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ADVANCED PROBABILISTIC METHODS FOR QUANTIFYING THE EFFECTS OF VARIOUS UNCERTAINTIES IN STRUCTURAL RESPONSE*

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A probabilistic structural analysis of a space shuttle main engine (SSME) turbopump blade has been underway at the Lewis Research Center for the last 3 years. The immediate objectives of this study are to evaluate the effects of actual variations, also called uncertainties, in geometry and material properties on the structural response of the turbopump blades. In this study a normal distribution has been assumed to represent the uncertainties statistically. Uncertainties were assumed to be totally random, partially correlated, and fully correlated. The magnitudes of these uncertainties have been represented in terms of mean and variance. These two quantities were selected such that the absolute magnitudes of uncertainty at any point were either less than or equal to 10 percent of their original values.

Many studies have demonstrated that probabilistic analysis methods for components under random loading are more reliable than deterministic approaches. Probabilistic methods have been predominantly used in fatigue, fracture mechanics, and structural reliability analyses under ramdon vibrations (e.g., studies reported by Dover, 1979; Yang, 1981; Wirshing, 1981; Kawamoto, 1982; and Huang and Nagpal, 1984). Particularly in fatigue, improvements in estimating fatigue life ranged up to several hundred percent in comparison with a widely used deterministic approach developed by Miner (1945). Because of the potential, the application of probabilistic methods has been accepted in the other fields of engineering in recent years. These fields include input loading, finite elements, and metallurgy. A probabilistic approach to develop a composite loading for SSME components is underway at the Battelle Laboratories under the technical guidance of R. Kurth (1985) and has been discussed by Shinozuka and Lin (1981). Other studies using probabilistic finite-element methods have been briefly reviewed by DasGupta (1986). A variational approach for developing the probabilistic finite elements is under development by Belytschko and Liu (1985). The usefulness and importance of the probabilistic approach, especially for turbopump blades, has been summarized by Chamis (1986).

Blade response, recorded in terms of displacements, natural frequencies, and maximum stress, was evaluated and plotted in the form of probabilistic distributions under combined uncertainties. These distributions provide an estimate of the range of magnitudes of the response and probability of occurrence of a given response. Most importantly, these distributions provide the information needed to estimate quantitatively the risk in a structural design.
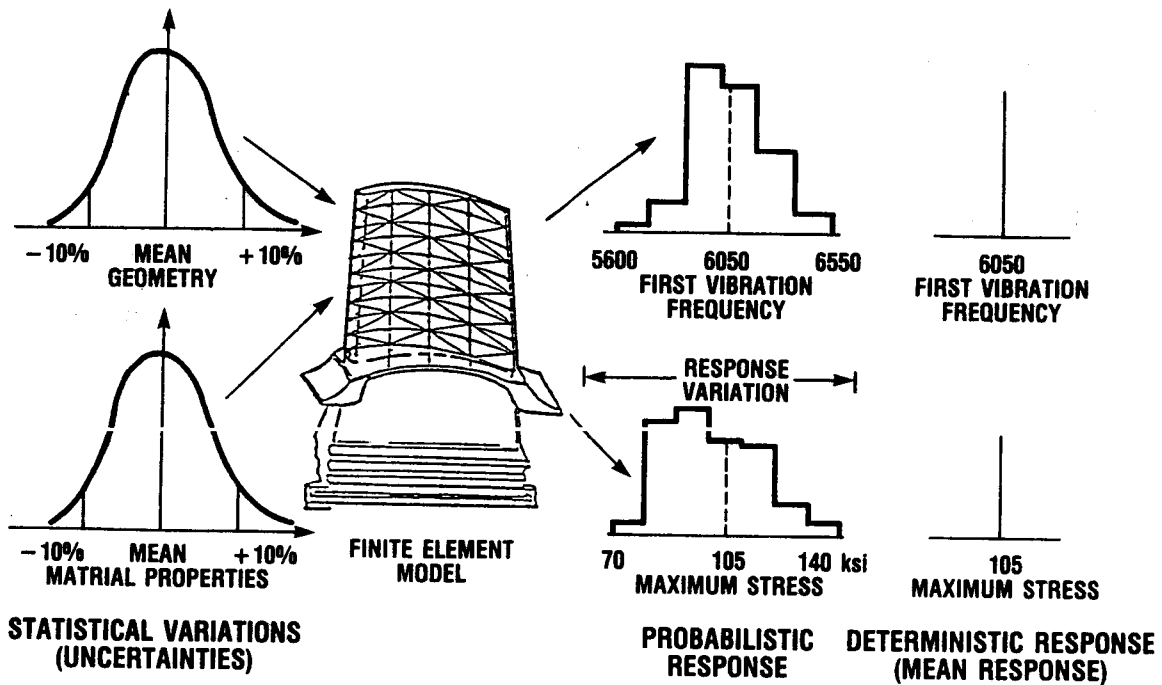

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A wide range of response, such as a maximum stress distribution, implies that small natural uncertainties in the geometry and material properties can cause a large variation in the response. Consequently, designs based on deterministic methods will not be as safe as probability-based designs. Higher stresses, which have low but certain probability of occurrence, are not included in the deterministic designs. In most cases available experimental results are insufficient to provide an entire range of stresses and their probabilities of occurrence. Limited experimental results as well as deterministic methods provide estimates of the mean response but do not cover the scatter. Consequently, the risk in design even after using the factor of safety may or may not be within acceptable limits.

The results of the study indicate that an uncertainty up to 10 percent in the material properties had no significant effect on the response. However, uncertainty up to 10 percent in geometry, only through the thickness, has significant effect on the structural response. Based on the results, a number of probabilistic models has also been developed using regression analysis. These models predict the response for given uncertainties in the geometry and material property parameters. The models for uncertainties in temperature combined with those in geometry and material properties are under development.

PROBABILISTIC STRUCTURAL ANALYSIS ESTIMATES OF RESPONSE

The probabilistic approach provides the scatter in the response and also provides the probability of occurrence. This information helps assess the risk quantitatively for all levels of response. Contrarily, the deterministic response provides an estimate of mean value and does not cover the scatter. Consequently, the risk even after using a safety factor may or may not be within the acceptable range.


STATISTICAL VARIATIONS (UNCERTAINTIES)

- −10% MEAN +10% GEOMETRY
- −10% MEAN +10% MATRIAL PROPERTIES

FINITE ELEMENT MODEL

5600 6050 6550 FIRST VIBRATION FREQUENCY

6050 FIRST VIBRATION FREQUENCY

RESPONSE VARIATION

70 105 140 ksi MAXIMUM STRESS

105 MAXIMUM STRESS

PROBABILISTIC RESPONSE

DETERMINISTIC RESPONSE (MEAN RESPONSE)

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OBJECTIVE

The analysis was performed on the airfoil of a typical SSME turbine blade. Nominal dimensions of the airfoil are 1 to 1½ inches long, about 1 inch wide, and a ¼ inch thick. Generally, these blades are made of anisotropic materials. The number of blades per turbine stage is about 65. The other parts of the blade are the platform, shank, and fir tree.

QUANTIFY UNCERTAINTIES ASSOCIATED WITH PROBABILISTIC RESPONSE

- **GEOMETRY**
  - **UNCORRELATED**
  - **CORRELATED**

- **MATERIAL PROPERTIES**

- **LOADING**

- **TEMPERATURE**

AIRFOIL

PLATFORM

SHANK

FIR TREE
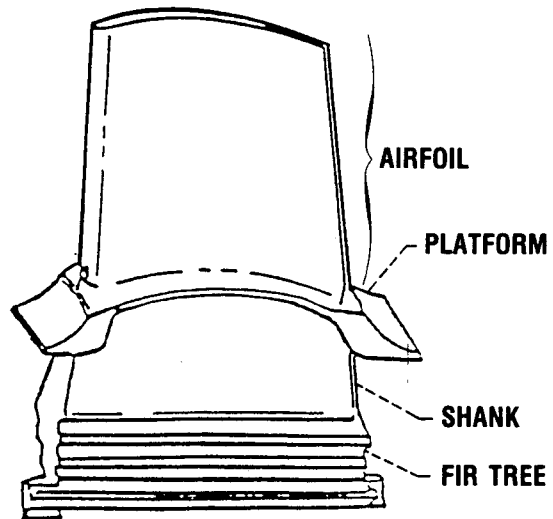
A TYPICAL TURBINE BLADE

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The probabilistic structural response of the blade was recorded in terms of first three natural frequencies, root stress, which is also the maximum stress, and blade tip displacements. The root stress is the stress at the junction of the airfoil and the platform. Since the blade airfoil is considered fixed at this junction for this analysis, stress at that location becomes the maximum. The tip displacements were estimated in three directions. Two displacements were in the plane of the airfoil, and one was perpendicular to the plane of the airfoil.

- **NATURAL FREQUENCIES**
  FIRST
  SECOND
  THIRD

- **ROOT (MAXIMUM) STRESS**

- **TIP DISPLACEMENTS**

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METHODOLOGY

The developed methodology is generic and can be used to analyze other structural components which may or may not be related to this problem. This chart describes the major steps used for performing this analysis. The first step is the generation of random numbers. Correlated or uncorrelated random numbers are used for perturbation of the geometry, spatial location, and/or material properties. Uncorrelated numbers indicate no bearing of the perturbation of one point on any other point on the airfoil.

The second step is modeling the blade with a finite-element model. Eighty triangular shell elements with 55 nodes were used to model the airfoil.

The third step involves setting up of the design to run simulations. Full factorial design was used because it is considered more economical than other available approaches.

The fourth step involved analyzing the response and developing its probabilistic distribution and models to predict the response for given uncertainties.

- **SELECTED RANDOM DISTRIBUTION TO SIMULATE UNCERTAINTIES IN GEOMETRY, SPATIAL LOCATION, AND MATERIAL PROPERTIES**
  - **NORMAL**

- **MODELED BLADE WITH FINITE-ELEMENT MODEL**
  - **80 ELEMENTS**
  - **55 NODES**

- **USED NUMERICAL EXPERIMENT DESIGN TO PERFORM SIMULATION**
  - **FULL FACTORIAL DESIGN**

- **ANALYZED RESPONSE**
  - **PROBABILISTIC MODELS**
  - **PROBABILISTIC DISTRIBUTIONS**
  - **STATISTICAL TESTS**

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SAMPLE RANDOMNESS USED IN PERTURBATIONS

Both uncorrelated and correlated random numbers are generated by the computer. They can be generated using any distribution and with preselected statistical properties such as mean, standard deviation, etc. In the present study a normal distribution with selected mean and variance was used. The random numbers have the property that they can be multiplied by any constant without losing their randomness. A plot of random numbers is shown.


SEQUENTIAL NUMBERS
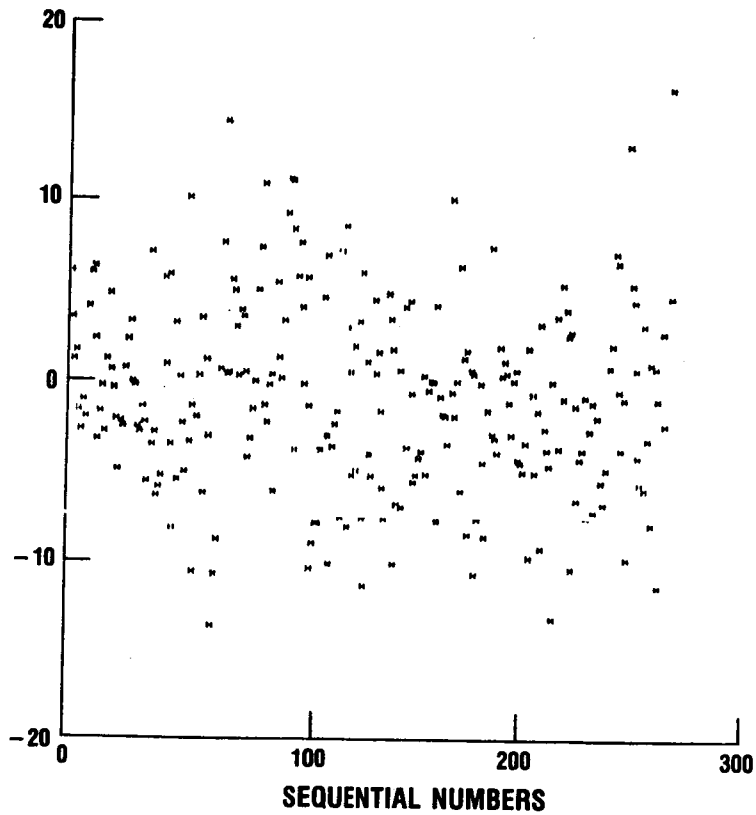
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FRACTORIAL DESIGN FOR THREE VARIABLES

A factorial design is an economical method for performing real experiments or computational simulations. Unlike a parametric approach, the influence of individual variables and their interactions can be estimated discretely. Another advantage of the factorial design is that the range of the variables can be extended by amending the design scientifically.

There are two limitations with the factorial design: it can be used only for the independent variables, and the results of the study are only applicable within the range studied.

A representation of the factorial design for three variables is shown in the attached figure. The solid circles at the corners or in the center of the box represent the experiment or simulation points. A range of variables is defined as the difference between its two extreme levels.
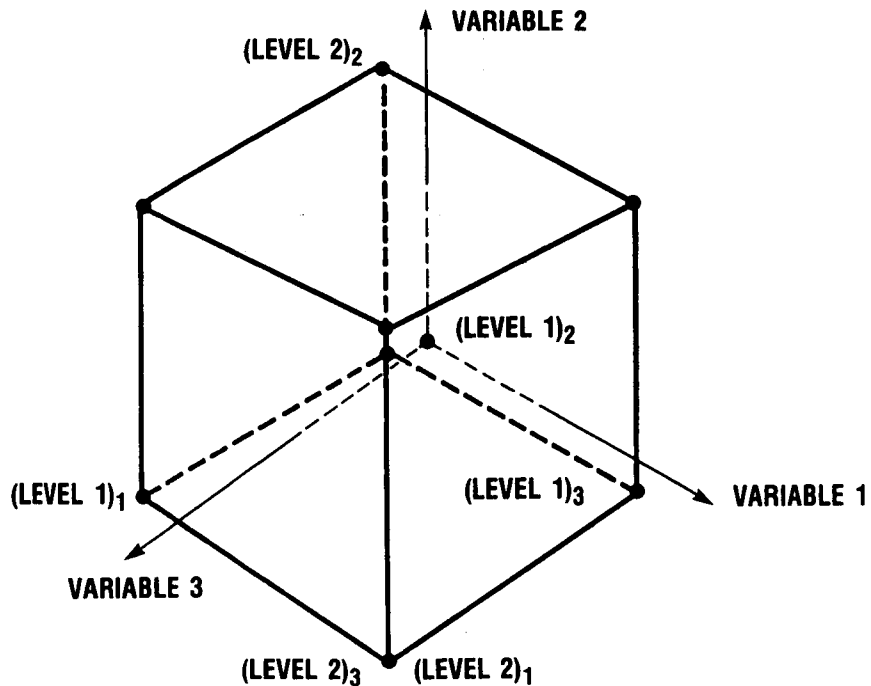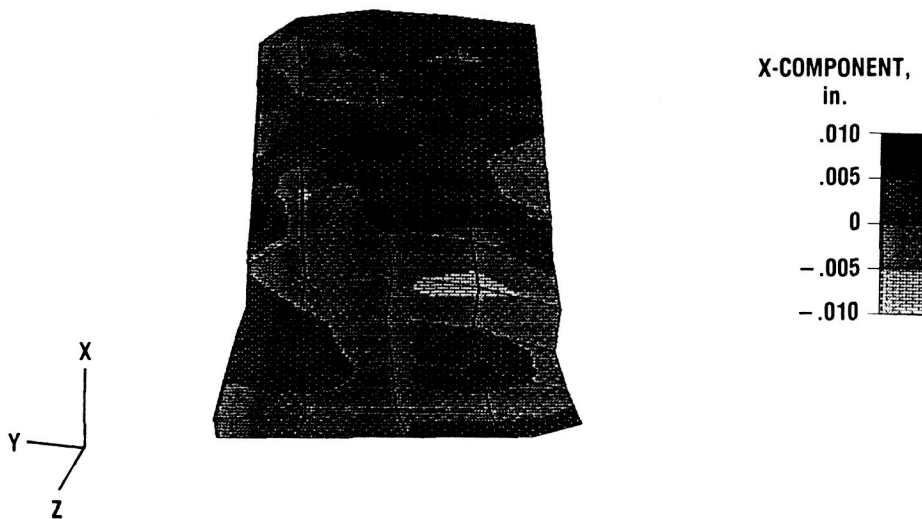

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RANDOM COORDINATE PERTURBATION OF SSME BLADE

A geometric perturbation is done by perturbing the nodal coordinates in all three dimensions. The figure shows an exaggeration of perturbation of nodal coordinates in the x direction only. Perturbation in all three directions is difficult to show in graphical form. The geometric perturbation in this study is intended to simulate realistic geometric variation due to manufacturing tolerances and the errors in finite-element modeling. The variations in geometry at any point were limited to less than 10 percent.

Perturbations were created using random numbers. Uncorrelated and partially correlated random numbers were used to simulate truly uncorrelated or patterned variations in the geometry.

X-COMPONENT,
in.

.010
.005
0
−.005
−.010

X
Y
Z

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RANDOM YOUNG'S MODULUS DISTRIBUTIONS IN SSME BLADE

Material properties perturbation is done by perturbing the properties of each finite element. The attached figure is a demonstration of the variation of modulus of elasticity for a blade that is made of an isotopic material. For a blade made of anisotropic material the entire material property matrix is perturbed at the same time. Material properties, like geometry, were perturbed using uncorrelated and partially correlated random numbers.

These perturbations are to represent realistic variation in the material properties. The maximum variation at any location was limited to 10 percent. The "10 percent" number was selected based on expert opinion.
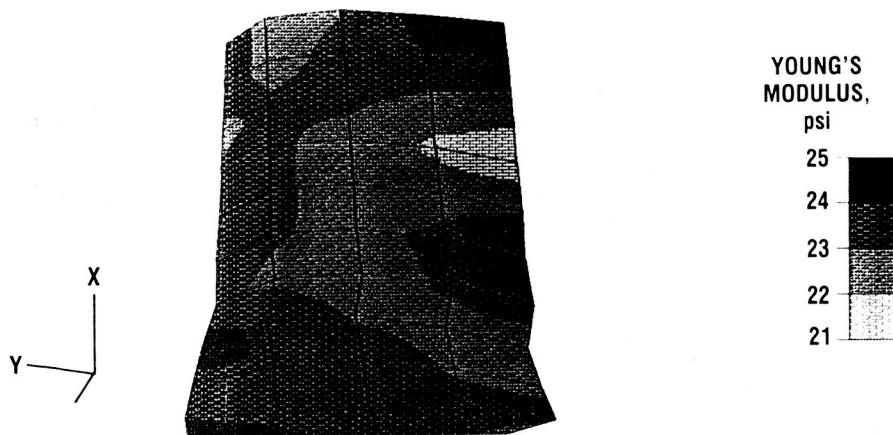
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Probabilistic models were developed using regression analysis. The general form of the model is represented at the top of the following figure. The left side of the model represents dependent variables, which can be any variable in the response. The right side of the model consists of means and standard deviations of uncertainties and their coefficients. Over 20 models were developed. For demonstration purposes coefficients of only one of the models are plotted. This model is for estimating the variation first natural frequency due to combined effect of uncertainties in geometric and material properties.
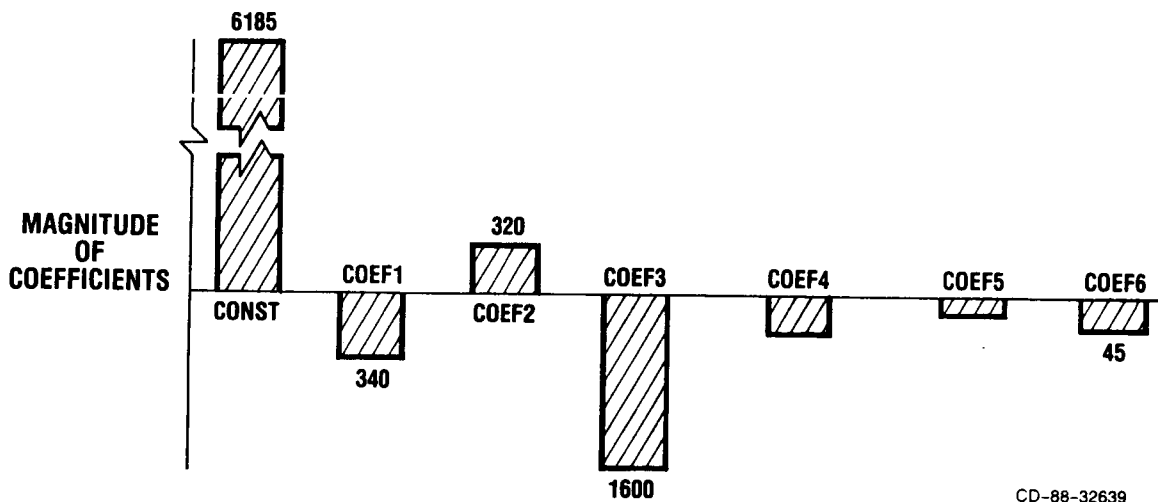
MODEL

DEPENDENT
VARIABLE

$$\hat{Y} = (CONST) + (COEF)_1\,\sigma_1 + \ldots + (COEF)_6\,\sigma_{C_{33}} + \ldots + (COEF)_{21}\,\sigma_3\,\sigma_{C_{33}}$$

WHERE

σ_i **STANDARD DEVIATION OF PERTURBATION IN i DIRECTION**

σC_{jj} **STANDARD DEVIATION OF PERTURBATIONS OF MATERIAL PROPERTY MATRIX ELEMENT C_{jj}**
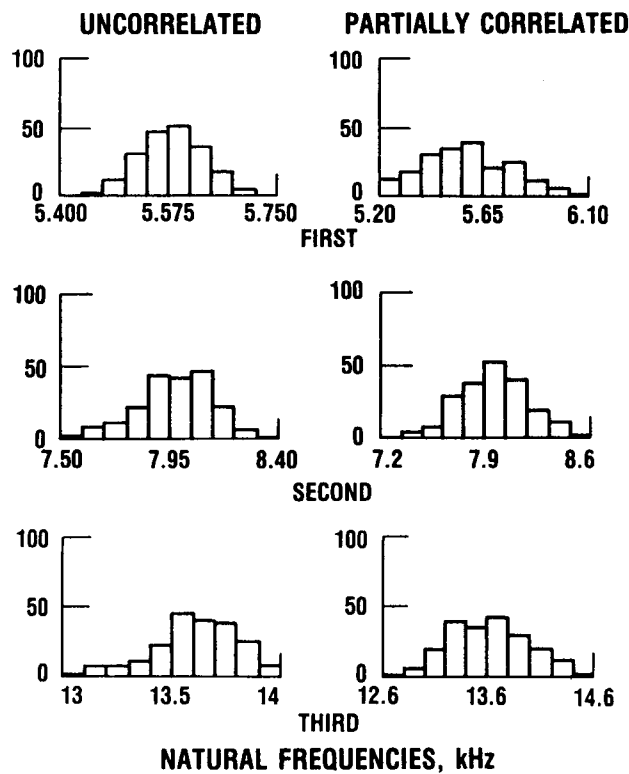

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INFLUENCE OF MATERIAL PERTURBATIONS ON NATURAL-FREQUENCY DISTRIBUTION

The response variables were evaluated for both uncorrelated and correlated random variations in geometry and material properties. The variation in the natural frequencies for partially correlated variations in material properties was slightly wider than for uncorrelated variations. However, the increment in variations was not significant. The response for variation in geometry alone and combined variation in geometry and material properties was studied.

Sample histograms of variations in the first three natural frequencies are shown.

NATURAL FREQUENCIES, kHz

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- METHODOLOGY TO QUANTIFY UNCERTAINTIES 1N THE DESCRIPTORS OF SSME BLADES HAS BEEN DEVELOPED AND APPLIED.

- METHODOLOGY IS GENERIC AND EXTENDABLE TO ANY STRUCTURAL COMPONENT AND ALL ASPECTS OF STRUCTURAL ANALYSIS AND DESIGN.

- GEOMETRIC UNCERTAINTIES SHOWED SIGNIFICANT EFFECT ON COMPONENT STRUCTURAL RESPONSE.

- MATERIAL PROPERTIES UNCERTAINTIES AND THEIR INTERACTIONS HAVE INSIGNIFICANT EFFECTS ON COMPONENT STRUCTURAL RESPONSE.

- RANGE OF VARIATION IN STRUCTURAL RESPONSE QUANTIFIED AND EXPLICIT PROBABILISTIC MODELS HAVE BEEN DEVELOPED.

- INFLUENCE OF UNCORRELATED AND PARTIALLY CORRELATED VARIATIONS IN GEOMETRY AND MATERIAL PROPERTIES ON THE RESPONSE HAVE BEEN STUDIED.

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REFERENCES

Belytschko, T., and Liu, W.K., 1985, "Probabilistic Finite Elements: Variational Theory," Structural Integrity and Durability of Reusable Space Propulsion Systems, NASA Report CP-2381.

Box, G.E.P., Hunter, W.G., and Hunter, J.S., 1982, An Introduction to Design, Data Analysis, and Model Building, John Wiley and Sons.

Chamis, Christos C., 1986, "Probabilistic Structural Analysis Methods for Space Propulsion System Components," presented in 3rd Space System Technology AIAA Conference, San Diego, California.

DasGupta, G., 1986, "Literature Review on Probabilistic Structural Analysis and Stochastic Finite Element Methods," Probabilistic Structural Analysis Methods for Select Space Propulsion System Components, Southwest Research Institute, Annual Report.

Dover, W.D., 1979, "Fatigue Crack Growth in Tubular Welded Connections," Second International Conference on Behavior of Offshore Structures, held at Imperial College, London, England.

Huang, T.C., and Nagpal, Vinod K., 1984, "Probabilistic Factors, Experimental Design and Statistical and Variance Analyses for Fatigue Under Random Vibrations," edited by T.C. Huang and P.D. Spanos, New Orleans.

Kawamoto, James, et al., 1982, "An Assessment of Uncertainties in Fatigue Analysis of Steel Jacket Offshore Platform," Applied Ocean Research, Vol. 4, No. 1.

Kurth, R., 1985, "Composite Load Spectra For Select Space Propulsion Structural Components, Structural Integrity and Durability of Reusable Space Propulsion Systems," NASA Report CP-2381.

Lin, Y.K., 1981, "Random Vibrations of Civil Engineering Structures," Proceedings of Symposium on Probabilistic Methods in Structural Engineering, HSCE, St. Louis, Missouri.

Miner, M.A., 1945, "Cumulative Damage in Fatigue," Journal of Applied Mechanics, Vol. 12.

Shirozuka, M., and Tan, R., 1981, "Probabilistic Load Combinations and Crossing Rate," Proceedings of Symposium Probabilistic Methods in Structural Engineering, ASCE, St. Louis, Missouri.

Wirshing, Paul H., 1981, "Fatigue Reliability Analysis in Offshore Structures," Proceedings of Symposium on Probabilistic Methods in Structural Engineering, ASCE, St. Louis, Missouri.

Yang, J.N., 1981, "Reliability Analysis of Aircraft Structures," Proceedings of Symposium on Probabilistic Methods in Structural Engineering, ASCE, St Louis, Missouri.