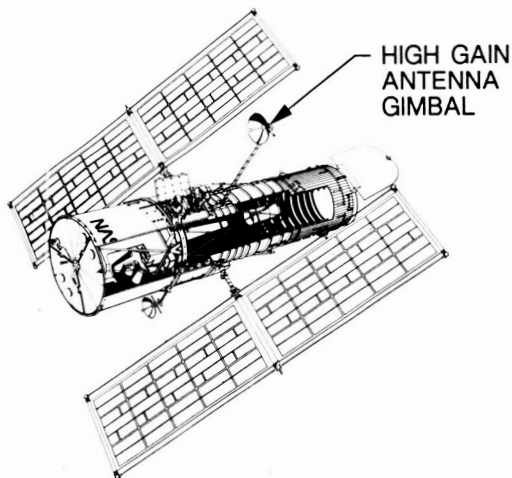


Fig. 6 Hubble Space Telescope
Two Axis Gimbal

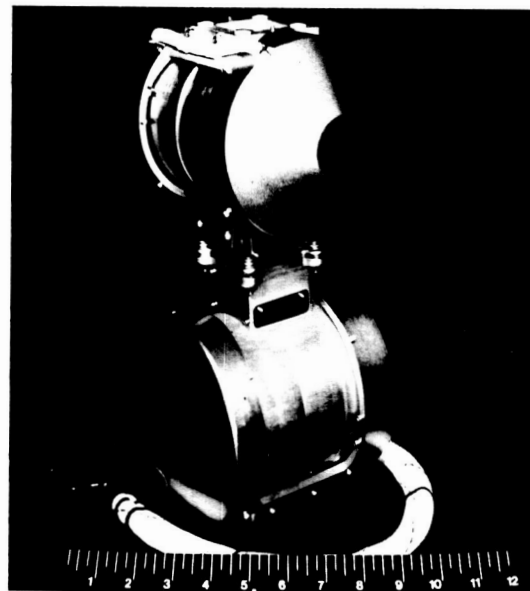


Fig. 7 High Gain Antenna Drive

Bearings - The HGA gimbal is supported by a hard preloaded, pair of A541 size, duplex angular contact ball bearings face-to-face mounted. They contained 24 balls of 3.18mm (0.125 inch) diameter, having an inner and outer race conformity of 51.8%. The one-piece, inner land riding, phenolic, laminated cage and balls were lubricated with a Kendal super refined mineral oil, KG-80.

Torque Anomaly

During life tests under repeated limit-to-limit cycling, the gimbal drag torque increased precipitously from a nominal 14 N-mm (2 in-oz) to as high as 127 N-mm (18 in-oz). This drag approached the stall torque of the motor. There were several curious aspects to this problem. The

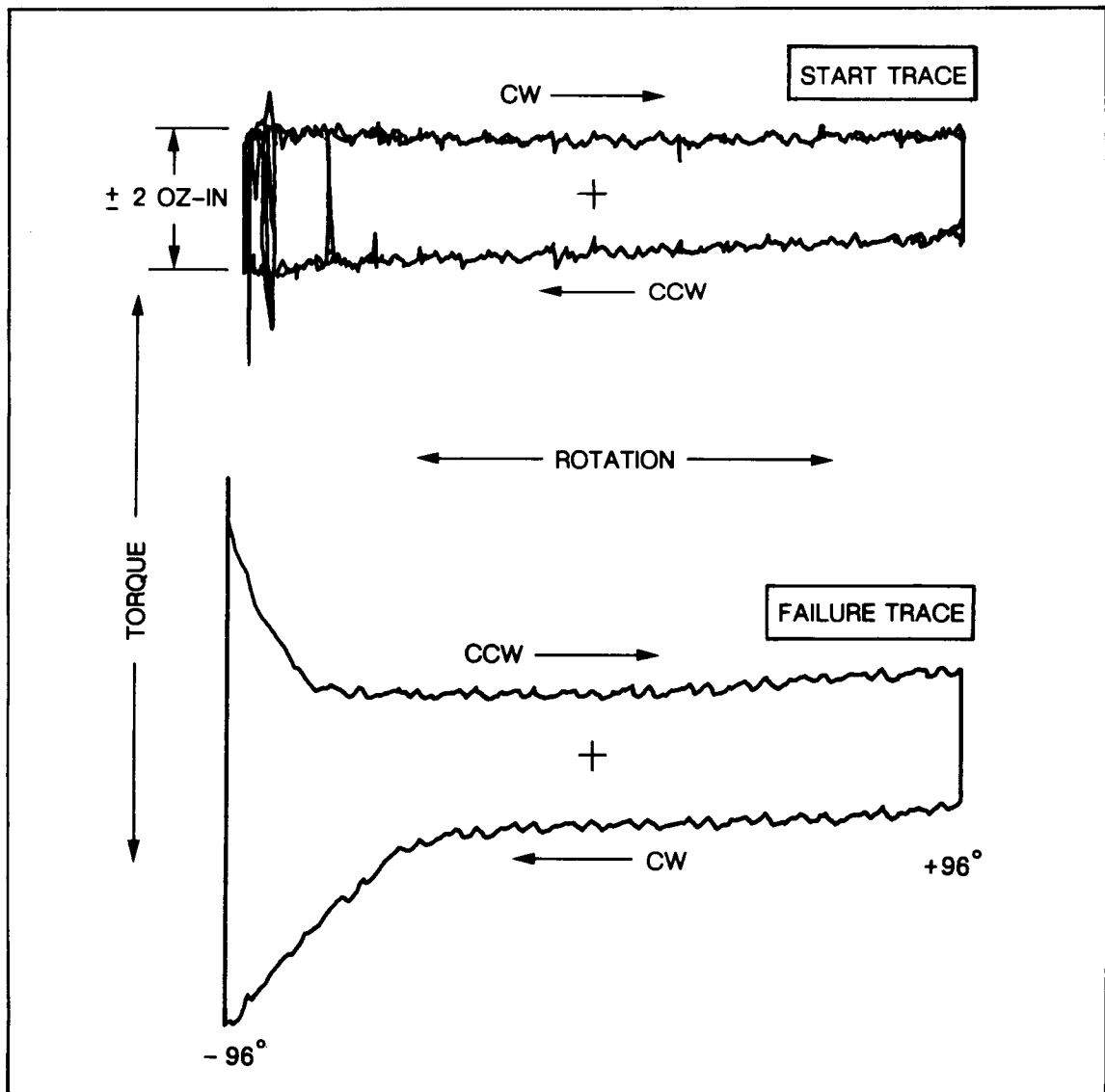


Fig. 8 Bearing Torque Signature at Failure

anomalous drag torque always took some operating time to develop, from a few hours to a few weeks at the 0.5 deg/sec cycling rate. The highest torques only occurred at the end-of-travel reversal point. The torque trace always assumed a characteristic saddle-shape that could be symmetric, but was more often nonsymmetric, as shown in Figure 8. Tapping and/or inverting the gimbal caused the torque to temporarily disappear, but it would eventually return. Finally, torque anomalies were observed with all six of the HGA gimbals, even though their design was essentially identical to an earlier gimbal which exhibited no such problem.

The Possibilities

The only possible contacting sources of drag were the flex capsule which contains contacting flex tape wraps and the bearings. Eliminating the flex capsule from the gimbal did not resolve the problem, so the bearings were the prime suspect. The fact that the high torques occurred only at the end of travel and only after many cycles of operation were the two biggest mysteries.

Ball Speed Variation - One hypothesis was that the phenomena often referred to as "cage wind up" was occurring. This refers to the cage pinching forces that can occur when individual balls are orbiting at different speeds due to small variations in contact angle or ball size. This *ball speed variation* (BSV) and its effects on cage loading, have been known for sometime as discussed in detail by Barrish [5]. The most common cause of BSV is due to misalignment as illustrated in Figure 9 from reference [5]. The inner race contact pattern shows that the balls at the top of the bearing must move faster to cover a greater circumferential distance in the same period of time as that of the bottom balls which have a shorter distance to travel. Thus, the balls advance or retard from the average speed and can "squeeze" the cage's ball pockets, increasing drag. Under severe loading conditions, excessive BSV have been observed to cause rapid cage failure.

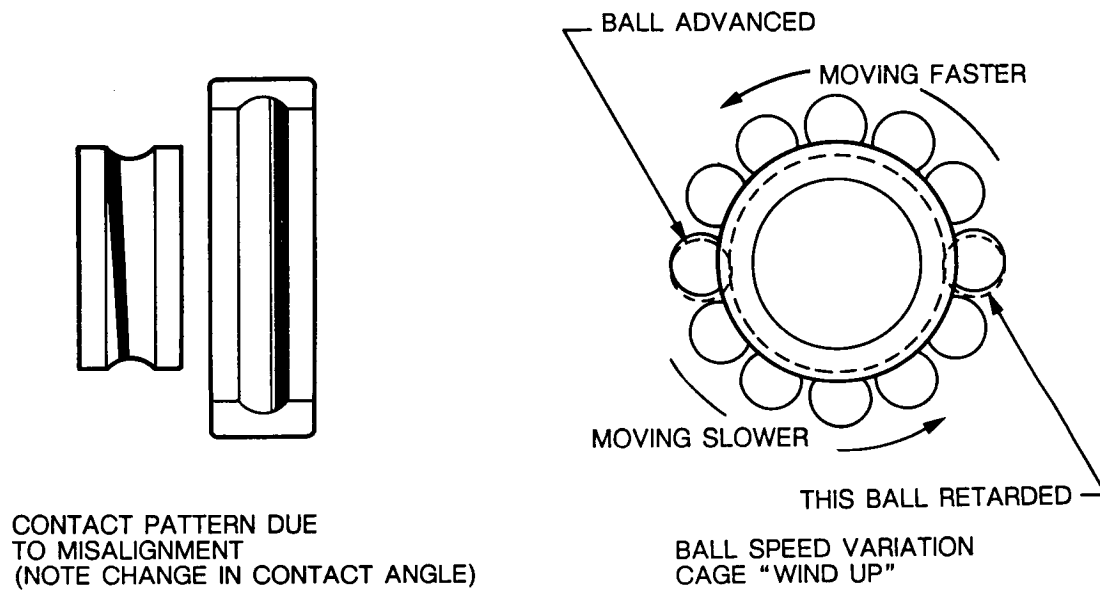


Fig. 9 Effect of Misalignment on Cage-Windup

In an oscillatory bearing, as described here, the distance errors between the balls and hence, the severity of the cage loading ("wind up") would increase with rotation, reach a maximum at

end-of-travel, and then decrease as rotation is reversed. This tends to explain the saddle-shaped torque trace that was observed. Furthermore, this torque buildup due to BSV would be more severe with: (a) a one-piece cage rather than individual toroids or spring separators; (b) low speed, boundary friction rather than the much lower friction which accompanies elastohydrodynamic lubrication; and (c) thrust bearings where all the balls are always loaded rather than a radially loaded bearing where ball spacing can freely adjust in the unloaded zone. The fact that all of these conditions were present with the HGA bearings suggests that BSV is at least a major contributing factor.

A common approach to minimize BSV is to use alternating toroid ball separators to permit the balls to more freely adjust their spacing. A switch was then made from the baseline one-piece phenolic cage to ball toroids as shown in Figure 10. While this change eliminated the "runaway" torque at end-of-travel, drag torque continued to increase over time by about 60%, on the average.

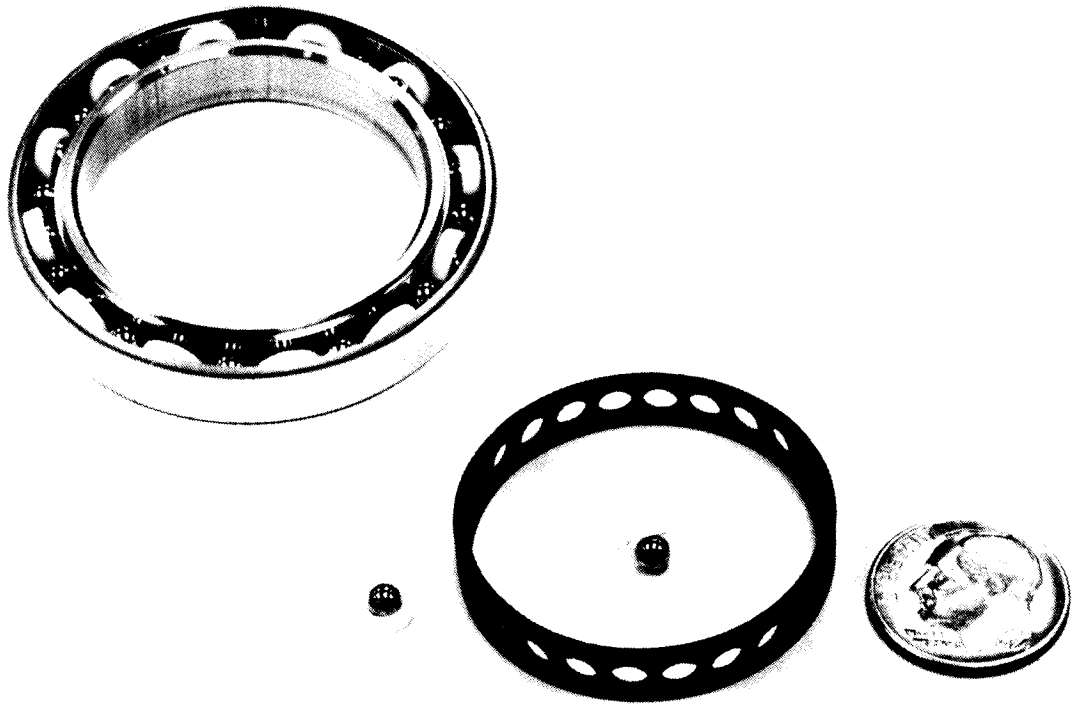


Fig. 10 Comparison of Toroid Ball Separator with Baseline Phenolic Cage

Blocking - Another possible source contributing to the torque anomaly is the effect of contact spin. The European Space Tribology Laboratory (ESTL) reported [6] progressive torque increases with hard-preloaded pairs of ball bearings that were oscillated over an arc of 90 degrees. ESTL referred to this phenomena, which bears a strong resemblance to that observed here, as *blocking*. One of the mechanisms attributed to cause *blocking* was Ball Speed Variation, as just discussed. In the ESTL tests, *blocking* occurred with the "tight" (51.8%) conformity bearing under significant preload (200 N) but was never observed with the "loose" (57%) conformity bearing, otherwise of identical in geometry [6]. It so happens that the HGA gimbal bearings had

the identical "tight" (51.8%) conformity as the ESTL bearings where "tight" and "loose" conformity is illustrated in Figure 11. The second explanation offered by ESTL for *blocking* was due to transverse ball creep, a phenomena first investigated by Prof. Ken Johnson [7].

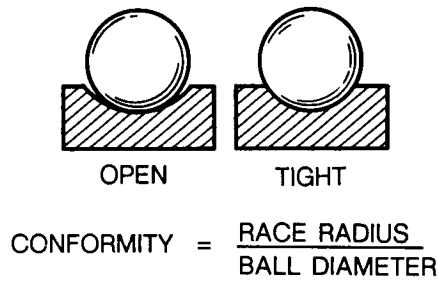


Fig. 11 Bearing Race Conformity

In Prof. Johnson's now-classic experiment with a simple counter rotating thrust bearing, (see Figure 12), the balls were observed to advance radially outward a small amount each revolution. This occurred at extremely slow velocities where centrifugal force effects were negligible. The explanation offered by Prof. Johnson was that circumferential slip (*spin*) velocities normal to the contact produced surface tractions predominantly in the rear of the contact which give rise to transverse ball creep. Thus, the ball radially creeps outward up the groove until, after a few revolutions, the outward *spin* transverse creep force is balanced by the inward axial component of the contact force, as shown in Figure 12. It is this same phenomena which causes a spinning and rolling bowling ball to "hook" into the pins.

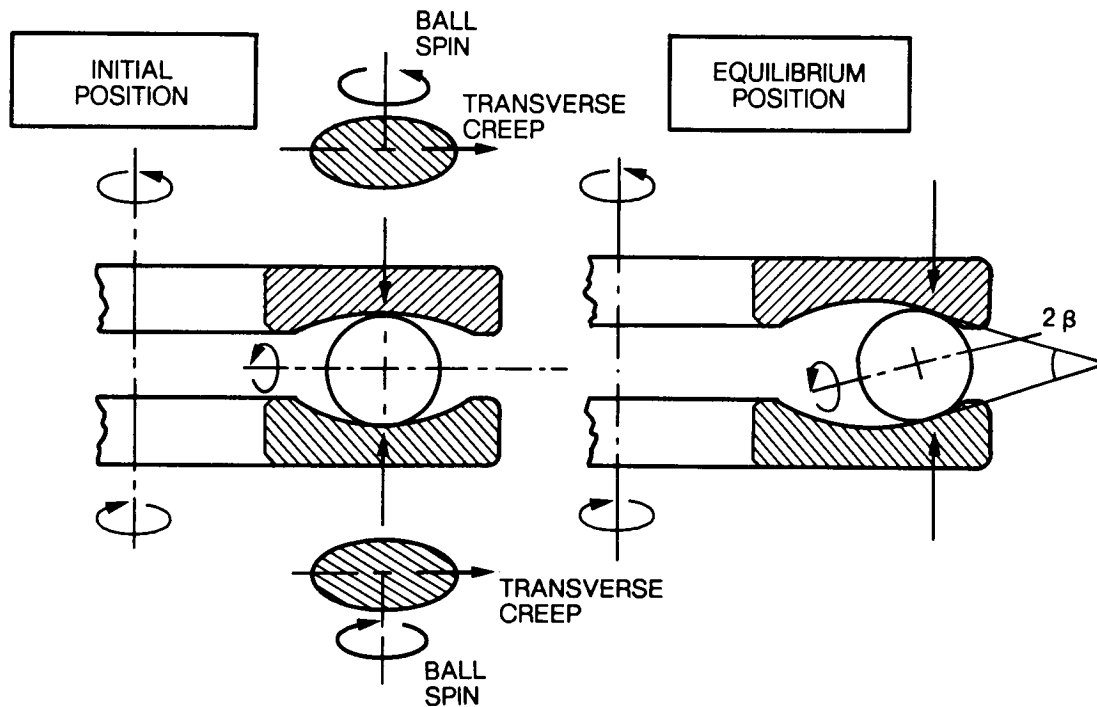


Fig. 12 K.L. Johnson's Spin Experiment with Ball Thrust Bearing

Spin and Stress Analysis - At first blush, transverse creep appeared to be a plausible explanation for the time-buildup of torque as the balls attempted to climb the race with time, thereby increasing the contact angle. However, a question still remained as to how large a transverse creep force would be generated and how much more open must bearing conformity be made to eliminate or minimize this *blocking* phenomena. A real concern was that the ball/race contact stresses at liftoff would exceed recommended limits if %-conformity had to be greatly increased (made looser).

A contact stress calculation, using a large general purpose bearing code, indicated that conformity could be safely increased from the baseline 51.8% to 54% under the "worst case" combination of gimbal loads at liftoff. Next a *spin* analysis was conducted.

The spin velocities at each contact, as shown in Figure 13, were calculated according to the methods of reference [7]. These spin velocities are due to the presence of a higher velocity at the upper edge of the contact than the lower one, as illustrated by the vectors in Figure 13. The contact spin engenders transverse creep forces which tend to increase the contact angle with time as shown. The increase in contact angle would aggravate the situation further since the spin velocity is proportional to the sine of the contact angle. It is unclear what resulting ball motion would occur with time, except there would be a tendency for the ball to spiral upward as it rolled along the race. However, it is instructive to note that the direction of the transverse creep force will be the same even if the bearing rotation is reversed. This illustrated in Figure 14 in which the ball rolling rearward is experiencing a hook from a "right-handed" bowler while the ball rolling into the foreground is experiencing a hook from a "left-handed" bowler. This unidirectionality would tend to support the monotonically increasing torque observed with the HGA gimbal bearings as they rotated back and forth between limits.

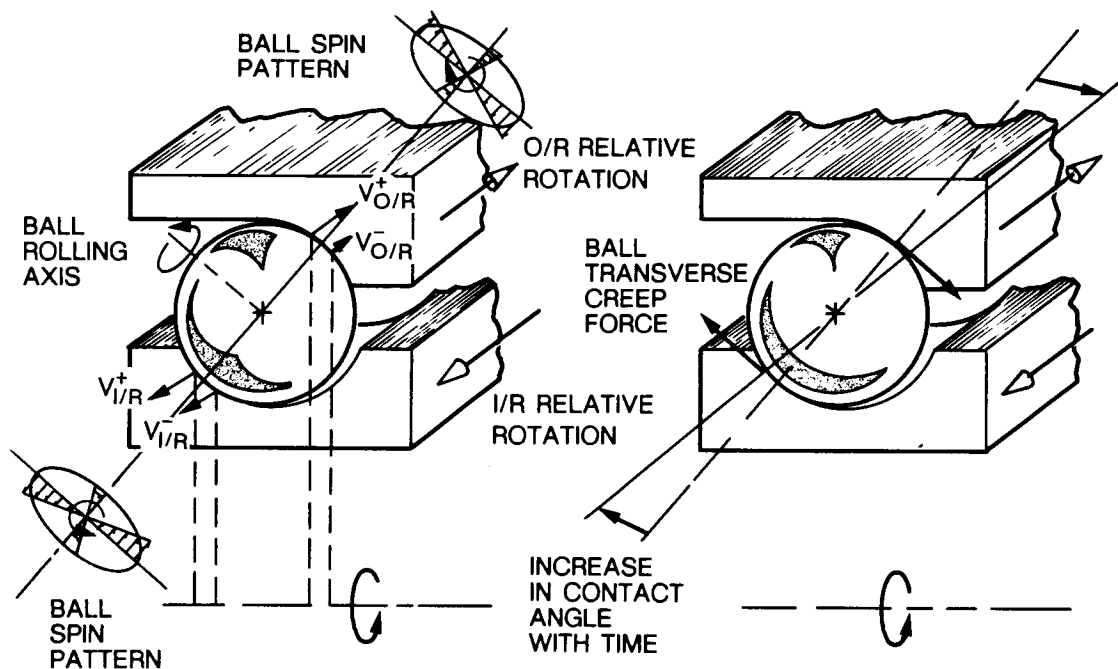


Fig. 13 Spin Generated Transverse Creep Force

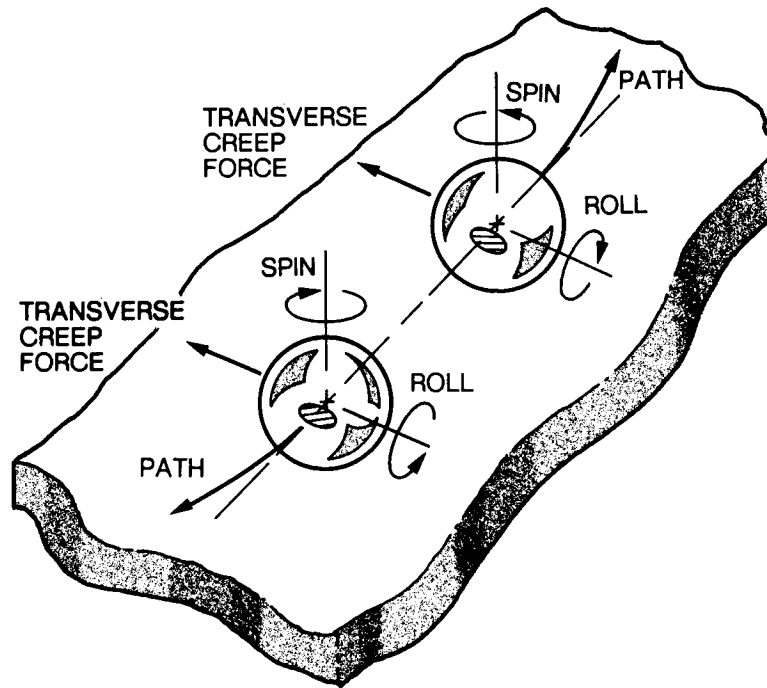


Fig. 14 Transverse Creep Unaffected by Rolling Direction

A proper analysis of these effects would require a time dependent individual ball/race motion analysis under the presence of misalignment. To simplify this sort of full-up analysis, a quasi-steady analysis was performed under uniform axial loading ignoring the effects of misalignment. The analysis is based on the constitutive traction equations of Johnson and Tevaarwerk [8]. Briefly, the *J&T theory* considers the lubricant film in the contact to behave elastically at low strain rates, until the shear stress reaches some limiting value. At this point, the film shears like a plastic material. Their constitutive model relates shear strain and shear strain rate at each point within the contact to the local shear stress (traction force). By integrating these local traction forces across the contact, the net traction force in the direction of rolling, as well as in the transverse direction, can then be computed.

In the case of the HGA bearings, it was necessary to estimate the shear modulus and limiting shear strength of the KG-80 oil film from property data of similar lubricants. Figure 15 summarizes the results of this analysis for different levels of race conformity. The spin torque is a measure of the torque normal to the contact (twisting force). The drag torque considers the combined effects of the spin torque and rolling torque (Heathcoate slip) as discussed in the first *case study*.

As expected, increasing %-conformity, reduced both spin torque and drag torque at the expense of increased contact stress. At 54% conformity, which provides an acceptable contact stress at launch, the predicted spin torque is 45% less and the predicted drag torque is 39% less, than the 51.8% conformity, baseline bearing. The measured torque reduction of a corresponding test bearing was even greater than these predicted values. This data, spotted on Figure 15 for comparison, shows that the drag torque reduction for the 53% inner race/54% outer race test bearing was larger for the ball toroid separators than the one-piece cage. This %-reduction is relative to the baseline bearing's starting torque. However, it is clear that a relatively subtle

increase in the race's transverse groove radii, on the order of 0.05 mm (.002 in), has a profound effect on bearing drag.

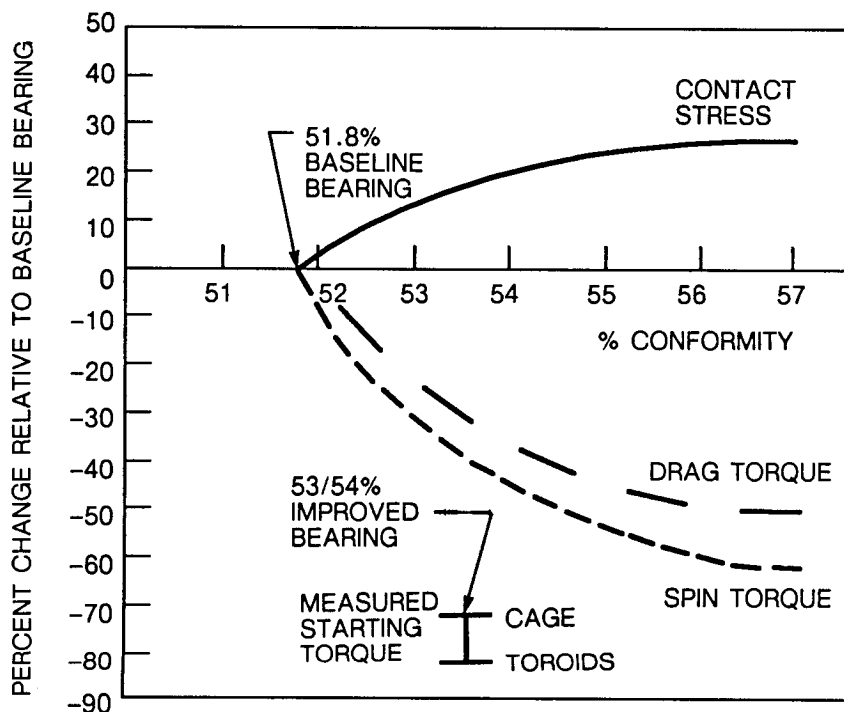


Fig. 15 Predicted Effect of Conformity on Bearing Stress and Friction

Test Results - This dramatic improvement in the torque characteristics of the HGA bearing from the seemingly small increase in %-conformity is readily apparent in Figure 16. In fact, the improvement is even more dramatic when one looks back at the baseline torque failure trace, appearing in Figure 8, rather than the intermediate trace of Figure 16 (a).

The flat trace in Figure 16 (b) shows that the switch to the toroidal separators at the baseline 51.8% conformity definitely got rid of the cage wind-up problem due to BSV. However, as Figure 17 shows, the ball toroids by themselves did not prevent the drag torque from escalating with cycling by about 60% over the starting levels. This cycle effect was hypothesized to be the result of spin generated, transverse ball creep which increases the ball's tendency to climb the shoulder.

It is clear from both Figures 16 and 17, that the dominant factor is the loosening of bearing conformity. The looser bearings not only have flat, small torque traces, but they are also far less sensitive to cycling (25 to 35% torque rise), *no matter what the cage*. This is not totally in concert with the spin analysis, which predicted that although there would be a substantial reduction in the spin torque with a looser conformity, as observed, the transverse creep force would actually increase by about 15%. If this is so, the cycle dependent torque due to transverse creep should be more noticeable. Despite this quirk, predicted gimbal bearing pair torque levels of 1.0 N-mm (0.65 oz-in) compare reasonably well with the measured 0.8 N-mm and 1.3 N-mm (0.5 oz-in and 0.8 oz-in) for the toroid and phenolic cage bearings, respectively.

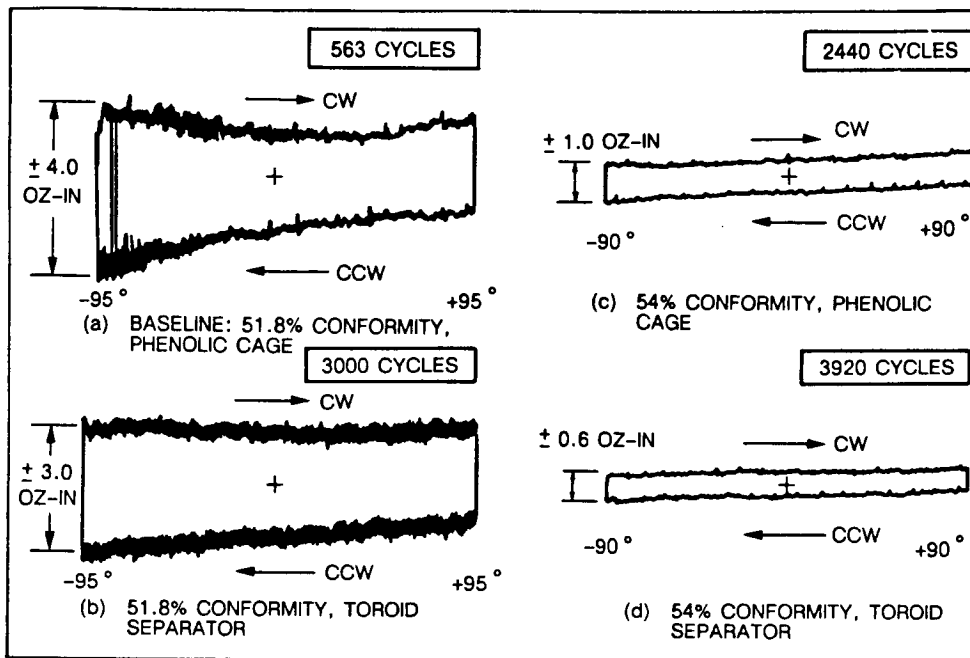


Fig. 16 Gimbal Bearing Torque Trace Summary

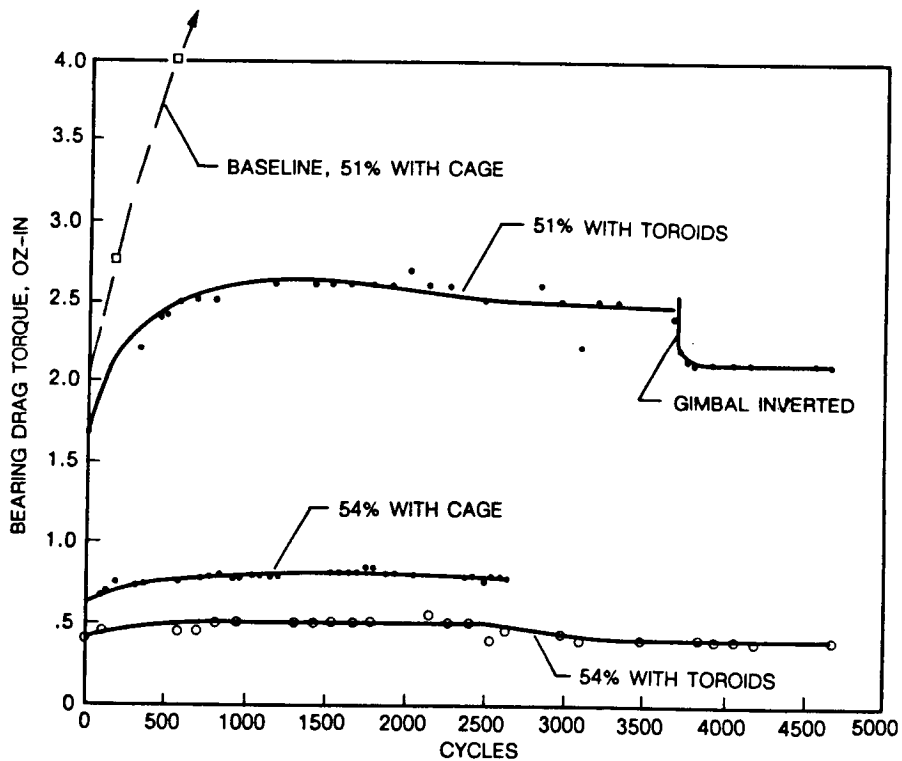


Fig. 17 Effect of Cycles on Bearing Drag Torque

Lessons Learned

The *blocking* torque anomaly was best combatted by increasing %-conformity, although switching to individual ball toroids did tend to lessen cage wind-up. The physical mechanisms responsible for blocking have not been conclusively identified. However, *ball speed variation* does provide a plausible explanation for the high end-of-travel torque. *Transverse ball creep due to spin* appears to be connected with the observed torque/time dependency, although this connection is still a bit illusive at this point.

TABLE I - FACTORS TENDING TO INCREASE BLOCKING

FACTOR INCREASED	EFFECT
• CONFORMITY (TIGHTER)	INCREASES SPIN - HIGHER SPIN TORQUE & DRAG
• CONTACT ANGLE	INCREASES SPIN - HIGHER SPIN TORQUE & DRAG
• ONE-PIECE CAGE	RESTRICTS BALL SPEED SPACING - INCREASES CAGE "WIND-UP"
• MISALIGNMENT	INCREASES BALL SPEED VARIATION - INCREASES " CAGE WIND-UP"
• PRELOAD	INCREASES TRACTION FORCES - INCREASES ANOMOLOUS TORQUE
• FRICTION COEFFICIENT	INCREASES TRACTION FORCES - INCREASES ANOMOLOUS TORQUE
• CONTACT ANGLE VARIATION	INCREASES BALL SPEED VARIATION
• BALL DIAMETER TOLERANCE	INCREASES BALL SPEED VARIATION
• THRUST VS. RADIAL BEARING	THRUST BEARING HAS ALL BALLS LOADED - NO OPPORTUNITY FOR BALL SPACING TO READJUST

Encountering the *blocking* phenomena for a gimbal bearing can be most disconcerting at any time, but particularly late in a flight program. Table I lists many of the likely factors that can contribute to either BSV and/or cage wind-up. They represent sort of a qualitative checklist on how to skirt *blocking*. The last item in the table points to the observation that *blocking* is not often observed in radially loaded bearings or those with a predominant radial load. Apparently, the balls traveling into the unloaded zone have a chance to relax their contact with the ball cage pocket. Perhaps a similar relaxation occurred with the HGA bearings when either tapping or inverting the gimbal temporarily restored lower torque levels.

SUMMARY AND CONCLUSIONS

This investigation focussed on two subtle, but not uncommon, torque problems that can be encountered with spacecraft gimbal bearings. The first example illustrated the surprisingly high torque - thermal sensitivity of a hard preloaded, thin section bearing. A thermal bearing performance code was useful in establishing the acceptable operating temperature envelope. Active heater control was then used to manage bearing torques.

In the second case study, a *blocking* torque anomaly was encountered with oscillatory gimbal bearings during life cycling tests. The remedy was slightly opening the race conformity and switching to alternating ball toroid separators. Bearing analysis, again, was illustrative in assessing the relative options to combat this problem.

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