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CONTINUOUS FIBER CERAMIC MATRIX COMPOSITES

FOR HEAT ENGINE COMPONENTS*

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ABSTRACT

High strength at elevated temperatures, low density, resistance to wear, and abundance of nonstrategic raw materials make structural ceramics attractive for advanced heat engine applications. Unfortunately, ceramics have a low fracture toughness and fail catastrophically because of overload, impact, and contact stresses. Ceramic matrix composites provide the means to achieve improved fracture toughness while retaining desirable characteristics, such as high strength and low density.

Unlike polymer matrix composites, where a strong fiber is added to a weak matrix to provide increased strength and stiffness, ceramic matrix composites add fibers to an already strong matrix to achieve improved toughness. The toughening mechanisms in ceramic matrix composites are crack bridging, debonding, fiber friction, and fiber pullout. The factors that increase toughness, such as large fiber diameter and low interfacial bond strength, decrease composite strength. Thus, ceramic matrix composites are very different from polymer matrix composites.

Materials scientists and engineers are trying to develop the ideal fibers and matrices to achieve the optimum ceramic matrix composite properties. A need, however, also exists for the development of failure models for the design of ceramic matrix composite heat engine components. Phenomenological failure models such as maximum stress, maximum strain, Tsai-Hill, and Tsai-Wu are currently the most frequently used in industry, but they are deterministic and do not adequately describe ceramic matrix composite behavior. Semi-empirical models have been proposed, such as Whitney and Nuismer (1974), which relate the failure of notched composite laminates to the stress a characteristic distance away from the notch. Shear lag models such as that proposed by Eringen and Kim (1974) describe composite failure modes at the micromechanics level. The enhanced matrix cracking stress predicted by Aveston, Cooper, and Kelly (1971) occurs at the same applied stress level as predicted by the two models of steady state cracking by Budiansky, Hutchinson, and Evans (1986), and Marshall, Cox, and Evans (1985). Finally, statistical models, such as

Wetherhold and Pipes (1984), take into consideration the distribution in composite failure strength.

The intent at the NASA Lewis Research Center is to develop these models into computer algorithms for the failure analysis of ceramic matrix composites under monotonically increasing loads. These algorithms will be included in a postprocessor to general purpose finite element programs.

SCOPE AND OBJECTIVES

Further developments in advanced heat engines are limited by the metallic materials currently available. For future applications (such as the National Aerospace Plane and automotive gas turbine engines) to become a reality, new materials capable of surviving the required stresses and temperatures for the life of the structure must become available. Not only, however, must those advanced materials systems be identified, but the necessary tools to design a structure with them must also be developed. The Structural Integrity Branch at NASA Lewis Research Center is identifying those ceramic matrix composite (CMC) systems currently being developed which are suitable for high-temperature applications and the failure models available to describe their behavior under monotonic loads. The results will be published in a survey later this year. Those models will then be selectively incorporated into a postprocessor for general purpose finite element programs, comparable to the SCARE postprocessor.

- **IDENTIFY CERAMIC MATRIX COMPOSITE SYSTEMS SUITABLE FOR ADVANCED HEAT ENGINE COMPONENTS**

- **IDENTIFY MODELS FOR THE FAST FRACTURE ANALYSIS OF CERAMIC MATRIX COMPOSITE LAMINATES**

- **INCORPORATE THOSE MODELS INTO A POSTPROCESSOR FOR GENERAL PURPOSE FINITE ELEMENT PROGRAMS SUCH AS MSC/NASTRAN**
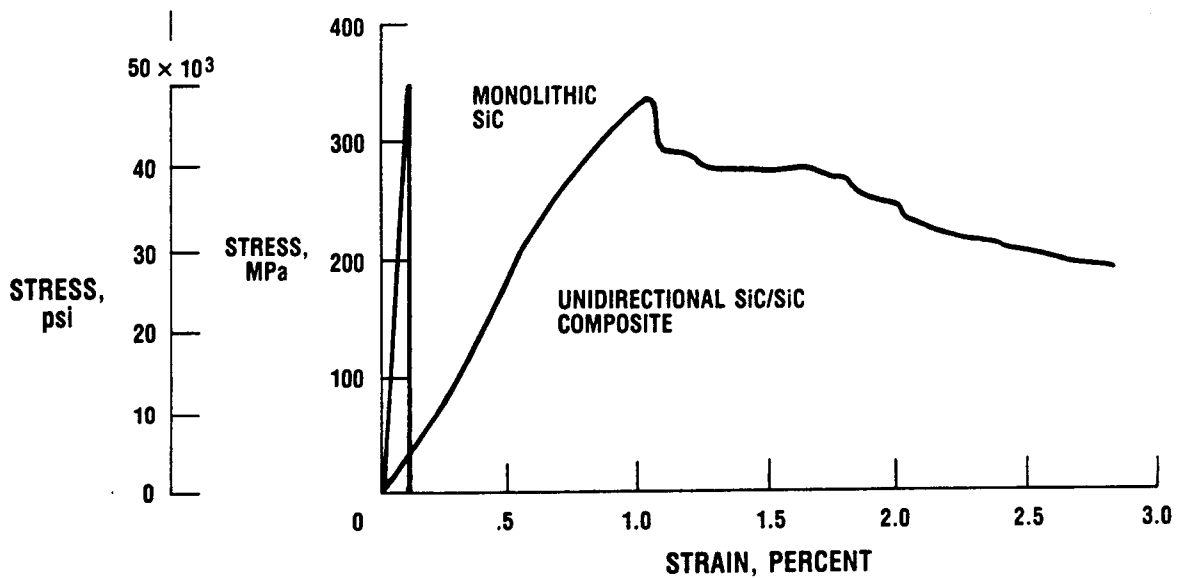
ADVANTAGES OF CERAMIC MATRIX COMPOSITES

Strong ceramic fibers have been added to reinforce low strength, typically glass-ceramic matrices, such as SiC/LAS, to achieve improved strength as in polymer matrix composites. These composites will not satisfy the high-temperature requirements of the applications we are interested in, but they may have other applications. The ceramic matrices attractive for advanced heat engine applications, such as SiC and Si_3N_4, already have adequate strength at elevated temperatures. Unfortunately, monolithic ceramics also have a low fracture toughness and fail catastrophically because of overload, impact, and contact stresses. Continuing improvements are being made in monolithic ceramics, and further reduction in critical flaw size could result in stronger ceramics. But in the past this has only resulted in increased strength without any appreciable increase in fracture toughness, and at a steadily increasing cost. Whisker reinforced composites provide improved fracture toughness and increased tolerance to flaws but still fail in a brittle manner. Continuous-fiber reinforced composites also have improved fracture toughness and increased tolerance to flaws but, in contrast to whisker reinforced composites, fail gracefully and are the answer to improved reliability.

- **THERE ARE LIMITED FUTURE IMPROVEMENTS IN MONOLITHIC PROCESSING AND POWDERS. ECONOMIC CONSTRAINTS HAVE BEEN REACHED ON IMPURITIES, DENSITIES, AND FLAW SIZES**

- **MONOLITHIC TOUGHNESS REMAINS VERY LOW. MONOLITHIC CERAMICS ARE INTRINSICALLY FLAW INTOLERANT AND FAIL CATASTROPHICALLY BECAUSE OF OVERLOAD, IMPACT, AND CONTACT STRESSES**

- **WHISKER REINFORCED COMPOSITES PROVIDE IMPROVED TOUGHNESS AND INCREASED FLAW TOLERANCE BUT REMAIN BRITTLE**

- **CONTINUOUS FIBER REINFORCED COMPOSITES PROVIDE INCREASED FLAW TOLERANCE, IMPROVED TOUGHNESS, AND GRACEFUL FAILURE—ANSWER TO IMPROVED RELIABILITY**

GRACEFUL FAILURE OF SiC/SiC

A typical stress-strain curve (Caputo et al., 1985) for a SiC/SiC composite
at room temperature demonstrates graceful failure. This specimen contained
58 vol % SiC fibers. The maximum flexural strength of 330 MPa was achieved at
a strain of 1.05 percent in a four-point flexure test. More significant, how-
ever, was the achievement of graceful failure. Unlike the monolithic SiC,
which failed catastrophically at a very low strain, the unidirectional SiC/SiC
composite is strain tolerant and sustained load after matrix crack initiation.
At a strain of 2.8 percent, the specimen maintained a stress of 188 MPa –
57 percent of its maximum strength. This gradual loss of strength as strain
increases, in contrast to the catastrophic failure of monolithic ceramics,
makes the use of advanced ceramic matrix composites attractive in heat engine
applications where catastrophic failure is unacceptable.

*ADAPTED FROM CAPUTO ET AL. (1984)

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TOUGHENING MECHANISMS IN CERAMIC MATRIX COMPOSITES

The toughening mechanisms (Harris, 1986) in ceramic matrix composites are described by considering an isolated fiber. A crack initiates in the matrix (fig. (b)) and starts to propagate normal to the load. The higher stiffness and strength of the fiber inhibits further extension of the crack when it reaches the fiber. As the load is increased (fig. (c)), local stress concentrations and Poisson contractions cause the fiber to debond from the matrix, provided the interfacial bond strength is weak enough. Outwater and Murphy (1970) gave an upper limit to the energy of debonding W_{db}. After debonding, the crack will open further as the load is increased. The term W_{fr} is an estimate of the work against frictional resistance as the fiber moves relative to the matrix. Upon further loading of the composite (fig. (e)), the fiber will break at some weak point. As the broken fibers are pulled out against the frictional resistance, they contribute to the work of pullout W_p.

(a)   (b)   (c)   (d)   (e)
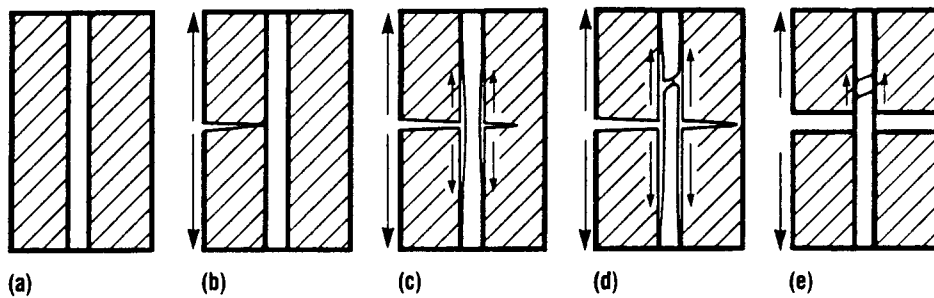
$$W_{db} = \frac{N \pi d_f^2 \sigma_f^2 y}{8 E_f}$$

$$W_{fr} = \frac{N \tau_f \pi d_f y^2 \epsilon_f}{2}$$

$$W_p = \frac{N \tau_f \pi d_f \ell_{cr}}{12}$$

*ADAPTED FROM HARRIS (1986)

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Aveston et al. (1971) showed that first matrix cracking for a brittle matrix composite will occur not at the nominal failure strain of the matrix but at an enhanced matrix cracking strain. According to their analysis, the strength of a brittle matrix composite is enhanced by a small fiber radius, a strong fiber-matrix interfacial shear strength, and a high matrix fracture surface energy. Conversely, fiber pullout increases fracture toughness. Cottrell (1964) and Kelly (1970) show that the pullout work of fracture is increased by a weak interfacial frictional shear stress, a large fiber diameter, and a large fiber failure strain. Toughness is gained at the expense of strength since large fiber diameter contributes to increased toughness but results in decreased strength. A similar relation holds for interfacial properties. Thus, optimal fiber diameters and interfacial properties exist for the desired combination of strength and toughness.

- **AVESTON, COOPER, AND KELLY (1971)—THEORY FOR ENHANCED MATRIX CRACKING**

$$\epsilon_{mu} = \left(\frac{12\tau\gamma_m E_f V_f^2}{E_c E_m^2 r_f V_m} \right)^{1/3}$$

- **COTTRELL (1964) AND KELLY (1970)—PULLOUT WORK OF FRACTURE**

$$\gamma_{fp} = \frac{\ell_{cr}}{\ell} \left(\frac{V_f^2 \sigma_{fu} r_f}{12\tau_f} \right)$$

- **CONCLUSION: FACTORS INCREASING TOUGHNESS MAY DECREASE STRENGTH**

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DESIRED FEATURES FOR ADVANCED CERAMIC MATRIX COMPOSITES

Ceramic matrix composites fracture by the low-strain propagation of cracks in the brittle matrix (DiCarlo, 1985). High composite fracture strain is achieved by a high volume fraction of fibers bridging the matrix cracks. The bridging fibers reduce crack openings under loading, requiring greater applied strains for matrix crack propagation than those needed in the unreinforced matrix. If the fiber-matrix interfacial bond is strong, the stress concentration on fibers at the crack tip generally will be high enough to fracture the fiber, resulting in a brittle composite fracture. However, if the interfacial bond is weak and the strength of the fibers is high enough to support the applied load, the matrix cracks will propagate around the fibers and not through them. The composite will not fracture catastrophically but will have a series of evenly spaced matrix cracks bridged by reinforcing fibers. Thus, ceramic matrix composites should contain a high volume fraction of fibers that are continuous, are stiffer than the matrix, and possess a small diameter. The high volume fraction and small diameter ensure that a sufficient number of fibers bridge the matrix crack to prevent crack propagation until higher strain levels are reached. The matrix and fibers should also be oxidation resistant to retain their strength at high temperatures. Compatible fiber and matrix thermal expansion coefficients prevent the formation of residual stresses that enhance matrix cracking.

- **FIBER SPACING SMALLER THAN CONTROLLING FLAWS IN MATRIX—TYPICALLY LESS THAN 100 μm**

- **FIBER DIAMETER MUCH SMALLER THAN MATRIX FLAW—TYPICALLY LESS THAN 20 μm**

- **FIBER YOUNG'S MODULUS GREATER THAN MATRIX YOUNG'S MODULUS FOR GREATER COMPOSITE STRENGTH**

- **OPTIMUM INTERFACIAL BONDING FOR TOUGHNESS AND STRENGTH**

- **DENSE, HIGH STRENGTH, HIGH TOUGHNESS, OXIDATION RESISTANT, REFRACTORY MATRIX**

- **COMPATIBLE FIBER AND MATRIX THERMAL EXPANSION COEFFICIENTS**

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MATRICES OF CURRENT INTEREST

A requirement of any composite system is compatibility of the matrix and fiber with each other and the environment. Fiber and matrix compatibility must result in optimal interfacial properties, but degradation by reaction or inter-diffusion must be avoided. To achieve a compromise, it may be necessary to coat the fibers to restrict interaction. Matrix materials include sintered powders, organometallic precursors, and materials deposited from the vapor phase (Phillips, 1983). The use of glass-ceramic matrices presents several advantages. The hot-pressing of viscous glass minimizes fiber damage which may occur with crystalline ceramics. The main disadvantage of glass-ceramics is that their temperature is limited compared to other ceramics. Silicon carbide and silicon nitride are regarded as the high-temperature materials of choice for most applications, but alumina and other oxides are also highly refractory. Fabrication from powders has the advantage of using materials which are inexpensive and available, but the formation of matrix agglomerates, inadequate infiltration of the reinforcement, and damage to the fibers by abrasion is a problem. Organometallic precursors can be used for oxide and non-oxide matrices and fiber coatings. A major advantage of this method is that damage to the fibers is less likely since the precursors used are in a fluid state, but densification is difficult to achieve.

	TENSILE MODULUS, GPa	TENSILE STRENGTH, MPa	DENSITY, g/cm^3
BOROSILICATE GLASS	60	100	2.3
LAS	100	100-150	2.0
Si_3N_4	310	410	3.2
Al_2O_3	360-400	250-300	3.9-4.0
SiC	400-440	310	3.2

FIBERS OF CURRENT INTEREST

Extensive research is being done in developing high-strength, oxidation resistant, thermally stable small-diameter fibers. As mentioned, fiber-matrix compatibility is critical in composite behavior. A weak fiber-matrix interface causes noncatastrophic failure; whereas a strong interface causes catastrophic failure. The fiber Nicalon, derived from a polymer precursor, is a β-SiC fiber containing excess carbon that forms a weak carbon-rich interface with many matrices (Mah et al., 1987). Nicalon, however, has limited thermal stability and loses significant strength above 1000 °C. The fiber-matrix strength increases, possibly because of oxidation of the fibers, resulting in catastrophic failure of the composite. The AVCO monofilament fiber is produced by chemical vapor deposition of SiC onto a carbon fiber core. A carbon-rich layer is then applied to the fiber, which provides weak interfacial bonding and promotes debonding and fiber pullout. AVCO fibers also experience significant strength degradation. The AVCO fiber is a large diameter fiber. The oxide fibers, Nextel 312 and FP, chemically bond to many matrices causing the composites to fail catastrophically. Fiber coatings, however, may provide optimal interfacial characteristics. None of these fibers are thermally stable above 1200 °C, and work continues on developing new fibers. The Tyranno fiber is produced from a polymer and is similar to Nicalon except for the addition of Ti, which is said to retard grain growth and is expected to preserve high-temperature strength. Nextel 440 and 480 are similar to Nextel 312, except for the reduction in B_2O_3, which is also expected to improve high-temperature properties.

DESIGNATION	COMPOSITION, wt %	TENSILE STRENGTH, MPa	MODULUS, GPa	DENSITY, g/cm	DIAMETER, μm
NICALON	59 Si, 31 C, 10 O	2520-3290	182-210	2.55	10-20
SCS-6	SiC ON CARBON CORE	3920	406	3.0	143
NEXTEL 312	62 Al_2O_3, 14 B_2O_3, SiO_2	1750	154	2.7	11
FP	>99 α-Al_2O_3	>1400	385	3.9	20
TYRANNO	Si, Ti, C, O	>2970	>200	2.3-2.5	8-10
NEXTEL 440	70 Al_2O_3, 28 SiO_2, 2 B_2O_3	2100	189	3.05	10-12
NEXTEL 480	70 Al_2O_3, 28 SiO_2, 2 B_2O_3	2275	224	3.05	10-12

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There are five characteristic values of strength for a unidirectional composite: (1) longitudinal tensile strength, (2) longitudinal compressive strength, (3) transverse tensile strength, (4) transverse compressive strength, and (5) in-plane shear. The maximum stress theory states that failure will occur in a lamina if any of the stresses in the principal material axes exceeds the corresponding allowable stress as determined from simple unidirectional stress tests (Nahas, 1986). Failure will occur in the maximum strain theory if any of the strains in the principal axes exceeds the corresponding allowable strain. The maximum strain theory is similar to the maximum stress theory and allowable strains can be directly related to the allowable strengths. Predictions of the two theories are quite close to each other. The differences are due to the Poisson ratio. The Tsai-Hill criteria (Azzi and Tsai, 1985) provides a single function to predict failure and takes into consideration the interaction between strengths. The Tsai-Hill criterion remains applicable for materials with properties different in tension and compression. Tsai and Wu (1971) have proposed a tensor polynomial failure criteria. Wu (1974) has shown the previous criteria are limit cases of this theory. A failure surface in stress space exists where F_i and F_{ij} are second- and fourth-order strength tensors. The noninteraction F terms are related to the engineering strengths. The interaction F terms are determined from biaxial tests and are constrained by the inequality $F_{ii}F_{jj} - F_{ij}^2 > 0$. According to Burk (1983), the maximum stress, maximum strain, Tsai-Hill, and Tsai-Wu failure criteria are the most widely used in industry. These failure criteria, however, are deterministic and do not describe the failure mechanisms. They also do not consider the scatter in ceramic composite strengths and are simply fail/no-fail criteria.

- **MAXIMUM STRESS**

$$\sigma_1 = \sigma_{1u} \qquad \sigma_2 = \sigma_{2u} \qquad \tau_{12} = \tau_{12u}$$

- **MAXIMUM STRAIN**

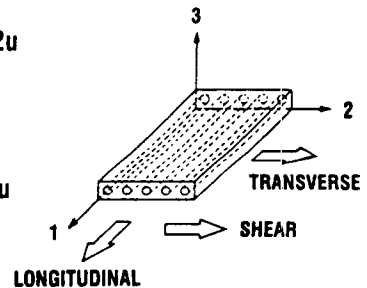$$\epsilon_1 = \epsilon_{1u} \qquad \epsilon_2 = \epsilon_{2u} \qquad \gamma_{12} = \gamma_{12u}$$

- **TSAI-HILL**

$$\frac{1}{\sigma_{1u}^2}\sigma_1^2 - \left(\frac{1}{\sigma_{1u}^2} + \frac{1}{\sigma_{2u}^2} - \frac{1}{\sigma_{3u}^2}\right)\sigma_1\sigma_2 + \frac{1}{\sigma_{2u}^2}\sigma_2^2 + \frac{1}{\gamma_{12}^2}\tau_{12}^2 = 1$$

- **TSAI-WU**

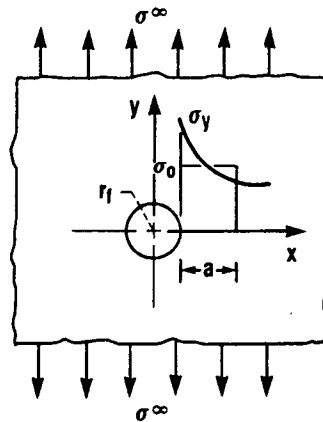$$F(\sigma) = F_i\,\sigma_i + F_{ij}\,\sigma_i\,\sigma_j = 1$$

*NAHAS (1986)

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Two stress criteria for predicting the tensile strength of notched composite laminates have been proposed by Whitney and Nuismer (1974). The point stress criteria and the average stress criteria assume that fracture occurs when the stress at some characteristic distance away from the discontinuity reaches the unnotched strength. The Whitney-Nuismer failure criteria were motivated by the hole size effect in which larger holes cause greater strength reduction than do smaller holes (Awerbuch and Madhukar, 1985). Although the stress concentration factor is independent of hole size, the normal stress σ_y is concentrated near the hole boundary for a smaller hole. It has been suggested that a larger area is subjected to high stress for a larger hole and, thus, has a higher probability of encountering inherent flaws, resulting in a lower strength. The point stress criteria assumes that failure occurs when the stress σ_y at a distance b away from the discontinuity is equal to the strength of the unnotched laminate. The average stress criteria assumes that failure occurs when the average stress σ_y over some distance a equals the unnotched laminate strength. Interest in the models is based on the assumption that the characteristic distance, b or a, is a material property of a particular laminate design. Experimental evidence suggests this may be true for epoxy systems. The applicability of these models to CMC is not known. Similar models have been proposed by Waddoups et al. (1971), Poe and Sova (1980), and Mar and Lin (1977).
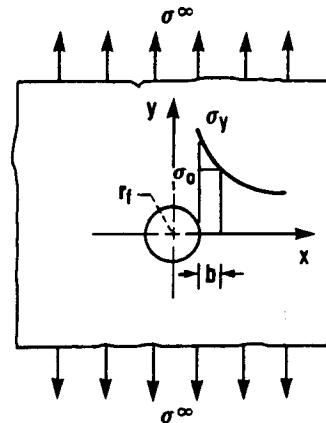
• WHITNEY-NUISMER MODELS
AVERAGE STRESS CRITERION   POINT STRESS CRITERION

$$\sigma_0 = \frac{1}{a} \int_{r_f}^{r_f+a} \sigma_y(x,0)dx \qquad \sigma_0 = \sigma_y(x,0)\, \big|_{x=r_f+b}$$

*AWERBUCH AND MADHUKAR (1985)

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Shear lag failure models examine failure modes at the micromechanics level of ceramic matrix composites. Cox (1952) introduced shear lag models and Hedgepeth (1961) applied them to filamentary structures. Hedgepeth's model considered filaments separated by a constant distance. The displacement of the nth filament is given by $u_n(x,t)$ and the force in the nth filament is given by $P_n(x,t)$. The fibers carry all the tensile load while the matrix carries only shear. Equilibrium of an element of the nth filament results in the partial differential difference equations shown. By applying the appropriate boundary conditions, we can solve the equations for the stress concentrations in the filamentary structure. Eringen and Kim (1974) generalized the model to include transverse loads in the matrix. Neither of these models can accurately describe ceramic matrix composites because they neglect the tensile load carrying capability of the matrix, but further generalizations may make these models applicable. Once such models are available they may be used to consider failure mechanisms, such as longitudinal yielding and matrix splitting, as did Goree and Gross (1979). They generalized Hedgepeth's model to include longitudinal yielding and matrix splitting and arrived at three partial differential difference equations to describe the stresses and displacements in a unidirectional-fiber-reinforced composite.

- **HEDGEPETH (1961)**
  FORCE IN nth FILAMENT

$$P_n = EA_f \frac{\partial U_n}{\partial x}$$

$$EA_f \frac{\partial^2 U_n}{\partial x^2} + \frac{G}{h}(U_{n+1} - 2U_n + U_{n-1}) = m \frac{\partial^2 U_n}{\partial t^2}$$
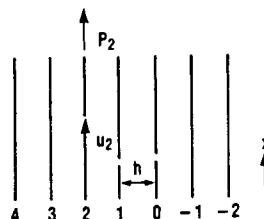
SHEAR FORCE

$$\frac{G(U_{n+1} - U_n)}{h}$$

- **ERINGEN AND KIM (1974)**

$$\frac{E_m}{h}(U_{n+1} - 2U_n + U_{n-1}) + \frac{gG_m}{2}\left[\frac{1}{2}\frac{d^2}{dy^2}(U_{n+1} + 2U_n + U_{n-1}) + \frac{1}{h}\frac{d}{dy}(v_{n+1} - v_{n-1})\right] = 0$$

$$gE_f \frac{d^2 v_n}{dy^2} + G_m\left[\frac{1}{2}\frac{d}{dy}(U_{n+1} - U_{n-1}) + \frac{1}{h}(v_{n+1} - 2v_n + v_{n-1})\right] = 0$$

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- **GOREE AND GROSS (1979)**
  ALL FIBERS EXCEPT n AND n + 1
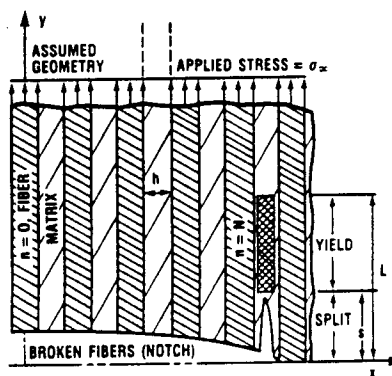
$$\frac{E_f A_f h}{G_m l}\left(\frac{d^2 U_n}{dy^2}\right) + U_{n+1} - 2U_n + U_{n-1} = 0$$

FIBER n

$$\frac{E_f A_f h}{G_m l}\left(\frac{d^2 U_n}{dy^2}\right) + U_{n+1} - U_n - \frac{h}{G_m}\tau \langle y - s \rangle = 0$$

FIBER n + 1

$$\frac{E_f A_f h}{G_m l}\left(\frac{d^2 U_{n+1}}{dy^2}\right) + U_{n+2} - U_{n+1} + \frac{h}{G_m l}\tau \langle y - s \rangle = 0$$

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FRACTURE MECHANICS MODELS

The first matrix crack marks the beginning of permanent damage and permits oxidation of the fibers through loss of protection by the matrix. As we have seen, Aveston et al. (1971) have shown that first matrix cracking occurs in a ceramic matrix composite at a higher strain than it does for the monolithic ceramic. For a crack to form, the stress in the matrix must be equal to its breaking stress. In addition, the energy condition shown by the inequality below must be satisfied. The inequality consists of energy terms for various failure mechanisms under tensile loads. The fracture surface work in forming a matrix crack is γ_m. The work in breaking the fiber-matrix bond, given by Outwater and Murphy (1969), is γ_{db}. Work as the matrix slides over the fibers against a frictional force is U_{fr}. The decrease in the elastic strain energy in the matrix as the matrix cracks is given by ΔU_m. Conversely, the elastic strain energy in the fibers increases and is given by ΔU_f. Finally, the work done by the applied stresses is ΔW. Substituting these terms into the inequality and assuming a frictional bond between the fiber and matrix yields the formula for the enhanced matrix cracking strain.

• AVESTON, COOPER, AND KELLY (1971)
A CRACK WILL FORM PROVIDED

$$2\gamma_m V_m + \gamma_{db} + U_{fr} + \Delta U_f \leq \Delta W + \Delta U_m \qquad\qquad c = \frac{E_m V_m}{E_f V_f}$$

WORK OF APPLIED STRESS $\qquad \Delta W = \dfrac{E_f E_m V_m}{2\tau} \epsilon_{mu}^3 \, cr_f \, (1 + c)$

REDUCTION IN MATRIX STRAIN ENERGY $\qquad \Delta U_m = \dfrac{E_f E_m V_m}{3\tau} \epsilon_{mu}^3 \, cr_f$

INCREASE IN FIBER STRAIN ENERGY $\qquad \Delta U_f = \dfrac{E_f E_m V_m}{2\tau} \epsilon_{mu}^3 \, cr_f \left(1 + \dfrac{c}{3}\right)$

WORK OF FRICTION $\qquad \Delta U_{fr} = \dfrac{E_f E_m V_m}{6\tau} \epsilon_{mu}^3 \, cr_f \, (1 + c)$

WORK OF DEBONDING $\qquad \gamma_{db} = \dfrac{2\sigma_{mu} V_m \mathcal{G}_{II}}{\tau_0}$

FOR A PURELY FRICTIONAL FIBER-MATRIX BOND, $\mathcal{G}_{II} = 0$

ENHANCED MATRIX CRACKING STRAIN $\qquad \epsilon_{mu} = \left(\dfrac{12\tau\gamma_m E_f V_f^2}{E_c E_m^2 r_f V_m}\right)^{1/3}$

Aveston et al. (1971) looked at the energy states in a crack before and after crack propagation. Budiansky et al. (1986) assumed that if a crack engulfs more than a few fibers the applied stress necessary for propagation is constant and steady state cracking occurs. The assumption of steady state cracking implies that the stresses at the crack front remain unchanged during crack growth and also that the upstream and downstream states, far ahead of and behind the crack, do not change. Equation (1) governs matrix cracking for the fiber slip and no-slip cases. A shear lag analysis is used to determine the upstream and downstream stresses. The matrix cracking stress predicted is essentially the same as that of predicted by Aveston et al. (1971) except for the initial stress term σ'_m. Another model for steady state cracking was proposed by Marshall et al. (1985). The analysis is of unbonded unidirectional lamina in which the sliding of the matrix over fibers is resisted only by frictional forces. The energy solution is derived from the earlier analysis by Aveston et al. (1971) but is expressed in terms of incremental crack extension. For an incremental crack extension work dU is done against frictional forces, the strain energy in the matrix decreases by dU_m, the strain energy in the fibers increases by dU_f, and the potential energy of the loading system decreases by dU_l. Again, the predicted first cracking stress agrees with the results of Aveston et al. (1971).

• BUDIANSKY, HUTCHINSON, AND EVANS (1986)

(1) $\dfrac{1}{2A_c} \displaystyle\int_{-L}^{L} \int_{A_c} (\sigma^u - \sigma^d):(\epsilon^u - \epsilon^d)\, dA_f\, dz = V_m \mathcal{G}_m$

(2) $\dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty}\left[\dfrac{V_f}{E_f}(\sigma^u_f - \sigma^d_f)^2 + \dfrac{V_m}{E_m}(\sigma^u_m - \sigma^d_m)^2\right]dz + \dfrac{1}{2\pi r^2 G}\int_{-\infty}^{\infty}\int_{a}^{F}(\tau^d_{rz})2\pi r_f\, dr_f\, dz = V_m \mathcal{G}_m$

(3) $\dfrac{\sigma_{cf}}{E_c} + \dfrac{\sigma'_m}{E_m} = \dfrac{\sigma_1}{E}$ $\sigma = \left(\dfrac{6V_f^2 E_f \tau_{fr}}{V_m E_m E}\right)^{1/3}\left(\dfrac{\mathcal{G}_m}{aE_m}\right)^{1/3}$

WHERE σ'_m IS INITIAL MATRIX STRESS

• MARSHALL, COX, AND EVANS (1985)

$dU = 2\gamma_m V_m\, dc + dU_{fr} + dU_f - dU_m - dU_L$

$dU_{fr} = \left[\dfrac{\sigma_\infty^3 r_f}{6\tau E_f V_f^2(1+c)^2}\right]dc$

$dU_m = \left[\dfrac{\sigma_\infty^3 r_f c}{3\tau E_f V_f(1+c)^3}\right]dc$

$dU_f = \left[\dfrac{\sigma_\infty^3 r_f(3c+1)}{6\tau E_f V_f^2(1+c)^3}\right]dc$

$dU_L = \left[\dfrac{\sigma_\infty^3 r_f}{2\tau E_f V_f^2(1+c)^2}\right]dc$

WHERE

$\sigma_0 = \left[\dfrac{6(1-\nu^2)K_{cr}^{m2}\tau E_f V_f^2 V_m(1+c)^2}{E_m r}\right]^{1/3}$

AND

$c = \dfrac{E_f V_f}{E_m V_m}$

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The previous models have ignored the statistical aspects of failure. Average strengths have been employed resulting in fail/no-fail decisions. The models considered here are weakest-link models where the failure of an element of the volume results in the failure of the volume. The principle of independent action considers the stress components to act separately in producing failure. The Weibull shape parameter is ß, the Weibull scale parameter is α, and σ is the stress component in the principal material coordinate system. The parameters α and ß are obtained from uniaxial strength tests. In not allowing the stresses to interact, the principle of independent action should give nonconservative results. Wetherhold and Pipes (1984) allow for interaction of stresses by incorporating the maximum distortional energy failure function into the probability of failure function. The probability density functions for the strengths X_1, X_2, and X (the strengths in the principal material directions) are substituted into the maximum distortional energy failure function. The reliability then is the probability that K is less than one. The resulting integral is analytically intractable. A Monte Carlo simulation is used to evaluate the reliability. Other models have been proposed by Batdorf (1982) and Harlow and Phoenix (1978). Macroscopic models were used in these failure criteria and the micromechanics of failure were not considered. The linking of micromechanics models and macromechanics models could result in better probabilistic models.

• PRINCIPLE OF INDEPENDENT ACTION

$$R = \exp \left\{ - \int_v \left[\left(\frac{\sigma_1}{\beta_1} \right)^{\alpha_1} + \left(\frac{\sigma_2}{\beta_2} \right)^{\alpha_2} + \left(\frac{\sigma_3}{\beta_3} \right)^{\alpha_3} \right] dv \right\}$$

• WETHERHOLD AND PIPES (1984)

$$K = \left(\frac{\sigma_1}{X_1} \right)^2 - \left(\frac{\sigma_1 \sigma_2}{X_1^2} \right) + \left(\frac{\sigma_2}{X_2} \right)^2 + \left(\frac{\sigma_3}{X_3} \right)^2$$

WHERE $K < 1$ = NO FAILURE AND $R = P\{K < 1\}$

$$R = \int_0^{i(1)} \int_0^{h(1,X_3)} \int_0^{g(1,X_2,X_3)} F_{X_1}(X_1)F_{X_2}(X_2)F_{X_3}(X_3)dX_1 \, dX_2 \, dX_3$$

WHERE F_{X_1}, F_{X_2}, F_{X_3} = PROBABILITY DENSITY FUNCTIONS FOR X_1, X_2, AND X_3

AND

$$i(1) = \frac{\sigma_3}{1} \qquad h(1,X_3) = \sqrt{\frac{\sigma_2^2}{1^2 - (\sigma_3/X_3)^2}} \qquad g(1,X_2,X_3) = \sqrt{\frac{\sigma_1^2 - \sigma_1 \sigma_2}{1^2 - (\sigma_2/X_2)^2 - (\sigma_3/X_3)^2}}$$

SUMMARY AND CONCLUSIONS

Monolithic ceramics have high strength at high temperatures but are very sensitive to flaws. Whisker composites have increased flaw tolerance but still fail in a brittle manner. Ceramic matrix composites have improved fracture toughness and fail noncatastrophically. In ceramic matrix composites, fibers are added to a matrix to improve fracture toughness; whereas in polymer matrix composites, a strong fiber is added to a weak matrix to improve strength. Consequently, designing with ceramic matrix composites is different from designing with polymer matrix composites, and different design criteria are needed. The four most commonly used failure criteria in industry – maximum stress, maximum strain, Tsai-Hill, and Tsai-Wu – do not consider the scatter in ceramic matrix composite strengths but describe phenomenologically the failure mechanisms. Shear lag models describe failure mechanisms at a micromechanics level but are currently not capable of describing ceramic matrix composites. Semi-empirical models fit equations to existing data and are applicable only to tensile loaded composites. Statistical models, such as Wetherhold, consider the scatter in ceramic composite strength but do not model failure mechanisms and are difficult to use. A survey of these failure models will be published later. Future work will involve selectively incorporating portions of these models into a postprocessor for reliability analysis.

- CERAMIC MATRIX COMPOSITES PROVIDE THE MEANS TO ACHIEVE IMPROVED FRACTURE TOUGHNESS AND GRACEFUL FAILURE WHILE RETAINING OTHER DESIRABLE PROPERTIES SUCH AS HIGH-TEMPERATURE STRENGTH AND LOW DENSITY.

- DESIRED FEATURES FOR ADVANCED CERAMIC MATRIX COMPOSITES ARE DIFFERENT FROM THOSE FOR POLYMER MATRIX COMPOSITES.

- VARIOUS FAILURE MODELS FOR MONOTONICALLY LOADED CERAMIC MATRIX COMPOSITES WERE REVIEWED.

- FUTURE WORK WILL INVOLVE SELECTIVELY INCORPORATING THESE MODELS INTO A POSTPROCESSOR FOR RELIABILITY ANALYSIS.

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A	area
a	characteristic material distance
b	characteristic material distance
d	diameter
E	Young's modulus
F	strength tensor
G	matrix shear modulus
$\mathscr{G}_m$	critical mode I matrix energy release ratio
$\mathscr{G}_{II}$	debonding energy of fiber matrix interface
g	thickness of filament
h	distance between filaments
K	stress intensity factor
L	composite length
ℓ	fiber load transfer length
m	mass
N	number of fibers bridging crack
P	force
R	reliability
r	radius
s	length of matrix split
t	thickness, time
U	energy
u	displacement of filament
V	volume fraction
v	displacement of matrix
W	work

x,y,z axes

y   mean debond length

α   Weibull scale parameter

β   Weibull shape parameter

γ   work of fracture

ε   strain

ν   Poisson's ratio

σ   stress

τ   infacial shear

Subscripts:

c   composite

cr   critical

db   debonding

f   fiber

fr   frictional force

L   potential energy

m   matrix

n   nth filament

o   unnotched strength

p   pullout

u   ultimate strength

x,y,z axes

1,2,3 principal material axes

Superscripts:

d   downstream

m   matrix

u   upstream

'   initial stress

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