MONOLITHIC CERAMIC ANALYSIS USING THE SCARE PROGRAM

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ABSTRACT

The SCARE (Structural Ceramics Analysis and Reliability Evaluation) computer program calculates the fast fracture reliability of monolithic ceramic components. The code is a post-processor to the MSC/NASTRAN general purpose finite element program. SCARE automatically accepts the MSC/NASTRAN output necessary to compute reliability. This includes element stresses, temperatures, volumes, and areas. The SCARE program computes two-parameter Weibull strength distributions from input fracture data for both volume and surface flaws. The distributions can then be used to calculate the reliability of geometrically complex components subjected to multiaxial stress states. Several fracture criteria and flaw types are available for selection by the user, including out-of-plane crack extension theories. The theoretical basis for the reliability calculations was proposed by Batdorf. These models combine linear elastic fracture mechanics (LEFM) with Weibull statistics to provide a mechanistic failure criterion. Other fracture theories included in SCARE are the normal stress averaging technique and the principle of independent action. The objective of this presentation is to summarize these theories, including their limitations and advantages, and to provide a general description of the SCARE program, along with example problems.

OUTLINE

Designing with ceramics requires a new approach involving statistics. Inherent to this method is the realization that any component will have a finite failure probability; that is, no design is fail-safe. Methods of quantifying this failure probability have been investigated and refined. These theories have been programmed into SCARE. The accuracy of the FORTRAN coding and the mathematical modeling has been verified by analytical and experimental methods. Using SCARE a design engineer can easily calculate the change in reliability due to a design change. This can lead to more efficient material utilization and system efficiency.

- **BASIC CONCEPTS OF BRITTLE MATERIAL DESIGN**
- **STATISTICAL FRACTURE THEORIES:**
  - **1. BASED ON STATISTICS AND OBSERVATIONS:**
    UNIAXIAL WEIBULL, NORMAL STRESS AVERAGING, PIA
  - **2. BASED ON STATISTICS AND FRACTURE MECHANICS: BATDORF**
- **THE SCARE PROGRAM**
- **VERIFICATION AND APPLICATION OF SCARE**
- **SUMMARY**

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Structural ceramics have been utilized for various test engine components since the early 1970's. This work has been sustained by the unique properties that ceramics offer in the areas of high-temperature strength, environmental resistance, and low density. These characteristics can result in large benefits in system efficiency and performance. However, the brittle nature of ceramics causes a high sensitivity to microscopic flaws and catastrophic fracture. The subsequent low reliability of ceramic components has limited their application in large scale engine production.
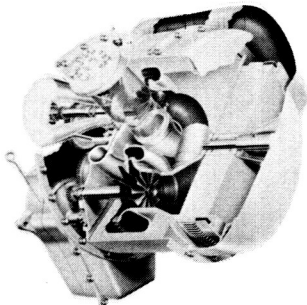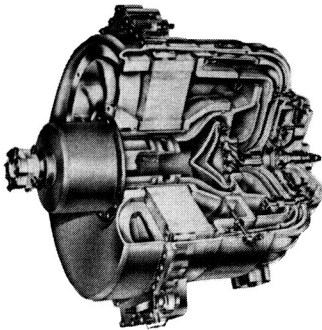
CERAMIC PROPERTIES
— **HIGH-TEMPERATURE STRENGTH**
— **ENVIRONMENTAL RESISTANCE**
— **LOW DENSITY**

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IMPROVED EFFICIENCY AND PERFORMANCE

BUT

• **HIGH CERAMIC SENSITIVITY TO FLAWS**
• **BRITTLE CATASTROPHIC FAILURE**

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• **LOW RELIABILITY**
• **LIMITED APPLICATION**

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The design of ceramics differs from that of ductile metals in that ceramic materials are unable to redistribute high local stresses induced by inherent flaws. Random flaw size and orientation require a probabilistic analysis. The first step of a probabilistic design methodology is the determination of a temperature-dependent fracture strength distribution from flexural or tensile test specimens. From this data, the reliability for a given component geometry and loading is then computed. An important characteristic of probabilistic design is that the stress distribution over the entire volume is needed. The design is not necessarily governed by the most highly stressed location, but by the entire stress field.

- **CERAMICS ARE BRITTLE AND HAVE MANY FLAWS**
- **RANDOM FLAW SIZE AND ORIENTATION REQUIRE PROBABILISTIC METHOD**
- **GENERAL APPROACH:**

SIMPLE TESTS   **COMPLEX PREDICTIONS**
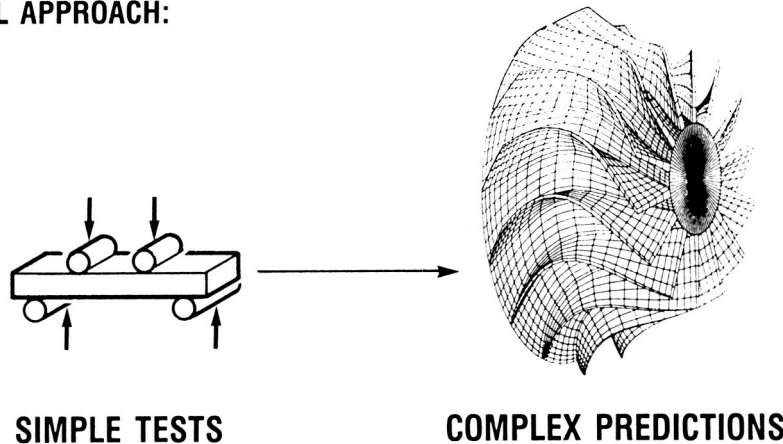
- **REQUIRES ENTIRE STRESS FIELD, NOT MAXIMUM STRESS POINT**

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STATISTICAL FRACTURE THEORIES

A common aspect of any weakest link theory is that the component volume and/or surface area of a stressed material will affect its strength, whereby larger components result in lower average strengths. This observation led Weibull (1939) to propose a phenomenological model to describe the scatter in brittle material fracture strengths. To predict material response under multiaxial stresses Weibull suggested averaging the tensile normal stress in all directions. As this approach is arbitrary and involves tedious numerical integration, other approaches have been subsequently introduced. The most simplistic is the principle of independent action (PIA) model (Barnett, 1967, and Freudenthal, 1968). The PIA theory assumes that each tensile principal stress contributes to the failure probability as if no other stress were present. Finally, Batdorf and Crose (1974) incorporated LEFM with the weakest link theory to predict failure on a mechanistic basis. The model was extended to account for mixed-mode fracture by Batdorf and Heinisch (1978). All of these models are available in the SCARE program.

WEAKEST LINK FRACTURE MODEL	SIZE EFFECT	STRESS STATE EFFECTS	COMPUTATIONAL SIMPLICITY	THEORETICAL BASIS
WEIBULL (1939)	YES	UNIAXIAL	SIMPLE	PHENOMENOLOGICAL
NORMAL STRESS AVERAGING (1939)	YES	MULTIAXIAL	COMPLEX	PHENOMENOLOGICAL
PRINCIPLE OF INDEPENDENT ACTION (1967)	YES	MULTIAXIAL	SIMPLE	MAXIMUM PRINCIPAL STRESS THEORY
BATDORF (SHEAR-INSENSITIVE, 1974) (SHEAR-SENSITIVE, 1978)	YES	MULTIAXIAL	COMPLEX	LINEAR ELASTIC FRACTURE MECHANICS

WEIBULL DISTRIBUTION

The Weibull distribution is a key ingredient for weakest link fracture theories. As the number of flaws present in a structure is proportional to its volume, the failure probability increases with both the applied stress and the material volume. The Weibull failure probability is dependent on three statistical parameters; the Weibull modulus m, the scale parameter σ_0, and the threshold strength σ_u. The Weibull modulus is indicative of strength variability, with smaller values representing a larger variation. The scale parameter, or normalizing stress, is related to the mean strength. The threshold strength is usually taken as zero because of limited experimental data and the mathematical simplicity of the resultant equation. A similar form of the Weibull distribution has been developed for surface flaw induced fracture, with corresponding surface material parameters.

## FAILURE PROBABILITY FOR VOLUMETRIC FLAWS IN UNIAXIAL TENSION

$$P_f = 1 - \text{EXP}\left[-\int_V \left(\frac{\sigma - \sigma_u}{\sigma_0} \right)^m dV \right] \quad (\sigma > \sigma_u)$$

$$P_f = 0 \qquad\qquad (\sigma \leq \sigma_u)$$

WHERE

σ **APPLIED TENSILE STRESS**

σ_0 **SCALE PARAMETER**

σ_u **THRESHOLD STRENGTH**

m **WEIBULL MODULUS**

V **VOLUME OF STRESSED MATERIAL**

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LIMITATIONS OF THE DIRECT STATISTICAL APPROACH

Several limitations are inherent to a purely statistical approach. One prob-
lem occurs when the design stress is below the range of experimental data.
Extrapolation of the Weibull distribution into this regime may yield erroneous
results if other phenomena are present. When two flaw populations exist con-
currently, but only one is active in the strength regime tested, the predicted
failure probability may be low (Johnson, 1983). Further, if the threshold
strength is not zero, the strength may be underestimated (Shih, 1980).
Finally, an approach based only on statistics can account for stress state
effects only in an empirical fashion.

BASIC DATA

NON-ZERO
THRESHOLD STRENGTH

UNDETECTED FLAW
POPULATIONS

STRESS STATE EFFECTS

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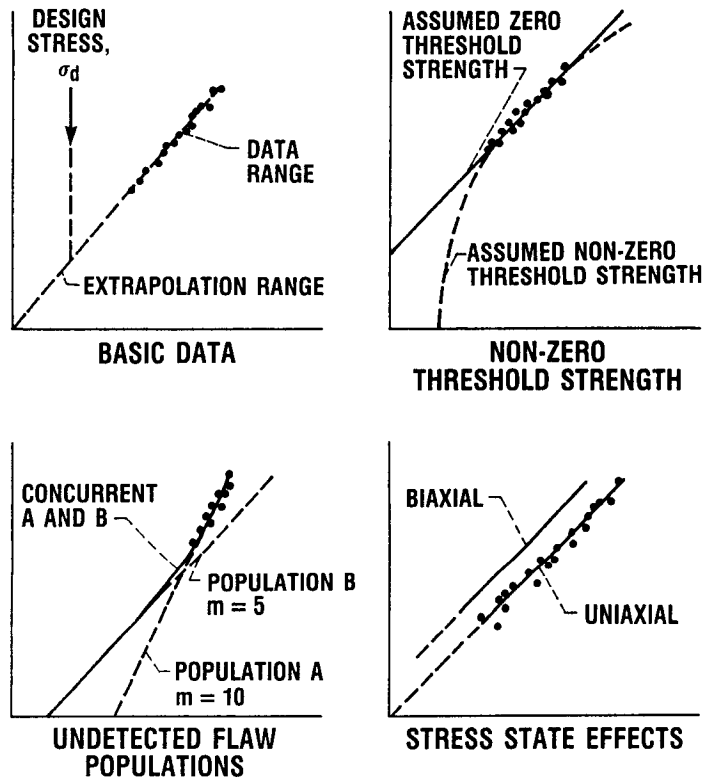
Recognizing that brittle fracture is governed by LEFM, Batdorf proposed that reliability predictions should be based on a combination of the weakest link theory and fracture mechanics. Conventional fracture mechanics dictates that both the size of the critical crack and its orientation relative to the applied loads determine the fracture stress. However, with ceramics the small critical flaw size and the large number of flaws prevent determination of the critical flaw, let alone its size and orientation. Instead, the combined probability of the critical flaw being within a certain size range and being oriented so that it may cause fracture is calculated. As flaw sizes correspond to strength levels and since strength is easier to measure than size for these microscopic flaws, the probability of a crack existing within a critical strength range is determined. This involves the derivative of the Batdorf crack density function which is expressed using the Weibull parameters obtained in uniaxial testing.

$$P_f = 1 - EXP\left[-\int_V dV \int_0^{\sigma_1} \left(\frac{\Omega}{4\pi}\right)\frac{dN}{d\sigma_{cr}}\, d\sigma_{cr} \right]$$

INVOLVES TWO PROBABILITIES:

(1) P [UNDERLINE]EXISTENCE[/UNDERLINE] OF A CRACK IN A GIVEN CRITICAL STRENGTH RANGE] $= dV\dfrac{dN}{d\sigma_{cr}}\, d\sigma_{cr}$

$N(m,\sigma_0,\sigma_{cr})$ **BATDORF CRACK DENSITY FUNCTION (MATERIAL PROPERTY)**

σ_{cr} **REMOTE, NORMALLY APPLIED, FRACTURE STRESS OF A CRACK**

(2) P [UNDERLINE]ORIENTATION[/UNDERLINE] OF A CRACK SUCH THAT $\sigma_e \geq \sigma_{cr}$] $= \dfrac{\Omega}{4\pi}$

σ_e **CRACK EFFECTIVE STRESS (FUNCTION OF CRACK GEOMETRY AND ORIENTATION, STRESS STATE, FRACTURE CRITERION)**

Ω **SOLID ANGLE IN STRESS SPACE WHICH INCLUDES ALL CRACK NORMALS FOR WHICH $\sigma_e \geq \sigma_{cr}$**

Fracture depends not only on the existence of a crack with a certain critical strength, but also on the crack orientation, the far-field stress state, and the crack shape. A collection of crack orientations can be described by a solid angle. A solid angle is measured by its subtended surface area on a sphere of unit radius. Therefore, the measure of the solid angle containing all possible crack orientations is 4π. The other solid angle of intest, Ω, is defined as that angle which includes all crack normals for which $\sigma_e \geq \sigma_{cr}$ (Batdorf and Crose, 1974). This assumes that fracture depends on an effective stress producing a singularity at the crack tip and that crack propagation occurs when the effective stress is greater than or equal to the critical stress. For uniaxial tension the effective stress is highest on cracks normal to the applied load. Therefore, the solid angle can be measured by the surface area of "polar caps" around the loading axis. The caps will decrease in size as the critical stress is increased, until $\sigma_{cr} = \sigma_1$ and $\Omega = 0$. A solid angle representative of equibiaxial tension is also shown.
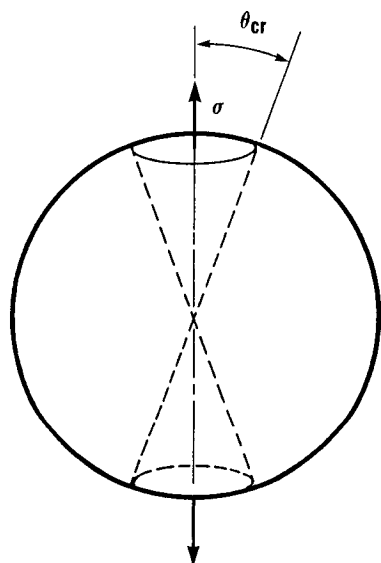
**UNIAXIAL TENSION—
TWO CONES**
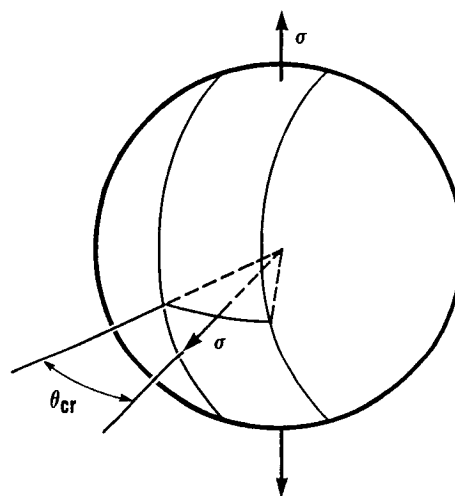
**EQUAL BIAXIAL TENSION—
EQUATORIAL BELT BETWEEN
TWO CONES**

AS THE CRITICAL STRESS INCREASES, THE SOLID ANGLE DECREASES

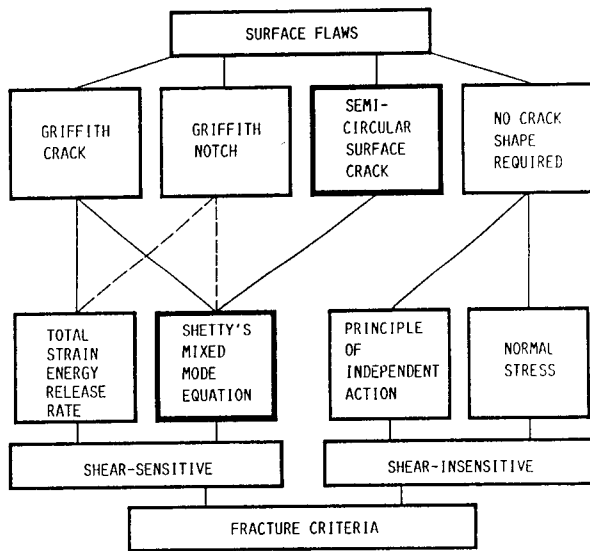CD-88-32568

SCARE PROCEDURAL DIAGRAM

The previously noted fracture theories have all been implemented in SCARE (Gyekenyesi, 1986, and Gyekenyesi and Nemeth, 1987). The bulk of the input data for SCARE comes from a finite element stress analysis and, if necessary, heat transfer analysis of the component. MSC/NASTRAN was chosen as the primary finite element package because of its extensive capabilities and widespread usage. The SCARE program can be readily modified to accept input from other methods of stress analysis. Input specifying fracture strength data from test specimens, usually MOR (modulus of rupture) bars, is also required. These specimens must be separated into two categories, those which failed because of surface flaws and those which fractured because of volume flaws. In addition, a flaw geometry and fracture criterion must be selected. The SCARE program then calculates the statistical parameters for both the Weibull and Batdorf fracture models. The survival probability is calculated for both volume and surface flaws for each element or subelement. Since the reliability of each subelement is independent, the failure probability for a given component under the specified load is computed.
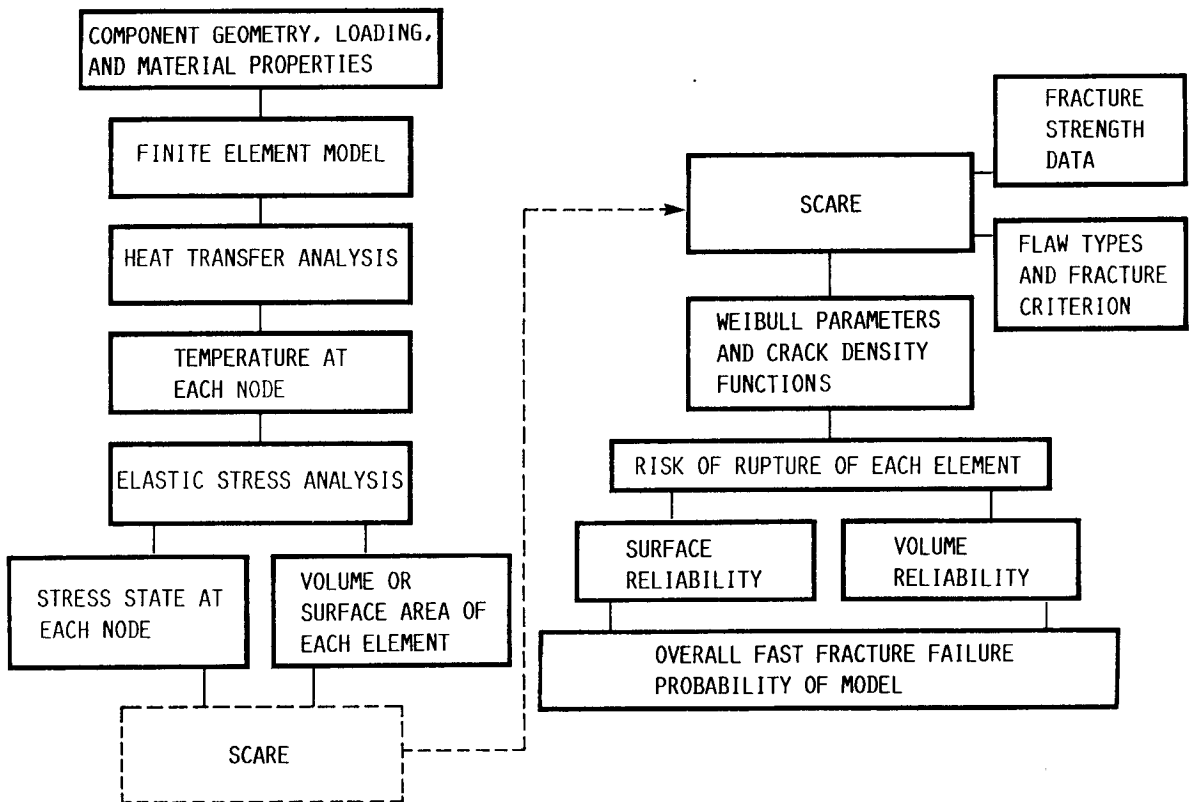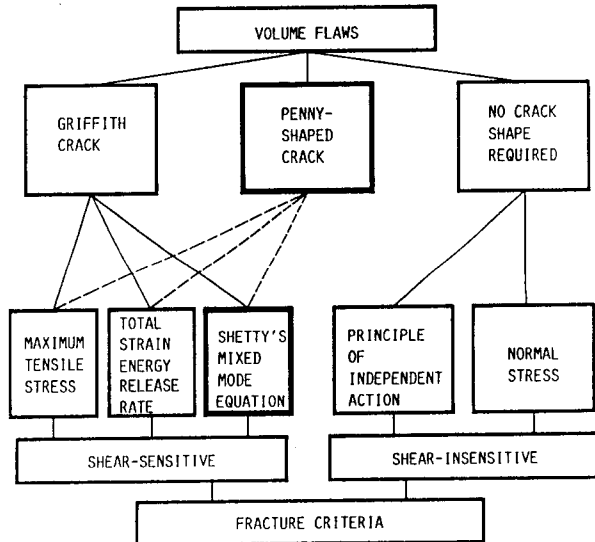

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AVAILABLE FLAW TYPES AND FRACTURE CRITERIA

The diagrams below depict the flaw types and fracture criteria available in the SCARE program. The simple statistical fracture theories, PIA and normal stress averaging, do not require a specific crack geometry since they are not based on fracture mechanics. On the other hand, Batdorf's fracture theory can be used with several different mixed-mode fracture criterion and crack geometries. The combination of a particular flaw shape and fracture criterion results in an effective stress equation involving far-field normal and shear stresses. Coplanar crack extension theories include the maximum tensile stress theory and the total strain energy release rate theory. In reality, a crack is not confined to grow within its plane, and out-of-plane crack extension criteria are more accurate. In SCARE, these criteria are approximated by a simple equation (Shetty, 1987). The approach involves a semi-empirical constant which is varied to model the maximum tangential stress theory, the maximum strain energy release rate theory, or experimental results. Because of the flexibility of this equation, it is the preferred model for both volume and surface flaws. Finally, with regard to crack geometry, semi-circular cracks are preferred for surface imperfections, whereas penny-shaped cracks best reflect the geometry of volume imperfections.

SURFACE FLAWS
- GRIFFITH CRACK
- GRIFFITH NOTCH
- SEMI-CIRCULAR SURFACE CRACK
- NO CRACK SHAPE REQUIRED

TOTAL STRAIN ENERGY RELEASE RATE | SHETTY'S MIXED MODE EQUATION | PRINCIPLE OF INDEPENDENT ACTION | NORMAL STRESS

SHEAR-SENSITIVE   SHEAR-INSENSITIVE

FRACTURE CRITERIA

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VOLUME FLAWS
- GRIFFITH CRACK
- PENNY-SHAPED CRACK
- NO CRACK SHAPE REQUIRED

MAXIMUM TENSILE STRESS | TOTAL STRAIN ENERGY RELEASE RATE | SHETTY'S MIXED MODE EQUATION | PRINCIPLE OF INDEPENDENT ACTION | NORMAL STRESS

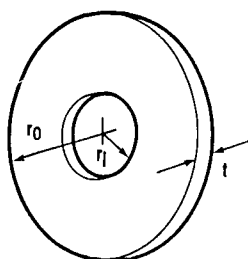SHEAR-SENSITIVE   SHEAR-INSENSITIVE

FRACTURE CRITERIA

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CODE VALIDATION

Several sample problems have been selected from the open literature to validate the SCARE program. For example, a silicon nitride disk was spun to fracture (Swank and Williams, 1981). The disk geometry is shown in the first figure, as well as the Weibull parameters from MOR bar testing. Reliability calculations from the SCARE code are compared to experimental results in the second figure. The predictions from a shear-sensitive Batdorf fracture criterion are closest to the experimental results. However, it should be noted that only seven disks were tested, compared to 85 MOR specimens. This leads to a large degree of statistical uncertainty in the disk data and may account for the greater difference between experimental and predicted P_f at lower failure probabilities. Rigorous testing is being conducted in an in-house program to gather more data for code validation purposes. Furthermore, the shear-insensitive fracture models have been favorably tested against proprietary codes with these capabilities.

ROTATING ANNULAR DISK
VOLUME FLAW ANALYSIS

DATA:

NC – 132 HOT PRESSED Si_3N_4

m   7.65

σ_0 74.82 MPa $METER^{(0.3922)}$

$N(\sigma_{cr})$ $16.30 \left(\dfrac{\sigma_{cr}}{74.82\ MPa} \right)^{7.65}$ PER CUBIC METER

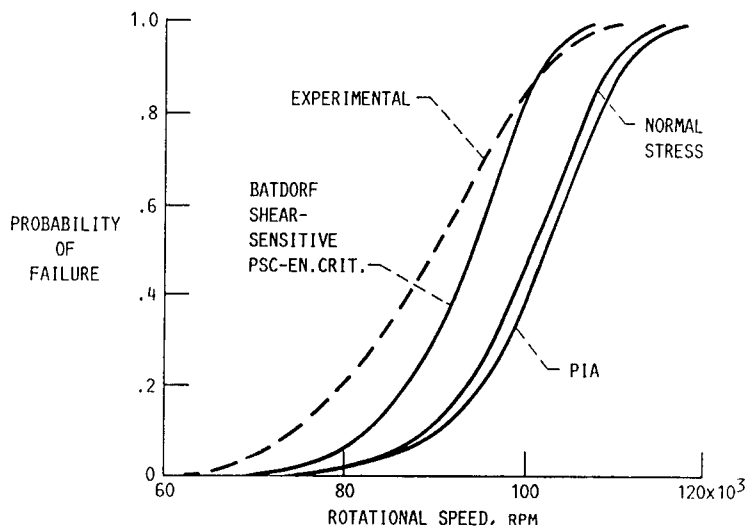SHEAR-INSENSITIVE CALCULATION OF N

r_i 6.35 mm (.25 in.)
r_0 41.275 mm (1.625 in.)
t   3.80 mm (.15 in.)
RPM   70 000 TO 114 000

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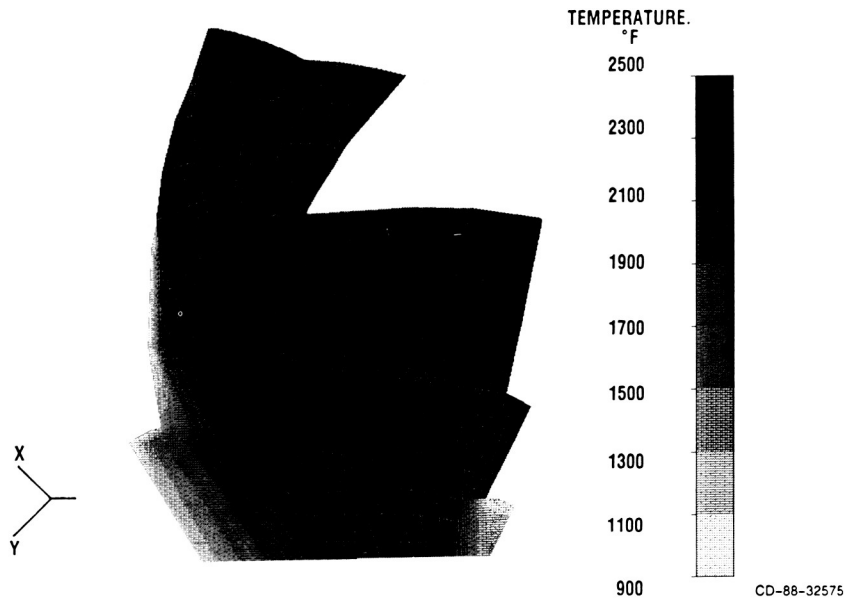## PROBABILITY OF FAILURE VERSUS DISK ROTATIONAL SPEED
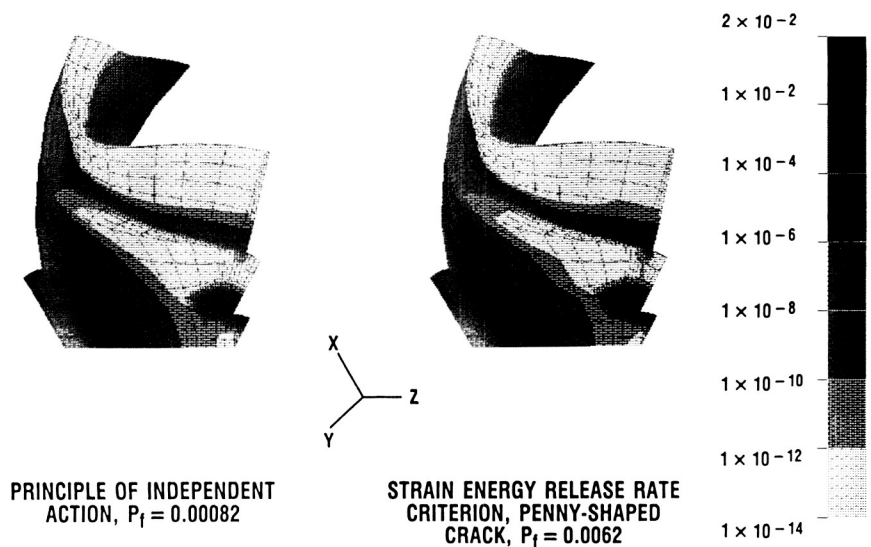

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DESIGN APPLICATION

SCARE has been used for the preliminary design of a silicon nitride mixed-flow
rotor for application in small, high temperature engines. A single blade and
a section of the rotor hub were analyzed using the cyclic symmetry option of
MSC/NASTRAN. The results from the heat transfer and reliability analyses are
shown below. Again, the shear-sensitive criterion yields a higher probability
of failure for the same applied load. However, the regions of low reliability
are the same for both models.

SILICON NITRIDE MIXED-FLOW ROTOR
TEMPERATURE DISTRIBUTION

COMPARISON OF RISK OF RUPTURE INTENSITIES
PER UNIT VOLUME

PRINCIPLE OF INDEPENDENT
ACTION, $P_f = 0.00082$

STRAIN ENERGY RELEASE RATE
CRITERION, PENNY-SHAPED
CRACK, $P_f = 0.0062$

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SCARE is a unique public-domain program which has been requested by numerous companies in various industries.

INITIAL VERSION OF SCARE RELEASED TO
(FEBRUARY 1, 1988)

GENERAL MOTORS CORPORATION	BOEING AEROSPACE COMPANY
DETROIT DIESEL ALLISON	GENERAL ELECTRIC COMPANY
FORD MOTOR COMPANY	BABCOCK AND WILCOX
CHRYSLER CORPORATION	AVCO LYCOMING DIVISION
TRW VALVE DIVISION	DOW CHEMICAL
NORTON-TRW CORPORATION	TRW SPACE & TECHNOLOGY GROUP
EATON CORPORATION	GARRETT TURBINE ENGINE COMPANY
PDA ENGINEERING	ALLISON GAS TURBINE DIVISION
NASA/COSMIC SOFTWARE CENTER	OAK RIDGE NATIONAL LABORATORY
SPARTA	SUNSTRAND-TURBOMACH CORPORATION
WLT CORPORATION	STRUCTURAL INTEGRITY ASSOCIATES

INDUSTRIES INVOLVED—AUTOMOBILE, AEROSPACE, NUCLEAR, & COMPUTER SOFTWARE

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SUMMARY

A statistical design methodology must be used with ceramics to account for not only the average strength but, also, the scatter in strength. However, statistics must be supplemented with LEFM to provide a mechanistic understanding of the pertinent phenomenon. The improvement in failure predictions when using a shear-sensitive fracture mechanics based failure criterion was shown in the rotating disk example. The fracture mechanics/statistics combination allows for sound predictions when considering multiaxial stress states or concurrent flaw populations. This has been accomplished in the public domain finite element post-processor SCARE. This framework will be built on as we begin research on ceramic fatigue due to slow crack growth.

- **PROBABILISTIC DESIGN APPROACH MUST BE USED FOR CERAMICS**
- **FRACTURE MECHANICS IS NEEDED TO ACCURATELY ACCOUNT FOR:**
  - **MULTIAXIAL EFFECTS**
  - **CONCURRENT VOLUME AND SURFACE FLAW POPULATIONS**
  - **SLOW CRACK GROWTH**
- **SHEAR-SENSITIVE FRACTURE CRITERIA GIVE BETTER RESULTS**
- **FAST FRACTURE PREDICTION CAPABILITIES ARE READILY ACCESSIBLE**

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